A formulation to study formation of Bose-Einstein condensation in cosmology at the level of particle physics processes

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Observations based on ΛCDM :

The present fluid content of the universe

$$\Omega_{rad} \sim 8 \times 10^{-5}, \quad \Omega_{mat}^{\text{Baryonic}} \sim 0.04, \quad \Omega_{mat}^{\text{Dark}} \sim 0.23, \quad \Omega_{\Lambda} \sim 0.72$$

- Λ **- problem** :
  The big difference $\sim 10^{123}$ between the particle theory and the observations.
- **CDM - problem** :
  The simulations based on ΛCDM differ from the observations at small scales.

Alternative scalar field models : $\not{\Lambda} \longrightarrow \phi$ and ~~CDM~~ $\longrightarrow$ SFDM

- **Quintessence**, **k-essence** models, etc.,
  where it is assumed that $\phi = \phi(t)$.

- **BEC scalar field :**

  Based on the relativistic GP equation :

$$\mathcal{L}_\phi = -g^{\mu\nu}\partial_\mu\phi^\star\partial_\nu\phi - m^2|\phi|^2 - \frac{\lambda}{2}|\phi|^4, \qquad (1)$$

  where it is naturally provided that $\phi = \phi(t)$.

Aim : **Formation of BEC scalar field in cosmology with particular emphasis on its microscopic description in particle physics**

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The metric :

$$ds^2 = -dt^2 + a^2(t)[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)] \qquad (2)$$

The action :

$$S = \int \sqrt{-g} d^4x \frac{1}{2}\{-g_{\mu\nu}[\partial_\mu\phi\partial_\nu\phi + \partial_\mu\chi\partial_\nu\chi] - m_\phi^2\phi^2 - m_\chi^2\chi^2 - \mu\phi^2\chi\} \qquad (3)$$

$$= \int d^3x d\eta \frac{1}{2}\{\tilde{\phi}'^{\,2} - (\vec{\nabla}\tilde{\phi})^2 + \tilde{\chi}'^{\,2} - (\vec{\nabla}\tilde{\chi})^2 - \tilde{m}_\phi^2\tilde{\phi}^2 - \tilde{m}_\chi^2\tilde{\chi}^2 - \tilde{\mu}\tilde{\phi}^2\tilde{\chi}\} \qquad (4)$$

with the following expressions

$$d\eta = \frac{dt}{a(t)}, \quad \tilde{\phi} = a\phi, \quad \tilde{\chi} = a\chi, \quad a' = \frac{da}{d\eta}, \quad \dot{a} = \frac{da}{dt}, \quad a'' = \frac{d^2a}{d\eta^2}, \quad \ddot{a} = \frac{d^2a}{dt^2},$$

$$\tilde{\mu} = a\mu, \quad \tilde{m}_i^2 = m_i^2 a^2 - \frac{a''}{a} = a^2\left(m_i^2 - \frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}\right), \qquad (5)$$

where the subscript i takes the values, $i = \phi, \chi$.

Introducing an effective Minkowski formulation would be useful to make use of the techniques and the relative simplicity of the usual (flat) Minkowski space quantum field theory

- We get rid of the contributions of gravitational particle production to transition amplitudes so that we may more easily identify the transition amplitudes that are directly due to particle physics processes.
- Calculation of transition amplitudes and cross sections are easier.
- Interpretation of the results are easier.

$\phi^2\chi$: $\chi\chi \to \phi\phi$ in $\triangle t = \frac{1}{n_\chi \beta\sigma v}$ with $\tilde{m}_\chi \ll \tilde{m}_\phi$.
The leading order contributions to the production of ϕ particles :

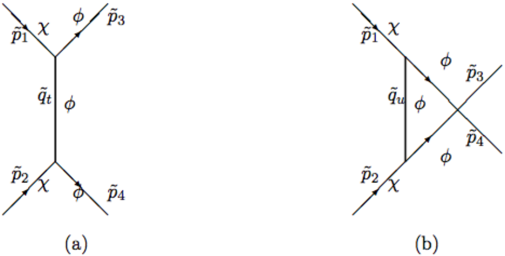

Figure 1. The leading order Feynman diagrams that may contribute to the production of ϕ particles. Here $\tilde{q}_t = \tilde{p}_1 - \tilde{p}_3 = \tilde{p}_4 - \tilde{p}_2$, $\tilde{q}_u = \tilde{p}_1 - \tilde{p}_4 = \tilde{p}_3 - \tilde{p}_2$ are the 4-momenta carried in the internal lines. Note that the s-channel is forbidden in this case by kinematics.

<u>Two conditions</u> during each process $\chi\chi \to \phi\phi$:

$$\left|\frac{\triangle\tilde{m}^2}{\tilde{m}^2}\right| = \left|\frac{\triangle t\left(\frac{da^2(m^2 - \dot{H} - 2H^2)}{dt}\right)}{a^2(m^2 - \dot{H} - 2H^2)}\right| \ll 1 \quad \text{and} \quad \left|\frac{\triangle\tilde{\mu}}{\tilde{\mu}}\right| = |H\triangle t| \ll 1 \qquad (6)$$

They are satisfied for a considerable range of parameters by letting $H = \xi a^{-s}$.

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The quantization of any field χ during each $\chi\chi \to \phi\phi$:

$$\tilde{\chi}^{(i)}(\vec{r},\eta) \simeq \int \frac{d^3\tilde{p}}{(2\pi)^{\frac{3}{2}}\sqrt{2w_p^{(i)}}}\left[a_p^{(i)-}e^{i\left(\vec{\tilde{p}}.\vec{r}-w_p^{(i)}(\eta-\eta_i)\right)} + a_p^{(i)+}e^{i\left(-\vec{\tilde{p}}.\vec{r}+w_p^{(i)}(\eta-\eta_i)\right)}\right],$$

where $\eta_i < \eta < \eta_{i+1}$, (i) : ith time interval between the ith and $(i+1)$th processes.

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where $\eta_i < \eta < \eta_{i+1}$, (i) : ith time interval between the ith and $(i+1)$th processes.

By these requirements, we get the effective Minkowski space

$$d\tilde{s}^2 = -d\eta^2 + d\tilde{x}_1^2 + d\tilde{x}_2^2 + d\tilde{x}_3^2 \qquad (7)$$

during each process $\chi\chi \to \phi\phi$.

In the center of mass frame : $\vec{\tilde{p}}^2 + \tilde{m}_\chi^2 = \vec{\tilde{k}}^2 + \tilde{m}_\phi^2$ for each $\chi\chi \to \phi\phi$

$$\Longrightarrow \quad \boxed{\vec{\tilde{p}}^2 - \vec{\tilde{k}}^2 = a^2(m_\phi^2 - m_\chi^2)}\,. \tag{8}$$

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Note that $|\vec{\tilde{p}}| = a|\vec{p}| \propto a\frac{1}{a} =$ constant :

(i) For a process $\chi\chi \to \phi\phi$ in $\triangle t_1$, a_1 : constant
(ii) For another process $\chi\chi \to \phi\phi$ in $\triangle t_2 > \triangle t_1$, a_2 : constant $> a_1$
(iii) So on...

$\Longrightarrow$ For a given $|\vec{\tilde{p}}|$, $|\vec{\tilde{k}}| \to 0$ by increasement of $a(t)$

$\Longrightarrow$ The curved space effect on condensation is provided by increasement of a(t)

The evolution of the distribution function for ϕ particles :

$$\frac{d\tilde{f}(\vec{\tilde{p_4}},\eta)}{d\eta} = \frac{1}{32(2\pi)^5\tilde{E}_4}\int\int\int \delta^{(4)}(\tilde{p_1}+\tilde{p_2}-\tilde{p_3}-\tilde{p_4})|\tilde{M}|^2 \times \{\tilde{f}_1\tilde{f}_2(1+\tilde{f}_3)(1+\tilde{f}_4)-\tilde{f}_3\tilde{f}_4(1+\tilde{f}_1)(1+\tilde{f}_2)\}\frac{d^3\vec{\tilde{p_1}}}{\tilde{E}_1}\frac{d^3\vec{\tilde{p_2}}}{\tilde{E}_2}\frac{d^3\vec{\tilde{p_3}}}{\tilde{E}_3}. \quad (9)$$

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$$\times\{\tilde{f}_1\tilde{f}_2(1+\tilde{f}_3)(1+\tilde{f}_4)-\tilde{f}_3\tilde{f}_4(1+\tilde{f}_1)(1+\tilde{f}_2)\}\frac{d^3\vec{\tilde{p}}_1}{\tilde{E}_1}\frac{d^3\vec{\tilde{p}}_2}{\tilde{E}_2}\frac{d^3\vec{\tilde{p}}_3}{\tilde{E}_3}. \qquad (9)$$

Our assumptions :

- We consider initial times of $\chi\chi\rightarrow\phi\phi \implies \tilde{f}^{(f)}(\vec{\tilde{p}}_j,\eta)\simeq 0$ with $j=3,4.$
- $0\leq|\vec{\tilde{p}}|_{\text{min}} = \sqrt{\tilde{m}_\phi^2-\tilde{m}_\chi^2} < |\vec{\tilde{p}}| < |\vec{\tilde{p}}|_{\text{max}} = \sqrt{\vec{\tilde{k}}^2_{\text{max}}+\tilde{m}_\phi^2-\tilde{m}_\chi^2}.$
  The spatial distributions of χ particles in this range are homogeneous and isotropic. $\tilde{n}_\chi(\eta)=\int d^3\tilde{p}\tilde{f}^{(i)}(\vec{\tilde{p}},\eta)$:
  $$\implies \tilde{f}^{(i)}(\vec{\tilde{p}}_j,\eta)\simeq\tilde{n}_\chi\frac{[\Theta(|\vec{\tilde{p}}|_{\text{max}}-|\vec{\tilde{p}}|_{\text{min}})-\Theta(|\vec{\tilde{p}}|_{\text{min}}-|\vec{\tilde{p}}|_{\text{max}})]}{4\pi(|\vec{\tilde{p}}|_{\text{max}}-|\vec{\tilde{p}}|_{\text{min}})|\vec{\tilde{p}}_j|^2}$$
  $$\times[\Theta(|\vec{\tilde{p}}_{\text{j}}|-|\vec{\tilde{p}}|_{\text{min}})-\Theta(|\vec{\tilde{p}}_{\text{j}}|-|\vec{\tilde{p}}|_{\text{max}})] \qquad \text{with} \quad j=1,2, \qquad (10)$$

$$\Longrightarrow \quad \frac{d\tilde{f}^{(f)}(\vec{\tilde{p}}_4,\eta)}{d\eta} \simeq \tilde{n}_\chi^2 \frac{\delta(\sqrt{\frac{1}{4}\vec{\tilde{p}}_4^2 + \tilde{m}_\phi^2 - \tilde{m}_\chi^2} - \sqrt{\tilde{m}_\phi^2 - \tilde{m}_\chi^2})}{(\sqrt{\frac{1}{4}\vec{\tilde{p}}_4^2 + \tilde{m}_\phi^2 - \tilde{m}_\chi^2} - \sqrt{\tilde{m}_\phi^2 - \tilde{m}_\chi^2})}\mathcal{B}(|\vec{\tilde{p}}_4|)$$

$\Longrightarrow \quad |\vec{\tilde{p}}_4| \to 0 \quad \Longrightarrow$ **Coherence** and **Correlation** $(\lambda \gg n_\phi^{-1/3})$

The curved space effect on the number density n_ϕ :

Using $\tilde{n} = \int \tilde{f} d^3\tilde{p}$ and $\tilde{f}^{(f)}(\vec{\tilde{p}}_j, \eta) \simeq 0$ with $j = 3, 4$ in the Eq.(9), we get

$$\frac{d\tilde{\eta}_4(\eta)}{d\eta} = \tilde{\beta}\tilde{n}_1\tilde{n}_2\tilde{\sigma}\tilde{v}. \tag{11}$$

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Since $\tilde{n}_i = a^3 n_i$, $\tilde{\sigma} = \sigma_0 a^{-1}$, $|\vec{\tilde{v}}| = v_0$, $n_i(t) = \frac{C_i(t)}{a^3(t)}$, $H = \xi a^{-s}$, we get

$$C_\phi \simeq \frac{C_1\beta\sigma_0 v_0}{(|s-2|)\xi} a^{|s-2|}\left[1 - \left(\frac{a_1}{a}\right)^{|s-2|}\right] \quad \text{for} \quad s-2>0, \tag{12}$$

$$C_\phi \simeq \frac{C_1\beta\sigma_0 v_0}{(|s-2|)\xi} a_1^{-|s-2|}\left[1 - \left(\frac{a_1}{a}\right)^{|s-2|}\right] \quad \text{for} \quad s-2<0. \tag{13}$$

At initial times C_ϕ reaches higher values with the increasement of a for $s < 2$ and it grows faster for $s > 2$ $\implies$ **Macroscopic nature of BEC is realized.**

Conclusion

- **Effective Minkowski space formulation** seems to be useful formalism to study particle physics processes in curved space-time provided that some conditions are satisfied, e.g. in the study of BEC for FRW metric.

- By using the effective Minkowski space formulation **we have shown** that curved space effects for FRW metric promote the formation of BEC scalar field in some simple models.

- **A separate study** is needed when $\tilde{f}^{(f)}(\vec{p}_j, \eta) \neq 0$ with $j = 3, 4$.

Thank you

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For more details :

R. Erdem and K. Gültekin,

A mechanism for formation of Bose-Einstein condensation in cosmology, arXiv :1908.08784 [gr-qc]

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