Plan for 3rd lecture: Perturbative calculations

This lecture will focus on perturbative calculations

- LO, NLO, NLO+MC, NNLO
- techniques, issue with divergences
- current status, sample results

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- Perturbative calculations = fixed order expansion in the coupling constant (or more refined expansions)
- Perturbative calculations are possible because the coupling is small at high energy
- In QCD (or generic QFT) coupling depends on the energy (ren.) scale
- So changing scale the result changes. By how much? What does this dependence mean?
- Let's consider some examples

Leading order n-jet cross-section

- Consider the cross-section to produce n jets. The leading order result at scale μ result will be

$$\sigma^{\mathrm{LO}}_{\mathrm{njets}}(\mu) = \alpha_s(\mu)^n A(p_i, \epsilon_i, \ldots)$$

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So the change of scale is an NLO effect ($\propto \alpha_s$), but this becomes more important when the number of jets increases ($\propto$n)

- Notice that at Leading Order the normalization is not under control:

$$\frac{\sigma^{\mathrm{LO}}_{\mathrm{njets}}(\mu)}{\sigma^{\mathrm{LO}}_{\mathrm{njets}}(\mu')} = \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu')}\right)^n$$

NLO n-jet cross-section

Now consider n-jet cross-section at NLO. At scale μ the result reads

$$\sigma^{\mathrm{NLO}}_{\mathrm{njets}}(\mu) = \alpha_s(\mu)^n A(p_i, \epsilon_i, \dots) + \alpha_s(\mu)^{n-1}\left(B(p_i, \epsilon_i, \dots) - n b_0 \ln\frac{\mu^2}{Q_0^2}\right) + \dots$$

- So the NLO result compensates the LO scale dependence. The residual dependence is NNLO.
- Scale dependence and normalization start being under control only at NLO, since compensation mechanism kicks in
- Notice also that a good scale choice automatically resums large logarithms to all orders, while a bad one spuriously introduces large logs and ruins the PT expansion
- Scale variation is conventionally used to estimate theory uncertainty, but the validity of this procedure should not be overrated (see later)

Leading order: Feynman diagrams

Get *any* LO cross-section from the Lagrangian

1. draw all Feynman diagrams
2. put in the explicit Feynman rules and get the amplitude
3. do some algebra, simplifications
4. square the amplitude
5. integrate over phase space + flux factor + sum/average over outgoing/incoming states

Automated tools for (1-3): FeynArts/Qgraf, Mathematica/Form etc.

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Bottlenecks

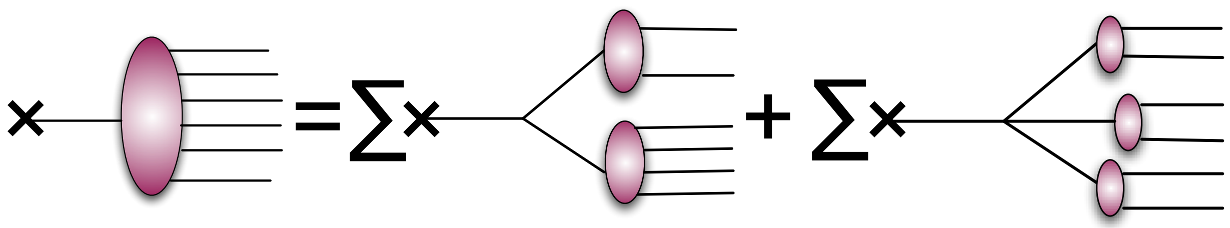
a) number of Feynman diagrams diverges factorially

b) algebra becomes more cumbersome with more particles

But given enough computer power everything can be computed at LO

Techniques beyond Feynman diagrams

✓ Berends-Giele relations: compute helicity amplitudes recursively using off-shell currents



Berends, Giele '88

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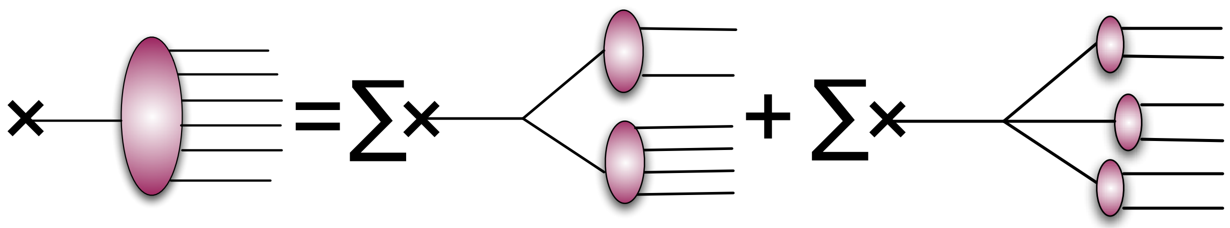

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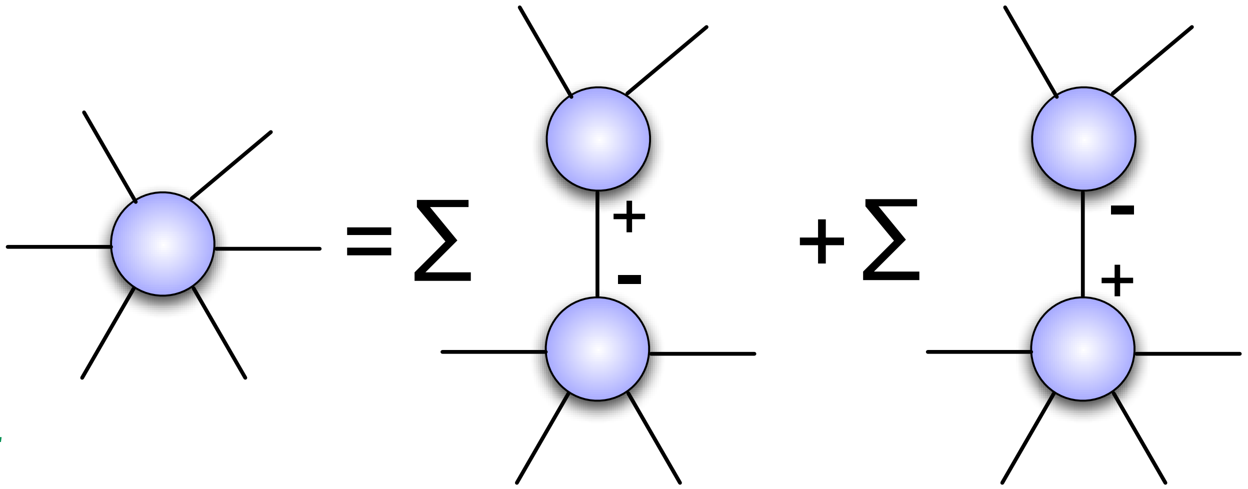
✓ BCF relations: compute helicity amplitudes via on-shell recursions (use complex momentum shifts)



Britto, Cachazo, Feng ’04

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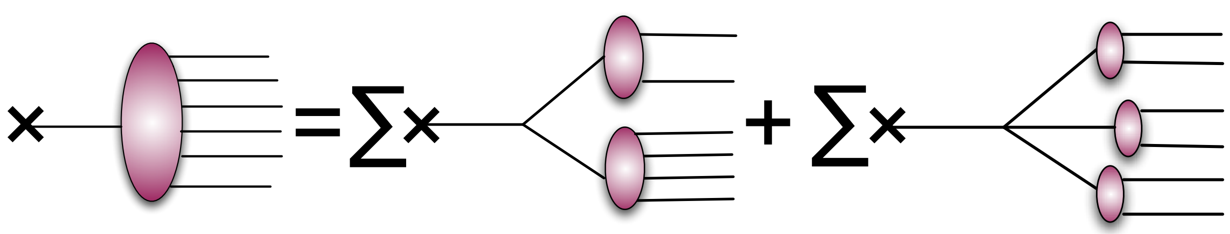

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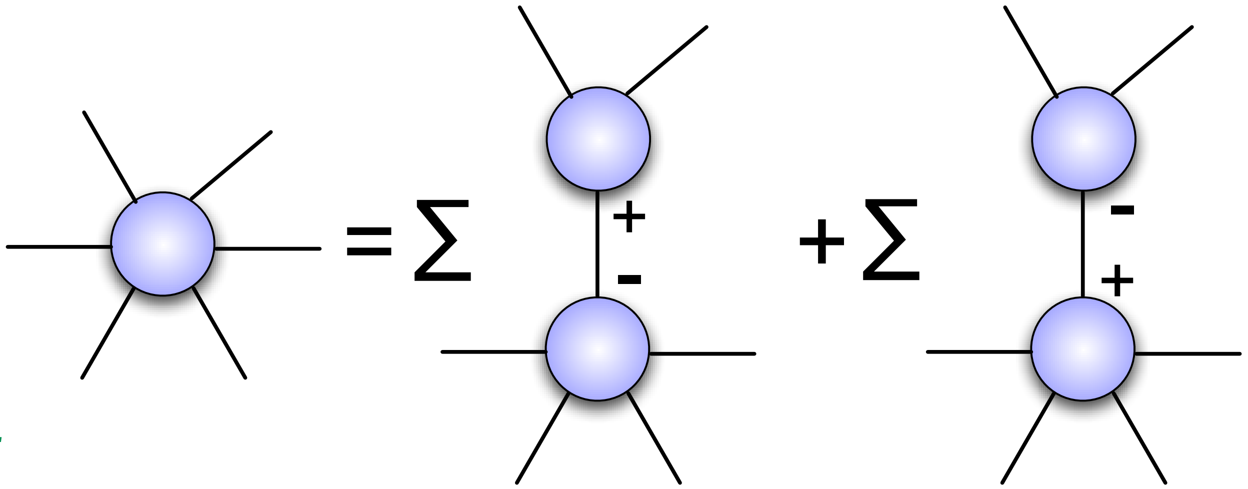
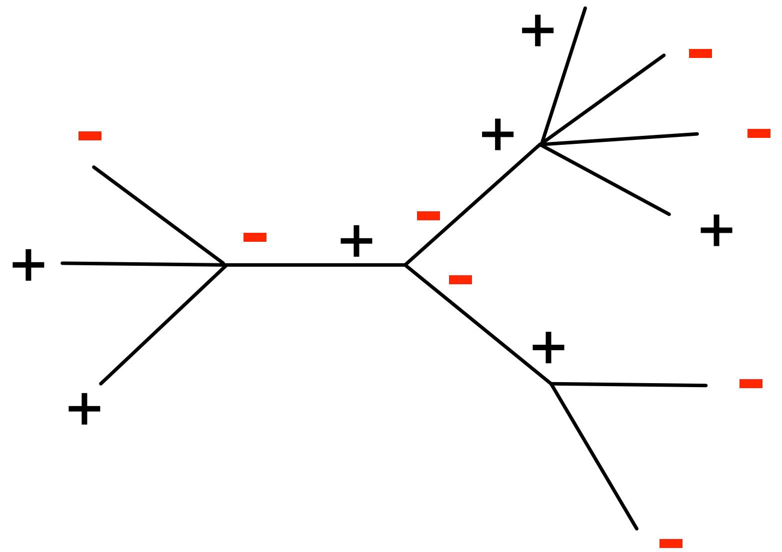


Britto, Cachazo, Feng '04

✓ CSW relations: compute helicity amplitudes by sewing together MHV amplitudes [- - + + ... +]



Cachazo, Svrcek, Witten '04

Matrix element generators

Fully automated

- generation of tree level matrix elements
  - Feynman diagrams [CompHEP/CalcHEP, Madgraph/Madevent, HELAS, Sherpa, ...]
  - Helicity amplitudes + off-shell Berends-Giele recursion [ALPHA/ALPGEN, Helac, Vecbos]
  - From twistors: on-shell recursion (BCF) / MHV vertices (CSW) (no public code)
- phase space integration
- interface to parton showers

Many well tested public available codes

Benefits and drawbacks of LO

<u>Benefits of LO:</u>

- fastest option; often the only one
- test quickly new ideas with fully exclusive description
- many working, well-tested approaches
- highly automated, crucial to explore new ground, but no precision

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Drawbacks of LO:

- large scale dependences, reflecting large theory uncertainty
- no control on normalizaiton
- poor control on shapes
- poor modeling of jets

Example: W+4 jet cross-section $\propto \alpha_s(Q)^4$

Vary $\alpha_s(Q)$ by ±10% via change of Q $\Rightarrow$ cross-section varies by ±40%

Next-to-leading order

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- through loop effects get indirect information about sectors not directly accessible

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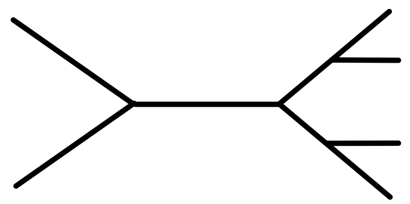

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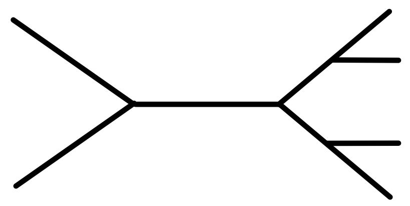

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  ➔ divergence from loop integration,
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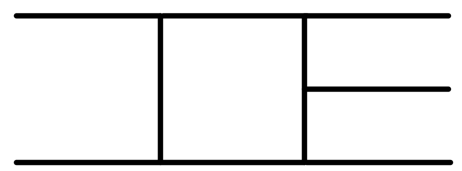

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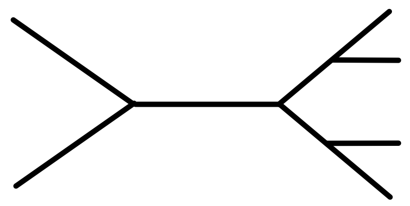

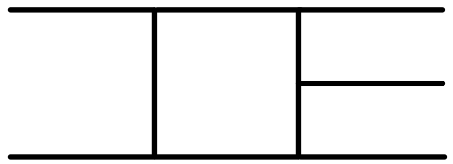

- tree graph rates with N+1 partons
  ➔ soft/collinear divergences
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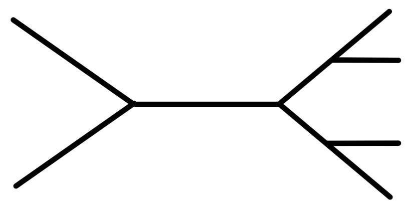

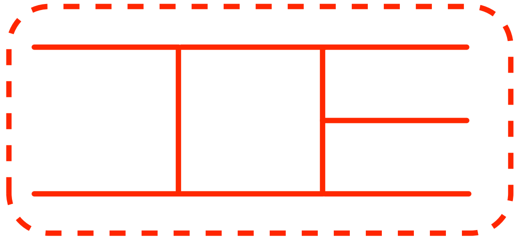

bottleneck

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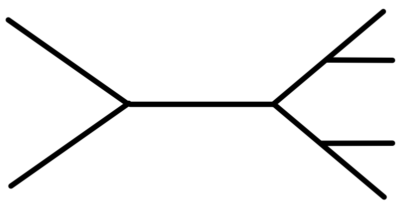

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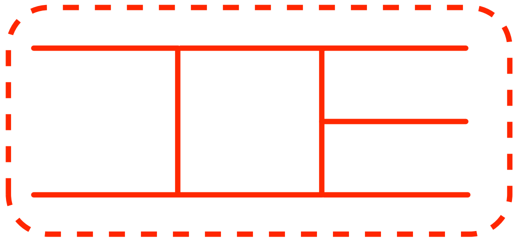

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We won't have time to do detailed NLO calculations, but let's look a bit more in detail at the issue of divergences/subtraction

Regularization procedures in QCD

Regularization: a way to make intermediate divergent quantities meaningful

- In QCD dimensional regularization is today the standard procedure, based on the fact that d-dimensional integrals are more convergent if one reduces the number of dimensions.

$$\int \frac{d^4 l}{(2\pi)^4} \to \mu^{2\epsilon} \int \frac{d^d l}{(2\pi)^d}\,, \quad d = 4 - 2\epsilon < 4$$

- N.B. to preserve the correct dimensions a mass scale μ is needed
- Divergences show up as intermediate poles 1/ε $\int_0^1 \frac{dx}{x} \to \int_0^1 \frac{dx}{x^{1-\epsilon}} = \frac{1}{\epsilon}$
- This procedure works both for UV divergences and IR divergences

Alternative regularization schemes: photon mass (EW), cut-offs, Pauli-Villard ...
Compared to those methods, dimensional regularizatiom has the big virtue that it leaves the regularized theory Lorentz invariant, gauge invariant, unitary etc.

Renormalization schemes

Renormalization: a global redefinition of couplings and masses which absorbs all UV divergences. Several schemes are possible (MS, $\overline{\text{MS}}$, OS ...).

- Take two different renormalization schemes of the QCD bare coupling as

$$\alpha_s^{\rm ren,A} = Z^A \alpha_s^0 \,, \quad \alpha_s^{\rm ren,B} = Z^B \alpha_s^0$$

- Infinite parts of renormalization constants must be the same, therefore renormalized constants must be related by a finite renormalization

$$\alpha_s^{\rm ren,B} = \alpha_s^{\rm ren,A}(1 + c_1 \alpha_s^{\rm ren,A} + \dots)$$

- Note that as a consequence the first two β-function coefficients do not change under such a transformation, i.e. they are scheme independent. This it not true for higher order coefficients.

The $\overline{\text{MS}}$ scheme

- Today standard scheme is the modified minimal subtraction scheme, $\overline{\text{MS}}$
- After regularizing integrals via the dimensional regularization, poles appear always in the combination

$$\frac{1}{\epsilon} + \ln(4\pi) - \gamma_E$$

- Therefore in the $\overline{\text{MS}}$-scheme, instead of subtracting poles minimally, one always subtracts that combination, and replaces the bare coupling with the renormalized one

- It is then standard to quote the coupling and Λ_{QCD} in this scheme, the current value is

$$206\text{MeV} < \Lambda_{\overline{\text{MS}}}(5) < 231\text{MeV}$$

- Uncertainties in this quantity propagate in the QCD cross-sections

Subtraction and slicing methods

- Consider e.g. an n-jet cross-section with some arbitrary infrared safe jet definition. At NLO, two divergent integrals, but the sum is finite

$$\sigma^J_{\rm NLO} = \int_{n+1} d\sigma^J_{\rm R} + \int_n d\sigma^J_{\rm V}$$

- Since one integrates over a different number of particles in the final state, real and virtual need to be evaluated first, and combined then
- This means that one needs to find a way of removing divergences before evaluating the phase space integrals
- Two main techniques to do this
  - *phase space slicing* ⇒ obsolete because of practical/numerical issues
  - *subtraction method* ⇒ most used in recent applications

Subtraction method

- The real cross-section can be written schematically as

$$d\sigma_R^J = d\phi_{n+1} |\mathcal{M}_{n+1}|^2 F_{n+1}^J(p_1, \ldots, p_{n+1})$$

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$$|\mathcal{M}_{n+1}|^2 = \frac{1}{x}\mathcal{M}(x)$$

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- IR divergences in the loop integration regularized by taking D=4-2ε

$$2\,\mathrm{Re}\{\mathcal{M}_V \cdot \mathcal{M}_0^*\} = \frac{1}{\epsilon}\mathcal{V}$$

Subtraction method

- The n-jet cross-section becomes

$$\sigma^J_{\mathrm{NLO}} = \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{M}(x) F^J_{n+1}(x) + \frac{1}{\epsilon} \mathcal{V} F^J_n$$

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- One can then add and subtract the analytically computed divergent part

$$\sigma_{\mathrm{NLO}}^{J} = \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{M}(x) F_{n+1}^{J}(x) - \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{V} F_n^{J} + \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{V} F_n^{J} + \frac{1}{\epsilon} \mathcal{V} F_n^{J}$$

Subtraction method

- This can be rewritten exactly as

$$\sigma_{\mathrm{NLO}}^{J} = \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{M}(x) \left(F_1^J(x) - \mathcal{V} F_0^J \right) + \mathcal{O}(1) \mathcal{V} F_0^J$$

$\Rightarrow$ Now both terms are finite and can be evaluated numerically

- Subtracted cross-section must be calculated separately for each process (but mostly automated now). It must be valid everywhere in phase space
- Systematized in the seminal papers of Catani-Seymour (dipole subtraction, '96) and Frixione-Kunszt-Signer (FKS method, '96)
- Subtraction used in all recent NLO applications and public codes (Event2, Disent, MCFM, NLOjet++, ...)

Approaches to virtual part of NLO

Two complementary approaches:

- Numerical/traditional Feynman diagram methods:
  use robust computational methods [integration by parts, reduction techniques...], then let the computer do the work for you

  Bottleneck:
  factorial growth, $2 \to 4$ barely touched, very difficult to go beyond

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  improve understanding of field theory [e.g. twistor methods, unitarity, supersymmetry, recursions ...]

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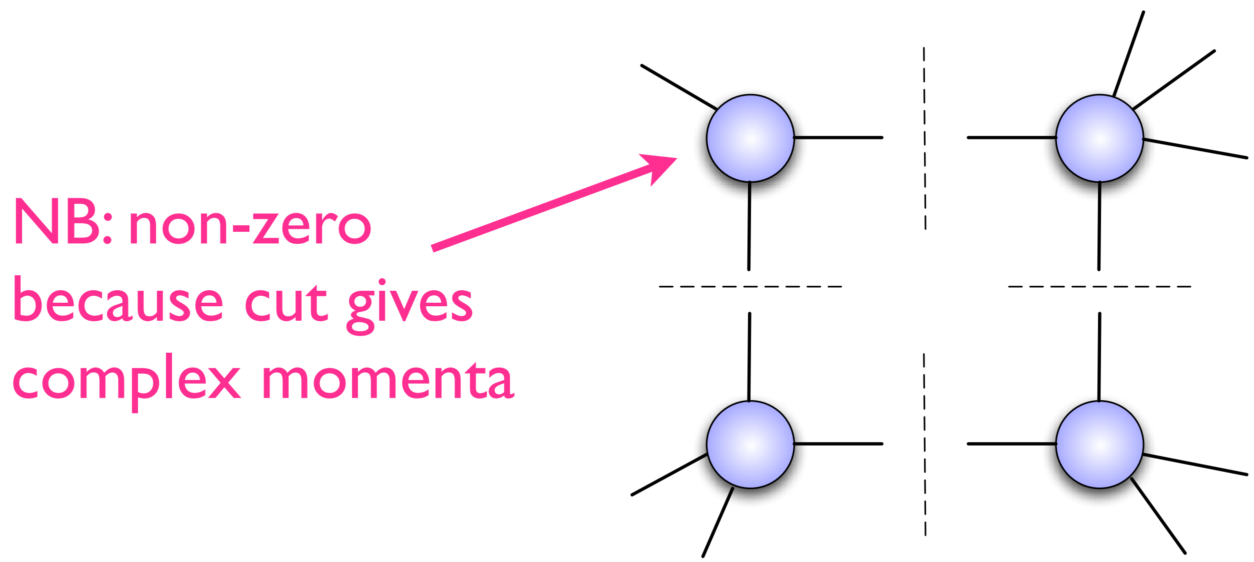
Recently: unified approaches as a winning strategy ?

Two breakthrough ideas

Aim: NLO loop integral without doing the integration

1) "... we show how to use generalized unitarity to read off the (box) coefficients. The generalized cuts we use are quadrupole cuts ..."



Britto, Cachazo, Feng '04

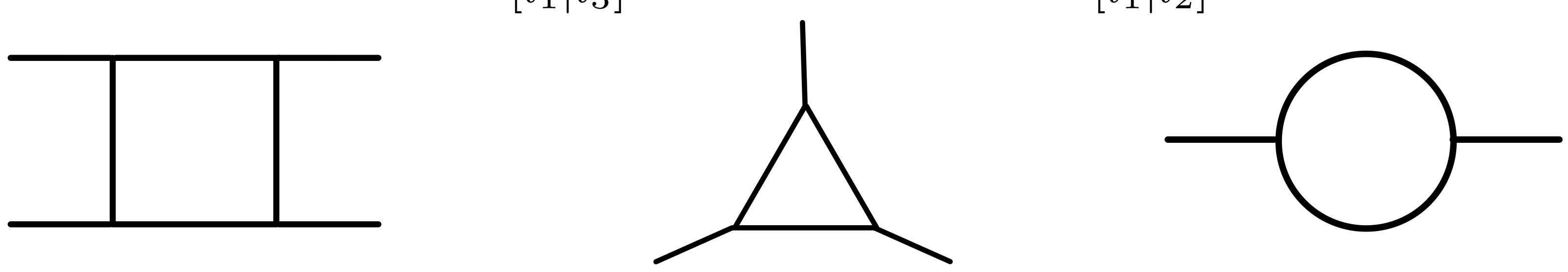
Quadrupole cuts: 4 on-shell conditions on 4 dimensional loop momentum) freezes the integration. But rational part of the amplitude, coming from $D=4-2\varepsilon$ not 4, computed separately

Two breakthrough ideas

Aim: NLO loop integral without doing the integration

2) The OPP method: "We show how to extract the coefficients of 4-, 3-, 2- and 1-point one-loop scalar integrals...."

$$\mathcal{A}_N = \sum_{[i_1|i_4]} \left(d_{i_1 i_2 i_3 i_4} \; I^{(D)}_{i_1 i_2 i_3 i_4} \right) + \sum_{[i_1|i_3]} \left(c_{i_1 i_2 i_3} \; I^{(D)}_{i_1 i_2 i_3} \right) + \sum_{[i_1|i_2]} \left(b_{i_1 i_2} \; I^{(D)}_{i_1 i_2} \right)$$



Ossola, Pittau, Papadopolous '06

Coefficients can be determined by solving system of equations: no loops, no twistors, just algebra!

Status of NLO

<u>Status of NLO:</u>

☑ $2 \to 2$: all known (or easy) in SM and beyond

☑ $2 \to 3$: very few processes left

[but: often do not include decays, newest codes private]

☐ $2 \to 4$: barely touched ground. Only three LHC processes computed at NLO up to now

The 2005 Les Houches wish-list

Table 42: The LHC "priority" wishlist for which a NLO computation seems now feasible.

process ($V \in \{Z, W, \gamma\}$)	relevant for
1. $pp \to V\,V$ jet	$t\bar{t}H$, new physics
2. $pp \to t\bar{t}\,b\bar{b}$	$t\bar{t}H$
3. $pp \to t\bar{t} + 2$ jets	$t\bar{t}H$
4. $pp \to V\,V\,b\bar{b}$	VBF$\to H \to VV$, $t\bar{t}H$, new physics
5. $pp \to V\,V + 2$ jets	VBF$\to H \to VV$
6. $pp \to V + 3$ jets	various new physics signatures
7. $pp \to V\,V\,V$	SUSY trilepton

The QCD, EW & Higgs Working group report hep-ph/0604120

The 2007 update

Process ($V \in \{Z, W, \gamma\}$)	Comments
Calculations completed since Les Houches 2005	
1. $pp \to VV$jet	WWjet completed by Dittmaier/Kallweit/Uwer [3]; Campbell/Ellis/Zanderighi [4] and Binoth/Karg/Kauer/Sanguinetti (in progress)
2. $pp \to$ Higgs+2jets	NLO QCD to the gg channel completed by Campbell/Ellis/Zanderighi [5]; NLO QCD+EW to the VBF channel completed by Ciccolini/Denner/Dittmaier [6, 7]
3. $pp \to V\,V\,V$	ZZZ completed by Lazopoulos/Melnikov/Petriello [8] and WWZ by Hankele/Zeppenfeld [9]
Calculations remaining from Les Houches 2005	
4. $pp \to t\bar{t}\,b\bar{b}$	relevant for $t\bar{t}H$
5. $pp \to t\bar{t}$+2jets	relevant for $t\bar{t}H$
6. $pp \to VV\,b\bar{b}$,	relevant for VBF $\to H \to VV$, $t\bar{t}H$
7. $pp \to VV$+2jets	relevant for VBF $\to H \to VV$ VBF contributions calculated by (Bozzi/)Jäger/Oleari/Zeppenfeld [10–12]
8. $pp \to V$+3jets	various new physics signatures
NLO calculations added to list in 2007	
9. $pp \to b\bar{b}b\bar{b}$	Higgs and new physics signatures
Calculations beyond NLO added in 2007	
10. $gg \to W^*W^*$ $\mathcal{O}(\alpha^2\alpha_s^3)$	backgrounds to Higgs
11. NNLO $pp \to t\bar{t}$	normalization of a benchmark process
12. NNLO to VBF and Z/γ+jet	Higgs couplings and SM benchmark
Calculations including electroweak effects	
13. NNLO QCD+NLO EW for W/Z	precision calculation of a SM benchmark

Table 1: The updated experimenter's wishlist for LHC processes

} with Feynman diagrams

← '09 with old techniques

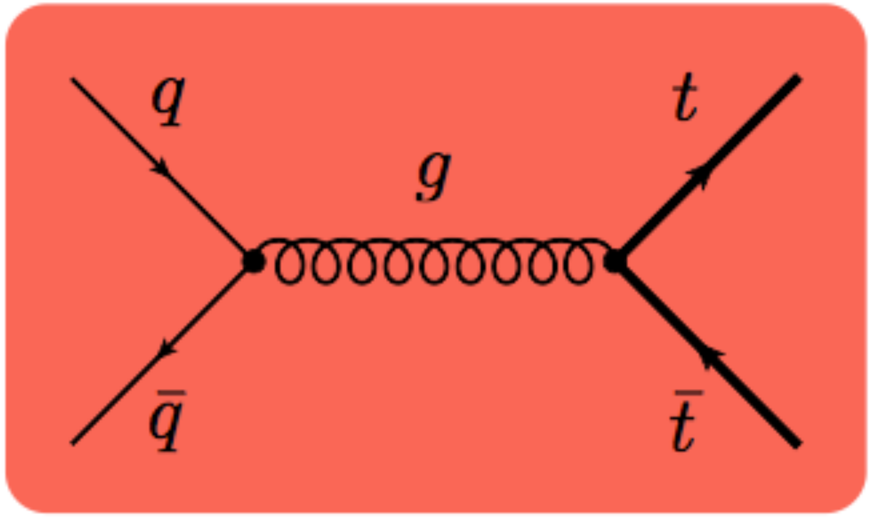
← '10 with new techniques

← '09 with new techniques

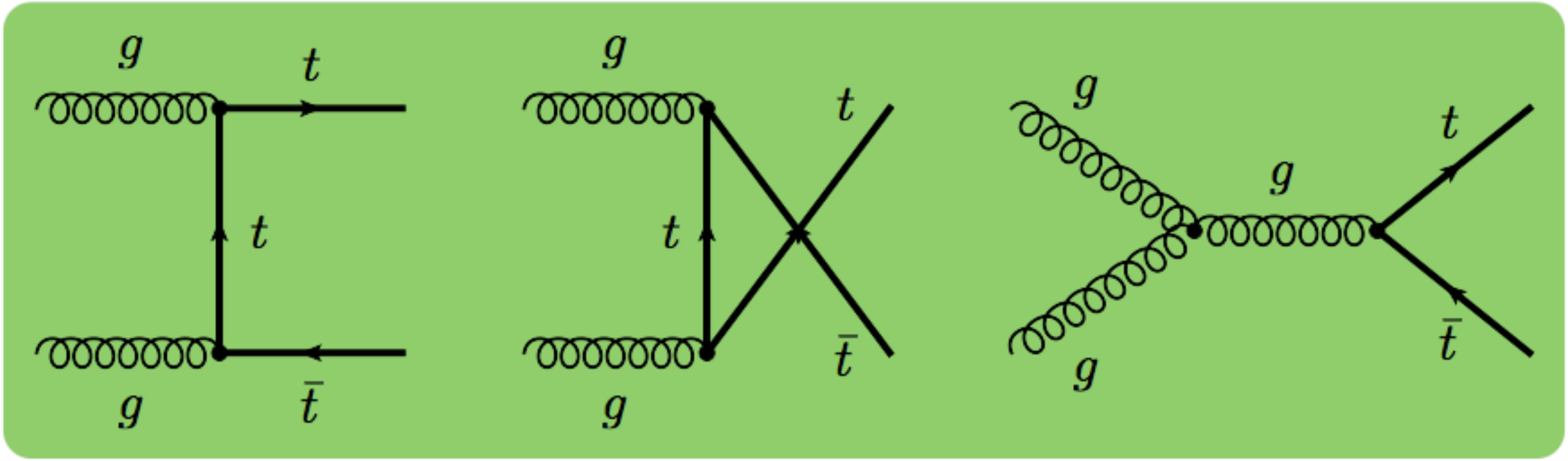
The NLO multi-leg Working group report 0803.0494

Top-pair production

Basic production mechanisms: initiated form quarks or gluons



What is the dominant production mechanism, at the Tevatron / LHC ? [And why ?]



Top-pair production: Tevatron

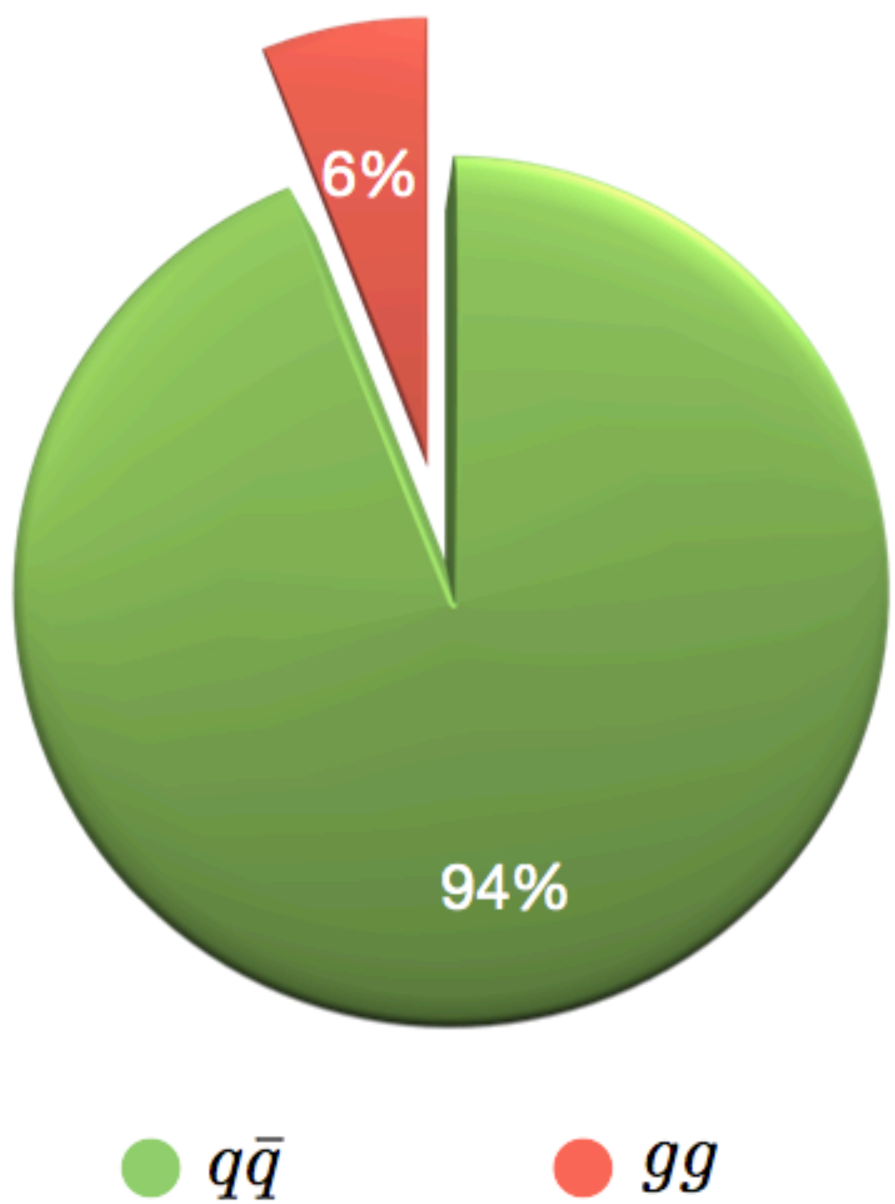
Running the program MCFM gives

```
Value of final lord integral is     9334.461 +/-     3.530 fb

 Total number of shots          :      200000
 Total no. failing cuts         :           0
 Number failing jet cuts        :           0
 Number failing process cuts    :           0

   Jet efficiency : 100.00%
   Cut efficiency : 100.00%
 Total efficiency : 100.00%

 Contribution from parton sub-processes:
 ---------------------------------------
       GG      |          563.36203     6.04%
       GQ      |            0.00000     0.00%
       GQB     |            0.00000     0.00%
       QG      |            0.00000     0.00%
       QBG     |            0.00000     0.00%
       QQ      |            0.00000     0.00%
       QBQB    |            0.00000     0.00%
       QQB     |         8723.36136    93.45%
       QBQ     |           47.73759     0.51%
 ---------------------------------------
```



Top-pair production: pp @ 1.96 TeV

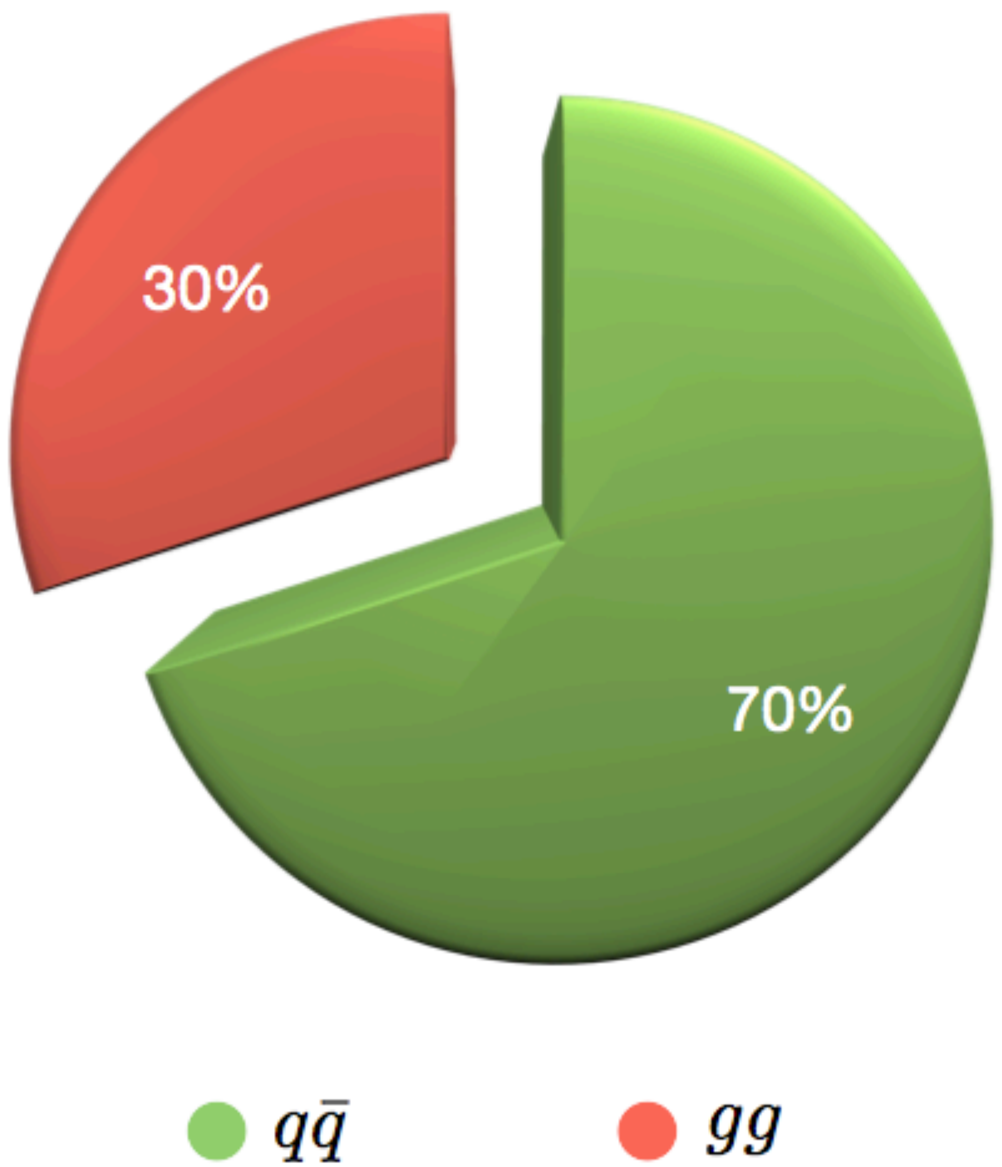
Running the program MCFM gives

```
Value of final lord integral is     1889.320 +/-     0.723 fb

 Total number of shots          :       200000
 Total no. failing cuts         :            0
 Number failing jet cuts        :            0
 Number failing process cuts    :            0

   Jet efficiency : 100.00%
   Cut efficiency : 100.00%
 Total efficiency : 100.00%

 Contribution from parton sub-processes:
 ---------------------------------------
       GG     |           563.26857   29.81%
       GQ     |             0.00000    0.00%
       GQB    |             0.00000    0.00%
       QG     |             0.00000    0.00%
       QBG    |             0.00000    0.00%
       QQ     |             0.00000    0.00%
       QBQB   |             0.00000    0.00%
       QQB    |           662.81972   35.08%
       QBQ    |           663.23143   35.10%
 ---------------------------------------
```



Top-pair production: LHC

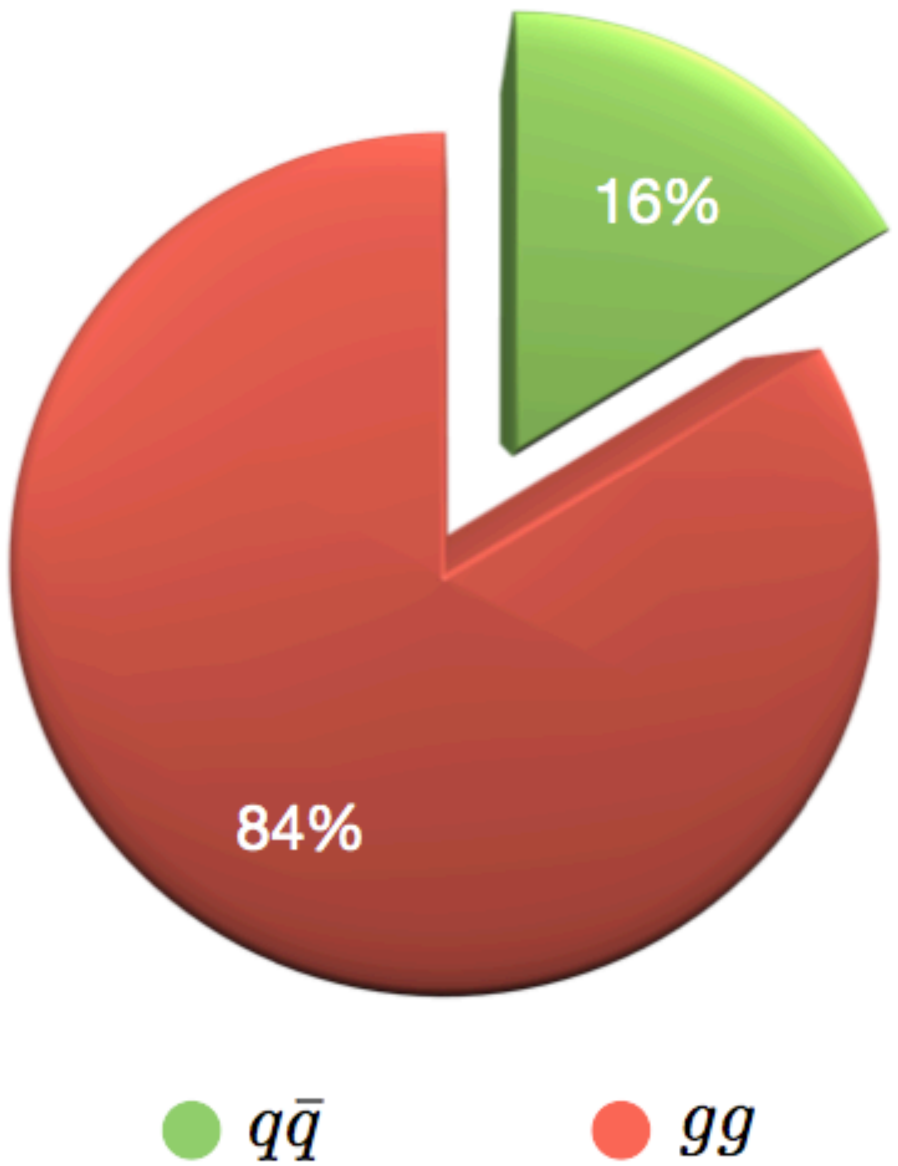
Running the program MCFM gives

```
Value of final lord integral is   373635.066 +/-   148.259 fb

 Total number of shots          :       200000
 Total no. failing cuts         :            0
 Number failing jet cuts        :            0
 Number failing process cuts :               0

   Jet efficiency : 100.00%
   Cut efficiency : 100.00%
 Total efficiency : 100.00%

 Contribution from parton sub-processes:
 ---------------------------------------
       GG     |       312453.03253    83.63%
       GQ     |            0.00000     0.00%
       GQB    |            0.00000     0.00%
       QG     |            0.00000     0.00%
       QBG    |            0.00000     0.00%
       QQ     |            0.00000     0.00%
       QBQB   |            0.00000     0.00%
       QQB    |        30598.98764     8.19%
       QBQ    |        30583.04606     8.19%
 ---------------------------------------
```



Top-asymmetry

At the Tevatron, one interesting top measurement is its asymmetry

$$A_{fb} = \frac{N_{\rm top}(\eta > 0) - N_{\rm top}(\eta < 0)}{N_{\rm top}(\eta > 0) + N_{\rm top}(\eta < 0)}$$

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At O(α_s^3) the asymmetry is non-zero, an NLO calculation gives

$$A_{fb}^{\rm NLO} = 0.050 \pm 0.015$$

Kuehn et al. '99

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Kuehn et al. '99

But CDF & D0 measurements give

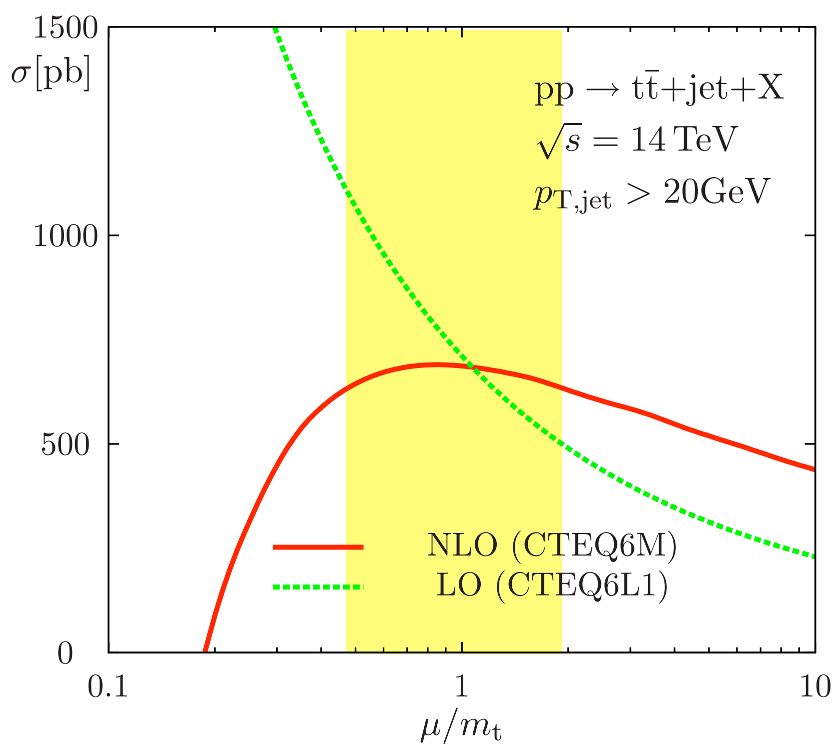
$$A_{fb}^{exp.} = 0.193 \pm 0.065\,(stat.) \pm 0.024\,(syst.)$$

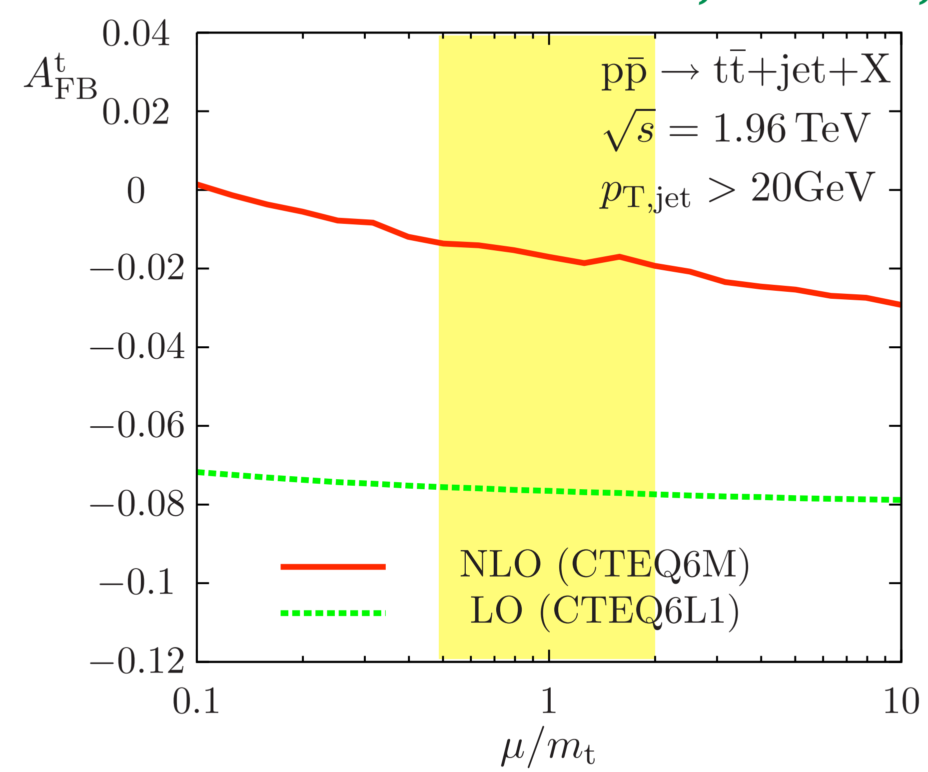
⇒ more than 2-sigma deviation from NLO. *New physics ?*

One advanced NLO example: tt+1jet

Calculation done with traditional methods

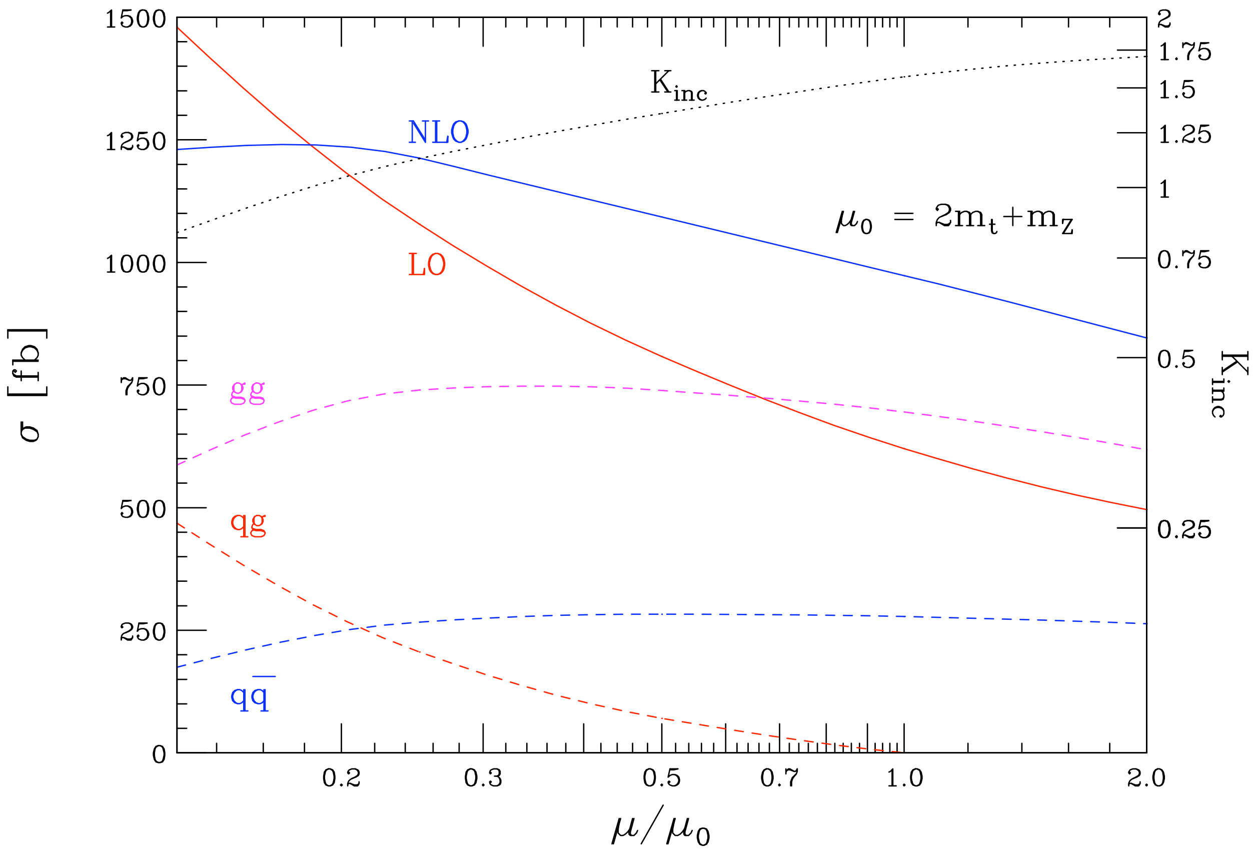
Dittmaier, Kallweit, Uwer '07-'08

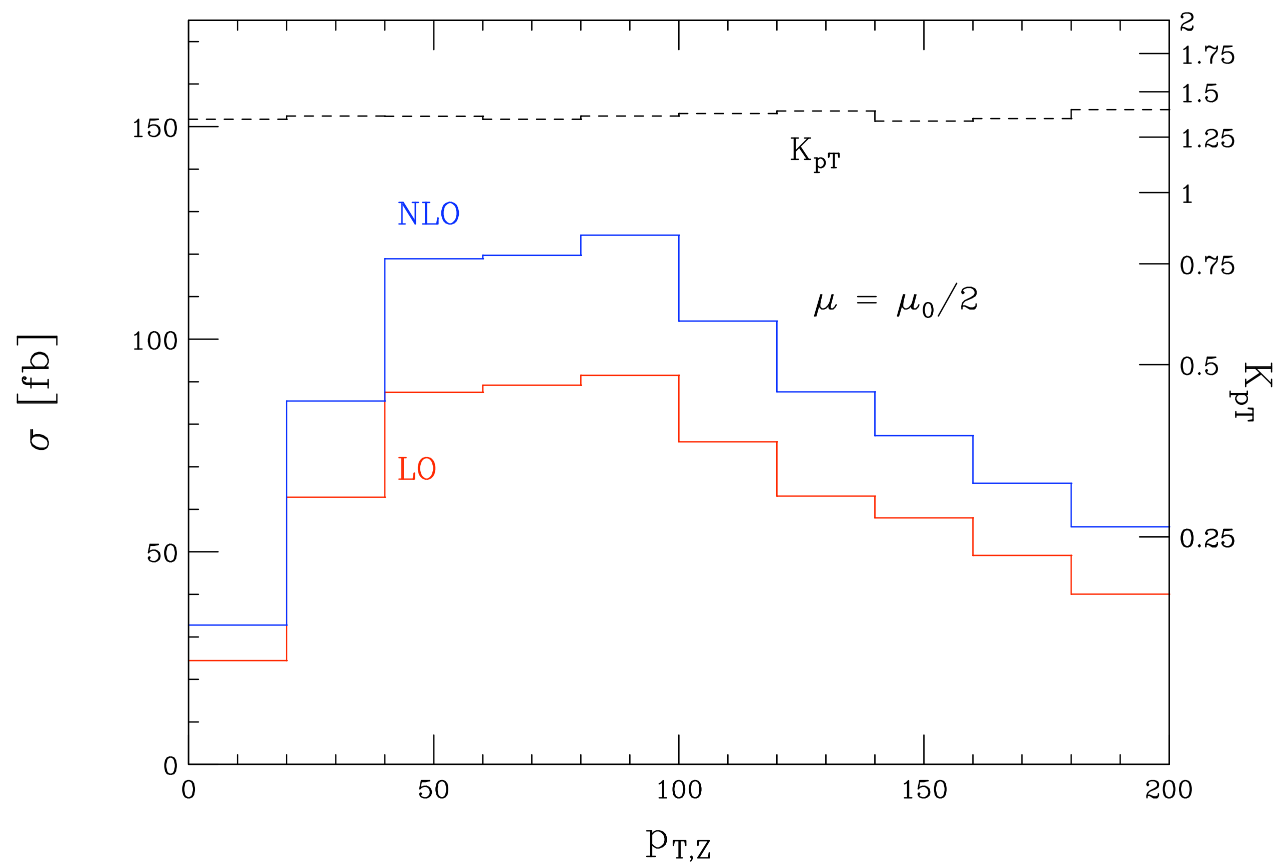




- improved stability of NLO result [but no decays]
- forward-backward asymmetry at the Tevatron compatible with zero
- essential ingredient of NNLO tt production (hot topic)

2nd NLO example: tt +Z

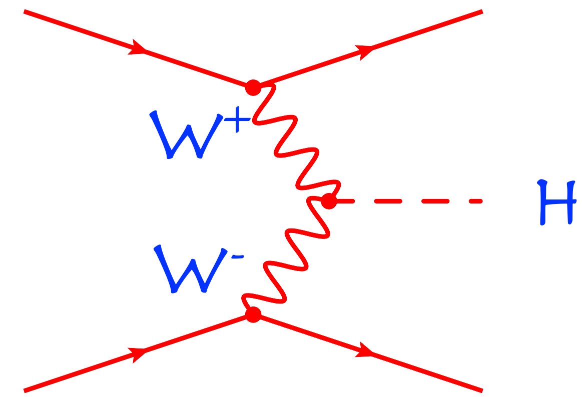




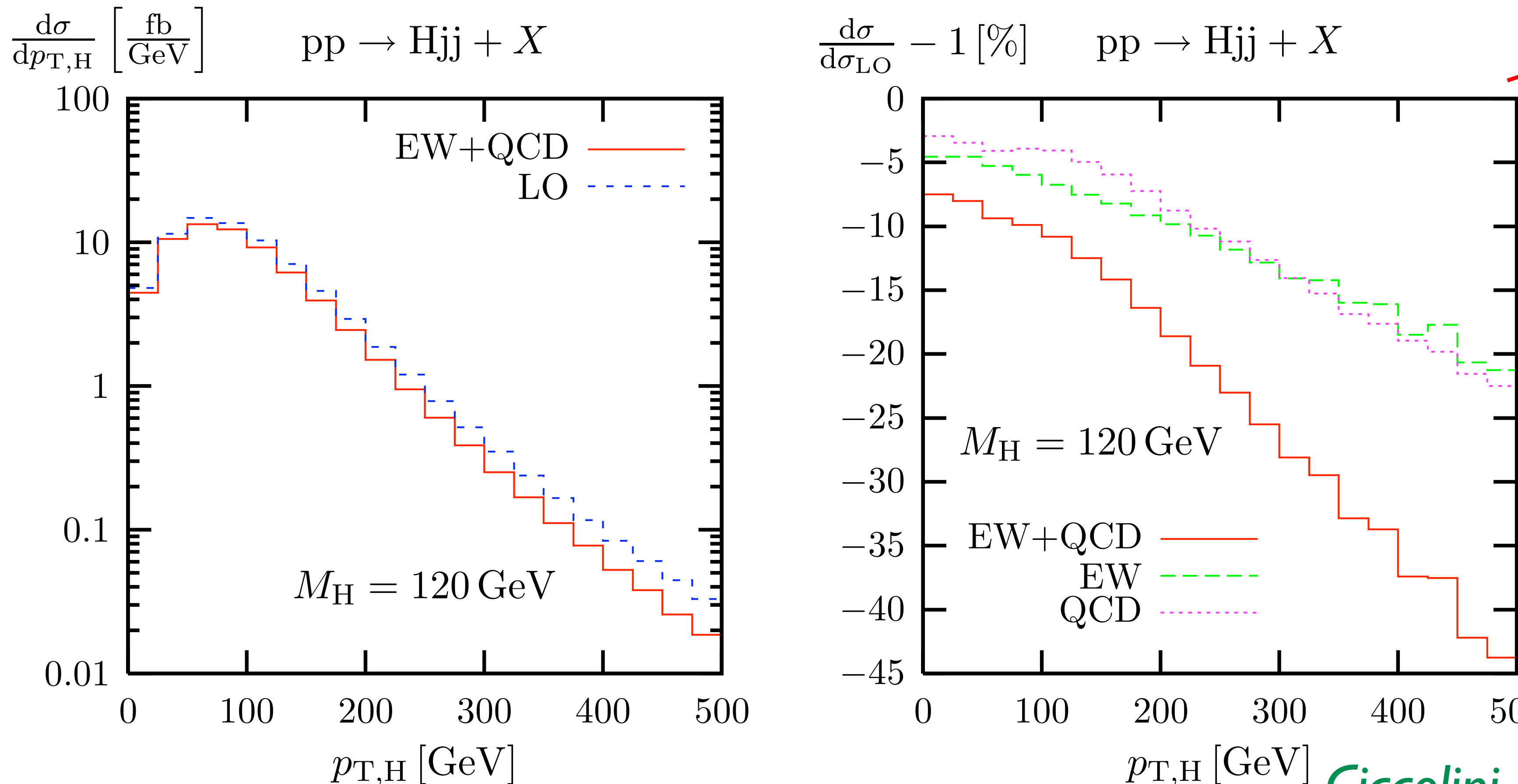
Lazopoulos, Melnikov, Petriello ’08

- NLO increase cross section by 35% (residual 10% uncertainty)
- factor of 1.5-2 improvement on ttZ measurement (probe BSM)
- no significant change in distributions

The “not so weak” electro-weak



Vector boson fusion Higgs production:



Ciccolini, Denner, Dittmaier ’07

☛ EW and QCD of the same size, both distort shapes!

Be aware of EW corrections for precision studies [peaks] and in tails of distributions [large EW logarithms]

General NLO features?

Process	Typical scales μ_0	μ_1	Tevatron K-factor $\mathcal{K}(\mu_0)$	$\mathcal{K}(\mu_1)$	$\mathcal{K}'(\mu_0)$	LHC K-factor $\mathcal{K}(\mu_0)$	$\mathcal{K}(\mu_1)$	$\mathcal{K}'(\mu_0)$
W	m_W	$2m_W$	1.33	1.31	1.21	1.15	1.05	1.15
W+1jet	m_W	p_T^{jet}	1.42	1.20	1.43	1.21	1.32	1.42
W+2jets	m_W	p_T^{jet}	1.16	0.91	1.29	0.89	0.88	1.10
WW+jet	m_W	$2m_W$	1.19	1.37	1.26	1.33	1.40	1.42
$t\bar{t}$	m_t	$2m_t$	1.08	1.31	1.24	1.40	1.59	1.48
$t\bar{t}$+1jet	m_t	$2m_t$	1.13	1.43	1.37	0.97	1.29	1.10
$b\bar{b}$	m_b	$2m_b$	1.20	1.21	2.10	0.98	0.84	2.51
Higgs	m_H	p_T^{jet}	2.33	–	2.33	1.72	–	2.32
Higgs via VBF	m_H	p_T^{jet}	1.07	0.97	1.07	1.23	1.34	1.09
Higgs+1jet	m_H	p_T^{jet}	2.02	–	2.13	1.47	–	1.90
Higgs+2jets	m_H	p_T^{jet}	–	–	–	1.15	–	–

$$\mathcal{K} = \frac{NLO}{LO}$$

[NLO report 0803.0494]

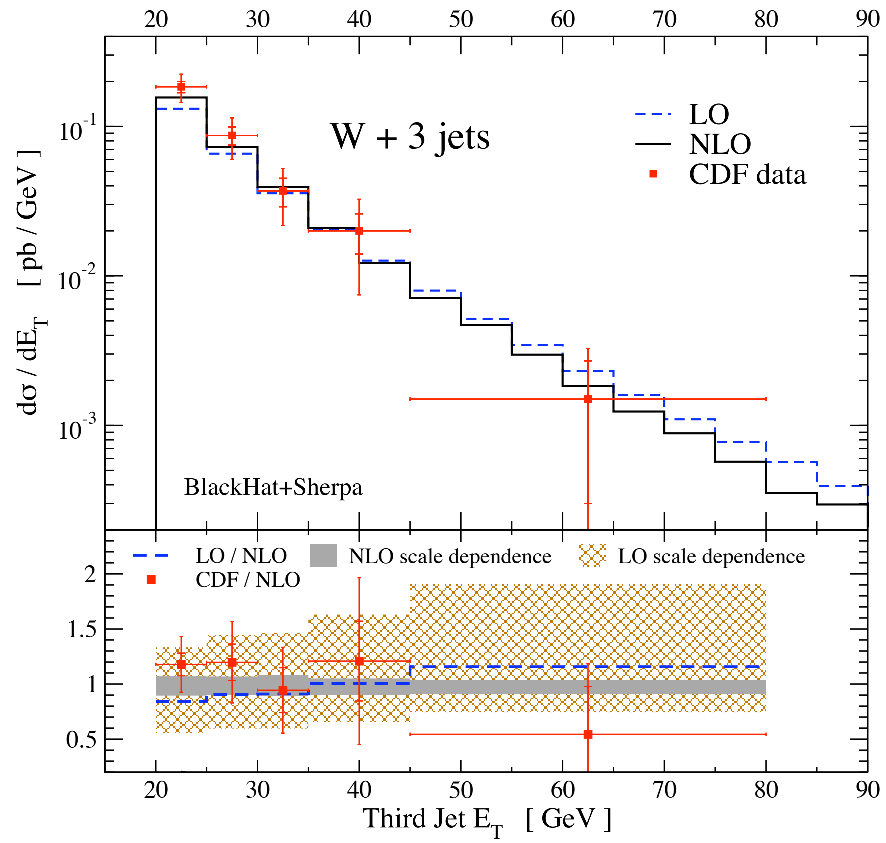
<u>General features:</u>

- color annihilation, gluon dominated ⇒ large K factors ?
- extra legs in the final state ⇒ smaller K-factors ?

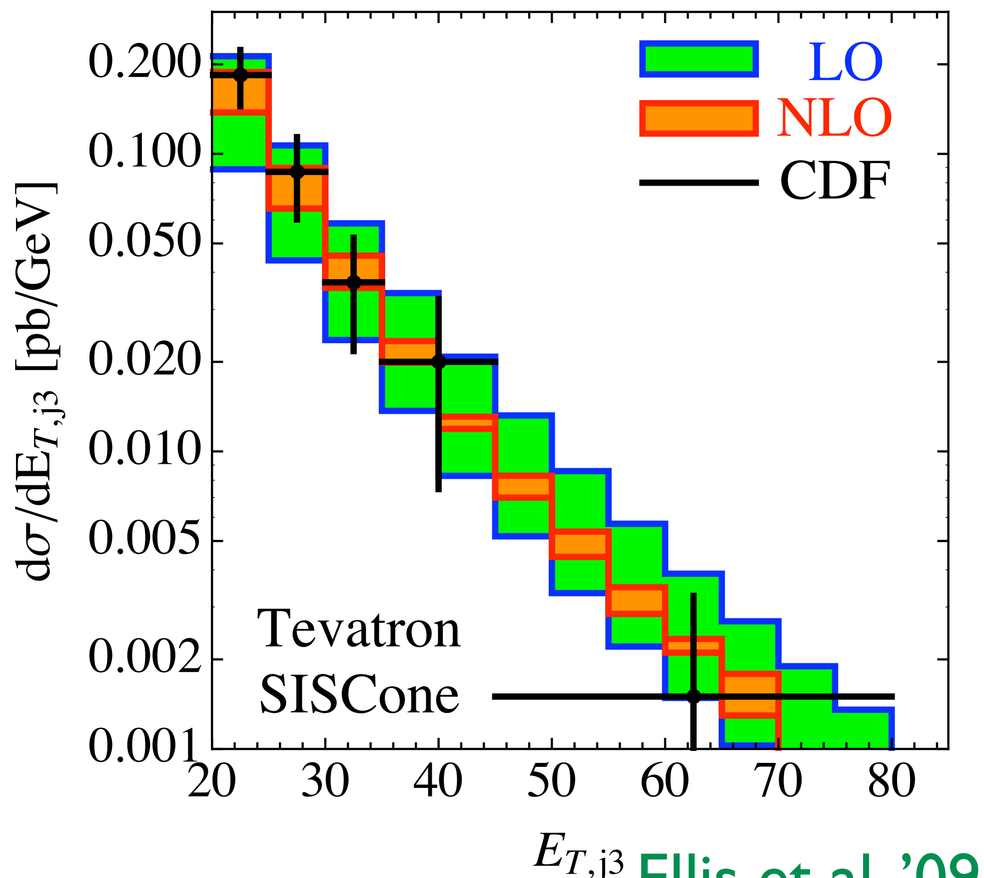
But be careful, only full calculations can really tell!

W + 3jets

Measured at the Tevatron + of primary importance at the LHC: background to model- independent new physics searches using jets + MET



Berger et al. '09

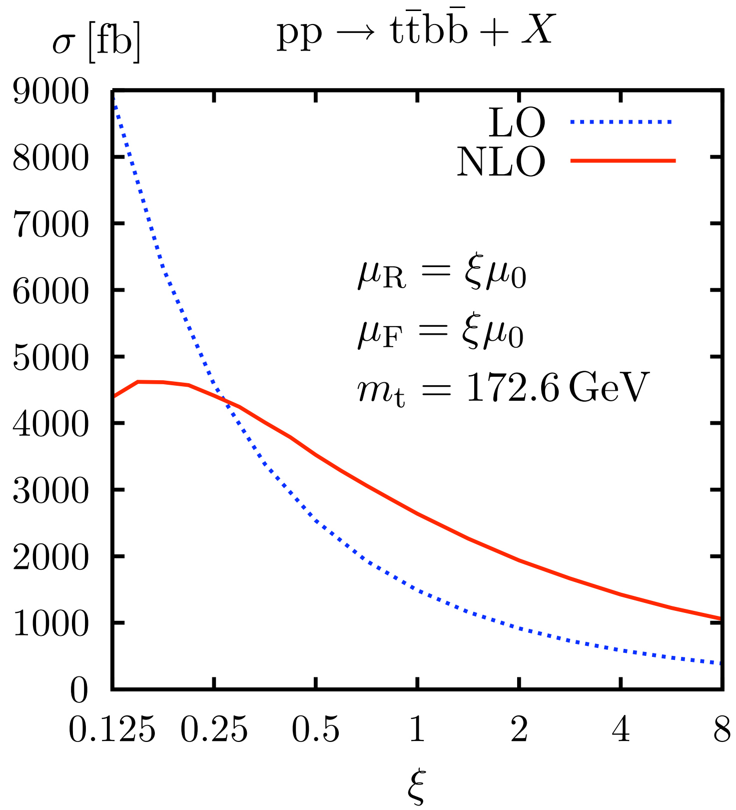


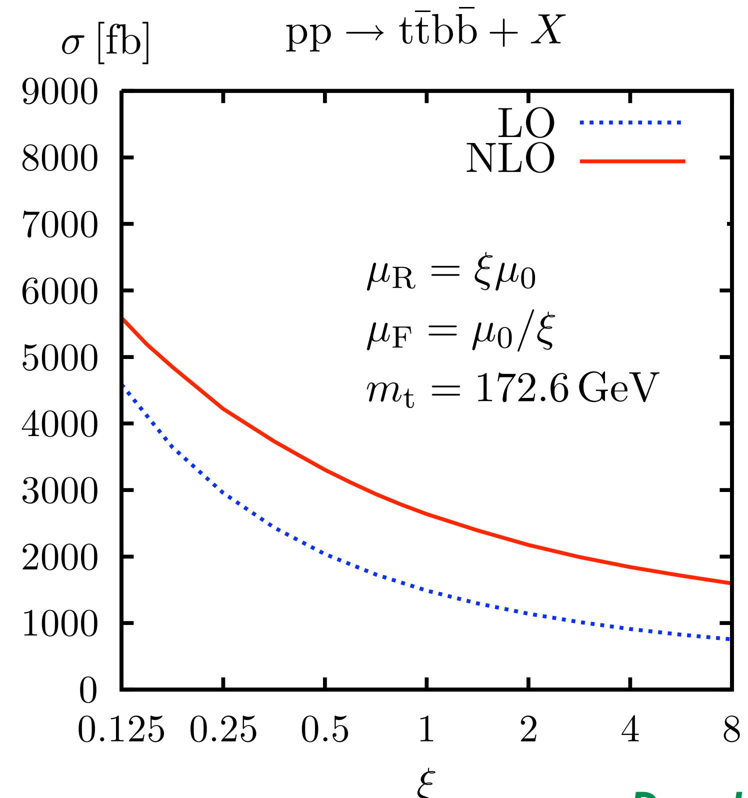
Ellis et al. '09

- ☺ Small K=1.0-1.1, reduced uncertainty: 50% (LO) → 10% (NLO)
- ☺ First applications of new techniques to 2 → 4 LHC processes

pp → tt bb

Measurement of ttH impossible without knowledge of pp →tt bb at NLO (need also pp →tt jj) + interesting per se





Bredenstein et al. '09

- ☹ Large K=1.8, large residual uncertainties: 70% (LO) → 35% (NLO)
- ☺ Demonstrates feasibility of Feynman diagrams calculation for $2 \to 4$ LHC processes

NLO + parton shower

Combine best features:

Get correct rates (NLO) and hadron-level description of events (PS)
Difficult because need exact NLO subtraction and remove it from PS

Only two working examples:

- MC@NLO
  - W/Z boson production
  - WW, WZ, ZZ production
  - inclusive Higgs production
  - heavy quark production
  - single-top

Frixione&Webber '02 and later refs.

- POWHEG
  - ZZ production
  - heavy quark production
  - W/Z production
  - Higgs, single top ... in progress

Nason '04 and later refs.

Other recent progress:

Shower with quantum inteference [Nagy, Soper], Geneve (SCET) [Bauer, Schwartz, Tackmann], Vincia (antenna factorization) [Giele et al.], Dipole factorization [Schumann]

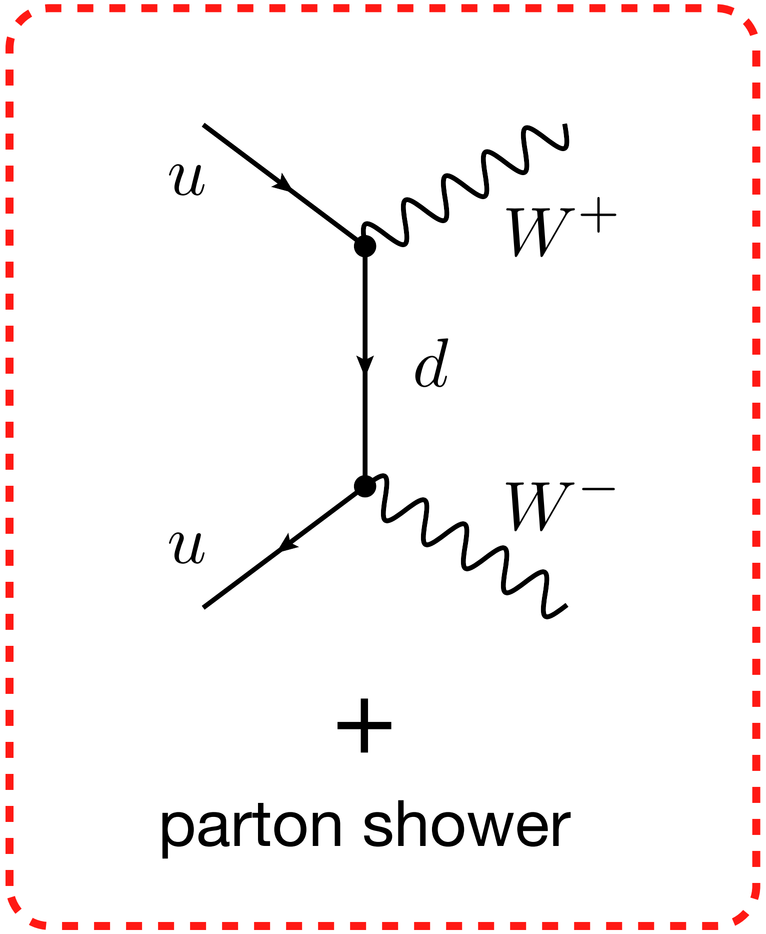
MC@NLO

IPROC	IV	IL$_1$	IL$_2$	Spin	Process
–1350–IL				✓	$H_1H_2 \to (Z/\gamma^* \to) l_{\rm IL}\bar{l}_{\rm IL} + X$
–1360–IL				✓	$H_1H_2 \to (Z \to) l_{\rm IL}\bar{l}_{\rm IL} + X$
–1370–IL				✓	$H_1H_2 \to (\gamma^* \to) l_{\rm IL}\bar{l}_{\rm IL} + X$
–1460–IL				✓	$H_1H_2 \to (W^+ \to) l^+_{\rm IL}\nu_{\rm IL} + X$
–1470–IL				✓	$H_1H_2 \to (W^- \to) l^-_{\rm IL}\bar{\nu}_{\rm IL} + X$
–1396				×	$H_1H_2 \to \gamma^*(\to \sum_i f_i\bar{f}_i) + X$
–1397				×	$H_1H_2 \to Z^0 + X$
–1497				×	$H_1H_2 \to W^+ + X$
–1498				×	$H_1H_2 \to W^- + X$
–1600–ID					$H_1H_2 \to H^0 + X$
–1705					$H_1H_2 \to b\bar{b} + X$
–1706		7	7	×	$H_1H_2 \to t\bar{t} + X$
–2000–IC		7		×	$H_1H_2 \to t/\bar{t} + X$
–2001–IC		7		×	$H_1H_2 \to \bar{t} + X$
–2004–IC		7		×	$H_1H_2 \to t + X$
–2030		7	7	×	$H_1H_2 \to tW^-/\bar{t}W^+ + X$
–2031		7	7	×	$H_1H_2 \to \bar{t}W^+ + X$
–2034		7	7	×	$H_1H_2 \to tW^- + X$
–2600–ID	1	7		×	$H_1H_2 \to H^0W^+ + X$
–2600–ID	1	i		✓	$H_1H_2 \to H^0(W^+ \to) l^+_i\nu_i + X$
–2600–ID	-1	7		×	$H_1H_2 \to H^0W^- + X$
–2600–ID	-1	i		✓	$H_1H_2 \to H^0(W^- \to) l^-_i\bar{\nu}_i + X$
–2700–ID	0	7		×	$H_1H_2 \to H^0Z + X$
–2700–ID	0	i		✓	$H_1H_2 \to H^0(Z \to) l_i\bar{l}_i + X$
–2850		7	7	×	$H_1H_2 \to W^+W^- + X$
–2860		7	7	×	$H_1H_2 \to Z^0Z^0 + X$
–2870		7	7	×	$H_1H_2 \to W^+Z^0 + X$
–2880		7	7	×	$H_1H_2 \to W^-Z^0 + X$

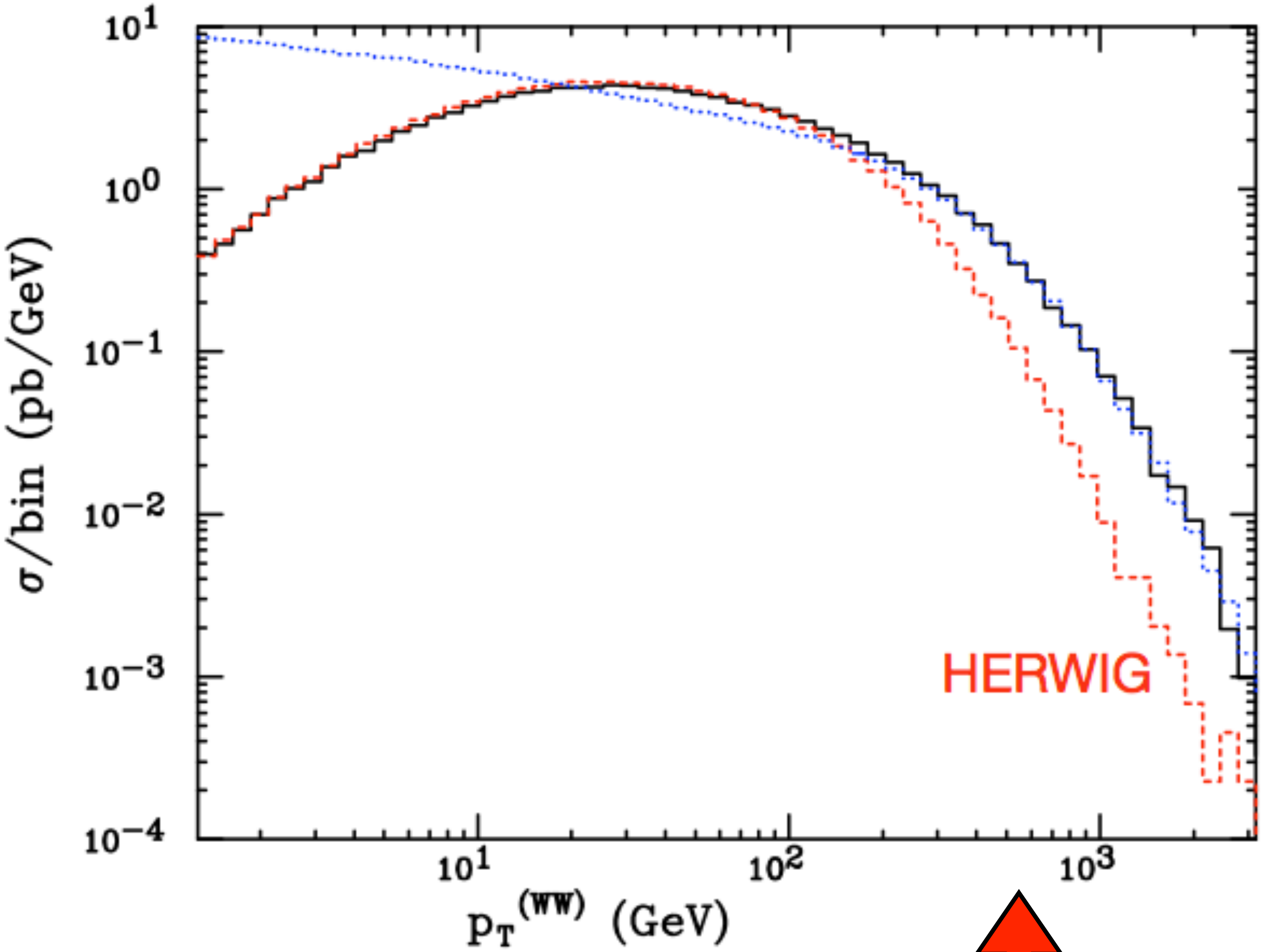
- $H_{1,2}$ denote nucleon and antinucleon
- The "Spin" indicates whether spin correlations in vector boson fusion or top decays are included (✓), neglected (×) or absent (void entry)
- The values of IV, IL, IL$_1$, and IL$_2$ control the identities of vector bosons and leptons

IPROC	IV	IL$_1$	IL$_2$	Spin	Process
–1706		i	j	✓	$H_1H_2 \to (t \to) b_k f_i f'_i (\bar{t} \to) \bar{b}_l f_j f'_j + X$
–2000–IC		i		✓	$H_1H_2 \to (t \to) b_k f_i f'_i / (\bar{t} \to) \bar{b}_k f_i f'_i + X$
–2001–IC		i		✓	$H_1H_2 \to (\bar{t} \to) \bar{b}_k f_i f'_i + X$
–2004–IC		i		✓	$H_1H_2 \to (t \to) b_k f_i f'_i + X$
–2030		i	j	✓	$H_1H_2 \to (t \to) b_k f_i f'_i (W^- \to) f_j f'_j /$ $(\bar{t} \to) \bar{b}_k f_i f'_i (W^+ \to) f_j f'_j + X$
–2031		i	j	✓	$H_1H_2 \to (\bar{t} \to) \bar{b}_k f_i f'_i (W^+ \to) f_j f'_j + X$
–2034		i	j	✓	$H_1H_2 \to (t \to) b_k f_i f'_i (W^- \to) f_j f'_j + X$
–2850		i	j	✓	$H_1H_2 \to (W^+ \to) l^+_i \nu_i (W^- \to) l^-_j \bar{\nu}_j + X$

MC@NLO: W^+W^- production (LHC)

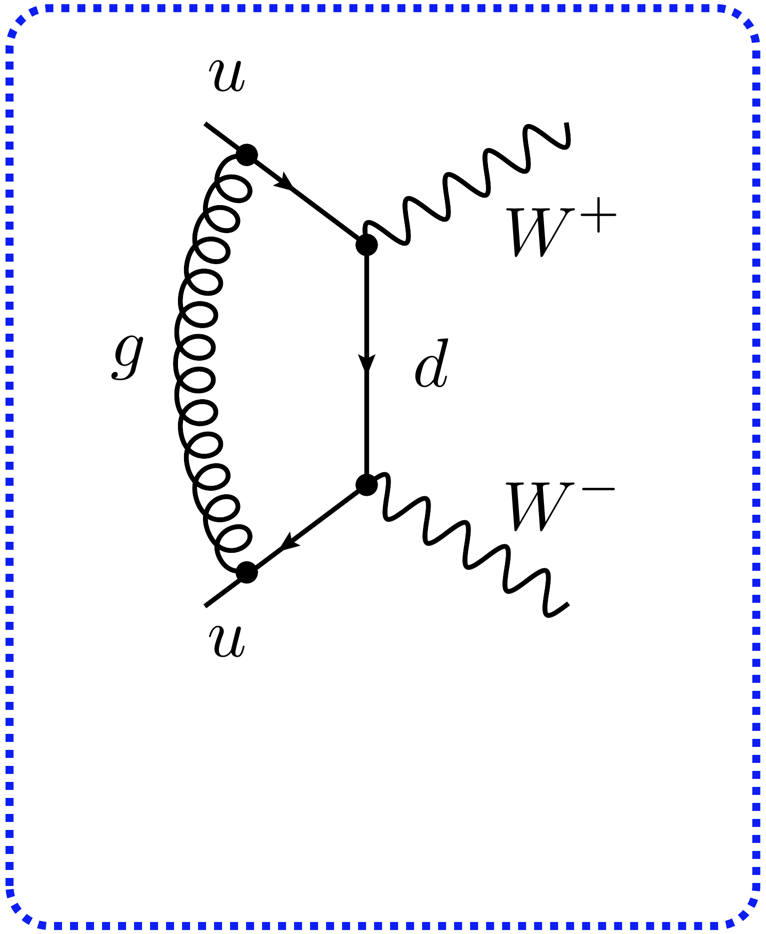


HERWIG

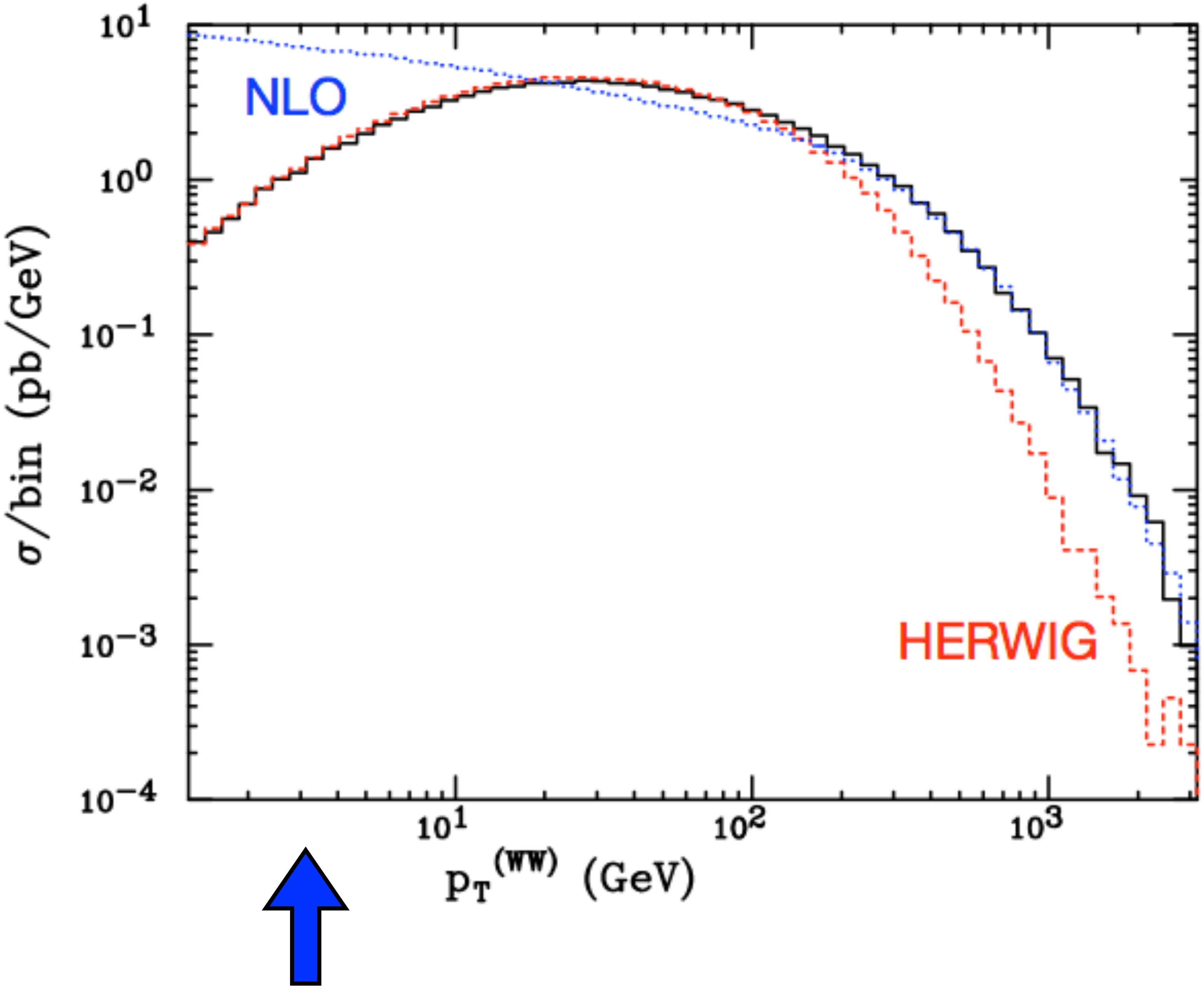


Herwig too soft in the high-p_t region

MC@NLO: W^+W^- production (LHC)

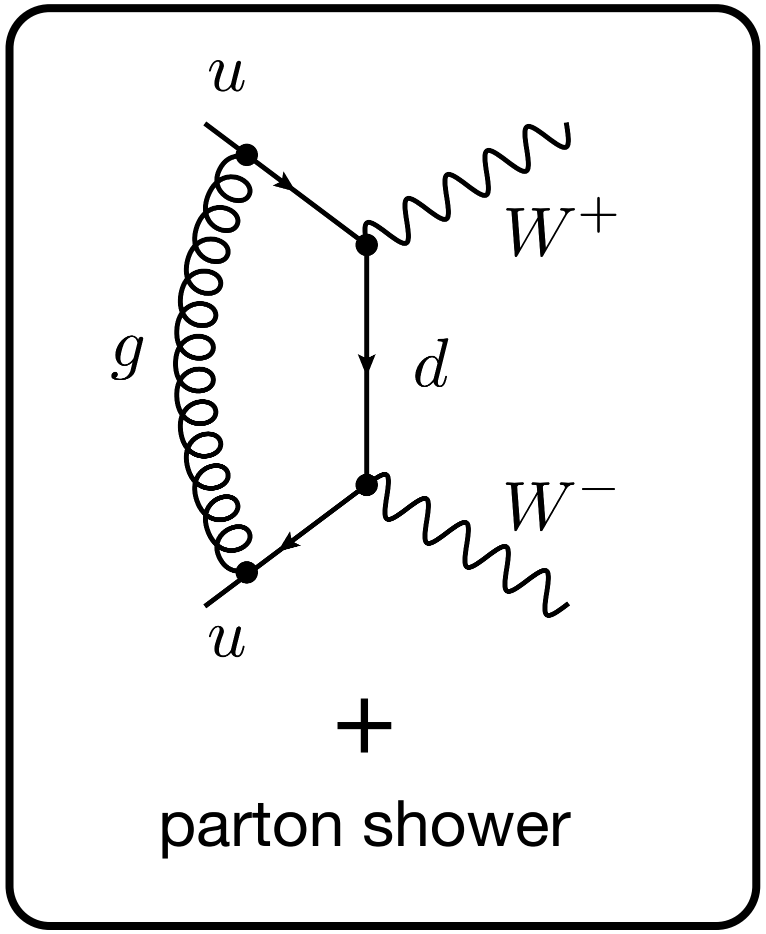


NLO

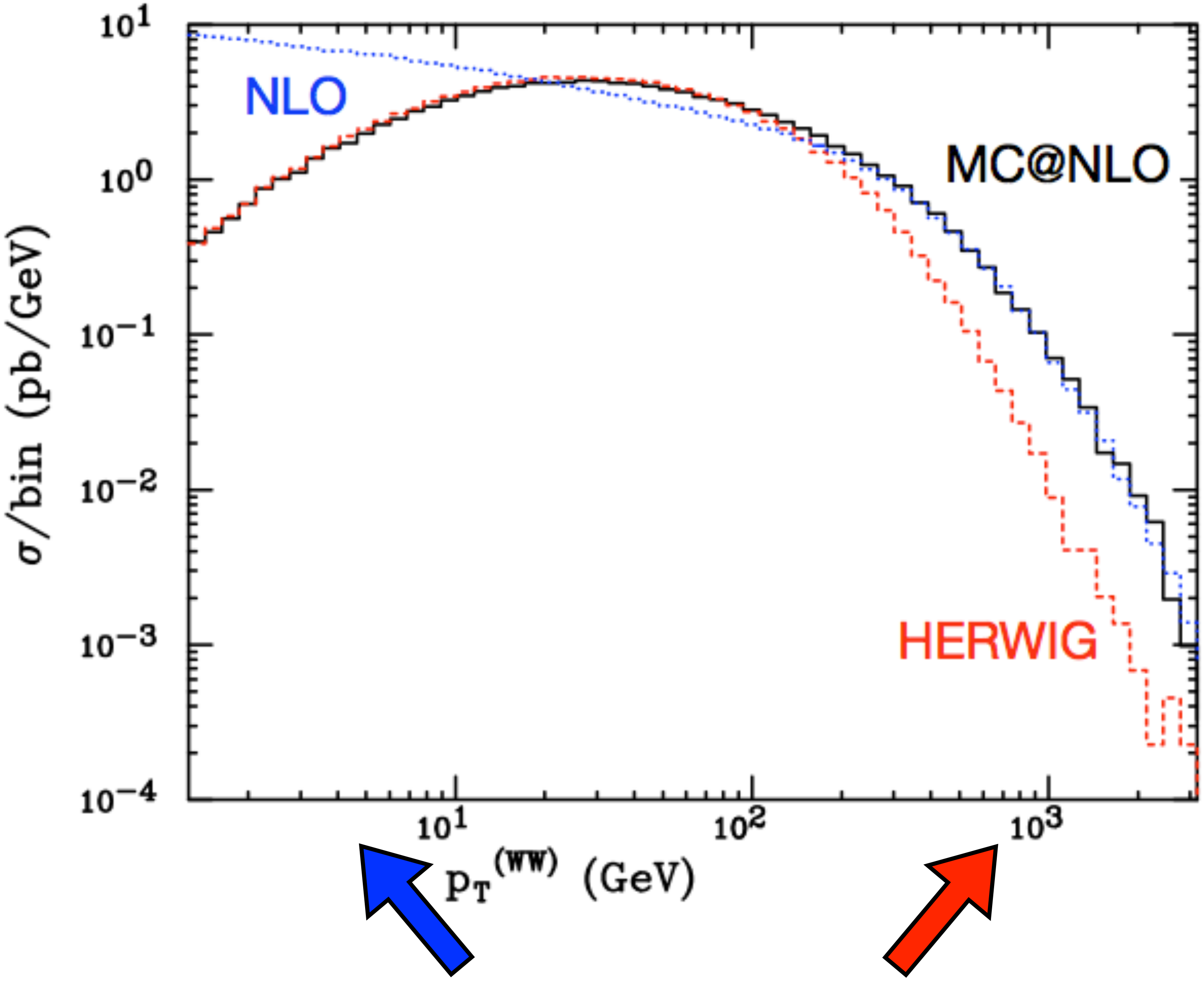


NLO divergent
in the soft region

MC@NLO: W^+W^- production (LHC)



MC@NLO



MC@NLO correctly interpolates between the two regimes

NNLO: when is NLO not good enough?

- when NLO corrections are large (NLO correction ~ LO)

  This may happens when

  - process involve very different scales → large logarithms of ratio of scales appear
  - new channels open up at NLO (at NLO they are effectively LO)
  - master example: Higgs production

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- when high precision is needed to match small experimental error

  - W/Z hadro-production, heavy-quark hadro-production, α_s from event shapes in e^+e^- ...

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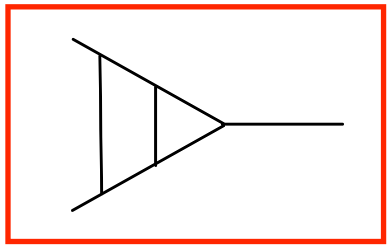
- when a reliable error estimate is needed



Collider processes known at NNLO

<u>Collider processes known at NNLO today:</u>

(a) Drell-Yan (Z,W)

(b) Higgs

(c) 3-jets in e+e-

Drell-Yan processes

Drell-Yan processes: Z/W production ($W \to l\nu$, $Z \to l^+l^-$)

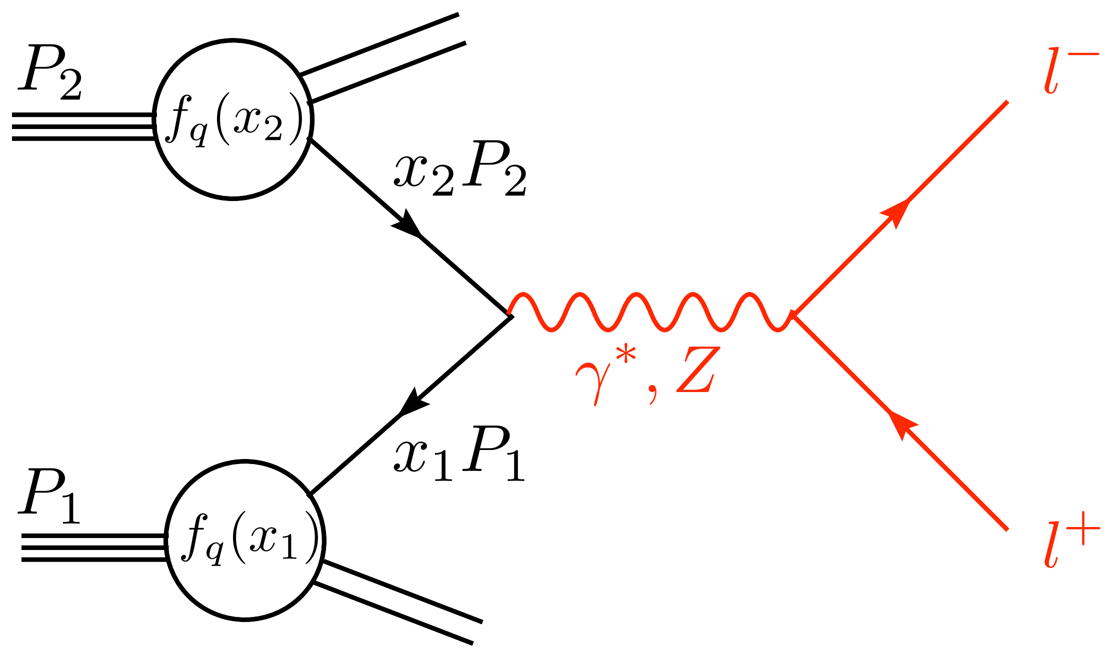
Very clean, golden-processes in QCD because

✓ dominated by quarks in the initial state

✓ no gluons or quarks in the final state (QCD corrections small)

✓ leptons easier experimentally (clear signature)

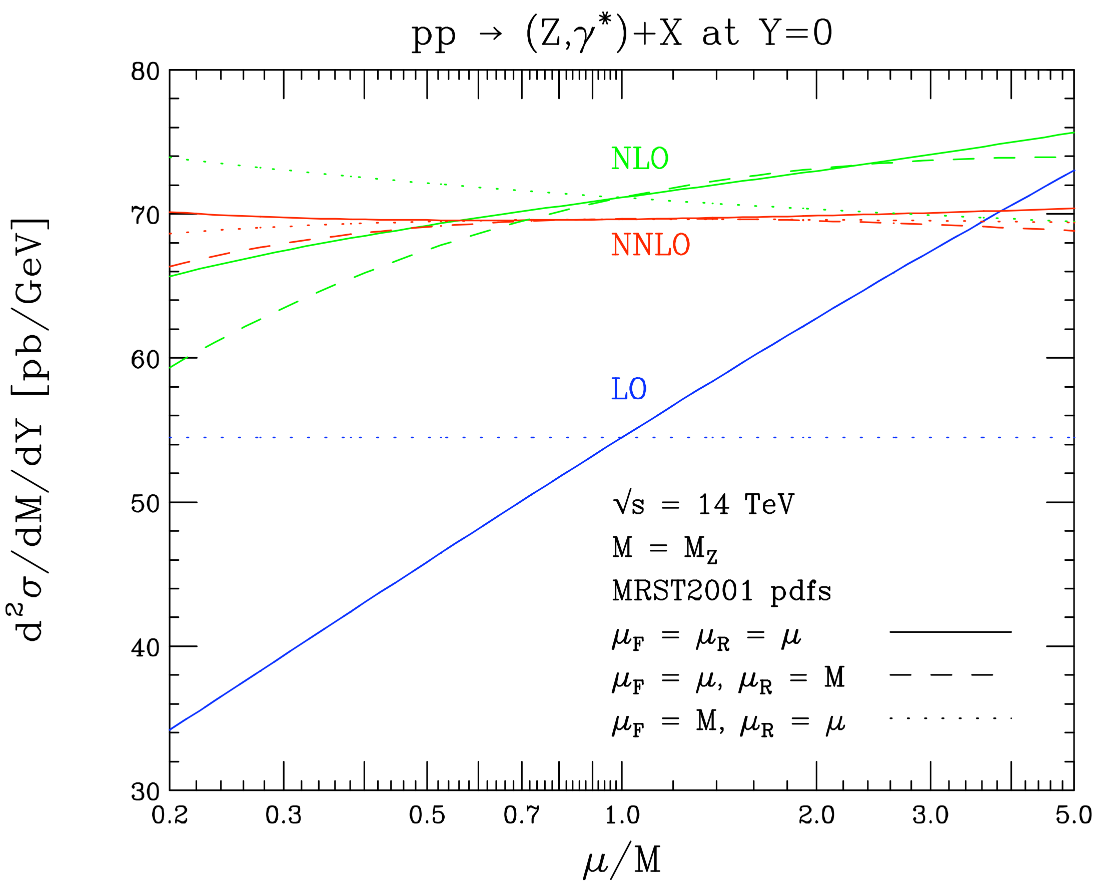
$\Rightarrow$ as clean as it gets at a hadron collider

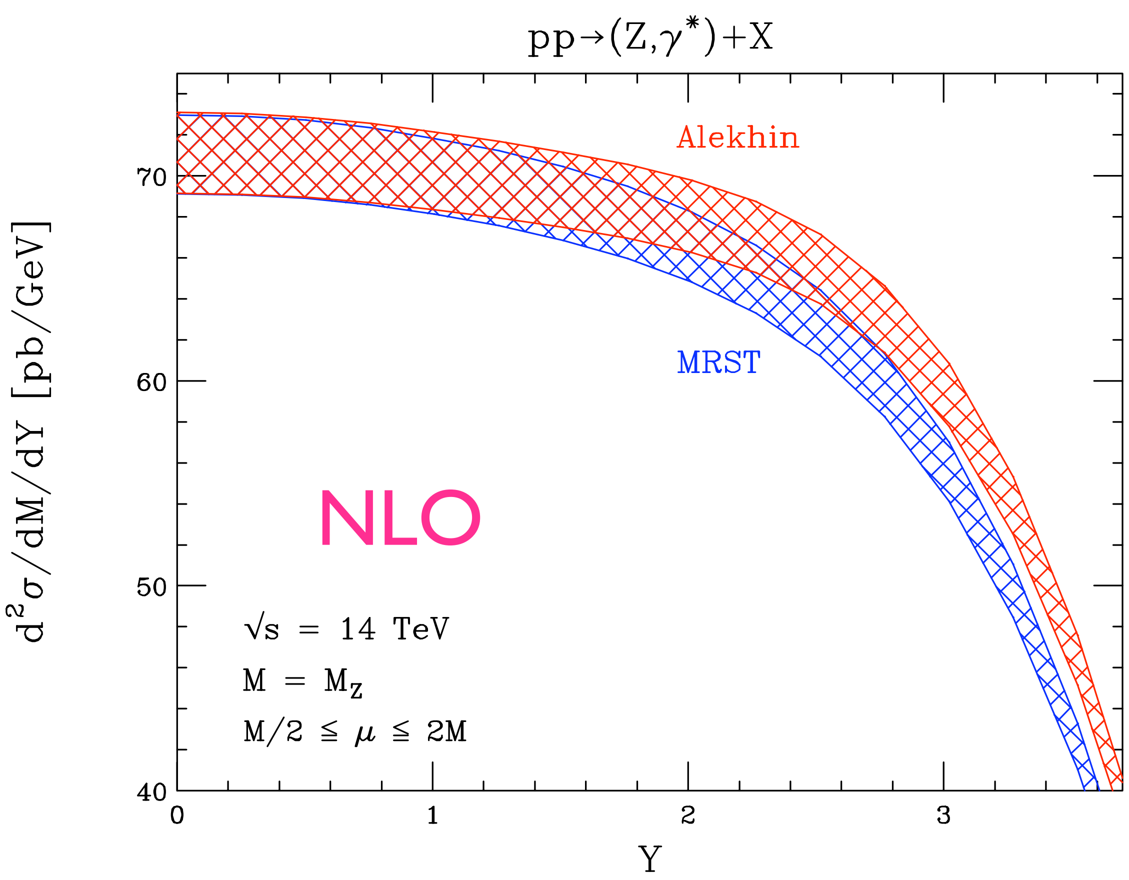


Drell-Yan

- most important and precise test of the SM at the LHC
- best known process at the LHC: spin-correlations, finite-width effects, γ-Z interference, fully differential in lepton momenta

<u>Scale stability and sensitivity to PDFs</u>





Anastasiou, Dixon, Melnikov, Petriello ’03, ’05; Melnikov, Petriello ’06

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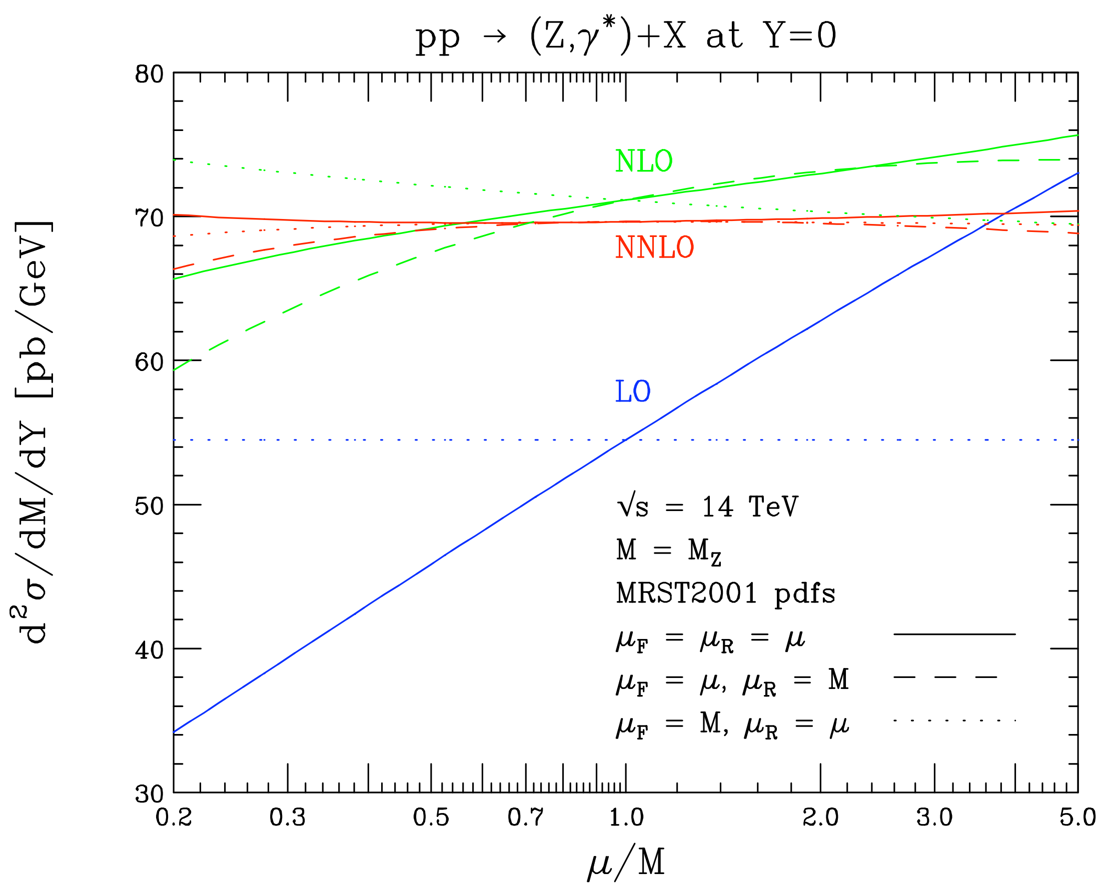
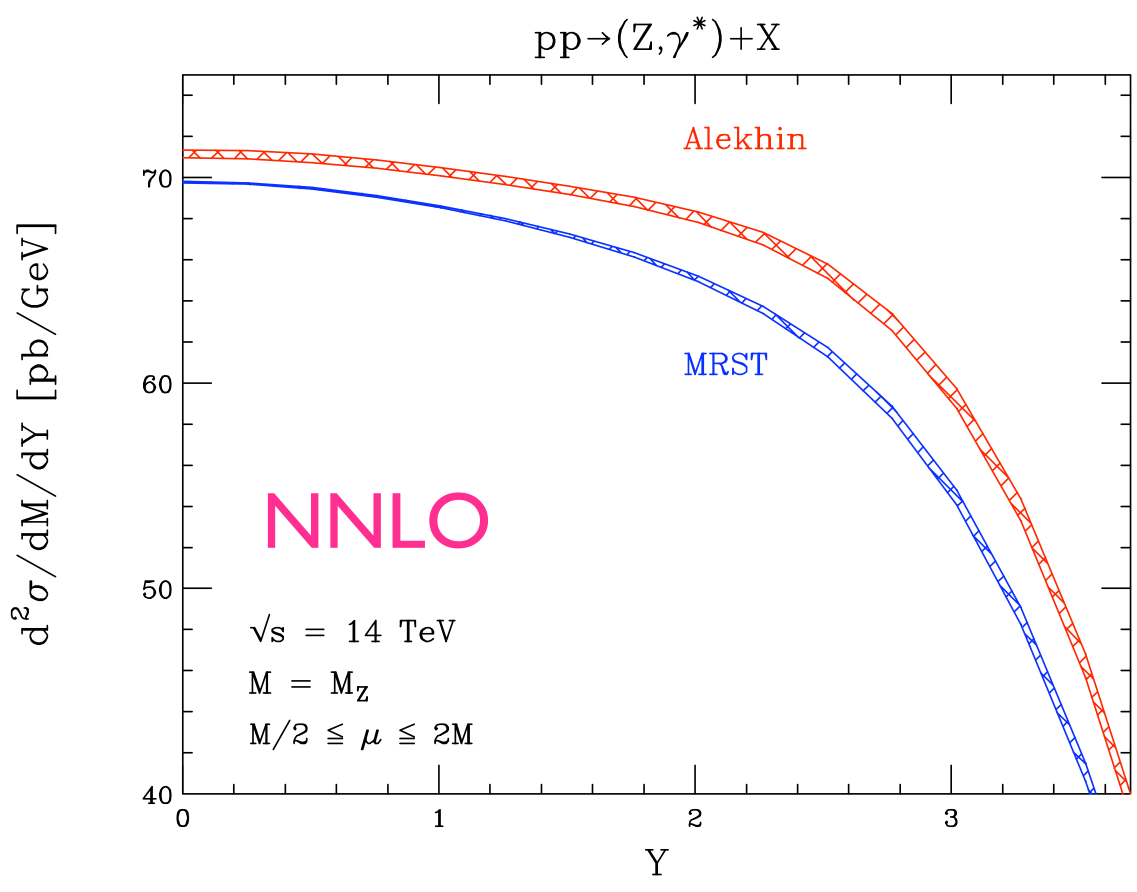
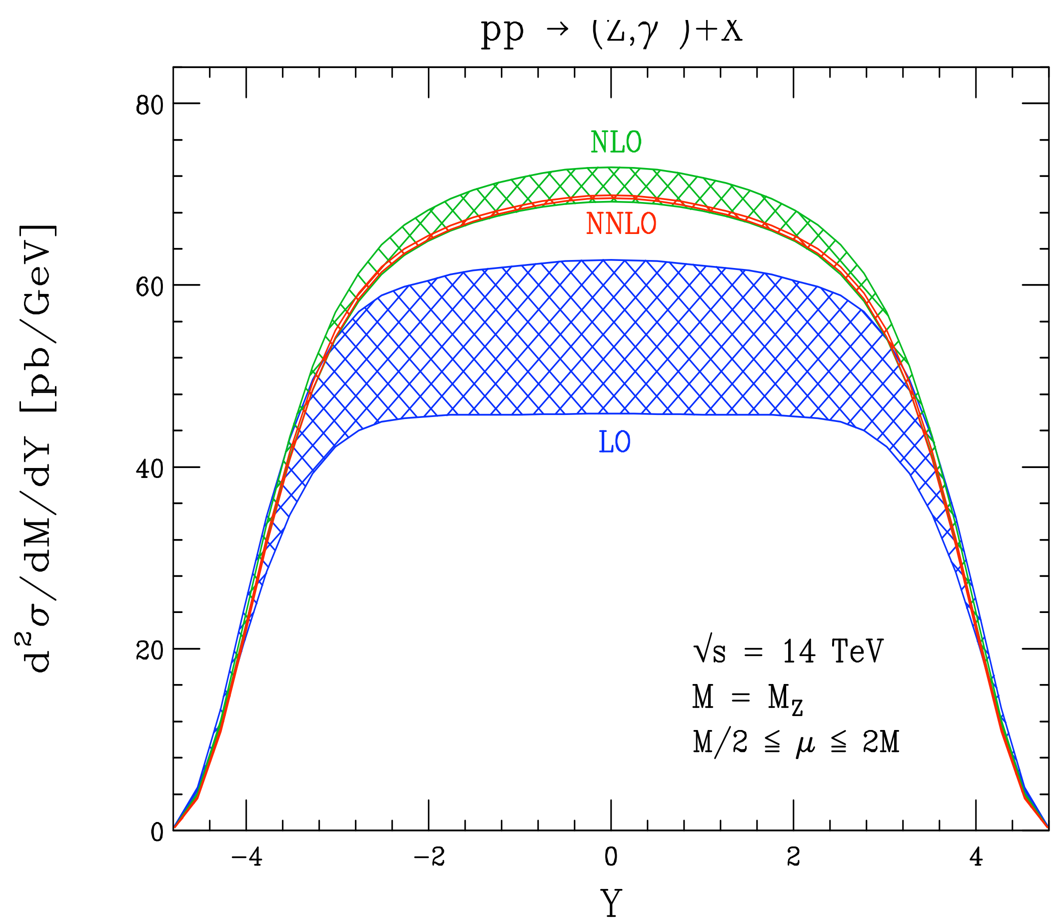
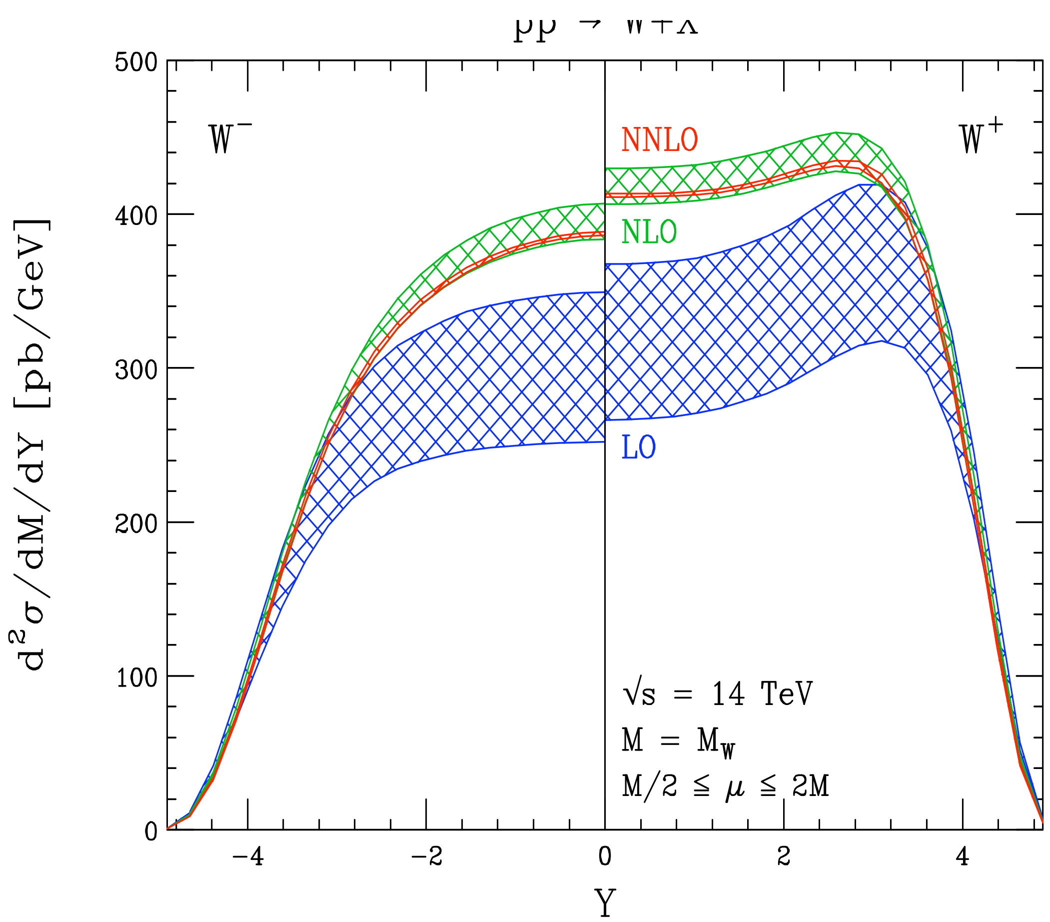




Anastasiou, Dixon, Melnikov, Petriello ’03, ’05; Melnikov, Petriello ’06

Drell-Yan: rapidity distributions



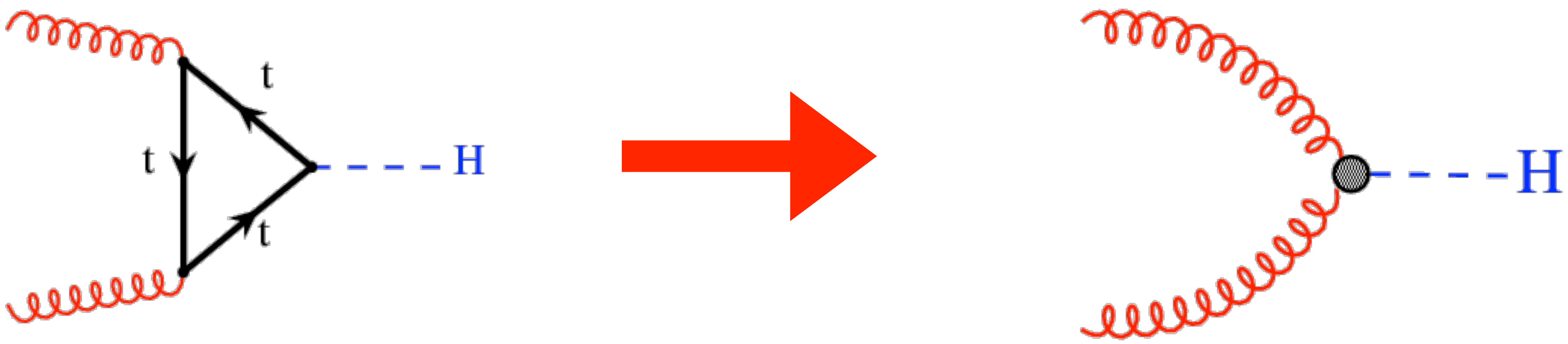


Anastasiou, Dixon, Melnikov, Petriello '03, '05; Melnikov, Petriello '06

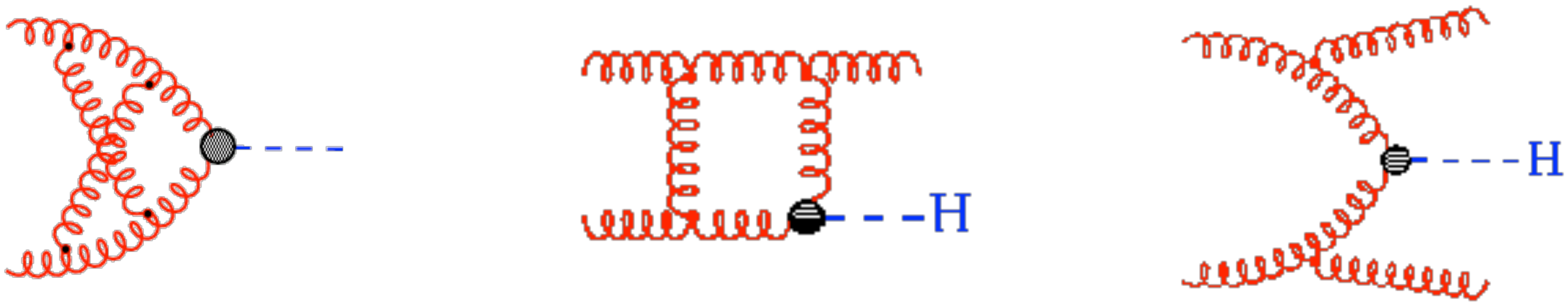
☛ at the LHC: perturbative accuracy of the order of 1%

Inclusive NNLO Higgs production

Inclusive Higgs production via gluon-gluon fusion in the large m_t-limit:

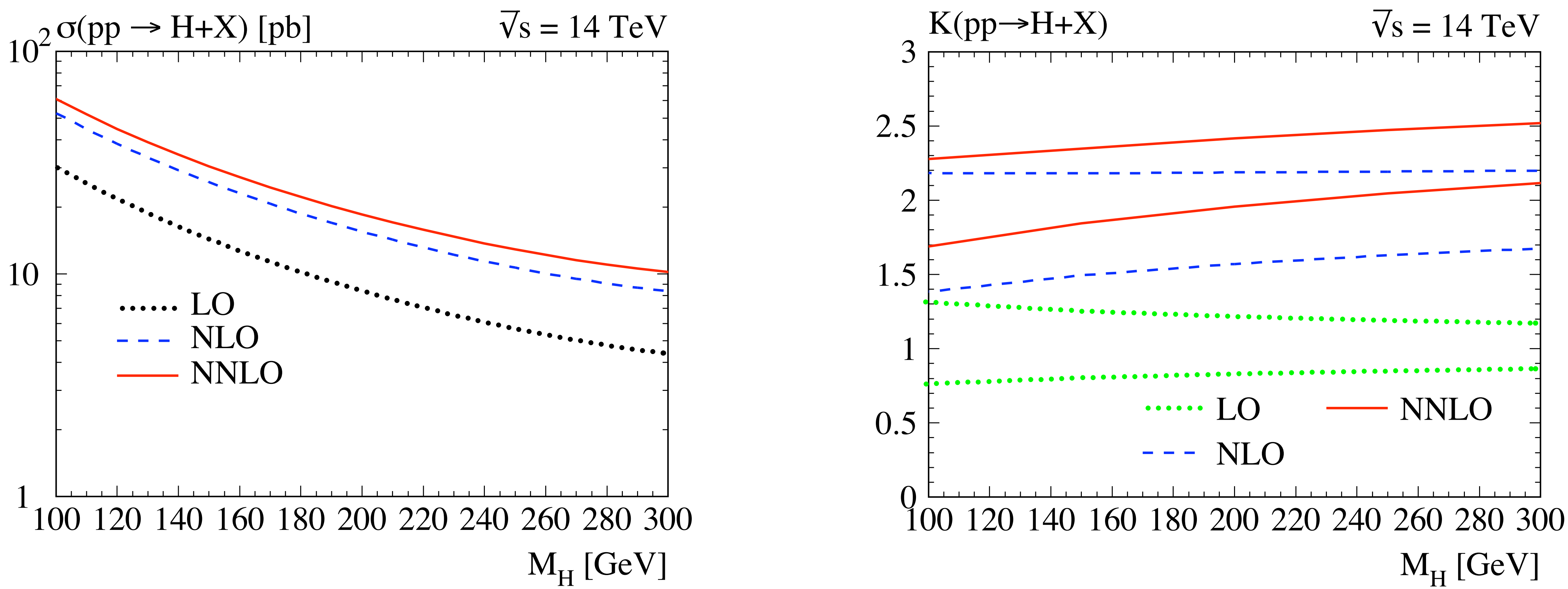


NNLO corrections knows since few years now:



virtual-virtual | real-virtual | real-real

Inclusive NNLO Higgs production

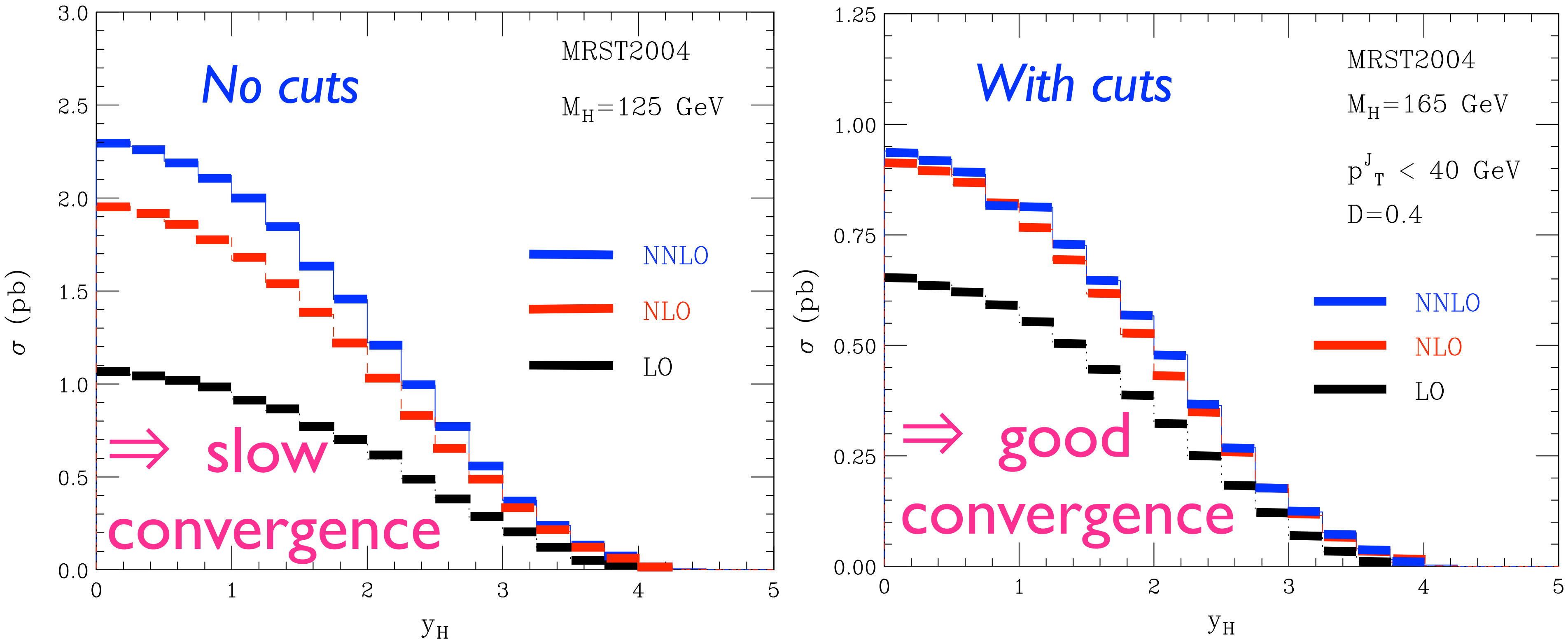


Kilgore, Harlander '02
Anastasiou , Melnikov '02

Exclusive NNLO Higgs production

First fully exclusive $H \to WW \to 2l\,2\nu$ NNLO calculation

FEHIP, Anastasiou, Dissertori, Stoeckli '07
also: HNNLO Catani, Grazzini '08



⇒ impact of NNLO dramatically reduced by cuts!

Very important to include cuts and decays in realistic studies!

NNLO 3-jets in e^+e^-

Motivation: error on α_s from jet-observables

$$\alpha_s(M_Z) = 0.121 \pm 0.001\,(\mathrm{exp.}) \pm 0.005\,(\mathrm{th.})$$

Bethke '06

➥ dominated by theoretical uncertainty

NNLO 3-jet calculation in e^+e^- completed in 2007

Method: developed antenna subtraction at NNLO

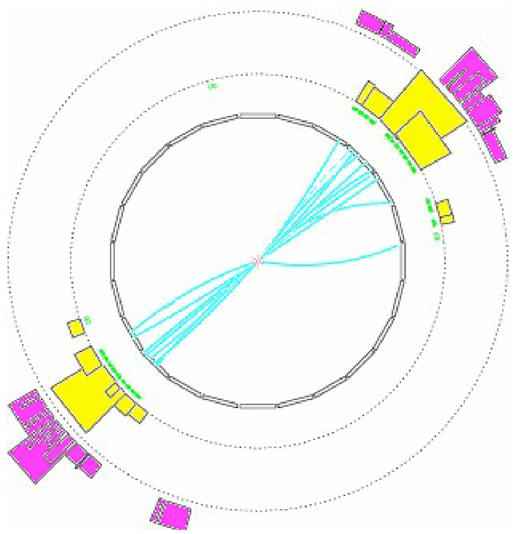
First application: NNLO fit of α_s from event-shapes

Event shapes

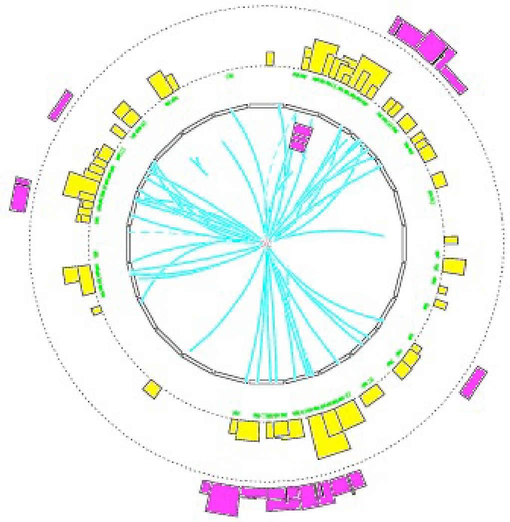
Event-shapes and jet-rates: infrared safe observables describing the energy and momentum flow of the final state.

Candle example in e^+e^-: The thrust $T = \max_{\vec{n}} \frac{\sum_i \vec{p}_i \cdot \vec{n}}{\sum_i |\vec{p}_i|}$

Pencil-like event: $1 - T \ll 1$

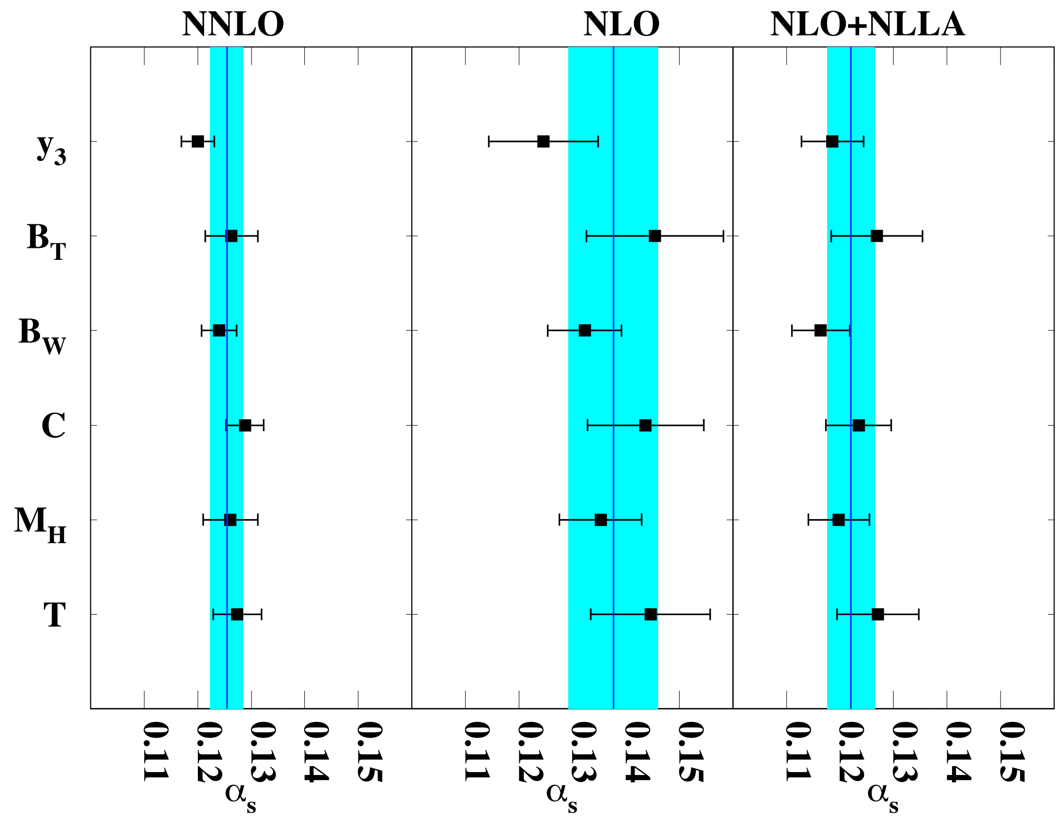


Planar event: $1 - T \sim 1$



α_s from event shapes at NNLO

- scale variation reduced by a factor 2
- scatter between α_s from different event-shapes reduced
- better χ^2, central value closer to world average



$$\alpha_s(M_Z^2) = 0.1240 \pm 0.0008\,(\mathrm{stat}) \pm 0.0010\,(\mathrm{exp}) \pm 0.0011\,(\mathrm{had}) \pm 0.0029\,(\mathrm{theo})$$

Dissertori, Gehrmann-DeRidder, Gehrmann, Glover, Heinrich, Stenzel '07
Gehrmann, Luisoni, Stenzel '08

NNLO on the horizon

Single-jet production

- needed to constrain gluon PDF and coupling constant
- matrix elements known for some time
- subtraction in progress

Anastasiou et al.; Bern et al.; Daleo et al.

Top pair production

- needed for more precise m_t determination
- possibly for further constraining PDFs
- matrix elements partially known

Czakon et al.; Bonciani et al.

Vector boson pair production

- study gauge structure of SM (triple gauge couplings)
- most important and irreducible background for Higgs production in intermediate mass region
- NLO corrections are large

Chachamis, Czakon, Eiras

Recap of 3rd Lecture

- Leading order
  - everything can be computed in principle today (practical edge: 8 particles in the final state), many public codes
  - techniques: standard Feynman diagrams or recursive BG, BCF, CSW ...
- Next-to-leading order
  - current status $2\to3$ in the final state. $2\to4$ very challenging, 1^{st} results
  - many new, promising techniques
  - improved NLO via MC@NLO or PowHeg, includes parton shower
- Next-to-next-to-leading order
  - few $2\to1$ processes available (Higgs, Drell-Yan)
  - recently 3jets in e^+e^-

Remember importance of decays and cuts in realistic studies