Spherical cosmological models

An alternative cosmology

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December 9, 2020

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Outline

1. Motivation
2. General properties (metric, dust content, time evolution, dark energy)
3. Connection with the mother universe (Novikov coordinates and metric, definition of a universe inside a BH)
4. Geometrical properties (embedding surfaces, boundary, the 2 sheets)
5. Light (Hubble relation, magnitude redshift relation)
6. Mass ejection from a black hole
7. Mach's principle and Newton's second law

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Motivation

- Tidal acceleration T in $G\mathcal{M}/R$ gravitational field

$$T = \frac{2G\mathcal{M}}{R^3}\Delta R = \frac{2c^6\Delta R}{G^2}\frac{1}{\mathcal{M}^2} \quad @ \quad R_S$$

- Density of a Black Hole

$$\rho \sim \frac{\mathcal{M}}{R_S^3} = \frac{c^6}{G^3}\frac{1}{\mathcal{M}^2}$$

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$$\rho \sim \frac{\mathcal{M}}{R_S^3} = \frac{c^6}{G^3}\frac{1}{\mathcal{M}^2}$$

1. A BH is nothing special if large enough: smooth crossing of R_S, no ripping apart
2. BH may be very empty, not at all dense object, if only large enough
3. Implies spherical symmetry!

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Lemaître(1933) - Tolman(1934) models (L-T models)
Bondi(1947)

$$ds^2 = dt^2 - X^2(r,t)\, dr^2 - R^2(r,t)\, d\Omega^2$$

$$d\Omega^2 = d\vartheta^2 + \sin^2\vartheta\, d\varphi^2$$

$$X(r,t) = \frac{\partial_r R(r,t)}{\sqrt{1-2e(r)}}, \qquad e(r) \le \frac{1}{2}$$

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   Spherically symmetric dependence on the angles
   Radius $R(r,t)$, to be determined
   Independent function $e(r)$, dimensionless, obvious constraints

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- $c = 1$, $t^{(10\,\mathrm{Ga})}$ or $t^{(3.07\,\mathrm{Gpc})}$
- Theorem t is a cosmic time all 'comoving' observers with $r = r_0$, $\vartheta = \vartheta_0$ and $\varphi = \varphi_0$ can agree on
- r is shell label of shell with surface $4\pi R^2(r,t)$
- radial distance between 2 shells is $X\Delta r = \frac{\partial_r R(r,t)}{\sqrt{1-2e(r)}}\Delta r$

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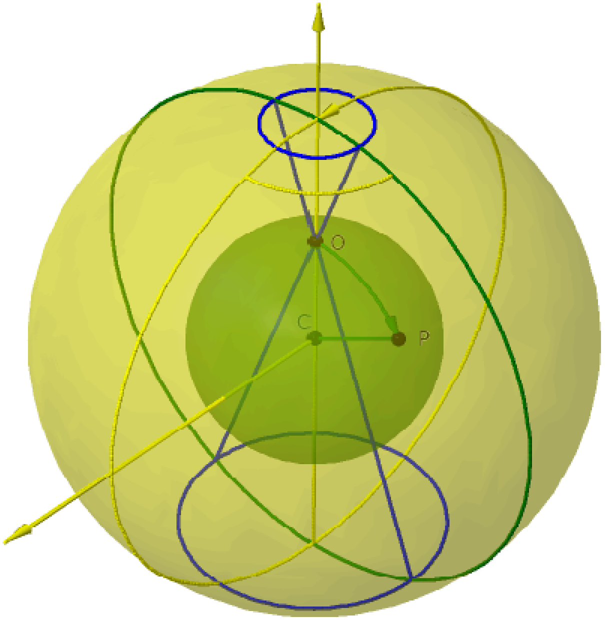
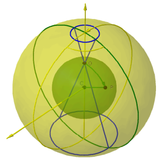
1. Diagonal
   Spherically symmetric dependence on the angles
   Radius $R(r,t)$, to be determined
   Independent function $e(r)$, dimensionless, obvious constraints
2. 2 of the 3 units to be chosen are relevant for metric: length and time
3. r has undetermined units (colour, whatever), label to a shell, by definition
   'radius' reserved to R because of the surface-relation





- Only slide with explicit picture of the geometry
- Universe has a center C

  Boundary: yellow spherical surface, general shell r: green spherical surface

  Observer O on a $\mathcal{Z}$-axis

  Orbital planes (green) through $\mathcal{Z}$-axis

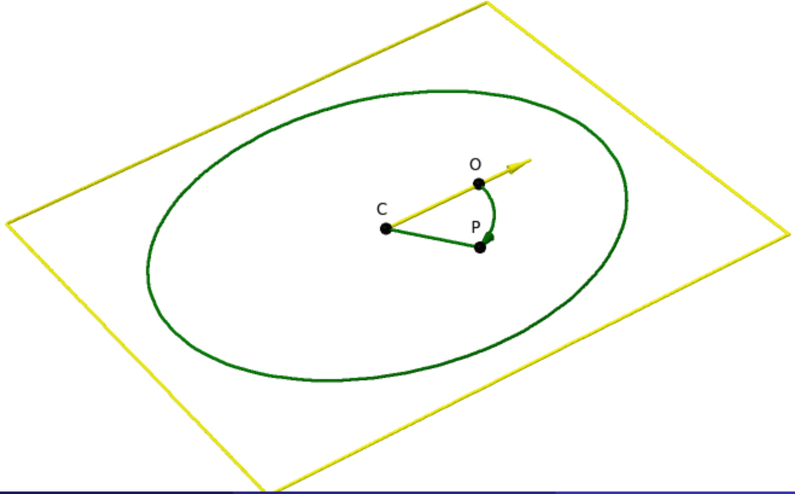
  General point P in a orbital plane, with angle $0 \leq OCP < 2\pi$

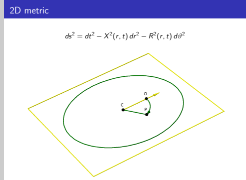
  Reference orbital plane, with $0 \leq$ inclination angle (yellow) $\leq \pi$
- Lines of sight: to center, and to anticenter

  Universe looks the same in directions given by small circles perpendicular to the $\mathcal{Z}$-axis that form cones with circular symmetry
- Sequel: suppression of inclination angle, everything in one particular orbital plane

2D metric

$$ds^2 = dt^2 - X^2(r,t)\,dr^2 - R^2(r,t)\,d\vartheta^2$$





- Meridional cut: universe is the disk, with boundary the circle

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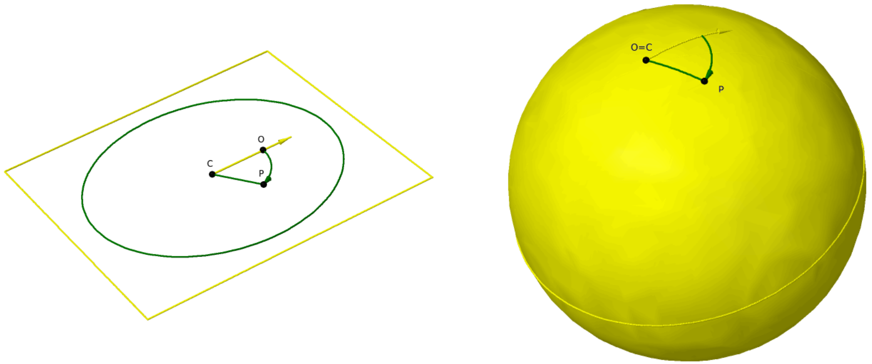

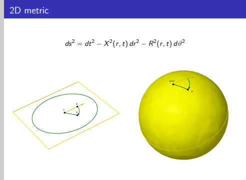

- Meridional cut: universe is the disk, with boundary the circle
- Isometric representation of Robertson Walker (R-W) metric in 2D: universe is the surface of a sphere (positive curvature)
- Isometric representations of spherical universes follow later!
- R-W universes are locally a special case of L-T universes
- Einstein equations determine the local geometry, not the global topology

Dust density $\rho(r, t)$

$$\kappa\rho(r,t) = \frac{4\pi G}{c^2}\rho(r,t) = \frac{d_r m(r)}{R^2(r,t)\partial_r R(r,t)} \geq 0$$

- $m(r)$ has the dimension of length: geometrical mass
- $\partial_r R(r,t) \geq 0$ and $d_r m(r) \geq 0$
  $r_1 < r_2$ if and only if $R(r_1, t) < R(r_2, t)$: 'inside' and 'outside'
- $$\rho(r,t) = 1.2\frac{d_r m(r)}{R^2(r,t)\partial_r R(r,t)} \times 10^{-26}\,\mathrm{kg\,m^{-3}}$$
  critical density: $3H_o/(8\pi G)$ equals $10^{-26}\,\mathrm{kg\,m^{-3}}$
- $\rho(r,t)R^2(r,t)\partial_r R(r,t)$ independent of t

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Choice of the unit for mass density completes the choice of the units.

Note the $\partial_r R(r, t)$ in the denominator.

Mass function $\mathcal{M}(r)$

volume element dV

$$dV(r,\varphi,\vartheta,t) = \frac{R^2(r,t)\partial_r R(r,t)}{\sqrt{1-2e(r)}} |\sin\vartheta|\, dr\, d\varphi\, d\vartheta$$

$$\begin{aligned}\mathcal{M}(r) &= 4\pi \int_0^r \frac{\rho(r',t)R^2(r',t)\partial_r R(r',t)}{\sqrt{1-2e(r')}}\, dr' \\ &= \frac{4\pi}{\kappa} \int_0^r \frac{d_{r'} m(r')}{\sqrt{1-2e(r')}}\, dr'\end{aligned}$$

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- Volume element is the square root of the determinant of the metric
- Mass inside shell with label r is constant
- κ has dimension of length over mass

Boundary shell r_b

- Definition:

$$\rho(r) = 0, \quad d_r m(r) = 0, \quad d_r \mathcal{M}(r) = 0 \qquad \text{for} \qquad r > r_b$$

$$0 < r_b \leq +\infty$$

- Notations:
  - $M = m(r_b)$ is the total effective geometrical mass
  - $\mathcal{M}_{\text{tot}} = \mathcal{M}(r_b)$ is the total mass
- Theorem

$$\frac{4\pi}{\kappa} M < \mathcal{M}_{\text{tot}} \quad \text{if} \quad e(r) \geq 0$$

$$\mathcal{M}_{\text{tot}} = \frac{4\pi}{\kappa} \int_0^{r_b} \frac{d_{r'}\, m(r')}{\sqrt{1 - 2e(r')}}\, dr'$$

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Time evolution

$$\left[\partial_t R(r,t)\right]^2 = -2e(r) + \frac{2m(r)}{R(r,t)} + \frac{1}{3}\Lambda\left[R(r,t)\right]^2$$

$$\frac{1}{2}V^2 - \frac{G\mathcal{M}(r)}{r} = -e$$

$m(r)$ is the effective gravitating (geometrical) mass function
$e(r)$ is the energy function
rectilinear 2-body motion (degenerate ellipses)

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$$\rho(r,t) = 1.2\,\frac{d_r m(r)}{R^2(r,t)\,\partial_r R(r,t)}$$

$\partial_r R(r_0, t_c) = 0$: collision of shells and $\rho(r_0, t_c) = +\infty$
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1. The validity of the L-T metric is the spherical volume inside the outermost massive shell (with label r_b) and a time interval starting just after the last collision at time $[t_c]_1$ until the next collision at time $[t_c]_2$

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Robertson-Walker: $[d_t\mathcal{R}(t)]^2 = -2e + 2M/\mathcal{R}(t) + \frac{1}{3}\Lambda\mathcal{R}^2(t)$

1. The validity of the L-T metric is the spherical volume inside the outermost massive shell (with label r_b) and a time interval starting just after the last collision at time $[t_c]_1$ until the next collision at time $[t_c]_2$
2. $\mathcal{R}(t) = R(r_b, t)$ yields Friedmann equation

Solution of the time equation

$$R(r,t) = p(r)\,\mathrm{cyc}\big[a(r), \epsilon(r), \psi(r,t)\big]$$

$$\text{shell parameter } p(r) = \begin{cases} \dfrac{m(r)}{e(r)}\epsilon(r) = \dfrac{m(r)}{|e(r)|} & \text{with} \quad \epsilon(r) = \mathrm{sign}\big(e(r)\big) \\ 2m(r) & \text{with} \quad \epsilon(r) = 2e(r) \end{cases}$$

$$\omega(r) = \sqrt{\frac{2m(r)}{p^3(r)}} \qquad\qquad a(r) = \frac{1}{3}\frac{\Lambda}{\omega^2(r)}$$

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$$\int_0^{\frac{R(r,t)}{p(r)} = \mathrm{cyc}(a,\epsilon,\psi)} \frac{\sqrt{\mathrm{cyc}'}\, d\mathrm{cyc}'}{\sqrt{1 - \epsilon(r)\mathrm{cyc}' + a(r)\mathrm{cyc}'^3}} = \omega(r)\big[t + \phi(r)\big] = \psi(r,t)$$

$\phi(r)$ phase function

$\psi(r,t)$ state of the shell with label r at cosmic time t

Solution of the time equation

1. Fuzzy boundary for when the 2 definitions of $p(r)$ apply
2. $\phi(r)$ is actually an integration constant as a function of t
   We store the mathematical details of the (implicit) solution into "cyc", instead of carrying along the equation

The phase function $\phi(r)$

$$R(r,t) = p(r)\,\mathrm{cyc}\big[a(r), \epsilon(r), \psi(r,t)\big]$$

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if $d_r\phi(r) > 0$, outer shells have a more evolved state than in the absence of $\phi(r)$, mimicking an acceleration without the need for dark energy

Robertson-Walker: $R(r,t) = p_c r\,\mathrm{cyc}(a, \epsilon, \psi(t))$

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Beyond the mass distribution

Notations

$$0 < P = p(r_b) < +\infty,$$

$$R_{\max}(t) = P\,\mathrm{cyc}[a(r_b), \epsilon(r_b), \psi(r_b, t)]$$

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Outside $R_{\max}$: Schwarzschild-Λ metric of the mother universe

$$ds^2 = \left(1 - \frac{2M}{R} - \frac{\Lambda R^2}{3}\right) d\bar{t}^{\,2} - \frac{dR^2}{1 - \dfrac{2M}{R} - \dfrac{\Lambda R^2}{3}} - R^2 d\Omega^2,$$

with coordinates $(\bar{t}, R, \vartheta, \varphi)$

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Radial motion in Schwarzschild-Λ metric

$$R_P(t) = P\,\mathrm{cyc}\left(a_P, \mathrm{sign}(\Lambda), \sqrt{\frac{2M}{P^3}}t + \bar{\psi}_{\mathrm{init}}\right)$$

$$a_P = \frac{\Lambda}{3}\frac{P^3}{2M} \qquad \tilde{E}^2_\infty = 1 - \mathrm{sign}(\Lambda)\frac{2M}{P}$$

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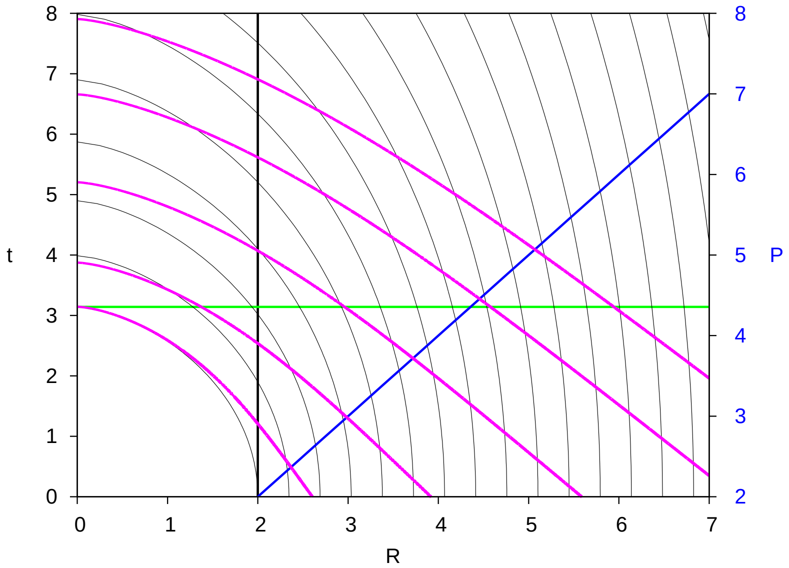
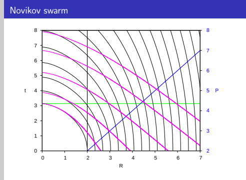
$$R_P(t) = P\,\mathrm{cyc}\left(a_P, \mathrm{sign}(\Lambda), \sqrt{\frac{2M}{P^3}}t + \bar{\psi}_{\mathrm{init}}\right)$$

$$a_P = \frac{\Lambda}{3}\frac{P^3}{2M} \qquad \tilde{E}^2_\infty = 1 - \mathrm{sign}(\Lambda)\frac{2M}{P}$$

1. It is very significant that the same symbol t is used that we used in the L-T metric: first link to spherical cosmology!
2. P and cyc are the second link to spherical cosmology!
3. The shell parameter P plays the role of a shell label: $P(\tilde{E}_\infty)$
4. The phase function re-appears: third link to spherical cosmology!
5. $\mathrm{sign}(0) = 1$ and thus $P \geq 2M$
6. $R_P(t)$ well defined during radial infall

Novikov swarm





1. $$R_P(t) = P\,\mathrm{cyc}\left(0, +1, \sqrt{\frac{2M}{P^3}}\,t + \frac{\pi}{2}\right)$$

   Black curves: $R_P(t)$ for infalling shells, start @ Schwarzschild-Λ radial velocity 0 for arbitrary $R \geq R_S$ (black vertical line), **at the same** $\bar{t}$ which is otherwise undefined but equated to $t = 0$
2. t is common to all shells (proper time)
3. None of the black curves intersect
4. After $t = \pi$ (green line) every radius R is covered by a black $R_P(t)$ at any time t
5. Blue line: the relation between start radius $R_P(0)$ and shell label P. In this case $P = R_P(0)$
6. (t, P) can be regarded as new coordinates that are nowhere singular

Transition between Schwarzschild-Λ and Novikov

$$ds^2 = dt^2 - \frac{\left[\partial_P R_P(t)\right]^2}{1 - \text{sign}(\Lambda)\dfrac{2M}{P}}\,dP^2 - R_P^2(t)\,d\Omega^2,$$

Schwarzschild-Λ	Novikov
$\bar{t},\, R,\, \Omega,\, d_{\bar{t}}R,\, d_{\bar{t}}\Omega$	$t,\, P,\, \Omega,\, d_t P,\, d_t\Omega$
$V_{\mathcal{X}} = \frac{d_{\bar{t}}R}{V_0(R)},\, V_{\mathcal{Y}} = \frac{R\, d_{\bar{t}}\Omega}{\sqrt{V_0(R)}}$	$V_{\mathcal{X}} = \frac{\partial_P R(P,t)}{\sqrt{1-\text{sign}(\Lambda)(2M/P)}} d_t P,\, V_{\mathcal{Y}} = R\, d_t\Omega$
$\upsilon(\bar{t}, R) = \frac{d_{\bar{t}}R_P}{V_0(R)} < 0$	$\upsilon(t, P) = \frac{\lvert d_t R_P \rvert}{\sqrt{1-\text{sign}(\Lambda)(2M/P)}} > 0$

$$|\upsilon| = \sqrt{1 - \frac{V_0}{\tilde{E}_\infty^2}} \leq 1$$

$$V_{\mathcal{X},N} = \frac{V_{\mathcal{X},S} + |\upsilon|}{1 + |\upsilon| V_{\mathcal{X},S}} \qquad V_{\mathcal{Y},N} = V_{\mathcal{Y},S}\frac{\sqrt{1-\upsilon^2}}{1 + |\upsilon| V_{\mathcal{X},S}}.$$

$$d_t R_p = \pm\sqrt{\tilde{E}_\infty^2 - V_0(R)} \qquad \tilde{E}_\infty^2 \geq V_0(R) \text{ where } V_0(R) \geq 0$$

Universes inside a BH: summary of the metrics

- Inner metric

$$ds^2 = dt^2 - \frac{(\partial_r R(r,t))^2}{1-2e(r)}\,dr^2 - R^2(r,t)\,d\Omega^2$$

$$2m(r_b) = 2M, \quad R(r,t) = p(r)\mathrm{cyc}(r,t), \quad p(r) \le p(r_b) = P = 2M,$$

$$e(r_b) = \frac{1}{2}, \qquad \lim_{r\to r_b} \frac{(\partial_r R(r,t))^2}{1-2e(r)} = \lim_{r\to r_b} X^2(r,t) = +\infty$$

- Outer metric

$$ds^2 = \left(1 - \frac{2M}{R} - \frac{\Lambda R^2}{3}\right) d\bar{t}^{\,2} - \frac{dR^2}{1-2M/R-\Lambda R^2/3} - R^2 d\Omega^2$$

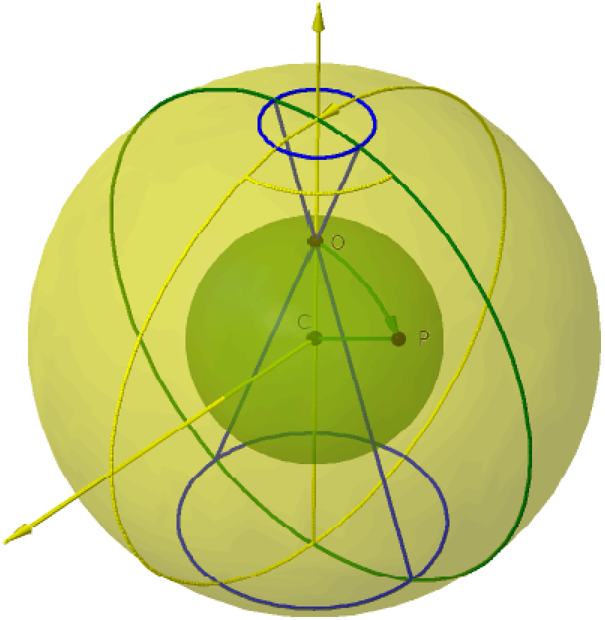
$$ds^2 = dt^2 - \frac{\left[\partial_P R(P,t)\right]^2}{1-\mathrm{sign}(\Lambda)\,2M/P}\,dP^2 - R^2(P,t)\,d\Omega^2, \quad P > 2M$$

Theorem

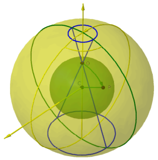
- @ maximum expansion: $R_{\max}(t)$ equals Schwarzschild radius R_S
- @ other cosmic times: spacetime between $R_{\max}(t)$ and R_S
- no space created in the expansion

Universes inside a BH: summary of the metrics
Inner metric
Outer metric
Theorem
@ maximum expansion: $R_{\max}(t)$ equals Schwarzschild radius R_S
@ other cosmic times: spacetime between $R_{\max}(t)$ and R_S
no space created in the expansion

- The **geometrical** total mass of the universe M is equal to what is normally called the mass of the black hole. The relation of that mass to the total mass of the universe depends on its internal state.
- $0 \le p(r) \le 2M < P < +\infty$. The function $p(r)$ plays the same function as P. In case $p(r)$ is monotonous, we could change shell label $r' = p$.
- Λ is not explicitly present in the inner metric, nor in the Novikov metric, but is present in $R(r,t)$ and $R(P,t)$



Spherical cosmological models



1. Motivation
2. General properties (metric, dust content, time evolution, dark energy)
3. Connection with the mother universe (Novikov coordinates and metric, definition of a universe inside a BH)
4. Geometrical properties (embedding surfaces, boundary, the 2 sheets)
5. Light (Hubble relation, magnitude redshift relation)
6. Mass ejection from a black hole
7. Mach's principle and Newton's second law

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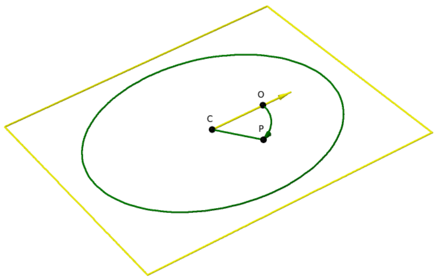
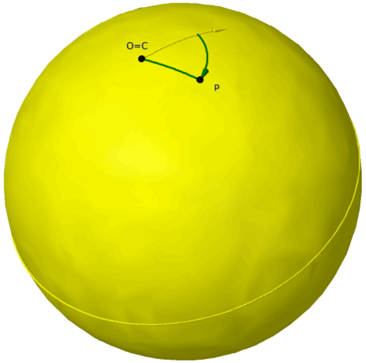
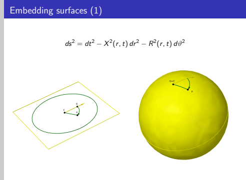
Embedding surfaces (1)

$$ds^2 = dt^2 - X^2(r,t)\,dr^2 - R^2(r,t)\,d\vartheta^2$$





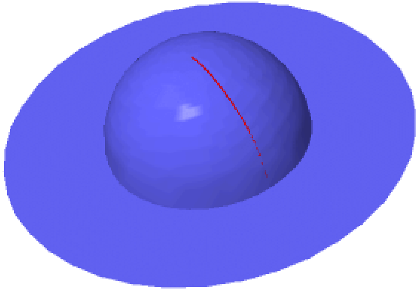
Spherical cosmological models
2020-12-09
Embedding surfaces (1)

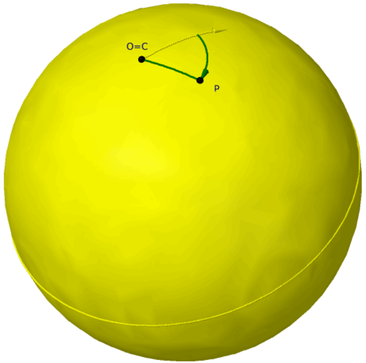


Embedding surfaces (2)

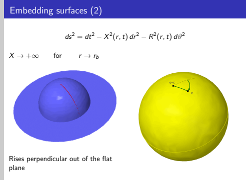
$$ds^2 = dt^2 - X^2(r,t)\,dr^2 - R^2(r,t)\,d\vartheta^2$$

$X \to +\infty$ for $r \to r_b$





Rises perpendicular out of the flat plane



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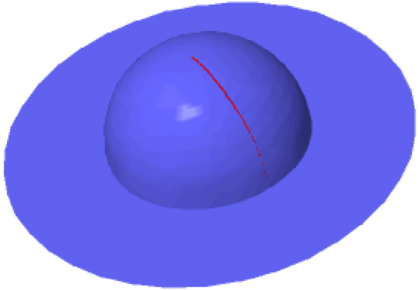
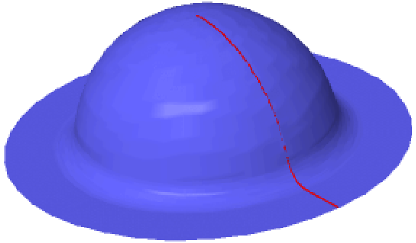
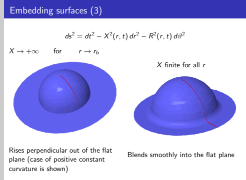


X finite for all r



Rises perpendicular out of the flat plane (case of positive constant curvature is shown)

Blends smoothly into the flat plane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$ds^2 = dt^2 - X^2(r,t)\,dr^2 - R^2(r,t)\,d\vartheta^2$
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Alternative definition of a universe

- Theorem
  - Locally the 4-metric can be made Lorentz, everywhere
  - Be χ a shell label, and nowhere $X(\chi, t) = +\infty$, then the metric is the metric of a universe if at the boundary $\partial_\chi \, R(\chi)|_{\chi=\chi_b} = 0$
  - $$\chi(r) = \int_0^r \frac{dr'}{\left[1-(r')^2\right]^{1-\delta}} = r \; {}_2F_1\left(1-\delta, \frac{1}{2}; \frac{3}{2}; r^2\right)$$
- Example: Robertson-Walker metric

$$ds^2 = dt^2 - [\mathcal{R}(t)]^2 \left[\frac{dr^2}{1-r^2} + r^2(d\vartheta^2 + \sin^2\vartheta \, d\varphi^2)\right]$$

$$r = |\sin\chi|, \quad R(r,t) = \mathcal{R}(t)|\sin\chi|, \quad X(r,t) = \frac{\mathcal{R}(t)}{|\cos\chi|}$$

$$ds^2 = dt^2 - [\mathcal{R}(t)]^2 \left[d\chi^2 + \sin^2\chi(d\vartheta^2 + \sin^2\vartheta \, d\varphi^2)\right],$$

$$\partial_\chi R(\chi)|_{\chi=\frac{\pi}{2}} = \mathcal{R}(t)\partial_\chi|\sin\chi||_{\chi=\frac{\pi}{2}} = 0$$

2020-12-09

└Alternative definition of a universe

Context: orbit integration. How do we elegantly deal with the singular radial coefficient in the metric at the boundary?

$$r = 1 - |\xi|^{1/\delta} \qquad -1 \le \xi \le 1, \qquad \delta^{-1} \ge 2 \qquad \epsilon_\xi = \mathrm{sign}(\xi)$$

$$1 - r = |\xi|^{1/\delta} \quad , \qquad |\xi| = (1-r)^\delta \, .$$

The disk $0 \le r \le 1$ is covered by 2 sheets:

- one for $\xi \ge 0$, or $\epsilon_\xi = +1$
- one for $\xi \le 0$, or $\epsilon_\xi = -1$

(follows from $d_\xi r = 0$ @ $\xi = 0$).

Theorem

- $w = \epsilon_\xi X \dot{r}$, (with $+\infty \times 0$), is the Lorentzian radial velocity
- equations of motion are regular in ξ at $r = 1$: $\xi(p) \sim p - p_b$, or $r(p) = 1 - |p - p_b|^{1/\delta}$
- in 2D universe, one passes, via the 1D circle $r = 1$, from one sheet to the other

Universes with 2 sheets (1)

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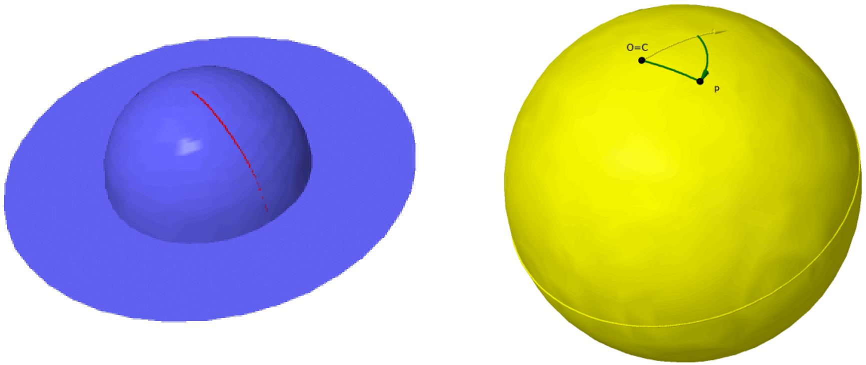
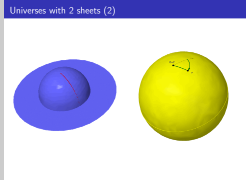
Universes with 2 sheets (2)





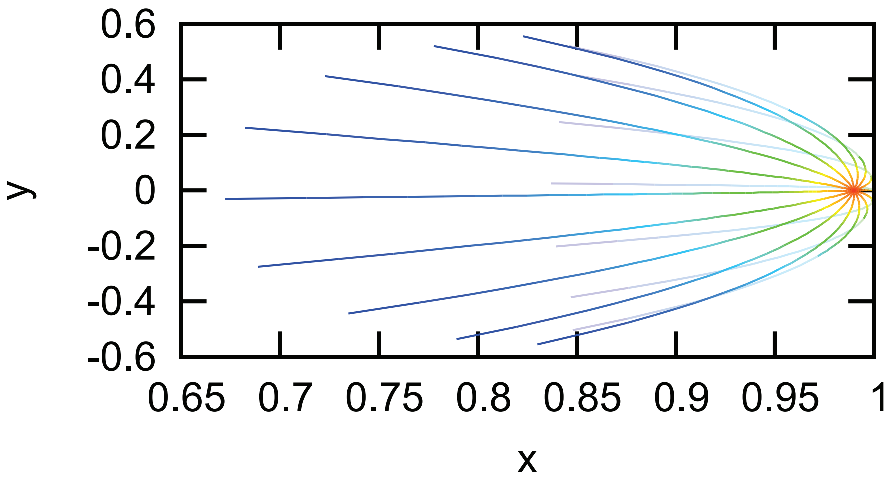
In de Robertson-Walker metric, the analogy of the boundary is the equator of the sphere. Crossing the equator is passing to the lower hemisphere. The surface of the 2D Robertson-Walker universe (right) is twice the surface of our universes (left).

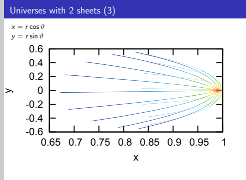
The surface of our 2D universe has 2 sides: the outer side of the sphere and the inner side of the sphere. The Robertson-Walker sphere has only an outer side.

Passing the boundary means passing from the outer side to the inner side, or from ‘above the surface’ to ‘underneath the surface’. This is analogous to a Möbius strip: no orientation.

Universes with 2 sheets (3)

$x = r \cos \vartheta$
$y = r \sin \vartheta$

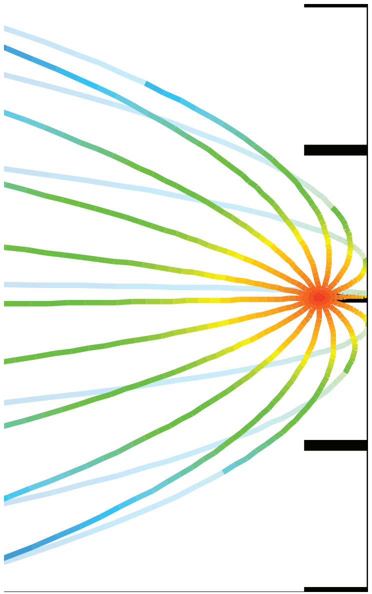


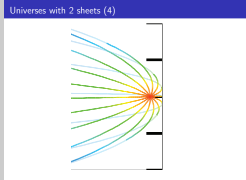


2D universe in (x, y) representation, with $r^2 = x^2 + y^2$

16 orbits are shown, starting from the red place close to the boundary, going off in 16 directions, outward and inward.

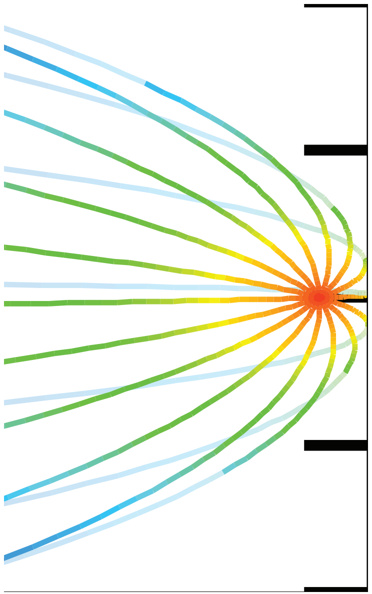
Universes with 2 sheets (4)

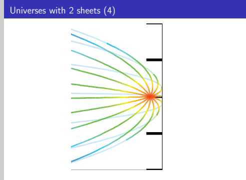




The 7 orbits that start off in the outward direction, hit the boundary somewhere (shown by changing the hue of the color, suggestive of being on the plane or under the plane).

Universes with 2 sheets (4)





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They keep their outward velocity:
$w = \epsilon_\xi X \dot{r}$, (with $+\infty \times 0$), is the Lorentzian radial velocity
$\dot{r}$ changes sign, but not w, because also ϵ_ξ changes sign.

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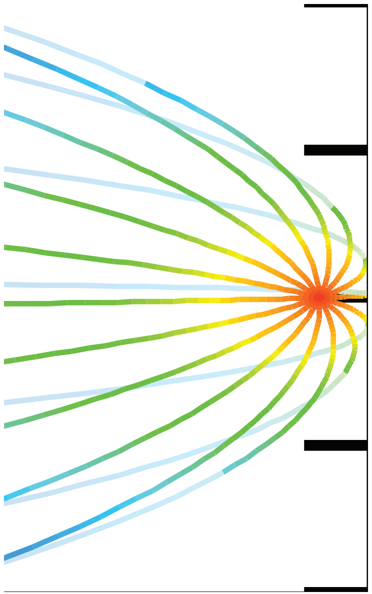

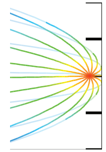

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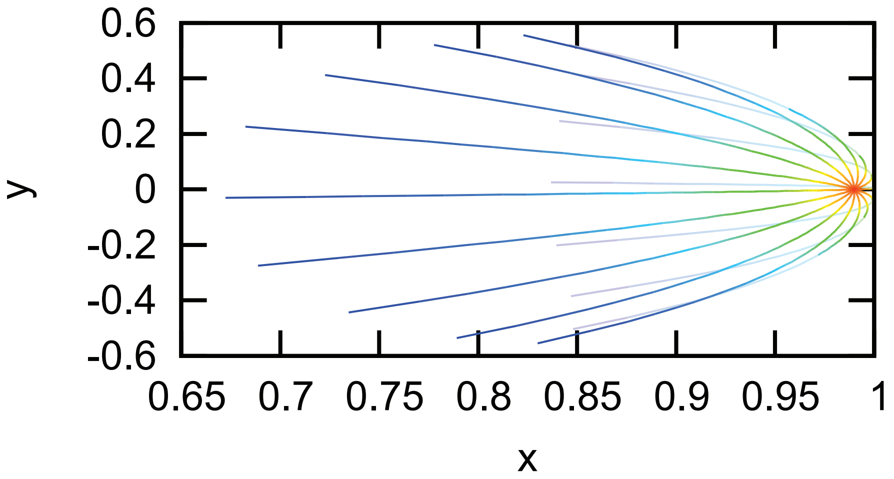
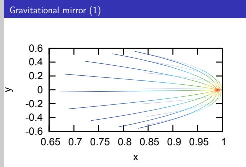
After touching the boundary, the traveler continues his/her journey, but travels back into the interior of the universe and revisits the shells he/she has come to leave. The traveler has never turned back, as he/she would have done in an ordinary orbit at a turning point.

Gravitational mirror (1)





1. Blue light travels 'from left to right', gradually being redshifted and arriving at the red dot. Colors are qualitative.

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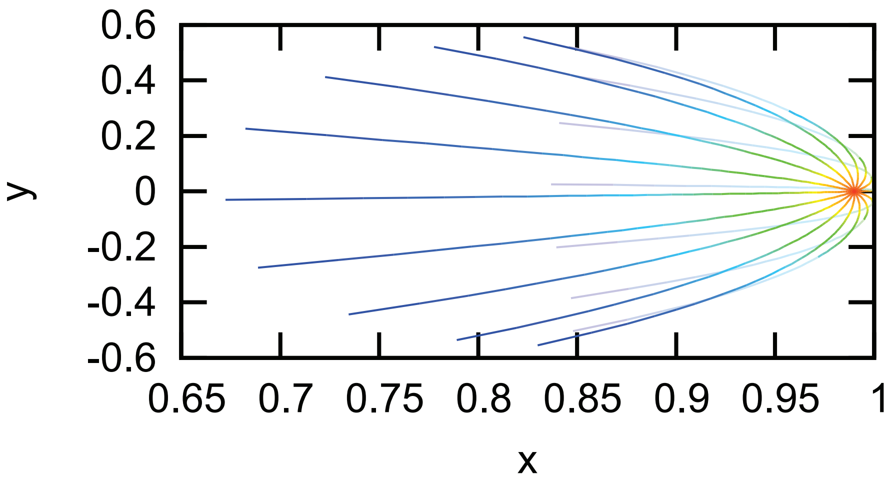

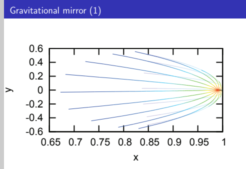

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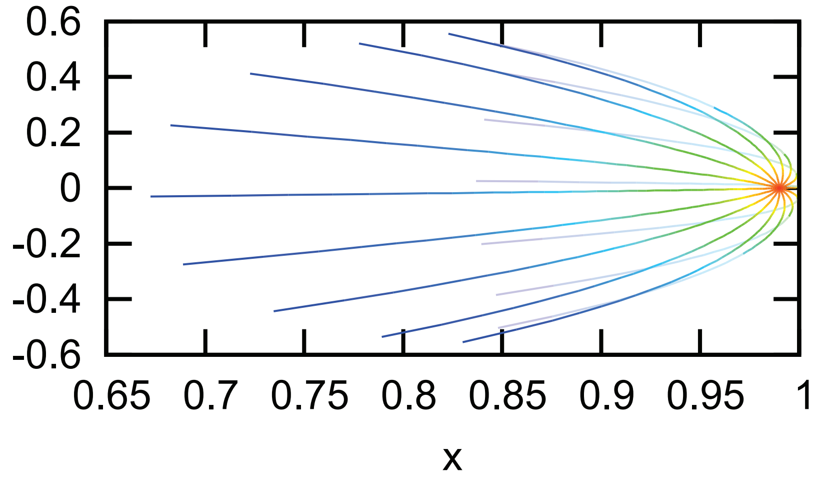
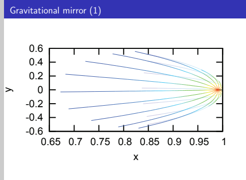
2. The 7 rays that start their journey in a light hue, ‘under the surface’, have a brush with the boundary and appear ‘above the surface’. They hit the red dot as an indirect image, since there is always the direct image, via the 7 rays that start ‘above the surface’ at the intersection with one of the light-hued rays.





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3. The observer (the red dot) sees a direct image in the hemisphere of the center, and a mirror image in the hemisphere of the anticenter, but with different aspects and ages!

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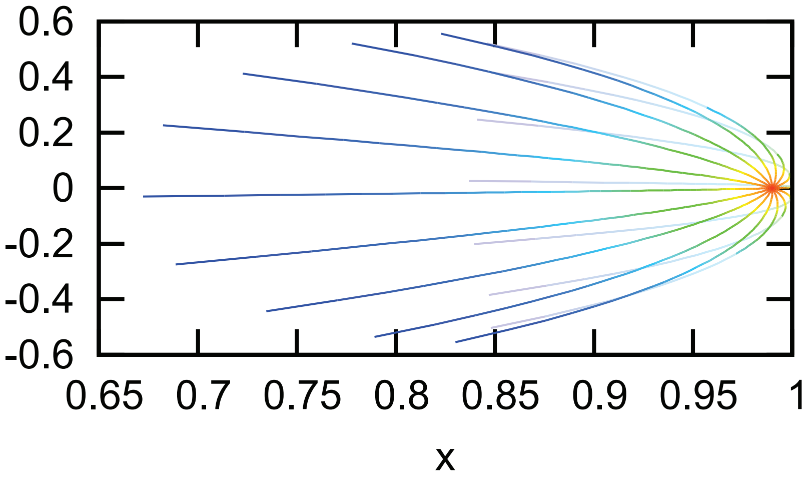

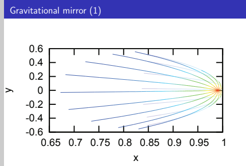

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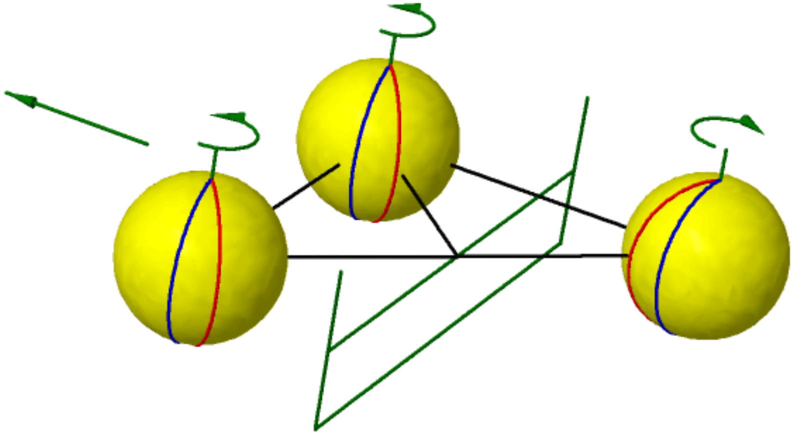
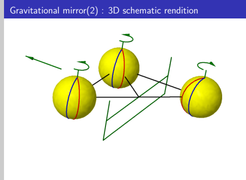
3. The observer (the red dot) sees a direct image in the hemisphere of the center, and a mirror image in the hemisphere of the anticenter, but with different aspects and ages!
4. When approaching the boundary, most of the sky of the observer will contain images of the hemisphere of the center, because there is less and less ‘stuff’ between the observer and the boundary. At the boundary, the sky of the hemisphere of the anticenter will be the mirror of sky of the hemisphere of the center.

Gravitational mirror(2) : 3D schematic rendition





1. The green frame represents the 2D boundary, acting as a mirror. The green arrow is the direction towards the center.

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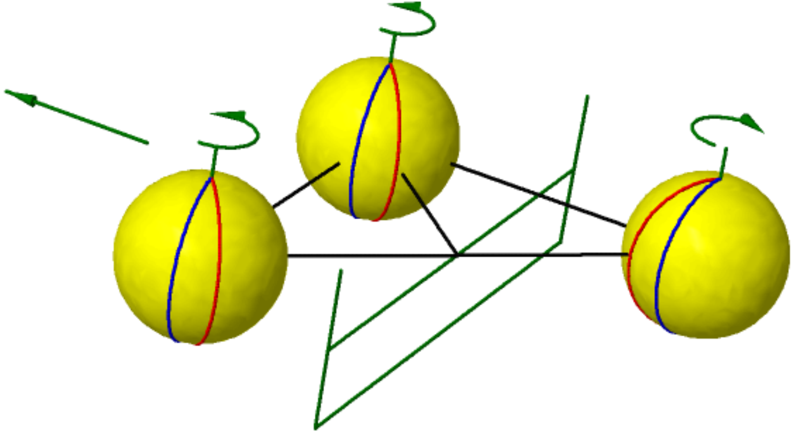

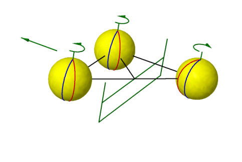

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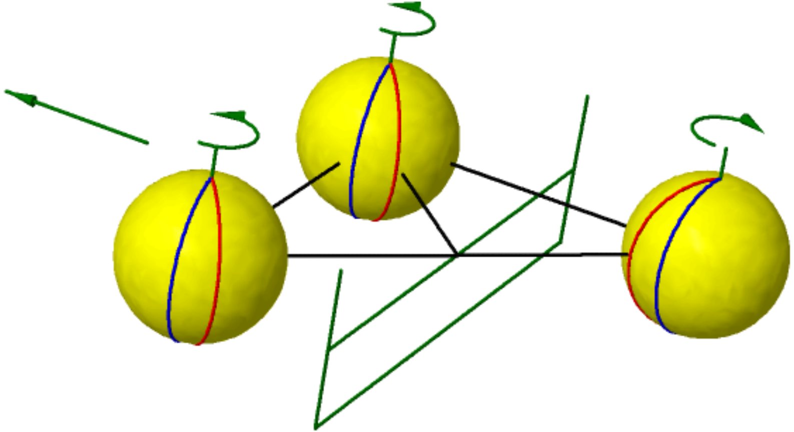
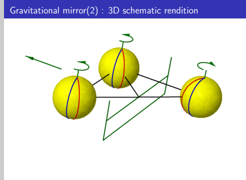
2. The black lines left from the mirror are real light paths. The 2 spinning balls on the left of the mirror are real balls. The left one represents an observer, the right ball is the object. They have the same spin.

Gravitational mirror(2) : 3D schematic rendition





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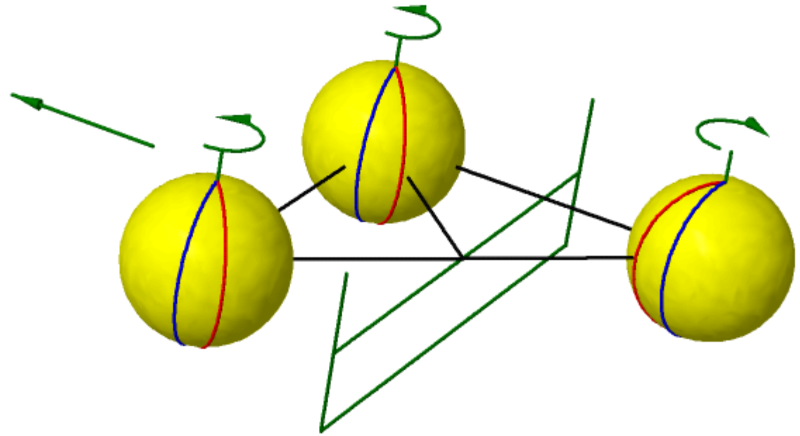
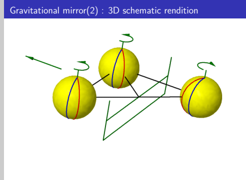
2. The black lines left from the mirror are real light paths. The 2 spinning balls on the left of the mirror are real balls. The left one represents an observer, the right ball is the object. They have the same spin.
3. The black lines right from the mirror are virtual light paths. The spinning ball on the right of the mirror is the mirror image of the object.

Gravitational mirror(2) : 3D schematic rendition





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4. Observer and mirror image have opposite spin. If the observer sets out to travel to the mirror image, he will arrive at it (he travels 'through' the mirror, his outward velocity does not change), but will find himself at the real object, since he followed the light path.

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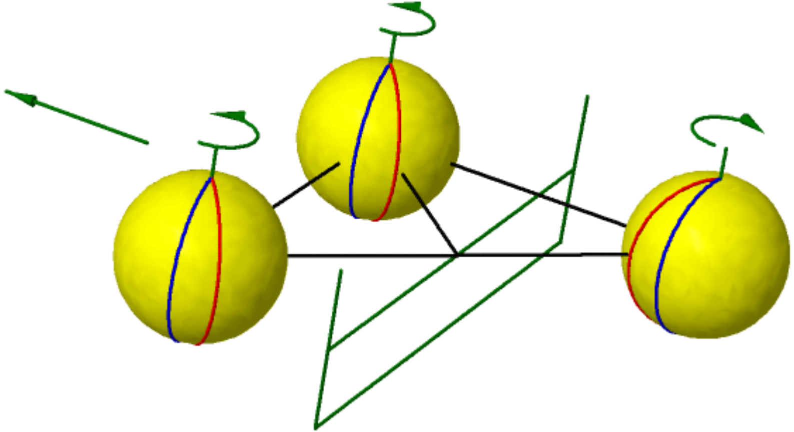

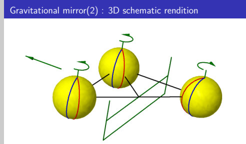

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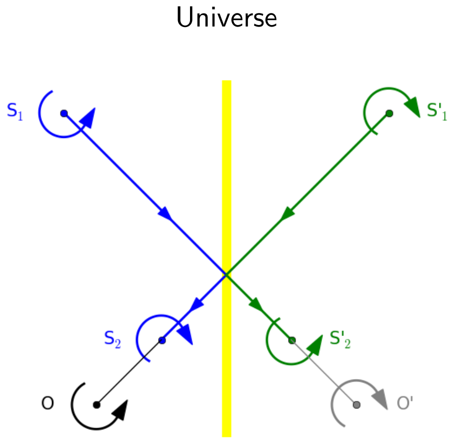
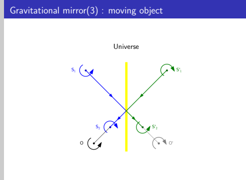
5. The observer does not know that he is traveling towards a mirror image. For him, object and mirror image are both real and different things he can choose to travel to.

Gravitational mirror(3) : moving object





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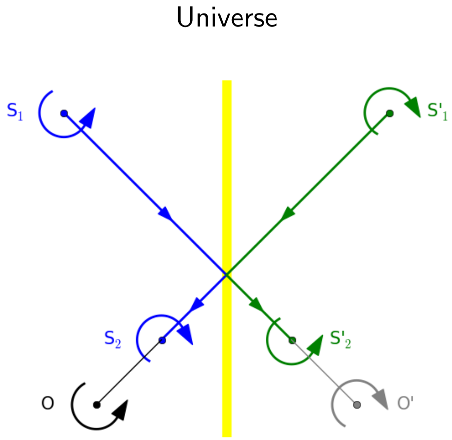

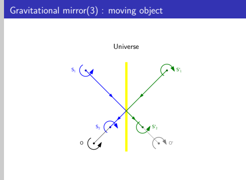

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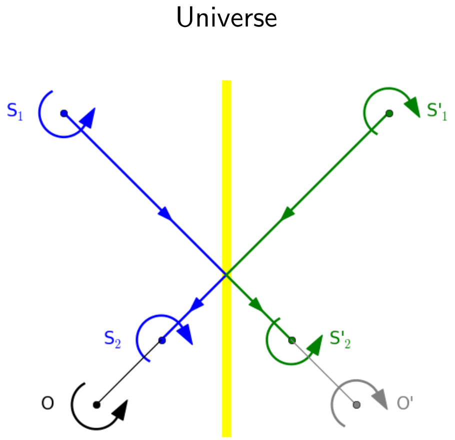

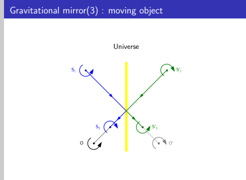

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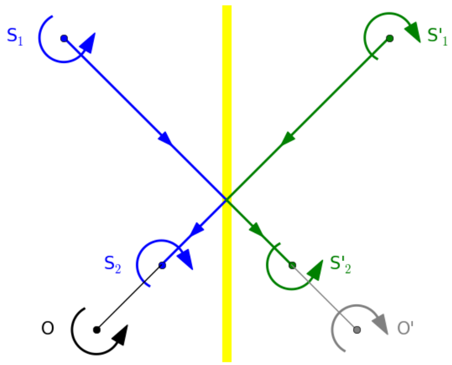

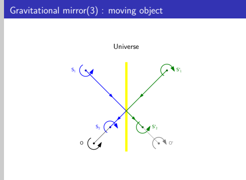

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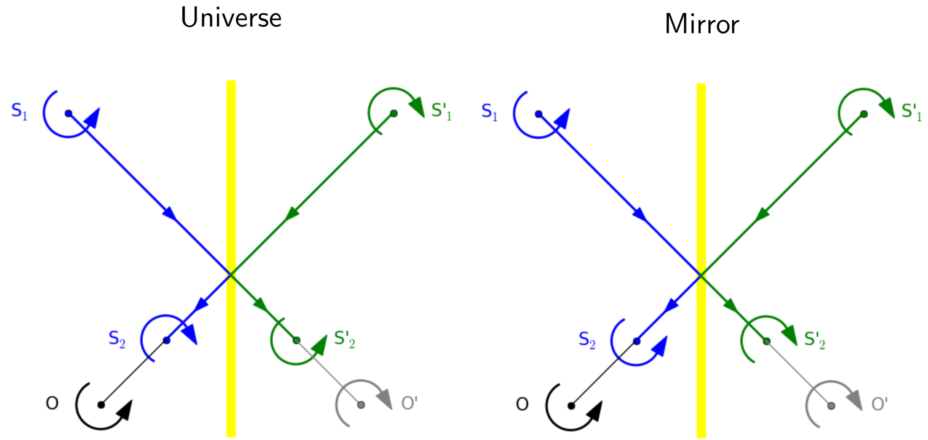

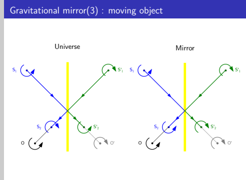

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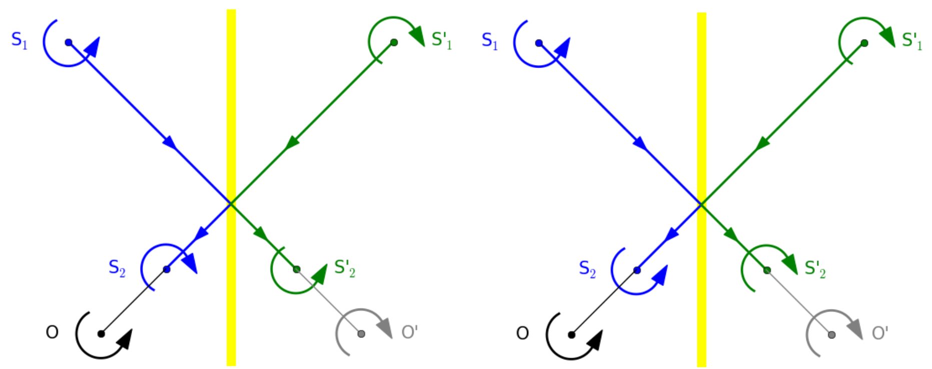

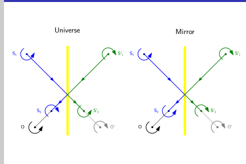

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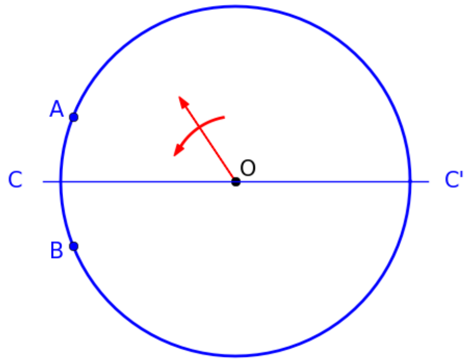
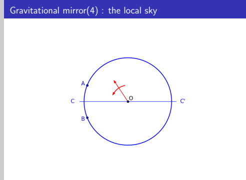
5. In the case of a mirror, there is no spin flip, and O must conclude that there was a mirror, a reflection, and virtual images S_1' and S_2'.
6. The gravitational mirror only manifests itself by the virtual collision at the mirror.

Gravitational mirror(4) : the local sky





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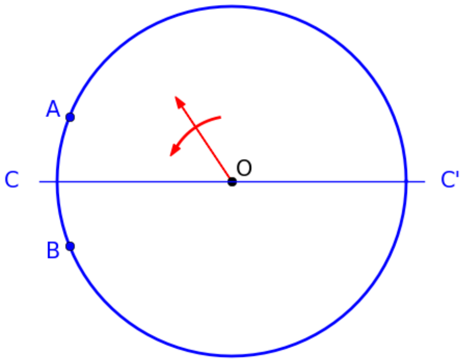

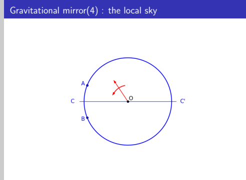

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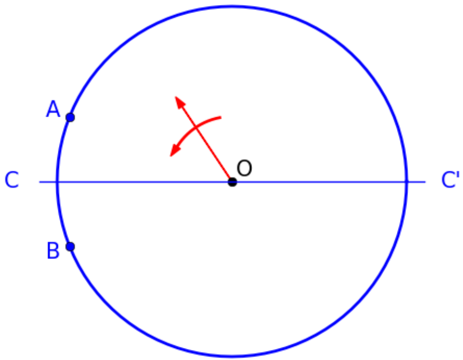

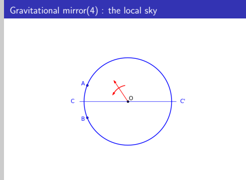

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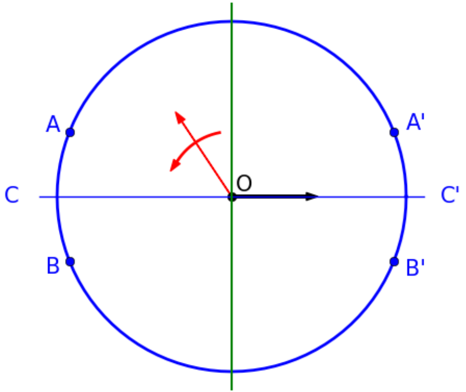

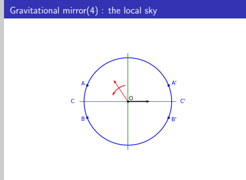

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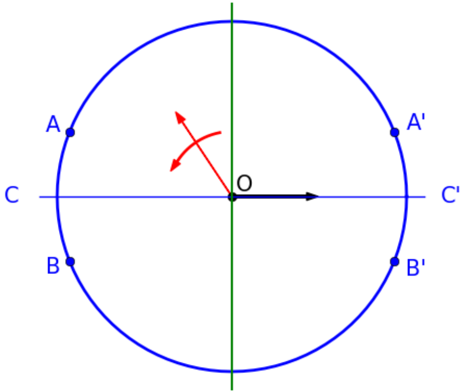

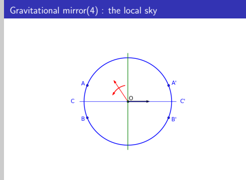

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5. O 'flies into the primed universe', and will find him/herself spinning clockwise with respect to the universe. O never flipped spin! At the boundary his/her spin is undefined with respect to the universe.

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2. General properties (metric, dust content, time evolution, dark energy)
3. Connection with the mother universe (Novikov coordinates and metric, definition of a universe inside a BH)
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Redshift and photometry (1)

$$z=\left[H_\mathrm{o}\sin^2\theta_\mathrm{o}+I_\mathrm{o}\cos^2\theta_\mathrm{o}\right]\,\ell\;=\;\left[H_\mathrm{o}+(I_\mathrm{o}-H_\mathrm{o})\cos^2\theta_\mathrm{o}\right]\,\ell$$

$$H^t=-\frac{R\partial_t^2R}{(\partial_tR)^2}=-\frac{\partial_t^2R}{RH^2}=q\qquad I^t=-\frac{X\partial_t^2X}{(\partial_tX)^2}=-\frac{\partial_t^2X}{XI^2}$$

$$H^{rt}=\frac{\partial_rR}{R}-\frac{\partial_{rt}^2R}{\partial_tR}\qquad I^{rt}=\frac{\partial_rX}{X}-\frac{\partial_{rt}^2X}{\partial_tX}.$$

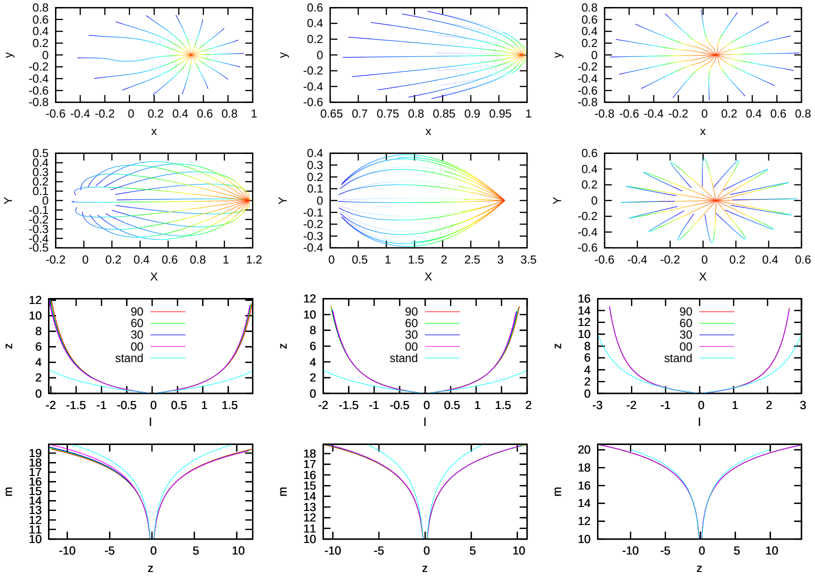
$$\begin{aligned}m_{X_\mathrm{o}}^{(\mathrm{mag})}(t_\mathrm{o},\mathbf{r}_\mathrm{o},z)\;&=\;5\log_{10}(3.066)+40+M_{X_\mathrm{e}}^{(\mathrm{mag})}(t_\mathrm{e},\mathbf{r}_\mathrm{e})-\\&\quad-5\log_{10}\left[I_\mathrm{o}\cos^2\theta_\mathrm{o}+H_\mathrm{o}\sin^2\theta_\mathrm{o}\right]+\\&\quad+5\log_{10}z+\frac{5}{\ln 10}\tilde{E}_{1,\mathrm{o}}^{-1}\left(\tilde{E}'_{2,\mathrm{o}}\tilde{E}_{1,\mathrm{o}}^{-1}-\tilde{E}_{11,\mathrm{o}}\right)z\end{aligned}$$

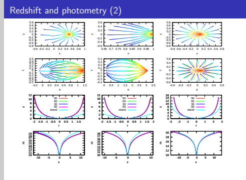
Redshift and photometry (1)

Tangential Hubble parameter, and radial Hubble parameter in the Hubble law

Instead of q, 3 more parameters

Redshift and photometry (2)





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Mass ejection from a BH

$$X(r_b, t) = \frac{\partial_r R(r_b, t)}{\sqrt{1 - 2e(r_b)}} = \frac{0}{0} \text{ and finite: dust ball}$$

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Until now, the no-collision condition $\partial_r R(r_b, t) > 0$ was assumed. The magnitude of $X(r_b, t)$ is undefined (de l'Hôpital).

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Condition

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$$\begin{aligned} ds^2 \ &= \left(1 - \frac{2m\rho}{\rho^2 + a^2\cos^2\theta}\right) c^2\,dt^2 - \frac{\rho^2 + a^2\cos^2\theta}{\rho^2 + a^2 - 2m\rho} d\rho^2 \\ &\quad - (\rho^2 + a^2\cos^2\theta)\,d\theta^2 - \left[(\rho^2 + a^2)\sin^2\theta + \frac{2m\rho a^2 \sin^4\theta}{\rho^2 + a^2\cos^2\theta}\right] d\varphi^2 \\ &\quad - 2\frac{2m\rho a \sin^2\theta}{\rho^2 + a^2\cos^2\theta} c\,dt\,d\varphi \end{aligned}$$

(Kerr solution, Boyer and Lindqvist form)

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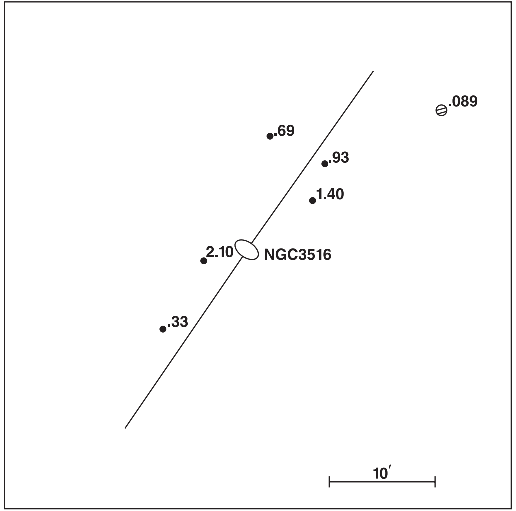
Material leaving the BH at the poles will have to overcome a gravitational well commensurate with the magnitude of $X(r_b, t)$. The larger $X(r_b, t)$, the more they will have lost kinetic energy and the more they will be redshifted.

Anomalous redshifts: NGC 3516



Seyfert Galaxy NGC 3516 @ z=0.009

H. Arp in "Current issues in cosmology", ed. J.-C. Pecker & J. Narlikar, 2006, Cambridge Univ. Press

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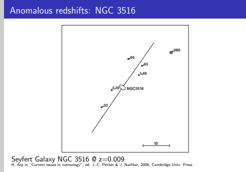

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Mach's principle and Newton's second law (1)

Mach's principle and Newton's second law (1)

Gottfried Wilhelm Leibniz (1646-1716) and Isaac Newton (1642-1727)
Ernst Mach (1838-1916)
Inertial forces are proportional to mass, but is inertial mass the same mass as gravitational mass?

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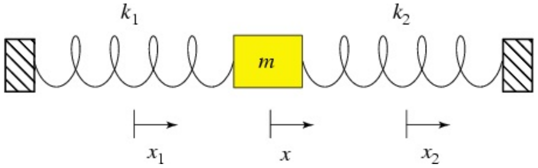
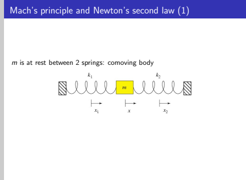
m is at rest between 2 springs: comoving body





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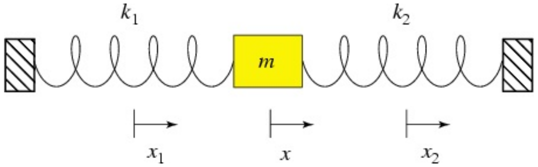

$$F = m_G \int (4\pi G) \frac{\rho}{(4\pi r^2)} r^2 \, dr \int_{-\pi/2}^{\pi/2} |\sin\theta| \cos\theta \, d\theta \int_0^{\pi} d\phi$$

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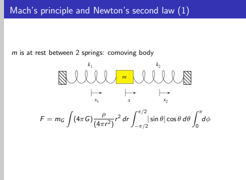

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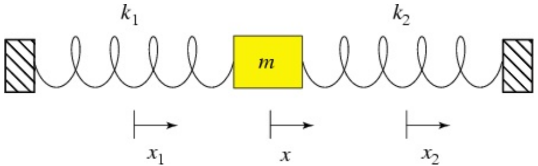

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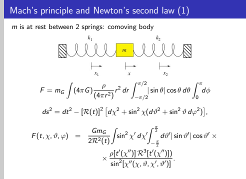
$$ds^2 = dt^2 - [\mathcal{R}(t)]^2 \left[d\chi^2 + \sin^2\chi (d\vartheta^2 + \sin^2\vartheta \, d\varphi^2) \right],$$

$$\begin{aligned} F(t,\chi,\vartheta,\varphi) &= \frac{Gm_G}{2\mathcal{R}^2(t)} \int \sin^2\chi' \, d\chi' \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\vartheta' |\sin\vartheta'| \cos\vartheta' \times \\ &\times \frac{\rho[t'(\chi'')] \, \mathcal{R}^3[t'(\chi'')])}{\sin^2[\chi''(\chi,\vartheta,\chi',\vartheta')]}. \end{aligned}$$



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$$F(t) = \frac{Gm_G\pi}{2\mathcal{R}^2(t)}\frac{\rho}{\omega^3}\int d\chi''.$$

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Result for closed universe and $\Lambda = 0$. One full expansion or one full contraction yields $\chi = \pi$ in timespan $\pi M(t)/c$. The horizon is twice the universe: every object produces 2 images within timespan t_b.
Once an object comes 'in sight', it remains 'in sight'. Images accumulate as time passes.

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The Hubble parameter provides a useful constraint, eliminating $\mathcal{R}(t)$ in favor of $M(t)$.

In the past, the inertial force must have been huge.

Wrap up

- Our universe is the interior of a BH in another 'mother' universe
- BHs in our universe are embryonic universes
- No need for a big bang of creation
- No need for dark energy
- Gravitational mirror
- Arp *et.al.* were partly right
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