Accelerated universe from non-extensive entropy

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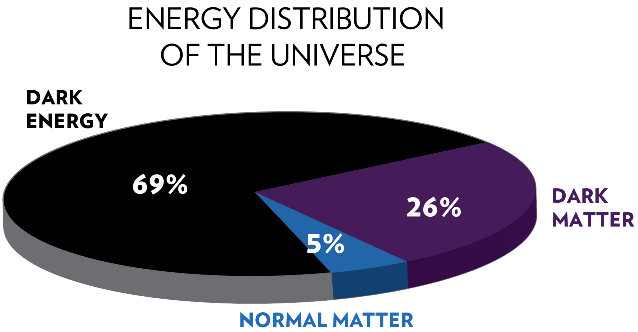
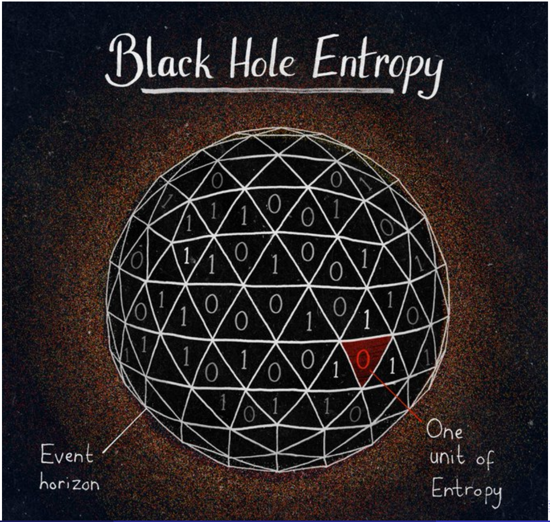
Dark side of the universe

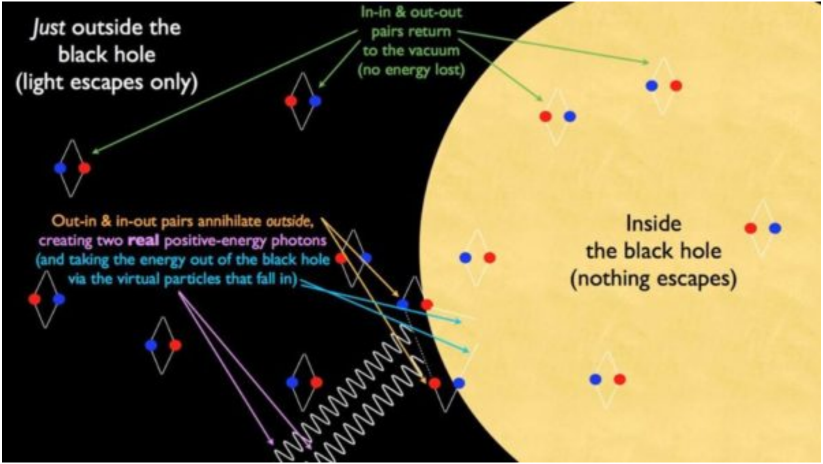


Thermodynamics of black holes: Bekenstein and Hawking in 1970's:
BHs have entropy: $S = A/(4L_p^2)$



Introduction

BHs have temperature : $T = \kappa/2\pi$ (Inspired by Hawking Radiation)



Introduction

BHs obey the law of thermodynamics:

LAW	Thermodynamics	Black Holes
Zero	T constant in equilibrium,	κ constant on horizon
First	$dE = TdS +$work terms	$dM = \frac{1}{8\pi}\kappa dA + \Omega_H dJ$
Second	$\delta S \geq 0$ in any process	$\delta A \geq 0$ in any process
Third	Impossible to achieve $T = 0$,	Impossible to achieve $\kappa = 0$

Introduction

Since the discovery of black hole thermodynamics, physicists have been speculating that there should be some deep relation between black hole thermodynamics and Einstein equation.

- Gravity (Geometry): horizon area, surface gravity
- Thermodynamics: entropy, temperature

$$S \propto A, \qquad T \propto \kappa,$$

Introduction

In 1995 Jacobson [PRL] showed explicitly that the hyperbolic second order partial differential Einstein equation for the spacetime metric has a predisposition to thermodynamic behavior.
Combining the fundamental Clausius relation, together with the entropy expression, he derived Einstein field equations.

$$\delta Q = T\delta S \quad \Rightarrow G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

δQ=energy flux crossing the horizon, and T is Unruh temperature.
This derivation is of great importance because it confirms that the Einstein field equations is nothing but the equation of state for the spacetime.

Introduction

In the framework of cosmology, it was shown that the Friedmann equations, in any gravity theory, can be written in the form of the first law of thermodynamics on the apparent horizon and vice versa.

The prescription:

$$dE = TdS + WdV \quad + \quad Entropy \;\; Expression \Leftrightarrow \;\; Friedmann \;\; Eq.$$

For example, in Gauss-Bonnet gravity the entropy of the 5D black hole is

$$S = \frac{A}{4L_p^2}\left(1 + \frac{6\tilde{\alpha}}{r_+^2}\right) \tag{1}$$

With replacement $r_+ \to \tilde{r}_A$, we write the entropy on the apparent horizon of FRW universe as

$$S = \frac{A}{4L_p^2}\left(1 + \frac{6\tilde{\alpha}}{\tilde{r}_A^2}\right) \tag{2}$$

Then, one can easily show

$$dE = TdS + WdV \rightleftarrows \left(H^2 + \frac{k}{a^2}\right) + \tilde{\alpha}\left(H^2 + \frac{k}{a^2}\right)^2 = \frac{4\pi L_p^2}{3}\rho, \tag{3}$$

Introduction

The deep connection between field equations of gravity and the first law of thermodynamics on the horizon, supports the idea of holography, since the field equation persists the information in the bulk and the first law of thermodynamics on the horizon contains the information on the boundary.

Tsallis Entropy

- In 1902 Gibbs pointed out that, in systems where the partition function diverges, the standard Boltzmann-Gibbs theory is not applicable, and large-scale gravitational systems are known to fall within this class.
- In 1988 Tsallis generalized standard thermodynamics to non-extensive one, which can be applied in all cases, and still possessing standard Boltzmann-Gibbs theory as a limit.
- One experimental verification of the predictions of Tsallis statistics concerned *cold atoms in dissipative optical lattices* predicted in 2003 and was verified in 2006 by a London team.

Tsallis Entropy

In 2013 Tsallis and Cirto showed that the entropy of a black hole does not obey the area law and can be modified as

$$S = \gamma A^{\beta}$$

where A is the horizon area, γ is an unknown constant, and β known as non-extensive parameter.
When $\beta = 1$ and $\gamma = 1/(4L_p^2)$, the area law is restored.
Here we set $k_B = 1 = c = \hbar$ for simplicity.

Modified Friedmann Equation from Tsallis Entropy

We assume the background spacetime is described by the line element

$$ds^2 = h_{\mu\nu} dx^\mu dx^\nu + \tilde{r}^2 (d\theta^2 + \sin^2\theta d\phi^2), \tag{4}$$

where $\tilde{r} = a(t)r$, $x^0 = t, x^1 = r$, and $h_{\mu\nu}$=diag $(-1, a^2/(1 - kr^2))$. The apparent horizon radius is

$$\tilde{r}_A = \frac{1}{\sqrt{H^2 + k/a^2}}. \tag{5}$$

while the temperature on the apparent horizon can be defined as

$$T_h = \frac{\kappa}{2\pi} = -\frac{1}{2\pi \tilde{r}_A}\left(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A}\right). \tag{6}$$

Modified Friedmann Equation from Tsallis Entropy

We assume the matter and energy content of the Universe is in the form of perfect fluid,

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu + p g_{\mu\nu}$$

where satisfy the conservation equation as

$$\nabla_\mu T^{\mu\nu} = 0 \Rightarrow \dot{\rho} + 3H(\rho + p) = 0 \tag{7}$$

We define the work density is

$$W = \frac{1}{2}(\rho - p)$$

The work density term is indeed the work done by the volume change of the Universe.

Modified Friedmann Equation from Tsallis Entropy

Taking differential of $E = \rho V$, where $V = \frac{4\pi}{3}\tilde{r}_A^3$, we find

$$dE = 4\pi\tilde{r}_A^2 \rho d\tilde{r}_A + \frac{4\pi}{3}\tilde{r}_A^3 \dot{\rho} dt \;\; \Rightarrow \tag{8}$$

$$dE = 4\pi\tilde{r}_A^2 \rho d\tilde{r}_A - 4\pi H \tilde{r}_A^3 (\rho + p) dt. \tag{9}$$

Also from $S = \gamma A^{\beta}$ we get

$$dS = \gamma\beta A^{\beta-1} dA = 8\pi\gamma\beta(4\pi r_A^2)^{\beta-1}\tilde{r}_A d\tilde{r}_A. \tag{10}$$

Modified Friedmann Equation from Tsallis Entropy

Substituting all quantities in $dE = TdS + WdV$, after some calculations, we get

$$\frac{1}{\tilde{r}_A^{4-2\beta}} = \frac{2\pi(2-\beta)}{3\gamma\beta}\left(4\pi\right)^{1-\beta}\rho, \tag{11}$$

which yields

$$\left(H^2 + \frac{k}{a^2}\right)^{2-\beta} = \frac{8\pi L_p^2}{3}\rho, \tag{12}$$

provided we define

$$\gamma \equiv \frac{2-\beta}{4\beta L_p^2}\left(4\pi\right)^{1-\beta}. \tag{13}$$

Since $\gamma > 0$ we have $\beta < 2$. For $\beta = 1$ it reduces to standard one.

Tsallis Cosmology

Combining the first Friedmann equation with continuity equation, $\dot{\rho} + 3H(\rho + p) = 0$, we can get

$$\frac{\ddot{a}}{a}\left(H^2 + \frac{k}{a^2}\right)^{1-\beta} = -\frac{4\pi L_p^2}{3(2-\beta)}\left[(2\beta - 1)\rho + 3p\right] \tag{14}$$

Since our Universe is currently accelerating ($\ddot{a} > 0$), thus

$$(2\beta - 1)\rho + 3p < 0 \quad \longrightarrow \quad \omega < \frac{1 - 2\beta}{3}, \tag{15}$$

where $\omega = p/\rho$ is the equation of state parameter.
When $\beta = 1$, we have the well-known condition $\omega < -1/3$ for accelerated universe in standard cosmology.

Tsallis Cosmology

From inequality $\omega < \frac{1-2\beta}{3}$, we see that:

- For $\beta \geq 1/2$, we always have $\omega < 0$
- For $\beta < 1/2$, we can have $\omega \geq 0$ in an accelerated universe!!

For example, for $\beta = 1/3$ the above inequality implies $\omega < 1/9$.

This indicates that in Tsallis cosmology, the late time accelerated universe can be achieved even in the presence of the ordinary matter with $\omega \geq 0$!!

For matter-dominated era:

$$a(t) = (C_2 t)^{(4-2\beta)/3}, \quad C_2 \equiv \frac{3}{2}\frac{C_1^{1/(4-2\beta)}}{(2-\beta)} > 0 \tag{16}$$

$$\ddot{a}(t) = \frac{C_2^{(4-2\beta)/3}}{9}(4-2\beta)(1-2\beta)\ t^{-(2+2\beta)/3} \Rightarrow \ddot{a}(t) > 0, \tag{17}$$

provided $\beta < 1/2$. For other parameters:

$$\rho(t) \propto \frac{1}{t^{4-2\beta}}, \tag{18}$$

$$H(t) = \frac{\dot{a}}{a} = \frac{4-2\beta}{3t}, \tag{19}$$

$$q(t) = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{2\beta-1}{4-2\beta}. \tag{20}$$

Again $q < 0$ for $\beta < 1/2$.

Tsallis Cosmology

Let us see how the above relation can alleviate the age problem in standard cosmology. The Hubble constant is usually written as $H_0 = 100 \ h \ km \ sec^{-1} Mpc$,
where $h = 0.72 \pm 0.08$. The Hubble time is $t_H = 1/H_0 = 9.78 \times 10^9 h^{-1}$ years and the range of the Universe age in standard cosmology is estimated as

$$8.2 Gyr < t_0 < 10.2 Gyr$$

which is inconsistent with the ages of the oldest globular clusters/stars.

For $\beta < 1/2$ we have

$$t_0|_T > \frac{3}{2} t_0|_S, \tag{21}$$

where subscript "T" and "S" stand for Tsallis and Standard cosmology, respectively.

As an example, taking $\beta = 2/5$, we find $t_0|_T = 1.6 \ t_0|_S$.

Consequently, the age of the accelerated universe in Tsallis cosmology is in the range

$$13.12 Gyr < t_0|_T < 16.32 Gyr \tag{22}$$

which is compatible with observations.

Let us consider the radiation-dominated era:

$$a(t) = B_1^{1/4} \left(\frac{2t}{2-\beta} \right)^{1-\beta/2} \Rightarrow \ddot{a}(t) < 0. \tag{23}$$

And the other parameters are:

$$\rho(t) \propto \frac{1}{t^{4-2\beta}}, \tag{24}$$

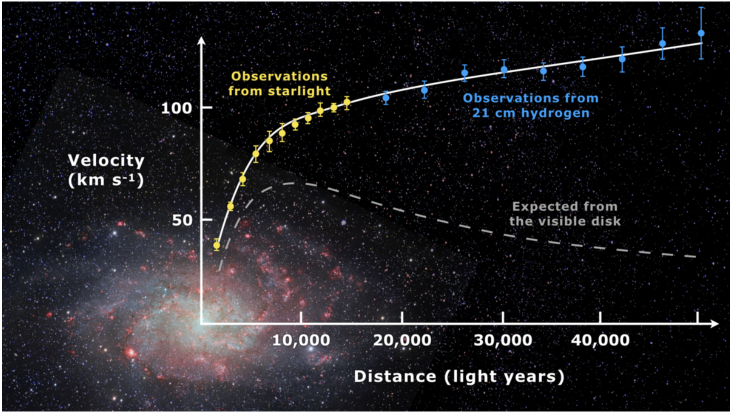
$$H(t) = \frac{\dot{a}}{a} = \frac{2-\beta}{2t}, \tag{25}$$

$$q(t) = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{\beta}{2-\beta}. \tag{26}$$

Therefore, for $\beta < 2$, we have always $q > 0$ ($\ddot{a}(t) < 0$), which confirms a decelerated phase for radiation dominated era, as expected.

Flat Galactic rotation curves

According to Newtonian gravity the luminous matter does not provide sufficient gravitation to explain the flat rotation curves of spiral galaxies and one need to take *dark matter* into account.



From Friedmann equation (12), we find

$$(2-\beta)\frac{\ddot{a}}{a}\left(\dot{a}^2+k\right)^{1-\beta}=-\frac{4\pi L_p^2}{3}\left[(2\beta-1)\rho+3p\right]a^{2-2\beta}. \tag{27}$$

We assume in the Newtonian cosmology the spacetime is Minkowskian with $k=0$, thus Eq. (27) reduces to

$$(2-\beta)\frac{\ddot{a}}{a}=-\frac{4\pi G}{3}\left[(2\beta-1)\rho+3p\right]\left(\frac{a}{\dot{a}}\right)^{2-2\beta} \tag{28}$$

Newton's Law of gravity at large scales

To transform from GR to Newtonian gravity, we replace active gravitational mass with the total mass,

$$\mathcal{M} = (\rho + 3p)\frac{4\pi}{3}R^3 \Rightarrow M = \frac{4\pi\rho}{3}R^3, \tag{29}$$

which leads to

$$(2-\beta)\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho(2\beta - 1)\left(\frac{a}{\dot{a}}\right)^{2-2\beta}. \tag{30}$$

We consider a compact spatial region V with a compact boundary $\mathcal{S}$, which is a sphere with physical radius $R = a(t)r = H^{-1}$.

On the other hand, the acceleration of a test particle m near the surface $\mathcal{S}$ can be written

$$\ddot{R} = \ddot{a}r = F/m. \tag{31}$$

where F is the gravitational force between m from M. Equating $\ddot{a}$ in Eqs. (30) and (31), we find

$$F = -\left(\frac{2\beta - 1}{2 - \beta}\right)\frac{4\pi G}{3}\rho m R \left(\frac{a}{\dot{a}}\right)^{2-2\beta} \tag{32}$$

Using the fact that $R = 1/H = a/\dot{a}$ and $\rho = M/V$, the above equation can be rewritten as

$$F = \left(\frac{1 - 2\beta}{2 - \beta}\right)\frac{GMm}{R^{2\beta}}. \tag{33}$$

Flat Galactic rotation curves

i) Inside the galaxy ($\beta = 1$) we have:

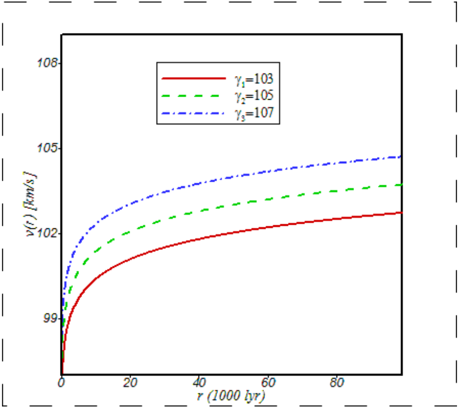
$$a = \frac{v^2}{r} = \frac{GM(r)}{r^2} \rightarrow \ v \propto r, \tag{34}$$

ii) Outside the galaxy ($\beta < 1/2$):

$$\frac{v^2}{r} = \left(\frac{1-2\beta}{2-\beta}\right) \frac{GM}{r^{2\beta}} \Rightarrow v(r) = \sqrt{\left(\frac{1-2\beta}{2-\beta}\right) GMr^{1-2\beta}}, \tag{35}$$

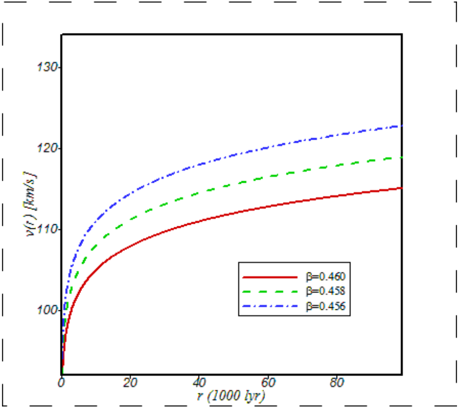
Flat Galactic rotation curves

$v(r)$ for $\beta = 0.49$ and different values of galaxy mass $M_i = \gamma_i \times 10^9 M_\odot$



Flat Galactic rotation curves

$v(r)$ for a typical galaxy with mass $M = 25 \times 10^9 M_\odot$



8. Conclusions

The cosmological model based on non-extensive Tsallis entropy can explain the late time accelerated expansion and flat rotation curves without needing to dark energy/matter. It can also address the age problem in standard cosmology.

Note that $\beta < 2$ is a necessary condition, while $\beta < 1/2$ is our choice for deriving accelerated expansion. There is still an open room to assume $1/2 < \beta < 2$, but in this case we must take into account the dark companion for matter/energy.

The main challenge may arise is whether or not one can justify $\beta \lesssim 1/2$, from physical point of view!

Thank you for listening

References:

- A. Sheykhi, Eur. Phys. J. C **80** (2020) 25