Anisotropic Cosmological Models under $f(R, T)$ Theory of Gravity

B. Mishra

Department of Mathematics
Birla Institute of Technology and Science-Pilani, Hyderabad Campus, India
bivu@hyderabad.bits-pilani.ac.in

December 11, 2020

Plan of the Talk

- Basic Formalism
- Physical Parameters
- RIP Cosmologies
- Wormhole Solutions and Big Trip
- Conclusion

The action for a geometrically extended theory with a matter-geometry coupling

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi}f(R,T) + \mathcal{L}_m\right], \qquad (1)$$

For a minimal matter-geometry coupling within the action, $f(R,T)$ can be splitted into two distinct functions $f_1(R)$ and $f_2(T)$ such that $f(R,T) = f_1(R) + f_2(T)$. Then the action for such minimal coupling becomes

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi}\left(f_1(R) + f_2(T)\right) + \mathcal{L}_m\right]. \qquad (2)$$

Basic Formalism

Variation of this action with respect to the metric $g_{\mu\nu}$ provides the modified field equation

$$f_{1,R}(R)R_{\mu\nu} - \frac{1}{2}f_1(R)g_{\mu\nu} = \left[(\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box)\, f_{1,R}(R)\, [8\pi + f_{2,T}(T)]\, T_{\mu\nu}\right] + \left[f_{2,T}(T)p + \frac{1}{2}f_2(T)\right] g_{\mu\nu}. \tag{3}$$

We assumed $\mathcal{L}_m = -p$ where p is the pressure of the cosmic fluid and

$$f_{1,R}(R) \equiv \frac{\partial f_1(R)}{\partial R}, \qquad f_{2,T}(T) \equiv \frac{\partial f_2(T)}{\partial T}. \tag{4}$$

The energy-momentum tensor $T_{\mu\nu}$ is related to the matter Lagrangian as

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\mathcal{L}_m\right)}{\delta g^{\mu\nu}}. \tag{5}$$

Basic Formalism

To develop an extended gravity theory from field equation (3), we consider a simple choice $f_1(R) = R$ which provides GR like field equations

$$G_{\mu\nu} = [8\pi + f_{2,T}(T)]\, T_{\mu\nu} + \left[f_{2,T}(T)p + \frac{1}{2}f_2(T)\right] g_{\mu\nu}, \qquad (6)$$

which can also be written as

$$G_{\mu\nu} = \kappa_T \left[T_{\mu\nu} + T^{int}_{\mu\nu}\right]. \qquad (7)$$

Here, $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$ is the Einstein tensor and $\kappa_T = 8\pi + f_{2,T}(T)$ is the redefined Einstein constant. κ_T become constants for a linear functional $f_2(T)$. However, κ_T evolves with time and dynamically mediates the coupling between the geometry and matter for any non linear choices of the functional $f_2(T)$. In (7), we have

$$T^{int}_{\mu\nu} = \left[\frac{f_{2,T}(T)p + \frac{1}{2}f_2(T)}{8\pi + f_{2,T}(T)}\right] g_{\mu\nu}, \qquad (8)$$

we consider a linear functional

$$\frac{1}{2}f_2(T) = \beta T + \Lambda_0, \tag{9}$$

so that

$$\kappa_T = 8\pi + 2\beta, \tag{10}$$

$$T^{int}_{\mu\nu} = \frac{g_{\mu\nu}}{\kappa_T}\left[(2p + T)\beta + \Lambda_0\right]. \tag{11}$$

Now, GR can be easily recovered for $\beta = 0$ and the responsibility of late time cosmic acceleration is shouldered by the constant Λ_0. In view of this, we may associate Λ_0 with the usual cosmological constant in GR.

Metric, Energy Momentum Tensor

$$ds^2 = dt^2 - A^2 dx^2 - B^2(dy^2 + dz^2), \tag{12}$$

where $A = A(t)$ and $B = B(t)$ are the directional scale factors that govern the rates of expansion along different spatial directions. We consider the universe to be filled with a cloud of one dimensional cosmic strings with string tension density ξ aligned along the x-axis. The energy-momentum tensor for such a fluid is given by

$$T_{\mu\nu} = (p + \rho)u_\mu u_\nu - p g_{\mu\nu} - \xi x_\mu x_\nu, \tag{13}$$

$$u^\mu u_\mu = -x^\mu x_\mu = 1 \tag{14}$$

$$u^\mu x_\mu = 0. \tag{15}$$

Here, ρ represents the energy density and is composed of the particle energy density ρ_p and the string tension density ξ so that $\rho = \rho_p + \xi$. For an isotropic universe, the string tension density ξ vanishes.

Field Equations

$$6(k+2)\dot{H} + 27H^2 = K^2\left[-\alpha(p-\xi) + \rho\beta + \Lambda_0\right], \quad (16)$$

$$3(k^2+3k+2)\dot{H} + 9(k^2+k+1)H^2 = K^2\left[-\alpha p + (\rho+\xi)\beta + \Lambda_0\right], \quad (17)$$

$$9(2k+1)H^2 = K^2\left[\alpha\rho - (p-\xi)\beta + \Lambda_0\right]. \quad (18)$$

Here $K = (k+2)^2$, $\alpha = 8\pi + 3\beta$.

$$\text{Hubble parameter:} \quad H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)$$

$$\text{Expansion scalar:} \quad \theta = u^l_{;l} = \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right),$$

$$\text{Deceleration parameter:} \quad q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right),$$

$$\text{Jerk parameter:} \quad j = \frac{\dddot{a}}{aH^3} = \frac{\ddot{H}}{H^3} - (2+3q).$$

Without putting any restriction on the parameter k, the physical properties of the model from (16)-(18) in terms of the Hubble parameter, the anisotropic parameter k and the coupling parameter β

$$p = -\frac{1}{\kappa_T(\kappa_T+2\beta)}\left[\phi_1(k,\beta)\dot{H}+\phi_2(k,\beta)H^2-\kappa_T\Lambda_0\right], \qquad (19)$$

$$\rho = \frac{1}{\kappa_T(\kappa_T+2\beta)}\left[\phi_3(k,\beta)\dot{H}+\phi_4(k,\beta)H^2-\kappa_T\Lambda_0\right], \qquad (20)$$

$$\xi = \frac{1}{\kappa_T}\left[\phi_5(k)\left(\dot{H}+3H^2\right)\right], \qquad (21)$$

$$\omega = -1+\frac{\left[\phi_3(k,\beta)-\phi_1(k,\beta)\right]\dot{H}+\left[\phi_4(k,\beta)-\phi_2(k,\beta)\right]H^2}{\phi_3(k,\beta)\dot{H}+\phi_4(k,\beta)H^2-\kappa_T\Lambda_0}. \qquad (22)$$

Physical Parameters, Anisotropic Case

$$\begin{aligned}
\phi_1(k,\beta) &= [8(k+1)\pi + 2(2k+1)\beta]\,\chi(k), \\
\phi_2(k,\beta) &= \left[8(k^2+k+1)\pi + (5k^2+3k+1)\beta\right]\chi^2(k), \\
\phi_3(k,\beta) &= -2\beta\chi(k), \\
\phi_4(k,\beta) &= [8(2k+1)\pi + (8k+1)\beta]\,\chi^2(k), \\
\phi_5(k) &= (1-k)\chi(k),
\end{aligned}$$

where $\chi(k) = \frac{3}{k+2}$.

$$\rho + p = \frac{1}{\kappa_T(\kappa_T + 2\beta)}\left[\left(\phi_3(k,\beta) - \phi_1(k,\beta)\right)\dot{H} + \left(\phi_4(k,\beta) - \phi_2(k,\beta)\right)H^2\right]. \tag{23}$$

When, $\beta = -2\pi$, $\phi_1(k,\beta) = \phi_3(k,\beta)$ and $\phi_2(k,\beta) = \phi_4(k,\beta)$.
When $\beta \to -2\pi$, a ΛCDM model is recovered with $p = -\rho$ and $\omega = -1$.
In phantom models, one can have $\dot{H} > 0, t > 0$ and hence the weak energy condition $\rho + p \geq 0$; $\rho \geq 0$ is not satisfied. We observed (23), violation of weak energy condition depends on the choice of the parameter k and β.

Physical Parameters, Anisotropic Case

In the GR limit with $\beta \to 0$,

$\phi_3(k,0) - \phi_1(k,0) = -8(k+1)\pi\chi(k), \phi_4(k,0) - \phi_2(k,0) = 8\pi k(1-k)\chi^2(k).$

$$\omega = -1 + \frac{(k+1)\chi^{-1}(k)\dot{H} + k(k-1)H^2}{\chi^{-2}(k)\Lambda_0 - (2k+1)H^2}. \tag{24}$$

In the absence of a cosmological constant it becomes

$$\omega = -1 - \frac{(k+1)\chi^{-1}(k)\dot{H} + k(k-1)H^2}{(2k+1)H^2}. \tag{25}$$

The EoS parameter depends on the anisotropic parameter k, the coupling constant β besides its dependence on the parameters appearing in the Hubble parameter.

The EoS parameter becomes a non evolving parameter in the absence of a cosmological constant for similar time dependence of $\dot{H}$ and H^2. In the presence of a cosmological constant, it evolves with time.

Physical Parameters, Isotropic Case

For $k = 1$,

$$\begin{aligned}
\phi_1(\beta) &= 2(8\pi + \beta), \\
\phi_2(\beta) &= \phi_4(\beta) = 3(8\pi + 3\beta), \\
\phi_3(\beta) &= -2\beta, \\
\phi_5 &= 0 \\
\omega &= -1 + 8(2\pi + \beta)\frac{\dot{H}}{2\beta\dot{H} - 3(8\pi + 3\beta)H^2 + \kappa_T\Lambda_0}
\end{aligned} \tag{26}$$

In the limit $\beta \to 0$ and $\Lambda_0 \to 0$, the ω reduces to that of the FRW model

$$\begin{aligned}
\omega &= -1 - \frac{2}{3}\frac{\dot{H}}{H^2} \\
\rho + p &= -\frac{2\dot{H}}{(\kappa_T + 2\beta)}
\end{aligned}$$

In phantom models, $\dot{H} > 0$ in positive time frame, violation of the weak energy condition is seen for positive coupling constant β.

Little Rip

The Hubble parameter and the scale factor for the LR model [1, 2]

$$H = H_0 e^{\lambda t}, \qquad H_0 > 0, \quad \lambda > 0$$
$$a = a_0 \; exp\left[\frac{H_0}{\lambda}\left(e^{\lambda t} - e^{\lambda t_0}\right)\right]$$

The Hubble rate increases exponentially with time and thereby produces strong inertial force. With the growth of cosmic time, the inertial force increases and any bound system tends to rip at an infinitely large time.

$$q = -1 - \frac{\lambda}{H_0} e^{-\lambda t} \qquad j = 1 + \frac{3\lambda}{H_0} e^{-\lambda t} + \left(\frac{\lambda}{H_0}\right)^2 e^{-2\lambda t}$$

The deceleration parameter and the jerk parameter asymptotically approach to -1 and 1 respectively. At the present epoch, the deceleration parameter q becomes $q_0 = -1 - \frac{\lambda}{H_0} e^{-\lambda t_0}$ which implies that $q_0 < -1$. In ΛCDM model, the jerk parameter at the present epoch is predicted to $j_0 = 1$. In LR model, $j_0 = 1 + \frac{3\lambda}{H_0} e^{-\lambda t_0} + \left(\frac{\lambda}{H_0}\right)^2 e^{-2\lambda t_0}$, which is greater than that predicted from ΛCDM model.

Since $\dot{H} = \lambda H > 0$, substitution the LR scale factor into (22), we obtain

$$\omega_{LR} = -1 + \frac{[\phi_3(k,\beta) - \phi_1(k,\beta)]\,\frac{\lambda}{H_0} e^{-\lambda t} + [\phi_4(k,\beta) - \phi_2(k,\beta)]}{\phi_3(k,\beta)\frac{\lambda}{H_0} e^{-\lambda t} + \phi_4(k,\beta) - \kappa_T \frac{\Lambda_0}{H_0^2} e^{-2\lambda t}}.$$

The evolution of the EoS parameter in the LR model depends on the anisotropic parameter k, the coupling constant β, the parameters of the scale factors λ and H_0. At an initial epoch, $t \rightarrow 0$, we have

$$\omega_{LR} = -1 + \frac{[\phi_3(k,\beta) - \phi_1(k,\beta)]\,\frac{\lambda}{H_0} + [\phi_4(k,\beta) - \phi_2(k,\beta)]}{\phi_3(k,\beta)\frac{\lambda}{H_0} + \phi_4(k,\beta) - \kappa_T \frac{\Lambda_0}{H_0^2}},$$

and at a late phase ($t \rightarrow \infty$)

$$\omega_{LR}(t \rightarrow \infty) = -\frac{\phi_2(k,\beta)}{\phi_4(k,\beta)}.$$

The model evolves in a phantom phase with $\omega_{LR} < -1$ at an initial epoch to $\omega_{LR} \rightarrow -1$ at late phase thereby holding the LR scenario. However, the asymptotic value of the EoS depends on k and β.

Little Rip

For an isotropic case, the evolutionary behaviour of the EoS parameter in the LR model is given by

$$\omega_{LR}^{iso} = -1 + \frac{8(2\pi+\beta)\frac{\lambda}{H_0}e^{-\lambda t}}{2\beta\frac{\lambda}{H_0}e^{-\lambda t} - 3(8\pi+3\beta) + \kappa_T\frac{\Lambda_0}{H_0^2}e^{-2\lambda t}},$$

which asymptotically approaches to -1 as $t \to \infty$. In the limit of GR with $\beta \to 0$ and $\Lambda_0 \simeq 0$, we have

$$\omega_{LR}^{iso(GR)} = -1 - \frac{2}{3}\frac{\lambda}{H_0}e^{-\lambda t}.$$

Pseudo Rip

Another phantom behaviour without singularity at finite time is speculated by a Hubble parametrization [2]

$$H = H_0 - H_1 e^{-\lambda t}, \qquad a = a_0 \, exp\left[H_0(t - t_0) + \frac{H_1}{\lambda}\left(e^{-\lambda t} - e^{-\lambda t_0}\right)\right]$$

H_0, H_1 and λ are positive constants and $H_0 > H_1$. Since, in the limit $t \to +\infty$, the Hubble parameter becomes a constant $H \to H_0$, this model evolves asymptotically to a de Sitter universe.

$$q = -1 - \frac{\lambda H_1 e^{-\lambda t}}{(H_0 - H_1 e^{-\lambda t})^2}, \qquad j = 1 - \frac{\lambda H_1 e^{-\lambda t}\left[\lambda + 3(H_0 - H_1 e^{-\lambda t})\right]}{(H_0 - H_1 e^{-\lambda t})^3}.$$

$q(t \to 0) = -1 - \frac{\lambda H_1}{(H_0 - H_1)^2}$, it approaches -1 at late times. The deceleration parameter in general evolves from a higher negative value to -1 at late epoch. On the otherhand, the jerk parameter evolves from a low value of $j = 1 - \frac{\lambda H_1[\lambda + 3(H_0 - H_1)]}{(H_0 - H_1)^3}$ to $j = 1$ at late times. However, these parameters will have singularities at $t = ln\left(\frac{H_1}{H_0}\right)^{\frac{1}{\lambda}}$.

Pseudo Rip

The EoS parameter for the PR model can be obtained as

$$\omega_{PR} = -1 + \frac{\left[\phi_3(k,\beta) - \phi_1(k,\beta)\right]\lambda H_1 e^{-\lambda t} + \left[\phi_4(k,\beta) - \phi_2(k,\beta)\right]\left(H_0 - H_1 e\right.}{\phi_3(k,\beta)\lambda H_1 e^{-\lambda t} + \phi_4(k,\beta)\left(H_0 - H_1 e^{-\lambda t}\right)^2 - \kappa_T \Lambda_0}$$

At an initial epoch, $t \to 0$, we have

$$\omega_{PR}(t \to 0) = -1 + \frac{\left[\phi_3(k,\beta) - \phi_1(k,\beta)\right]\lambda H_1 + \left[\phi_4(k,\beta) - \phi_2(k,\beta)\right]\left(H_0 - \right.}{\phi_3(k,\beta)\lambda H_1 + \phi_4(k,\beta)\left(H_0 - H_1\right)^2 - \kappa_T \Lambda_0}$$

and at a late phase $(t \to \infty)$

$$\omega_{PR}(t \to \infty) = -1 + \frac{\left[\phi_4(k,\beta) - \phi_2(k,\beta)\right] H_0^2}{\phi_4(k,\beta) H_0^2 - \kappa_T \Lambda_0}.$$

However, in the absence of a cosmological constant, it reduces to $\omega_{PR}(t \to \infty) = -\frac{\phi_2(k,\beta)}{\phi_4(k,\beta)}$. This pseudo rip model evolves in a phantom phase with $\omega_{LR} < -1$ at an initial epoch to $\omega_{LR} \to -1$ at late phase. Just like the little rip case, in this model also, the asymptotic value of the EoS depends on the anisotropic parameter k and the coupling constant β.

Pseudo Rip

For an isotropic case,

$$\omega_{PR}^{iso} = -1 + \frac{8(2\pi+\beta)\lambda H_1 e^{-\lambda t}}{2\beta\lambda H_1 e^{-\lambda t} - 3(8\pi+3\beta)\left(H_0 - H_1 e^{-\lambda t}\right)^2 + \kappa_T \Lambda_0},$$

which asymptotically approaches to -1 as $t \to \infty$. In the limit of GR with $\beta \to 0$ and $\Lambda_0 \simeq 0$, we have

$$\omega_{PR}^{iso(GR)} = -1 - \frac{2}{3}\frac{\lambda H_1 e^{-\lambda t}}{\left(H_0 - H_1 e^{-\lambda t}\right)^2}.$$

The phantom evolution of the EoS parameter is obvious. It evolves from $\omega_{PR} < -1$ to an asymptotic value of -1. This model has a ω-singularity at $t = t_\omega = ln\left(\frac{H_1}{H_0}\right)^{\frac{1}{\lambda}}$ in the framework of GR.

Another scale factor describing an emergent solution, Mukherjee et al.[3]

$$a(t) = a_i \left(\nu + e^{\mu t}\right)^{\gamma}, \qquad H(t) = \frac{\mu\gamma e^{\mu t}}{\nu + e^{\mu t}}$$

where a_i, μ, ν and γ are positive constants.
It is obvious that as $t \to \infty$, we have $a \to \infty$ and $H \to \mu\gamma$. This model asymptotically evolves to a de Sitter universe. Also we have

$$\dot{H} = \frac{\mu\gamma e^{\mu t}}{\nu + e^{\mu t}}\left[\mu - \frac{1}{\gamma}\frac{\mu\gamma e^{\mu t}}{\nu + e^{\mu t}}\right] = H\left(\mu - \frac{H}{\gamma}\right) \tag{27}$$

which in the limit $t \to \infty$ approaches to 0. The value of the parameters μ and γ are chosen in such a manner that, $\dot{H} > 0$ for $t > 0$.

$$q = -1 + \frac{1}{\gamma} - \frac{\nu + e^{\mu t}}{\gamma e^{\mu t}}, \qquad j = \left(1 - \frac{3}{\gamma} + \frac{2}{\gamma^2}\right) + \frac{\mu}{H} + \frac{\mu(\mu - 2/\gamma)}{H^2}$$

The deceleration parameter evolves from $q = -1 - \frac{\nu}{\gamma}$ to -1, the jerk parameter evolves from $j = 1 + \frac{\nu-2}{\gamma} + \frac{1}{\gamma^2}\left[2 + \frac{(\nu+1)^2(\mu-2/\gamma)}{\mu}\right]$ at an initial phase to $1 - \frac{2}{\gamma} + \frac{1}{\gamma^2}\left[2 + \frac{(\mu-2/\gamma)}{\mu}\right]$ at late phase of evolution.

$$\omega_{ELR} = -1 + \frac{[\phi_3(k,\beta) - \phi_1(k,\beta)]\left(\frac{\mu}{H} - \frac{1}{\gamma}\right) + [\phi_4(k,\beta) - \phi_2(k,\beta)]}{\phi_3(k,\beta)\left(\frac{\mu}{H} - \frac{1}{\gamma}\right) + \phi_4(k,\beta) - \kappa_T \Lambda_0 H^{-2}}.$$

This EoS parameter evolves in phantom phase with $\omega_{ELR} < -1$ and asymptotically reduces to

$$\omega_{ELR}(t \to \infty) = -1 + \frac{[\phi_4(k,\beta) - \phi_2(k,\beta)]}{\phi_4(k,\beta) - \frac{\kappa_T \Lambda_0}{\mu^2 \gamma^2}}$$

at a late epoch $(t \to \infty)$. In the absence of cosmological constant, the EoS parameter reduces to $\omega_{ELR}(t \to \infty) = -\frac{\phi_2(k,\beta)}{\phi_4(k,\beta)}$. For an isotropic universe,

$$\omega_{ELR}^{iso} = -1 + \frac{8(2\pi + \beta)\left(\frac{\mu}{H} - \frac{1}{\gamma}\right)}{2\beta\left(\frac{\mu}{H} - \frac{1}{\gamma}\right) - 3(8\pi + 3\beta) + \kappa_T \Lambda_0 H^{-2}},$$

which asymptotically approaches to -1 as $t \to \infty$. In the limit of GR with $\beta \to 0$ and $\Lambda_0 \simeq 0$, we have

$$\omega_{ELR}^{iso(GR)} = -1 - \frac{2}{3}\left(\frac{\mu}{H} - \frac{1}{\gamma}\right).$$

Bouncing with Little rip

Myrzakulov and Sebastini [4] have studied a scale factor in exponential form

$$a(t) = a_0 e^{(t-t_0)^{2n}} \qquad H(t) = 2n(t-t_0)^{2n-1},$$

where $a_0 > 0$ is the scale factor at time t_0. The exponent $n \neq 0$ decides the bouncing behaviour of the model.
The model bounces at $t = t_0$ when the bouncing scale factor becomes a_0. It is obvious that as $t \to \infty$, we have $a \to \infty$ and $H \to \infty$ for positive integral values of n.

$$q = -1 - \frac{2n-1}{2n(t-t_0)^{2n}} \qquad j = 1 + \frac{3(2n-1)}{2n(t-t_0)^{2n}} + \frac{(n-1)(2n-1)}{2n^2(t-t_0)^{4n}}$$

The deceleration parameter is a negative quantity for $n > \frac{1}{2}$ and evolves to an asymptotic value of $q = -1$. The jerk parameter evolves to $j = 1$ at late times.

For the BLR model we can calculate the EoS parameter as

$$\omega_{BLR} = -1 + \frac{[\phi_3(k,\beta) - \phi_1(k,\beta)]\left[\frac{2n-1}{2n(t-t_0)^{2n}}\right] + [\phi_4(k,\beta) - \phi_2(k,\beta)]}{\phi_3(k,\beta)\left[\frac{2n-1}{2n(t-t_0)^{2n}}\right] + \phi_4(k,\beta) - \frac{\kappa_T \Lambda_0}{4n^2(t-t_0)^{4n-2}}}$$

which asymptotically reduces to $\omega_{BLR}(t \to \infty) = -\frac{\phi_2(k,\beta)}{\phi_4(k,\beta)}$.
The EoS parameter for this BLR model in an isotropic universe can be expressed as,

$$\omega_{BLR}^{iso} = -1 + \frac{8(2\pi + \beta)\left[\frac{2n-1}{2n(t-t_0)^{2n}}\right]}{2\beta\left[\frac{2n-1}{2n(t-t_0)^{2n}}\right] - 3(8\pi + 3\beta) + \frac{\kappa_T \Lambda_0}{4n^2(t-t_0)^{4n-2}}},$$

The phantom energy accretion on to wormhole leads to an increase in the size of the wormhole throat which may eventually engulf the whole universe before the occurrence of any kind of rip. Such a phenomenon is called Big Trip. We will calculate the wormhole throat radius and its evolution under the phantom energy accretion. The throat radius $R(t)$ of Morris-Thorne wormhole can be calculated for phantom dark energy models from the evolution equation [5, 6]

$$\dot{R} = -CR^2(\rho + p).$$

Here C is a positive dimensionless constant. We will restrict to the isotropic case with vanishing cosmological constant.

For LR model, the wormhole throat radius becomes

$$\frac{1}{R_{LR}(t)} = -\frac{2C}{\kappa_T + 2\beta} H_0 e^{\lambda t} + k_1,$$

At Big Trip, $t = t_B$ and we have $k_1 = \frac{2C}{\kappa_T + 2\beta} H_0 e^{\lambda t_B}$. So,

$$R_{LR}(t) = \frac{\kappa_T + 2\beta}{2CH_0} \left[e^{\lambda t_B} - e^{\lambda t} \right]^{-1}.$$

Assuming the wormhole throat radius at $t = t_0$ to be R_0, the Big Trip at

$$t_B = ln \left[e^{\lambda t_0} + \frac{\kappa_T + 2\beta}{2CH_0 R_0} \right]^{\frac{1}{\lambda}}. \tag{28}$$

In the limit, $\beta \rightarrow 0$, i.e in the GR limit,

$$t_B = ln \left[e^{\lambda t_0} + \frac{8\pi}{2CH_0 R_0} \right]^{\frac{1}{\lambda}}. \tag{29}$$

Comparison of (28) and (29) shows that, the presence of a positive finite coupling constant increases the time of occurrence of the Big Trip.

Wormhole solutions and Big trip

For PR, the throat radius

$$R_{PR}(t) = \frac{\kappa_T + 2\beta}{2CH_1}\left[e^{-\lambda t} - e^{-\lambda t_B}\right]^{-1}$$

and consequently the Big Trip occurs at

$$t_B = ln\left[e^{-\lambda t_0} - \frac{\kappa_T + 2\beta}{2CH_1 R_0}\right]^{-\frac{1}{\lambda}}.$$

It is interesting to note that, Big Trip occurs for the wormholes if their radius at $t = t_0$ satisfy the condition

$$R_0 > \frac{\kappa_T + 2\beta}{2CH_1}e^{\lambda t_0}.$$

For ELR, the wormhole throat radius is

$$R_{ELR}(t) = \frac{\kappa_T + 2\beta}{2C\mu\nu\gamma}\left[\frac{1}{\nu + e^{\mu t}} - \frac{1}{\nu + e^{\mu t_B}}\right]^{-1}.$$

and the the Big Trip occurs at

$$t_B = ln\left[\left(\frac{1}{\nu + e^{\mu t_0}} - \frac{\kappa_T + 2\beta}{2C\mu\nu\gamma R_0}\right)^{-1} - \nu\right]^{\frac{1}{\mu}}.$$

The Big Trip will occur if the wormhole simultaneously satisfies the conditions

$$R_0 > \frac{(\kappa_T + 2\beta)(\nu + e^{\mu t_0})}{2C\mu\nu\gamma}; \qquad R_0 < \frac{\nu(\kappa_T + 2\beta)}{2C\mu\nu\gamma\left[\nu - (\nu + e^{\mu t_0})\right]}. \tag{30}$$

For BLR, the wormhole throat radius

$$R_{BLR}(t) = \frac{\kappa_T + 2\beta}{4Cn}\left[(t_B - t_0)^{2n-1} - (t - t_0)^{2n-1}\right]^{-1}, \qquad (31)$$

and the Big Trip occurs at

$$t_B = t_0 + \left[\left(t' - t_0\right)^{2n-1} + \frac{\kappa_T + 2\beta}{4CnR'}\right]^{\frac{1}{2n-1}}. \qquad (32)$$

Here t_0 is the bouncing epoch and t' is the time corresponding to the wormhole radius R'.

Conclusion

- Four different models of Little Rip or Pseudo Rip both for anisotropic and isotropic universe are investigated and obtained the dynamical evolution of the EoS parameter.
- The model parameters are found to affect substantially the dynamical behaviour of the EoS parameter.
- Wormhole solutions are obtained for the models. It is possible to obtain wormhole solutions in phantom models where Big Rip can be avoided.
- An extended gravity theory delays the time of occurrence of Big Trip in wormholes than that in GR.
- With a lot of observational data coming in recent times that favour a phantom world, we hope, our theoretical models within a simple extended gravity theory may be quite useful.

References

- P. H. Frampton, K. J. Ludwick and R. J. Scherrer, *Phys. Rev. D* **84**, 063003 (2011).
- P. H. Frampton, K. J. Ludwick, S. Nojiri, S. D. Odintsov and R. J. Scherrer, *Phys. Lett. B* **708**, 204 (2012).
- S. Mukherjee, B.C. Paul, N. K. Dadhich, S. D. Maharaj and A. Beesham, *Class.Quant. Gravit.* **23**, 6927 (2006).
- R. Myrzakulov and L. Sebastini, *Astrophys. Space Sci.* **352**, 281 (2014).
- E. Babichev, V. Dokuchaev and Y. Eroshenko, *Phys. Rev. Lett.* **93**, 021102 (2004).
- A. V. Astashenok, S. Nojiri, S. D. Odintsov, A. V. Yurov, *Phys. Lett. B* **709**, 396 (2012).

For more details please see arXiv:1904.01443v2

Thank You