The MEMe Model

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Why another modified gravity theory?

- Expect general relativity to break down (singularities, quantum gravity)
- Should classify and test all alternatives to GR
- Do the same for alternatives to ΛCDM in light of recent tensions.

Minimal **E**xponential **Me**asure (**MEMe**) model

- Generalized coupling models:[1] matter minimally coupled to $\mathfrak{g}_{\mu\nu}$

$$\mathfrak{g}_{\mu\nu} = \Xi(A_{\cdot}^{\cdot})\, A_{\mu}{}^{\alpha}\, A_{\nu}{}^{\beta}\, g_{\alpha\beta}$$

- $A_{\mu}{}^{\alpha}$ are auxiliary fields (no kinetic terms)
- The MEMe model:
  - Set $\Xi(A_{\cdot}^{\cdot}) = e^{(4-A)/2}$, with action

$$S[\phi, g^{\cdot\cdot}, A_{\cdot}^{\cdot}] = \int d^4x \left\{ \frac{1}{2\kappa} \left[R - 2\,\tilde{\Lambda} \right] \sqrt{-g} + \left[L_m[\phi, \mathfrak{g}^{\cdot\cdot}] - \frac{1}{q} \right] \sqrt{-\mathfrak{g}} \right\}$$

  - q like inverse vacuum energy

[1] JCF and Sante Carloni. "New class of generalized coupling theories". In: *PRD* 101 (6 Mar. 2020), p. 064002.

Minimal Exponential Measure (MEMe) model

- Field equations[2] ($\Lambda = \tilde{\Lambda} + \kappa/q$, $|A| = \det(A.^{\cdot})$)

$$A_\beta{}^\alpha - \delta_\beta{}^\alpha = q\left((1/4)\mathfrak{T}\, A_\beta{}^\alpha - \mathfrak{T}_{\beta\nu}\, \mathfrak{g}^{\alpha\nu}\right)$$

$$G_{\mu\nu} + \left[\Lambda - (1 - |A|)\,\kappa/q\right]\, g_{\mu\nu} = \kappa\,|A|\,\bar{A}^\alpha{}_\mu\,\bar{A}^\beta{}_\nu\,\mathfrak{T}_{\alpha\beta}$$

- Can solve for single perfect fluid $\mathfrak{T}_{\mu\nu} = (\rho + p)\,u_\mu u_\nu + p\,\mathfrak{g}_{\mu\nu}$:

$$A_\mu{}^\alpha = Y\,\delta_\mu{}^\alpha + Z\,u_\mu\,u^\alpha$$

  Y, Z functions of ρ, p, q

- Some remarks
  - Reduces to GR in vacuum or in limit $q \to 0$
  - Qualitatively similar to Eddington-Inspired Born Infeld gravity[3]

[2] $(1 - |A|) \sim O(q^2)$ for small q

[3] Máximo Bañados and Pedro G. Ferreira. In: *Phys. Rev. Lett.* 105 (1 July 2010), p. 011101.

Weak-field results (arXiv:2011.12305)

- Parameterized Post-Newtonian formalism needs modification[4]
  - Need extra potential Ψ ($\underline{\rho}$, $\underline{\rho}^*$ are mass densitites):

$$g_{00} = g_{00}^{PPN} + 2\nu\Psi \qquad\qquad \Delta\Psi := -4\pi G^2\, \underline{\rho}^*\, \underline{\rho}$$

  - Ψ also needed for EiBI gravity
  - For MEMe model (& auxiliary fields), also need additional counterterms.
- Masses of compact objects different than the sum of its parts
  - In principle, can constrain q with measurements of G.
- Speed of light differs within matter distributions
  - Weakly constrain q with timing uncertainty measurements:[5][6] $|q| < 2 \times 10^{-14}\ \mathrm{m}^3/\mathrm{J}$

[4] JCF, Shinji Mukohyama, and Sante Carloni. In: (2020). arXiv: 2011.12305 [gr-qc].

[5] JCF and Carloni, "New class of generalized coupling theories".

[6] Collider experiments: $10^{-37}\ \mathrm{m}^3/\mathrm{J}$ $((14\mathrm{GeV}/\mathrm{fm}^3)^{-1})$ [(Roman Pasechnik and Michal Sumbera. In: *Universe* 3.1 [Jan. 2017], p. 7)]

Limits of MEMe model

- Explicit expression for determinant

$$|A| = \frac{256\,(1 - p\,q)^3 (q\,\rho + 1)}{(4 - q\,(3p - \rho))^4}$$

  In limit $|A| \to 0$, metric $\mathfrak{g}_{\mu\nu}$ fails to be invertible ($\mathfrak{g} = g|A|^2$)

- Breakdown of field theory over $\mathfrak{g}_{\mu\nu}$ at scale $\rho \sim 1/|q|$
  - Consistent with vacuum energy interpretation for $1/|q|$
  - In principle, $1/|q|$ can be independent of Planck scale!

- $|A| \to 0$ in limit $pq \to 1$ ($q > 0$), or in limit $\rho q \to -1$ ($q < 0$), and:

$$G_{\mu\nu} \approx (\kappa/q - \Lambda) g_{\mu\nu}$$

  De-Sitter vacuum for $q < 0$: Can MEMe do inflation?

MEME cosmology: Dynamical systems analysis

1. Define set of dimensionless variables (S is scale factor):

$$\chi = \frac{qH^2}{\kappa} \qquad K = \frac{k}{H^2 S^2} \qquad \Omega = \frac{\kappa\rho}{3H^2} \qquad L = \frac{\Lambda}{3H^2}$$

2. Rewrite cosmological eqs. as 1st order ODEs + constraint:

$$\begin{aligned}
\chi' &= F_\chi(\chi, K, \Omega, L) \\
\Omega' &= F_\Omega(\chi, K, \Omega, L) \\
L' &= F_L(\chi, K, \Omega, L) \\
1 &= L - K - R(\Omega, \chi) - \frac{1}{3\chi}
\end{aligned}$$

Scale factor S_J measured by comoving observers is

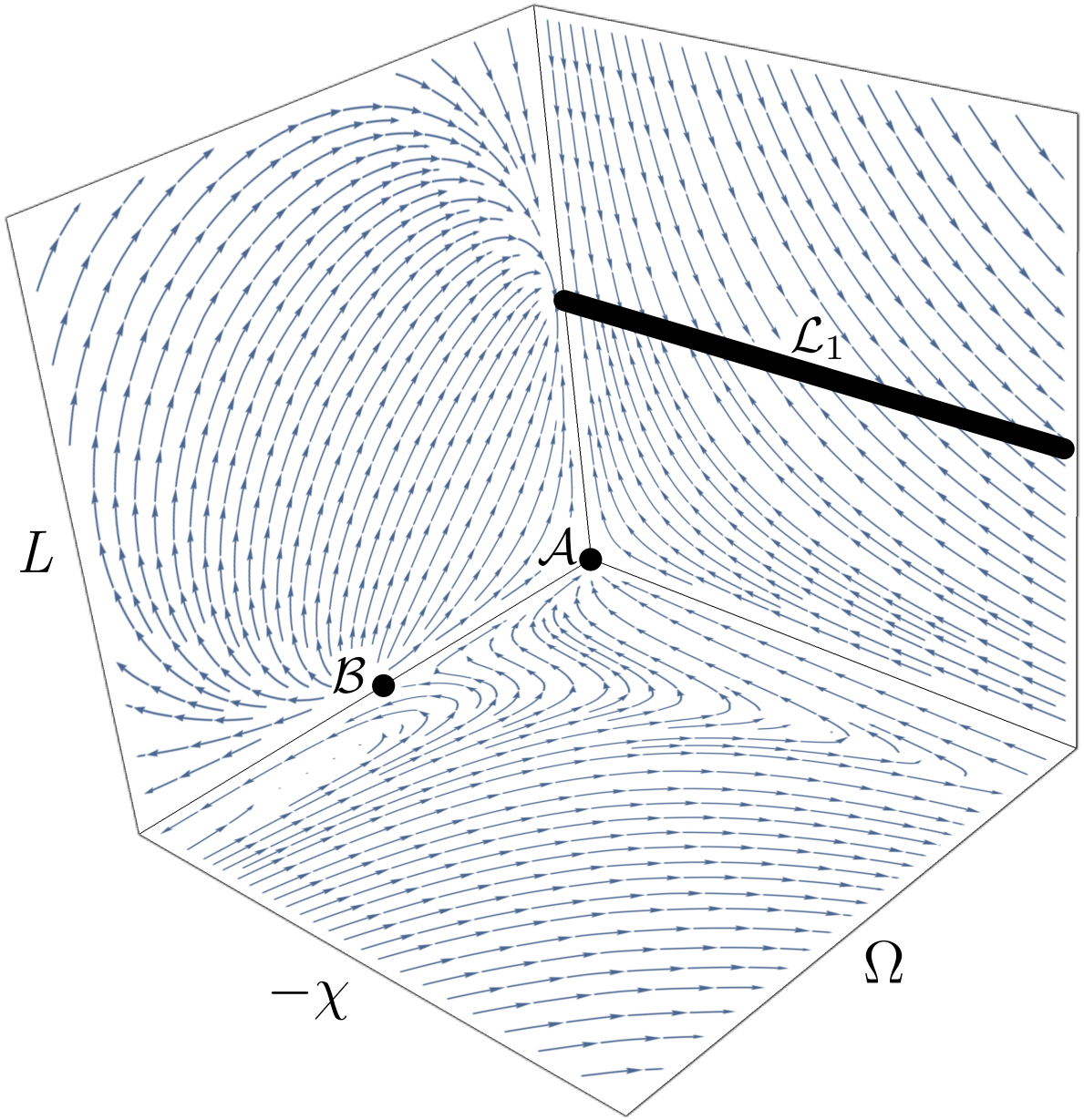
$$S_J = \Psi Y^2 S = \frac{16(1-pq)^2}{[4+q(\rho-3p)]^2} S.$$

For all relevant cases, factor is of order one.

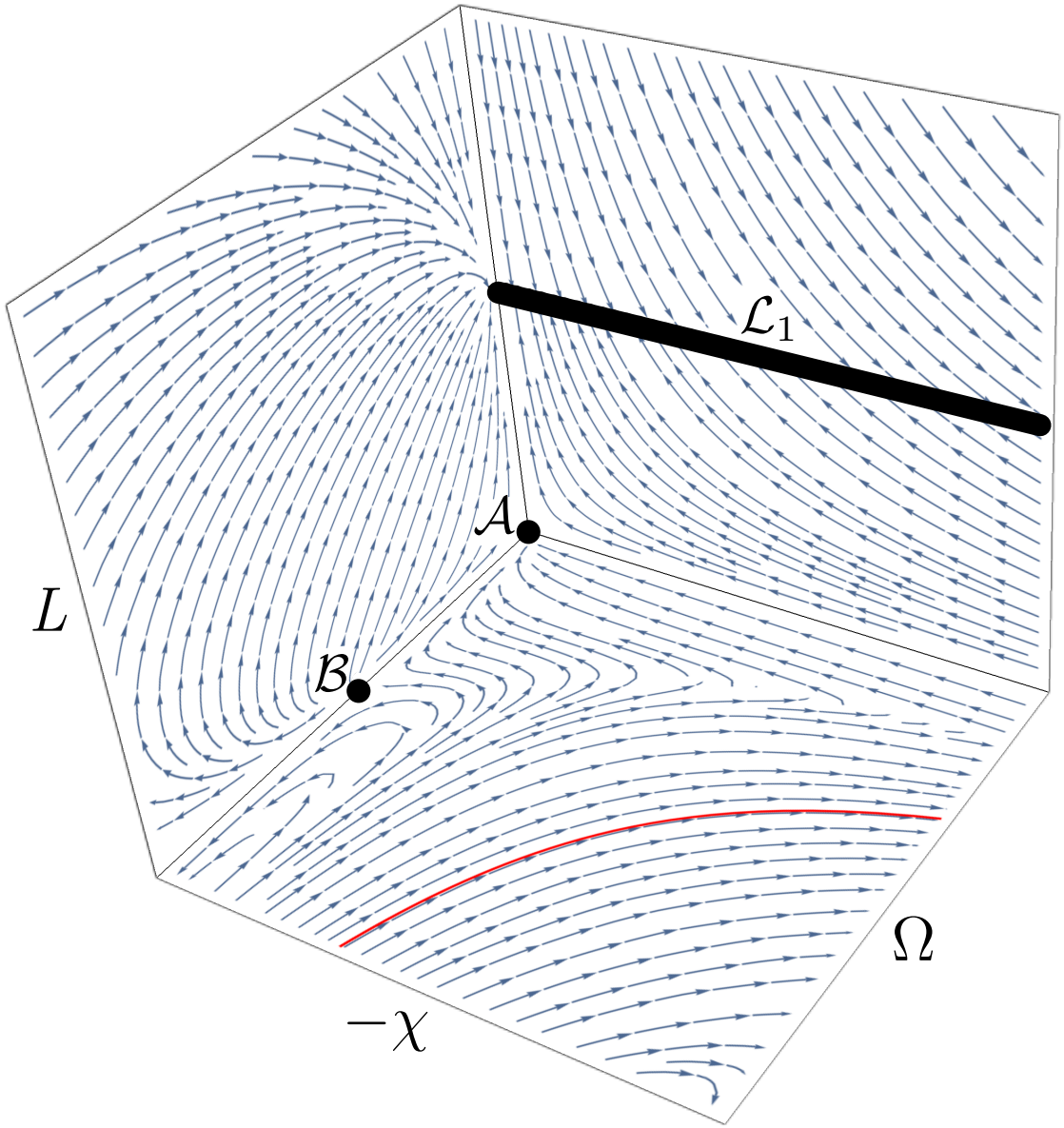
3 Fixed point Analysis

Point	$\{\chi, \Omega, L, K\}$	Attractor	Repeller	Scale Factor
$\mathcal{A}$	$\{0, 0, 0, -1\}$	Never	Never	$S = S_0(t - t_0)$
$\mathcal{B}$	$\{0, 1, 0, 0\}$	Never	Never	$S = S_0(t - t_0)^{\frac{2}{3(w+1)}}$
$\mathcal{L}_1$	$\{\chi_0, 0, 1, 0\}$	$0 \leq w \leq 1$	Never	$S = S_0 e^{\gamma_1 (t - t_0)}$
$\mathcal{L}_2$	$\{\frac{1}{3w\Omega_0}, \Omega_0, 1 + w\Omega_0, 0\}$	$0 < w \leq 1$	Never	$S = S_0 e^{\gamma_2 (t - t_0)}$

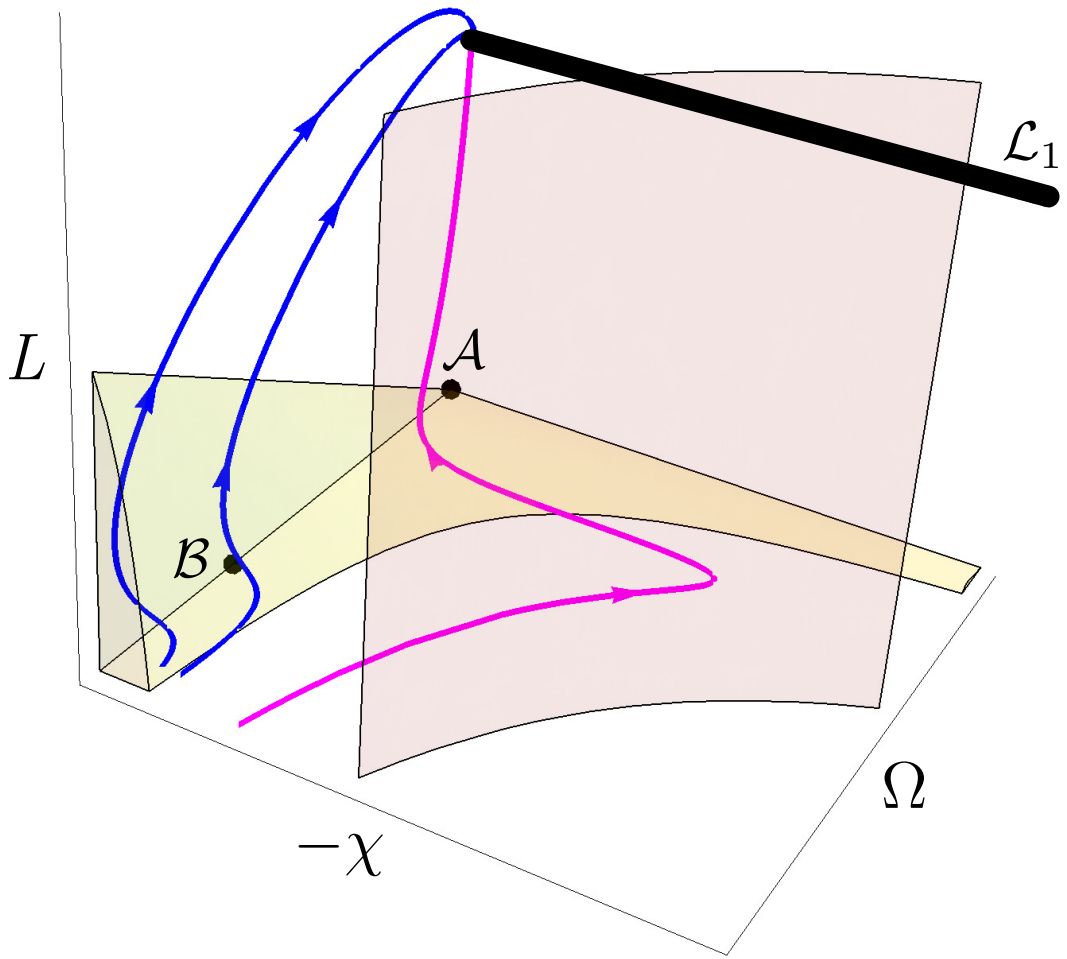
Phase space diagram: Radiation ($w = 1/3$)



Phase space diagram: Dust ($w = 0$)



Dynamical systems analysis: Dust ($w = 0$)



Summary

- **M**inimal **E**xponential **Me**asure (**MEMe**) model:

$$\mathfrak{g}_{\mu\nu} = e^{(4-A)/2}\, A_\mu{}^\alpha\, A_\nu{}^\beta\, g_{\alpha\beta}$$

$$S[\phi, g^{\cdot\cdot}, A.^{\cdot}] = \int d^4x \left\{ \frac{1}{2\kappa}\left[R - 2\,\tilde{\Lambda}\right]\sqrt{-g} + \left[L_m[\phi, \mathfrak{g}^{\cdot\cdot}] - \frac{1}{q}\right]\sqrt{-\mathfrak{g}} \right\}$$

  - Gravitational and minimally coupled metric differ
  - Same vacuum phenomenology as GR, differs in matter
  - Simple solution for single perfect fluid
  - Can qualitatively describe cosmic history

The End.

MEMe: Perfect fluid

- Energy-momentum tensor for a perfect fluid

$$\mathfrak{T}_{\mu\nu} = (\rho + p)\, u_\mu u_\nu + p\, \mathfrak{g}_{\mu\nu}$$

- Define $\varepsilon := u^\mu\, u_\mu$ (generally $\varepsilon \neq -1$) and use ansatz

$$A_\mu{}^\alpha = Y\, \delta_\mu{}^\alpha + Z\, u_\mu\, u^\alpha$$

- Solution:

$$Y = \frac{4(1 - p\, q)}{4 - q\,(3\, p - \rho)}$$

$$\varepsilon = -\frac{16\,(q\,\rho + 1)^2}{(4 - q\,(3\, p - \rho))^2} \qquad Z = -\frac{q\,(p + \rho)(4 - q\,(3\, p - \rho))}{4\,(q\,\rho + 1)^2}$$

- Gravitational eqs. $(|A| = \det(A.^{\cdot}) = Y^3\,(Y + \varepsilon\, Z))$,

$$G_{\mu\nu} = \kappa \left[-\varepsilon\, |A|\, (p + \rho)\, u_\mu\, u_\nu + \left(\frac{|A|\,(p\, q - 1) + 1}{q} - \frac{\Lambda}{\kappa} \right) g_{\mu\nu} \right]$$

GW propagation in matter

- GWs propagate according to $g_{\mu\nu}$, light propagates according to $\mathfrak{g}_{\mu\nu}$.
  No issues with causality as long as system is well-posed[7]

- Relative propagation speed of GWs c_g/c satisfies:

$$(c_g/c)^2 \approx 1 - q\ (p + \rho) + O(q^2)$$

- Can use timing uncertainty in GW170817[8] to get rough upper bound (scale: $\sim 6 \times 10^{-22}$eV):

$$|q| < 2 \times 10^{-14}\ \mathrm{m}^3/\mathrm{J}$$

- Highest densities probed in colliders[9] $\sim 14\mathrm{GeV}/\mathrm{fm}^3$, expect $|q| < 5 \times 10^{-37}\ \mathrm{m}^3/\mathrm{J}$.

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[7] Eugeny Babichev, Viatcheslav Mukhanov, and Alexander Vikman. "k-Essence, superluminal propagation, causality and emergent geometry". In: *JHEP* 2008.02 (Feb. 2008), pp. 101–101.

[8] B. P. Abbott et. al. "Multi-messenger Observations of a Binary Neutron Star Merger". In: *ApJ* 848.2 (Oct. 2017), p. L12.

[9] Pasechnik and Sumbera,

MEME cosmology: Dynamical systems analysis

1. Define set of dimensionless variables:

$$\chi = \frac{qH^2}{\kappa} \qquad K = \frac{k}{H^2 S^2} \qquad \Omega = \frac{\kappa\rho}{3H^2} \qquad L = \frac{\Lambda}{3H^2}$$

2. Rewrite cosmological eqs. as 1st order ODEs:

$$\begin{aligned}
\chi' =& 2(L-1)\chi + \frac{\chi R(\Omega,\chi)[3\chi\Omega(1+3w)-2]}{3\chi\Omega+1} - \frac{2}{3} \\
\Omega' =& \frac{1}{3}\Omega\left\{\frac{2}{\chi} + 6(1-L) - \frac{\chi R(\Omega,\chi)[3\chi\Omega(1+3w)-2]}{3\chi\Omega+1} - Q(\Omega,\chi)\right\} \\
L' =& \frac{1}{3}L\left\{\frac{2}{\chi} + 6(1-L) - R(\Omega,\chi)\right\} \\
1 =& L - K - R(\Omega,\chi) - \frac{1}{3\chi}
\end{aligned}$$

where

$$\begin{aligned}
R(\Omega,\chi) =& \frac{256(3\chi\Omega+1)^2(3w\chi\Omega-1)^3}{3\chi[(3-9w)\chi\Omega+4]^4} \\
Q(\Omega,\chi) =& \frac{9(w+1)(3w\chi\Omega-1)[3(3w-1)\chi\Omega-4]}{4-3\left(3w^2+13w+2\right)\chi\Omega+9w(3w-1)\chi^2\Omega^2}
\end{aligned}$$