Scalar-field Dark Matter around supermassive Black Holes

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1909.02614
2001.06873

PONT (Avignon)

08/12/2020

Summary:

- Scalar-field dark matter.

- Steady state around a supermassive BH for a quartic self-interaction.

- Non-standard kinetic term.

Dark matter candidates

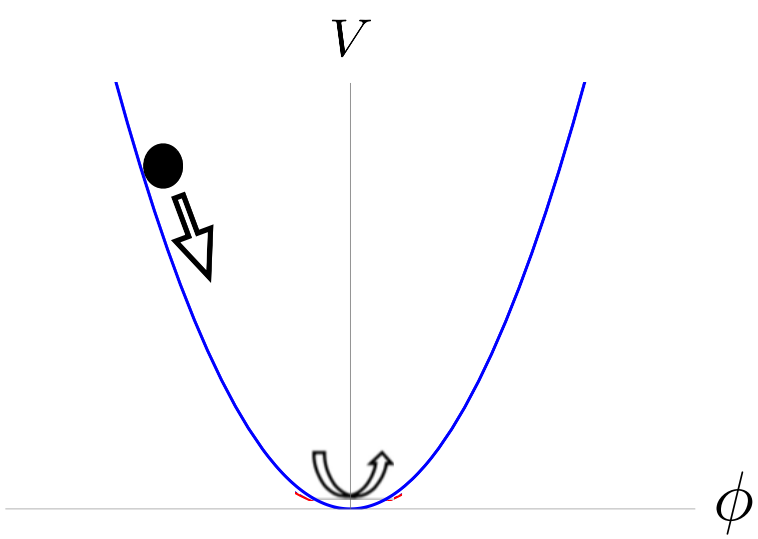
Bertone and Tait, 1810.01668

Scalar-field models

$$S_\phi = \int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right].$$

Background:

K.G. eq.: $\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$



e.g., no self-interactions: $V = \frac{1}{2}m^2\phi^2$

$H \ll m$ the scalar field oscillates with frequency m, and a slow decay of the amplitude:

$$\phi = \phi_0(a/a_0)^{-3/2}\cos(mt)$$

behaves like dark matter: $\rho \propto a^{-3}$

$V \propto \phi^n \Rightarrow w = \frac{\langle p_\phi \rangle}{\langle \rho_\phi \rangle} = \frac{n-2}{n+2}$

Brax et al. 2019

For a mostly quadratic potential with small self-interactions:

$$V(\phi) = \frac{1}{2}m^2\phi^2 + V_{\rm I}(\phi) \qquad V_{\rm I} \ll \frac{1}{2}m^2\phi^2$$

$$\bar{\phi}(t) = \bar{\varphi}(t)\cos(mt - \bar{S}(t)) \qquad \bar{\varphi} = \bar{\varphi}_0 a^{-3/2} \qquad \bar{S}(t) = \bar{S}_0 - \int_{t_0}^{t} dt\, m\Phi_{\rm I}\left(\frac{m^2\bar{\varphi}_0^2}{2a^3}\right)$$

Galactic-scale solitons:

Nonrelativistic regime

On the scale of the galactic halo we are in the nonrelativistic regime: the frequencies and wave numbers of interest are much smaller than m and the metric fluctuations are small.

Decompose the real scalar field ϕ in terms of the complex scalar field ψ

$$\boxed{\phi = \frac{1}{\sqrt{2m}}\left(\psi e^{-imt} + \psi^\star e^{imt}\right)}$$

factorizes (removes) the fast oscillations of frequency m

$$\dot{\psi} \ll m\psi, \quad \nabla\psi \ll m\psi$$

$\psi(x,t)$ evolves slowly, on astrophysical or cosmological scales.

Instead of the Klein-Gordon eq., it obeys the Schrödinger eq.:

$$\boxed{i\left(\dot{\psi} + \frac{3}{2}H\psi\right) = -\frac{\nabla^2\psi}{2ma^2} + m\Phi\psi + \frac{\partial\mathcal{V}_{\rm I}}{\partial\psi^\star}}$$

Newtonian gravitational potential

self-interactions

$$V_{\rm I}(\phi) = \Lambda^4 \sum_{p\geq 3} \frac{\lambda_p}{p}\left(\frac{\phi}{\Lambda}\right)^p$$

$$\Downarrow$$

$$\mathcal{V}_{\rm I}(\psi,\psi^\star) = \Lambda^4 \sum_{p\geq 2} \frac{\lambda_{2p}}{2p}\frac{(2p)!}{(p!)^2}\left(\frac{\psi\psi^\star}{2m\Lambda^2}\right)^p$$

Hydrodynamic picture

Madelung 1927, Chavanis 2012, ...

One can map the Schrödinger eq. to the hydrodynamical eqs.: $\psi = \sqrt{\frac{\rho}{m}} e^{iS}$ $\quad \vec{v} = \frac{\vec{\nabla} S}{ma}$

The real and imaginary parts of the Schrödinger eq. lead to the continuity and Euler eqs.:

$$\dot{\rho} + 3H\rho + \frac{1}{a}\nabla\cdot(\rho\vec{v}) = 0$$

conservation of probability for ψ ⇒ conservation of matter for ρ

$$\dot{\vec{v}} + H\vec{v} + \frac{1}{a}(\vec{v}\cdot\nabla)\vec{v} = -\frac{1}{a}\nabla(\Phi + \Phi_{\rm I} + \Phi_{\rm Q})$$

Newtonian gravitational potential

"quantum pressure": $\Phi_{\rm Q} = -\frac{\nabla^2\sqrt{\rho}}{2m^2a^2\sqrt{\rho}}$

comes from part of the kinetic terms in ψ

self-interactions:

$\mathcal{V}_I(\psi,\psi^\star)$ ⇒ $\mathcal{V}_I(\rho)$ ⇒ $\Phi_{\rm I}(\rho) = \frac{d\mathcal{V}_{\rm I}}{d\rho}$

$\lambda_4 > 0$ repulsive self-interactions

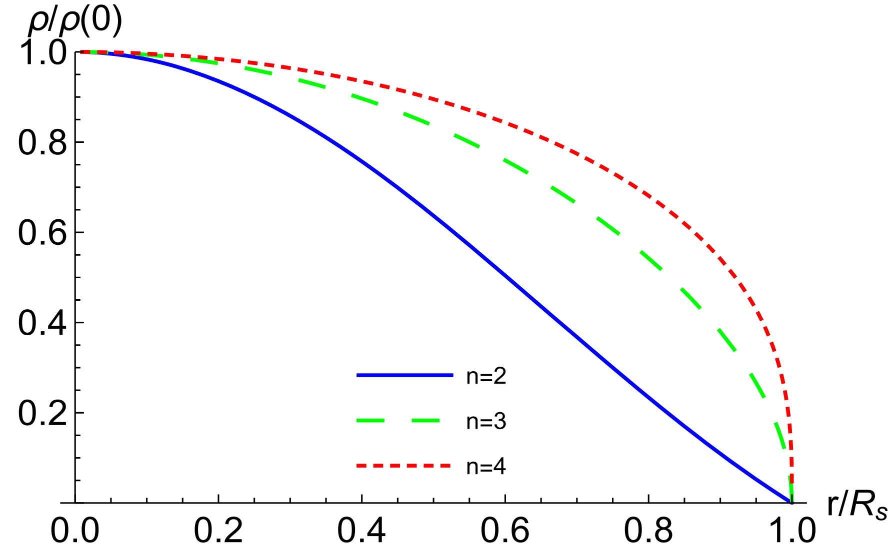
Hydrostatic equilibrium (also minimum of the total energy)

$\dot{\rho} = 0$
$\vec{v} = 0$ ⇒ $\Phi + \Phi_{\rm I} + \Phi_{\rm Q} = \alpha.$ ⇒ determines the radial density profile.

This compact (exponential tail) spherical solution is often called a "soliton".

P. Brax, J. Cembranos, PV, 1906.00730

$$V_I \propto \phi^{2n}, \quad \Phi_I \propto \rho^{n-1}$$



Ruffini and Bonazolla 1969,
Chavanis 2011,
Schiappacasse and Hertzberg 2018, ...

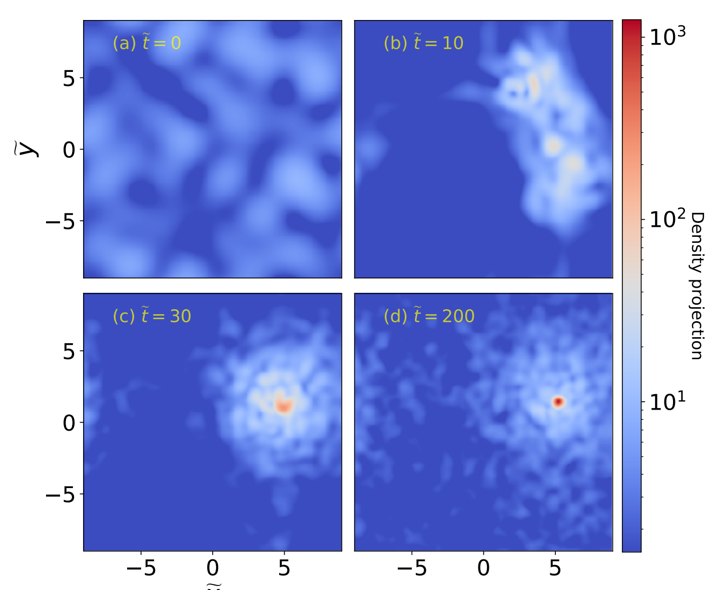
$m \gg 10^{-18}\mathrm{eV}$: galactic soliton governed by the balance between the repulsive self-interaction and self-gravity.

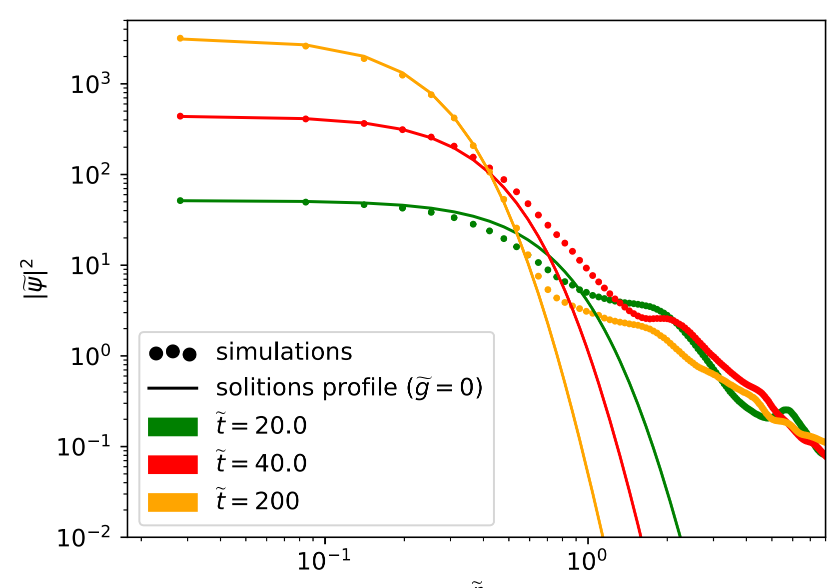
$m \sim 10^{-21}\mathrm{eV}$: Fuzzy Dark Matter (de Broglie wavelength of galactic size): galactic soliton governed by the balance between the quantum pressure and self-gravity.

Numerical simulations indeed find that solitons form, from gravitational collapse, within an extended NFW-like out-of-equilibrium halo.

Chen et al. 2020

Schive et al. 2014,
Veltmaat et al. 2018,
Mocz et al. 2019,
Amin and Mocz 2019, ...





Impact of the supermassive BH at the center of galaxies ?

- Does the scalar field falls onto the BH ?
- Is the soliton lifetime greater than the age of the Universe ?

case $V_{\rm I}(\phi) = \frac{\lambda_4}{4}\phi^4.$

quartic repulsive self-interaction

Relativistic regime:

- metric fluctuations are large close to the BH horizon

static spherical symmetry: $ds^2 = -f(r)dt^2 + h(r)(dr^2 + r^2 d\vec{\Omega}^2).$ (isotropic coordinates)

* Schwarzschild metric close to the BH: $\frac{r_s}{4} < r < r_{\rm NL}: \; f(r) = \left(\frac{1 - r_s/(4r)}{1 + r_s/(4r)}\right)^2,$

$$h(r) = (1 + r_s/(4r))^4,$$

* small metric fluctuations and self-gravity far from the BH, in the galactic-scale soliton:

$\Phi \ll 1, \quad f = 1 + 2\Phi, \quad h = 1 - 2\Phi \qquad r \gg r_{\rm sg}: \; \nabla^2\Phi = 4\pi\mathcal{G}\rho_\phi,$

- field oscillations are large and the cosine is significantly deformed by the self-interactions

→ nonlinear approach on the K.G. eq.

Nonlinear oscillator:

Nonlinear KG eq. of motion: $$\frac{\partial^2\phi}{\partial t^2} - \sqrt{\frac{f}{h^3}}\frac{1}{r^2}\frac{\partial}{\partial r}\left[\sqrt{fh}r^2\frac{\partial\phi}{\partial r}\right] + fm^2\phi + f\lambda_4\phi^3 = 0.$$

nonlinear term due to the self-interactions

In the large-mass limit, use a nonlinear local approximation: $$\phi = \phi_0(r)\mathrm{cn}[\omega(r)t - \mathbf{K}(r)\beta(r), k(r)],$$

$\mathrm{cn}(u,k)$ is a generalization of the cosine to the nonlinear (cubic) oscillator: $$\frac{\partial^2\mathrm{cn}}{\partial u^2} = (2k^2-1)\mathrm{cn} - 2k^2\mathrm{cn}^3,$$

$$k = 0: \quad \mathrm{cn}(u, k=0) = \cos(u)$$ (Jacobi elliptic function)

$\phi_0(r), \ \omega(r), \ \beta(r), \ \mathbf{K}(r), \ k(r)$ are slow functions of r

the frequency and the phase are of the order of m

$$\nabla_r \ll m$$

$$\omega \sim \beta \sim m$$

Substituting into the K.G. eq. determines all parameters $\{\phi_0, \ \omega, \ \beta, \ \mathbf{K}\}$ in terms of $k(r)$

(at leading order)

$k(r)$ is determined by a self-consistency constraint: the mean flux (averaged over the fast oscillations) must be constant over radius

$$\nabla_\mu T^\mu_0 = 0, \qquad \dot{\rho} - \frac{1}{\sqrt{fh^3}r^2}\frac{\partial}{\partial r}\left[\sqrt{fh^3}r^2 T^r_0\right] = 0,$$

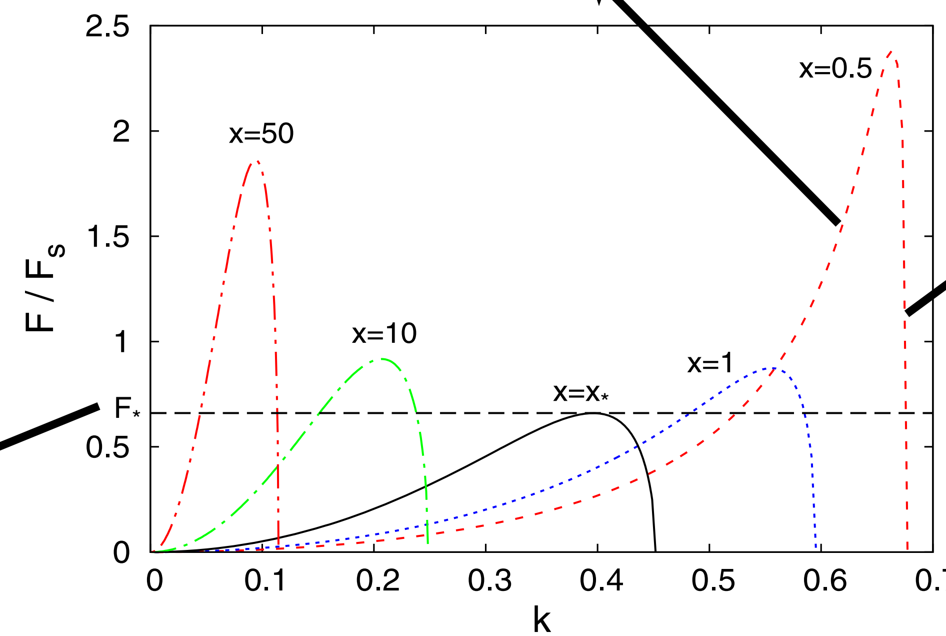
$$F = -\sqrt{fh^3}r^2\langle T^r_0\rangle = \sqrt{fh}r^2\phi_0^2\omega\mathbf{K}\beta'\left\langle\left(\frac{\partial\mathrm{cn}}{\partial u}\right)^2\right\rangle,$$

Critical flux:

Mean flux $F(k,x)$
as a function of parameter k,
at several radii x

high-velocity branch

low-velocity branch



critical value of the flux

$$x = \frac{r}{r_s} > \frac{1}{4}$$

$$F_s = -\frac{r_s^2 m^4 (1+\alpha)^2}{\lambda_4} \simeq -\frac{r_s^2 m^4}{\lambda_4},$$

At any radius x, the constant-flux constraint $F(k,x) = F_0$ selects 2 possible values of k, $\quad k_1 \leq k_2$

k_1 : high-velocity branch $\qquad$ k_2 : low-velocity branch

1) To have a solution to the constraint $F(k,x) = F_0$ at all radii, we need $F_0 \leq F_\star$

2) The boundary conditions require to be on the high-velocity branch close to the BH (~ free fall into the BH) and on the low-velocity branch at large radii (static soliton).

3) To switch in a continuous manner from the left (high-velocity) branch at low radii, to the right (low-velocity) branch at large radii, the flux must be equal to the critical value: $F_0 = F_\star$

This is similar to the hydrodynamic case, which selects a unique transonic solution.

Bondi 1952, Michel 1972

- at the horizon: nonlinear ingoing radial wave:

$$\tilde{r} \to r_s: \ \phi = \phi_s \mathrm{cn}\left[\frac{2\mathbf{K}_s}{\pi}(1+\alpha)m(\tilde{t}+\tilde{r}), k_s\right],$$

Schwarzschild radial coordinate,
Eddington time

The self-interactions remain relevant and determine higher-order harmonics.

characteristic density: $\rho_a \equiv \frac{4m^4}{3\lambda_4}.$

characteristic flux: $F_s = \frac{r_s^2 m^4}{\lambda_4}$

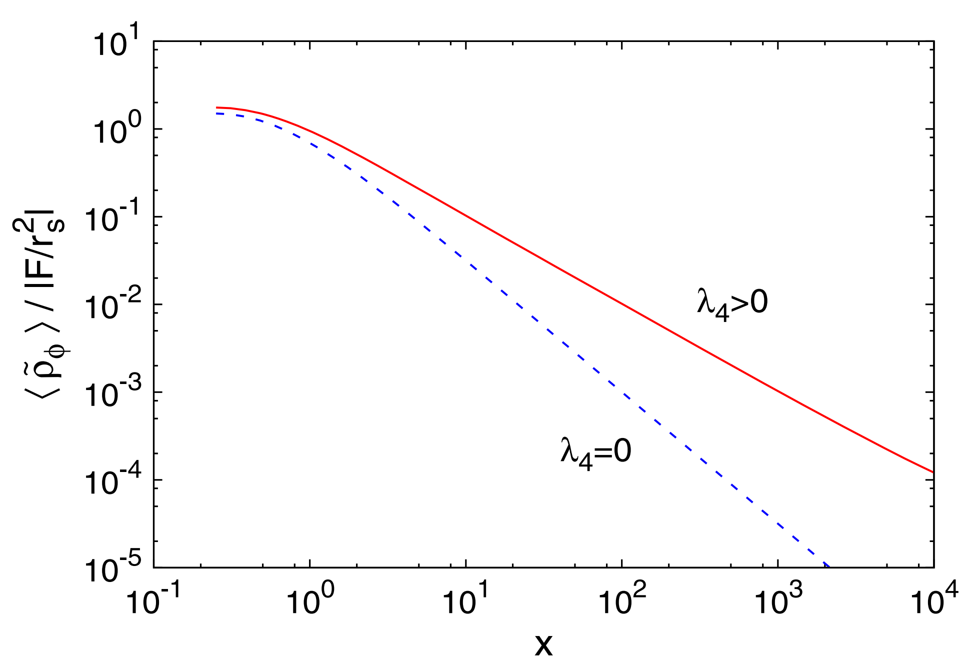
greater repulsive self-interactions decrease the scalar-field energy density and flux.

- intermediate radii (weak gravity dominated by the BH mass): $r_s \ll r \ll r_{\rm sg}: \ \langle \tilde{\rho}_\phi \rangle \propto r^{-1} \quad \text{and} \quad v_r \propto r^{-1}.$

- large radii (weak gravity dominated by the scalar-field soliton self-gravity):

$$r_{\rm sg} < r < R_s: \ \tilde{\rho}_\phi \sim \rho_s, \qquad v_r \sim -\frac{\rho_s}{\rho_a}\frac{r_{\rm sg}^2}{r^2}.$$

radial
scalar-field
energy-density
profile



Soliton lifetime:

From the value of the inward flux, we can estimate the soliton lifetime, until it completely falls into the BH:

$$t_c \equiv \frac{r_a}{|v_r|} \sim r_a \frac{\rho_s}{\rho_a} \frac{r_a^2}{r_s^2}.$$

$$t_c \sim 10^3 t_H \frac{\rho_s}{\bar{\rho}_c} \left(\frac{\rho_a}{1 \text{ eV}^4} \right)^{-5/2} \left(\frac{M}{10^8 \ M_\odot} \right)^{-2}.$$

To have a soliton of galactic scale (kpc) we need: $\rho_a \sim 1 \text{ eV}^4$ $\rho_s/\bar{\rho}_c \sim 10^5$

$\Rightarrow$ $t_c \gg t_H$ galactic solitons easily survive until today

k-essence models

Scalar-field models with a shift-symmetry only broken by the mass term.
K-essence model: only first-derivatives of the scalar field.

$$S_\phi = \int d^4x \sqrt{-g}\left[\Lambda^4 K(X) - \frac{m^2}{2}\phi^2\right], \qquad X = -\frac{1}{2\Lambda^4} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi,$$

Galactic-scale solitons:

Nonrelativistic regime

Small nonlinear corrections: $K(X) = X + K_{\rm I}(X).$ $\qquad X \ll 1: \; K_{\rm I}(X) = \sum_{n\geq 2} \frac{k_n}{n} X^n.$

In this regime, the model is equivalent to a theory with a standard kinetic term and a nonlinear self-interaction potential:

$$V_{\rm I}(\phi) = \Lambda^4 \sum_{n\geq 4} \frac{\lambda_n}{n}\frac{\phi^n}{\Lambda^n}, \qquad \lambda_{2n} = -2k_n\left(\frac{m^2}{2\Lambda^2}\right)^n. \qquad K_{\rm I}(X) = \frac{k_2}{2}X^2, \quad V_{\rm I}(\phi) = \frac{\lambda_4}{4}\phi^4, \quad \lambda_4 = -k_2\frac{m^4}{2\Lambda^4}.$$

→ formation of stable equilibrium solitons, where the self-gravity is balanced by a pressure term associated with the effective repulsive self-interaction.

Relativistic regime:

static spherical symmetry: $ds^2 = -f(r)dt^2 + h(r)(dr^2 + r^2 d\vec{\Omega}^2).$

Nonlinear KG eq. of motion:

$$\frac{\partial}{\partial t}\left[K'\frac{\partial\phi}{\partial t}\right] - \sqrt{\frac{f}{h^3}}\frac{1}{r^2}\frac{\partial}{\partial r}\left[\sqrt{fh}r^2K'\frac{\partial\phi}{\partial r}\right] + fm^2\phi = 0, \qquad K' = dK/dX \qquad X = \frac{1}{2\Lambda^4 f}\left(\frac{\partial\phi}{\partial t}\right)^2 - \frac{1}{2\Lambda^4 h}\left(\frac{\partial\phi}{\partial r}\right)^2.$$

In the large-mass limit, use again a nonlinear local approximation: $\phi(r,t) = \phi_0(r)\mathrm{ck}[\omega(r)t - \mathbf{Q}(r)\beta(r), \mu(r)].$

$\mathrm{ck}(u,\mu)$ is a generalization of the cosine to the nonlinear oscillator: $\frac{\partial^2 \mathrm{ck}}{\partial u^2} + \mathrm{ck} + \tilde{K}_{\mathrm{I}}\left[\mu\left(\frac{\partial \mathrm{ck}}{\partial u}\right)^2\right]\frac{\partial^2 \mathrm{ck}}{\partial u^2} \equiv 0,$

$\mathrm{ck}(u,0) = \cos(u).$

$\phi_0(r), \ \omega(r), \ \beta(r), \ \mathbf{Q}(r), \ \mu(r)$ are slow functions of r $\quad \nabla_r \ll m$

the frequency and the phase are of the order of m $\quad \omega \sim \beta \sim m$

Substituting into the K.G. eq. determines all parameters $\{\phi_0, \ \omega, \ \beta, \ \mathbf{Q}\}$ in terms of $\mu(r)$

(at leading order)

$\mu(r)$ is determined by a self-consistency constraint: the mean flux (averaged over the fast oscillations) must be constant over radius

Quartic Lagrangian:

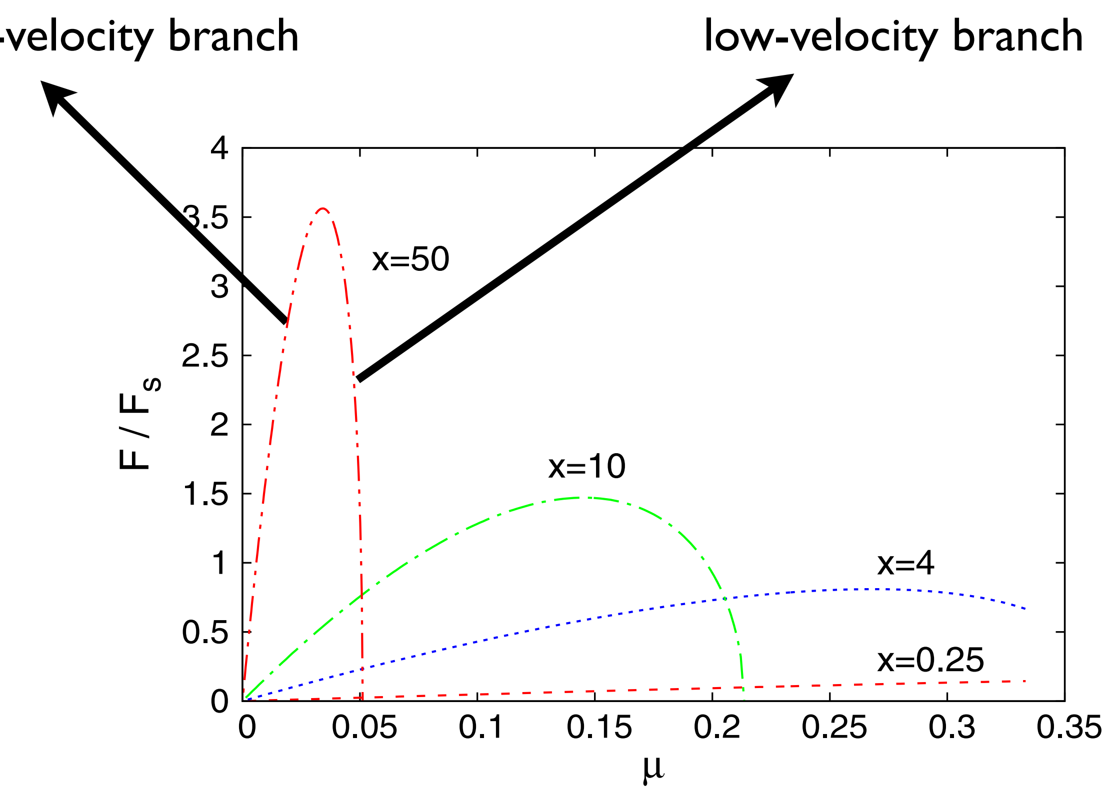
$$K_{\mathrm{I}}(X) = \frac{k_2}{2} X^2, \quad V_{\mathrm{I}}(\phi) = \frac{\lambda_4}{4}\phi^4, \quad \lambda_4 = -k_2 \frac{m^4}{2\Lambda^4}.$$

In the nonrelativistic regime, this is equivalent to the previous model, with a standard kinetic term and a quartic repulsive potential.

However, the behavior is very different in the relativistic regime !

No critical flux !

Mean flux $F(\mu, x)$
as a function of parameter μ ,
at several radii x



It is impossible to connect the high-velocity branch near the BH to the low-velocity branch at large radii.

These solutions describe the late stages when the scalar cloud has already mostly been eaten by the BH and the remaining scalar energy density is quickly falling into the BH.

Contrary to the quartic potential, the quartic derivative self-interaction X^2 is not able to support the scalar cloud against the BH gravity in the relativistic regime.

Conditions for a well-behaved solution:

Power-law behavior at large X $\qquad X \gg 1: \; K(X) \simeq a X^{\nu}, \qquad a > 0, \quad \nu > 0.$

- the solution of the anharmonic diff. eq. is a periodic function $\Rightarrow$ $K' + 2XK'' > 0$ $\Rightarrow$ $\nu > \frac{1}{2}$

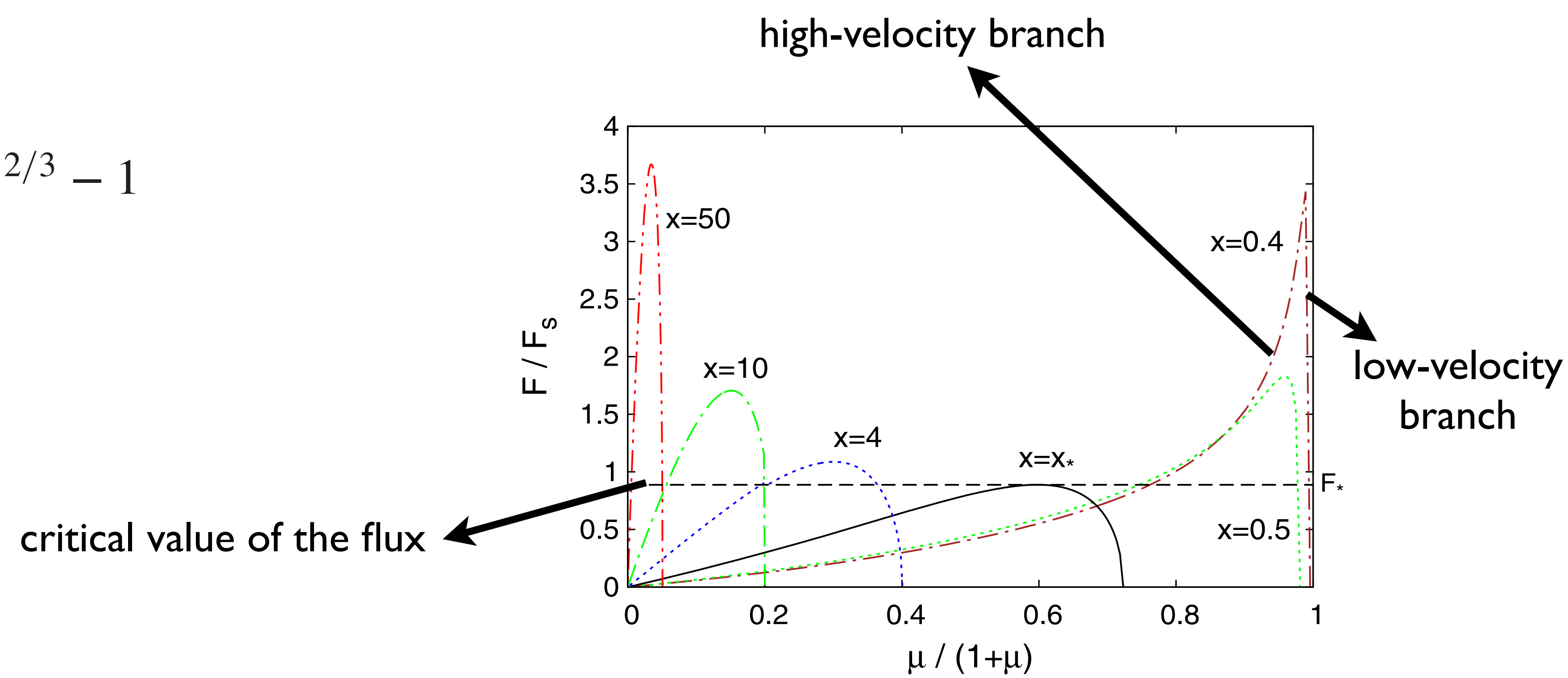
- the flux shows a peak that grows at low radius and vanishes at both ends of the range of μ

$\Rightarrow$ the period $4\,\mathbf{Q}$ must vanish at the horizon $\Rightarrow$ $\nu < 1$

$$\Rightarrow \quad \frac{1}{2} < \nu < 1$$

Example:

$$K(X) = (1 + 3X/2)^{2/3} - 1$$



We obtain again a unique solution, associated with a critical value of the constant flux.
It connects the low-velocity branch at large radius to the high-velocity branch at low radius.

- at the horizon: nonlinear ingoing radial wave: $\tilde{r} \to r_s: \ \phi = \phi_s \mathrm{ck}\left[\frac{2\mathbf{Q}_s}{\pi}(1+\alpha)m(\tilde{t}+\tilde{r}), \mu_s\right]$

Schwarzschild radial coordinate,
Eddington time

The self-interactions remain relevant and determine higher-order harmonics.

characteristic density: $\rho_a \equiv \frac{4m^4}{3\lambda_4} = \frac{8\Lambda^4}{3|k_2|}.$

greater self-interactions decrease the scalar-field energy density and flux.

Soliton lifetime:

From the value of the inward flux, we can estimate the soliton lifetime, until it completely falls into the BH:

$$t_c \equiv \frac{r_a}{|v_r|} \sim r_a \frac{\rho_{\rm sol}}{\rho_a} \frac{r_a^2}{r_s^2}$$

$$t_c \sim 10^3 t_H \frac{\rho_{\rm sol}}{\bar{\rho}_c} \left(\frac{\rho_a}{1 \ \mathrm{eV}^4}\right)^{-5/2} \left(\frac{M}{10^8 \ M_\odot}\right)^{-2}$$

To have a soliton of galactic scale (kpc) we need: $\rho_a \sim 1 \ \mathrm{eV}^4$ $\quad \rho_s/\bar{\rho}_c \sim 10^5$

$t_c \gg t_H$ galactic solitons easily survive until today

Quantum corrections:

Using background quantization, we can see that quantum corrections from higher-order Feynman diagrams are negligible.

Weak gravity regime: Minkowski spacetime, scale-independent background

$$\phi = \bar{\phi} + \hat{\phi} \qquad \hat{\mathcal{L}}^{(2)} = \frac{\bar{K}' + 2\bar{X}\bar{K}''}{2}\left(\frac{\partial\hat{\phi}}{\partial t}\right)^2 - \frac{\bar{K}'}{2}(\nabla\hat{\phi})^2 - \frac{m^2}{2}\hat{\phi}^2.$$

no ghosts, no small-scale instabilities: $\bar{K}' > 0, \quad \bar{K}' + 2\bar{X}\bar{K}'' > 0$

also required to have a periodic solution to the anharmonic diff. eq.

L-loop vacuum Feynman diagram contributing to the corrections to the classical action:

$$I_L = \int \prod_{\ell=1}^{L} d^4 p_\ell \prod_{n=1}^{N} \frac{1}{p^2 + \bar{m}^2} \prod_{v=1}^{V} \Lambda^4 c_{m_v} \prod_{s=1}^{m_v} \frac{p_s}{\Lambda^2} \qquad c_n = (\bar{K}' + 2\bar{X}\bar{K}'')^{-n/2} \sum_{m=[n/2]_+}^{n} \bar{K}^{(m)} \bar{X}^{m-n/2}.$$

The quantum corrections remain small, for any scalar-field background *X*, provided:

$$\delta\mathcal{L}^{(L)} \ll \mathcal{L}^{(0)} : \qquad \boxed{m \ll \Lambda, \quad m \ll 10^{-3}\ \mathrm{eV}, \quad \text{and} \quad \nu \geq \frac{2}{3} \ \text{at large } X}$$

Relativistic background: Schwarzschild metric, scalar field depends on time and radius

quadratic Lagrangian:

$$\hat{\mathcal{L}}^{(2)} = \frac{1}{2}\left[\mathcal{K}_{00}\left(\frac{\partial\hat{\phi}}{\partial\tilde{t}}\right)^2 + 2\mathcal{K}_{01}\frac{\partial\hat{\phi}}{\partial\tilde{t}}\frac{\partial\hat{\phi}}{\partial\tilde{r}} + \mathcal{K}_{11}\left(\frac{\partial\hat{\phi}}{\partial\tilde{r}}\right)^2\right] - \frac{\bar{K}'}{2\tilde{r}^2}(\partial_\Omega\hat{\phi})^2 - \frac{m^2}{2}\hat{\phi}^2$$

where $\mathcal{K}_{ij}$ are nonlinear oscillating factors.

$\det(\mathcal{K}_{ij}) = -\bar{K}'(\bar{K}' + 2\bar{X}\bar{K}'') < 0.$ The signature (+,-) is preserved: no ghosts, no gradient instabilities.

$\bar{X} \sim 1, \quad \bar{K}' \sim 1$

The quantum corrections remain small $\delta\mathcal{L}^{(L)} \ll \mathcal{L}^{(0)}$: $m \ll \Lambda$

We can work with classical backgrounds at density $\rho_\phi \sim \Lambda^4$

provided the conditions above for small quantum corrections are satisfied.

Conclusion:

Quartic repulsive potential:

- well defined steady state from the galactic radius to the BH horizon

- as for the hydrodynamical case of polytropic fluids, a unique solution that goes from a low-velocity branch at large radii to a high-velocity branch at low radii

- nonlinear ingoing wave with unit velocity at the BH horizon

- ρ and v decay as $1/r$ at intermediate radii

- ρ is constant and v decays as $1/r^2$ at larger radii, in the soliton self-gravity domain

- the soliton lifetime is much greater than the age of the Universe

k-essence models:

- equivalent to the models with standard kinetic term and nonlinear potential in the nonrelativistic regime

- the relativistic regime can be very different

- the quartic derivative self-interaction $-(\partial\phi)^4$ (equivalent to repulsive ϕ^4 in the NR regime) cannot support the scalar cloud against the BH gravity close to the horizon

need to go beyond the weak-gravity large-radius analysis to check the self-consistency of the system down to the horizon

- conditions for a well-behaved solution: $K' > 0, \quad K' + 2XK'' > 0$

$X \gg 1: \quad K \sim X^{\nu}, \quad 1/2 < \nu < 1$

- quantum corrections remain negligible: $m \ll \Lambda, \quad m \ll 10^{-3}\ \mathrm{eV}, \quad \text{and} \quad \nu \geq \frac{2}{3}$ at large X

further works:

- rotation (Kerr metric)

- dynamical friction of BH inside scalar clouds