

# The threshold for Primordial Black Holes: formation and cosmological impact

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- *C.Germani, IM* - PRL 122, 141302 (2019)
- *IM* - PRD 100, 123524 (2019) - editor suggestion
- *S. Young, IM, C. Byrnes* - JCAP 11, 012 (2019)
- *A. Kehagias, IM, A. Riotto* - JCAP 12, 029 (2019)
- *A. Kalaja, N. Bellomo, N. Bartolo, D. Bertacca, S. Matarrese, IM, A. Racanelli, L. Verde* - JCAP 10, 031 (2019)
- **IM**, V. De Luca, G. Franciolini, A. Riotto - arXiv:2011.03014 (2020)

# Introduction

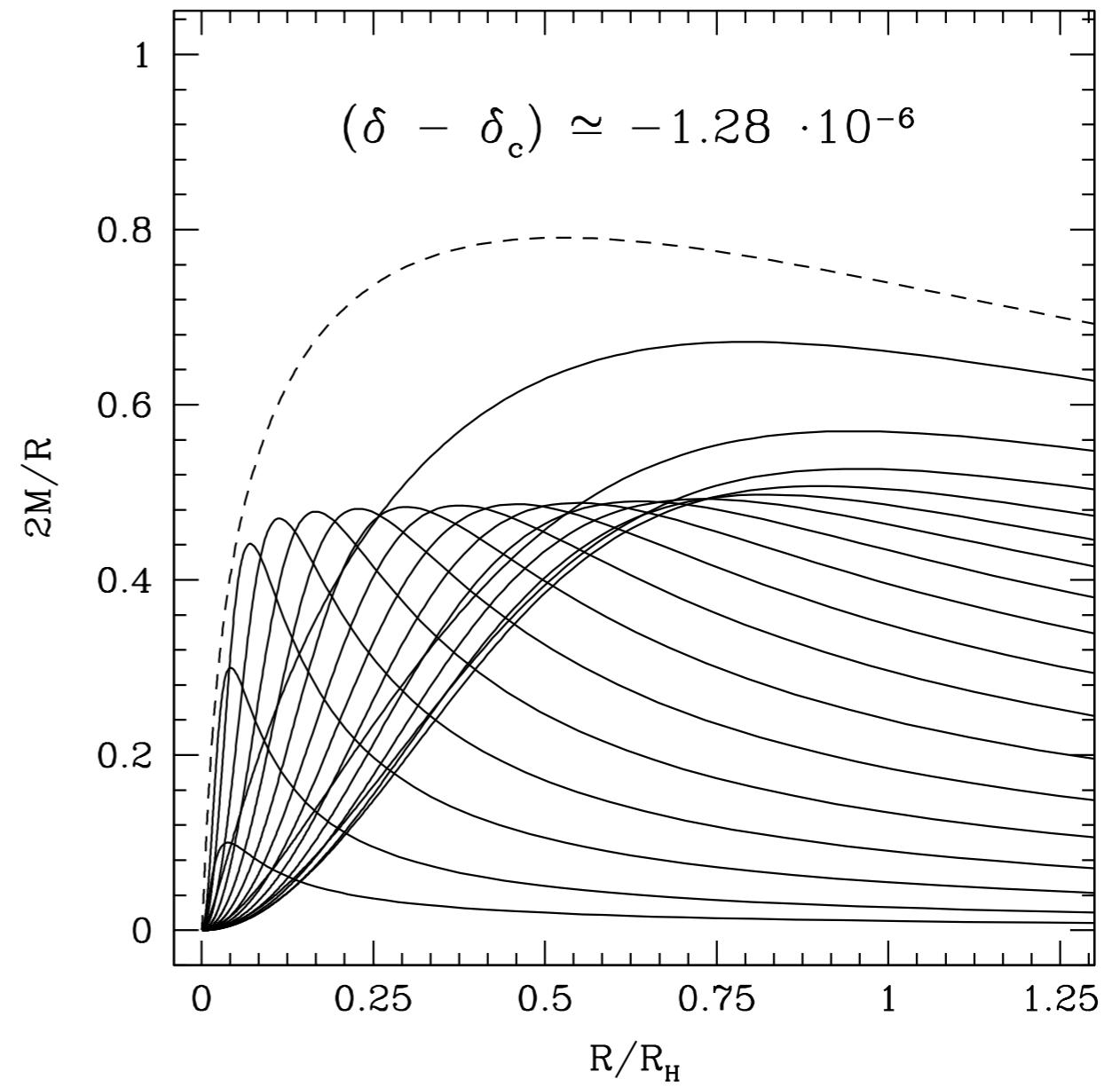
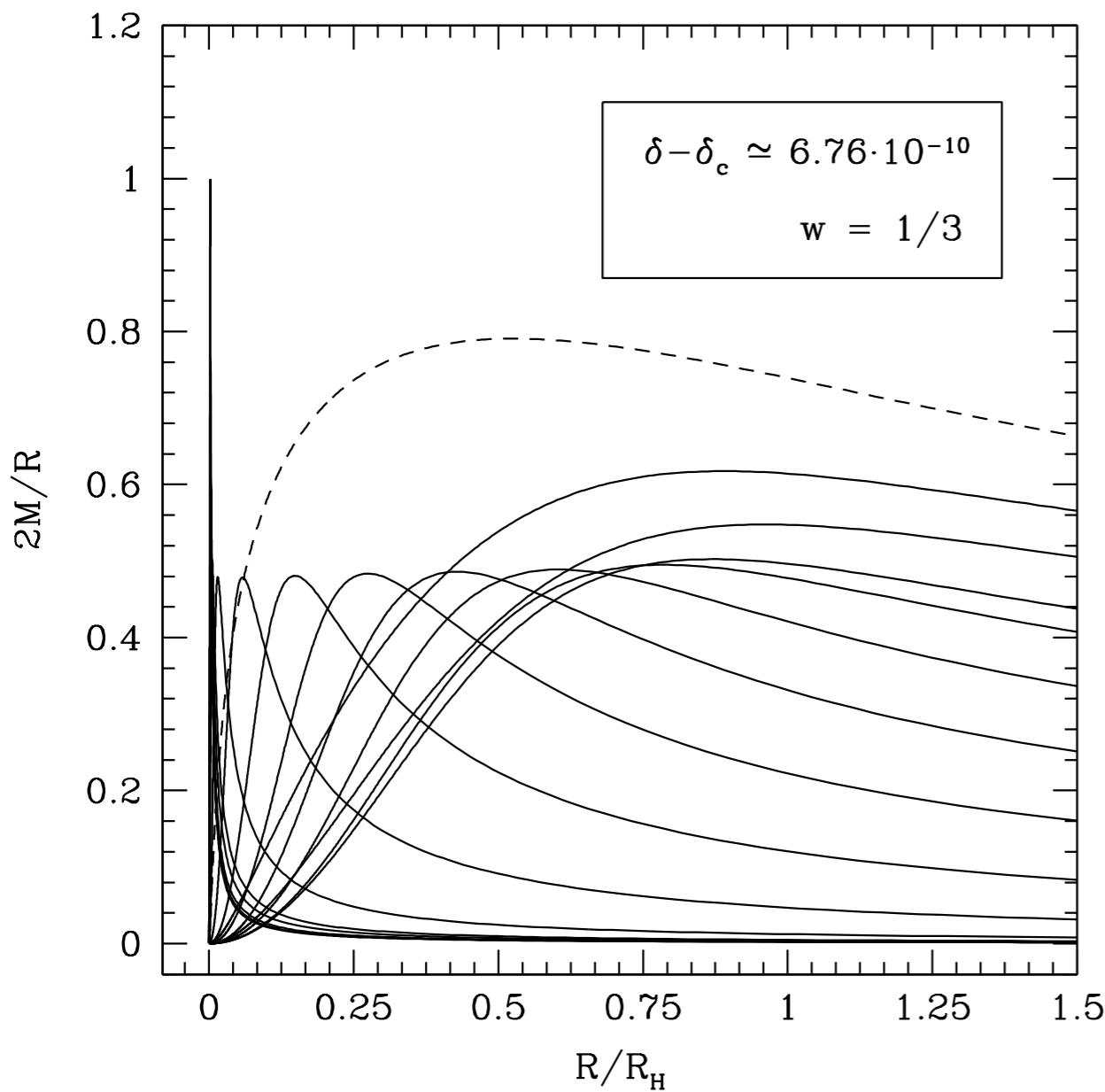
- Primordial Black Holes (PBHs) [Hawking (1971)] could form from the collapse of cosmological perturbation during the radiation dominated era.

$$p = \frac{\rho}{3}$$

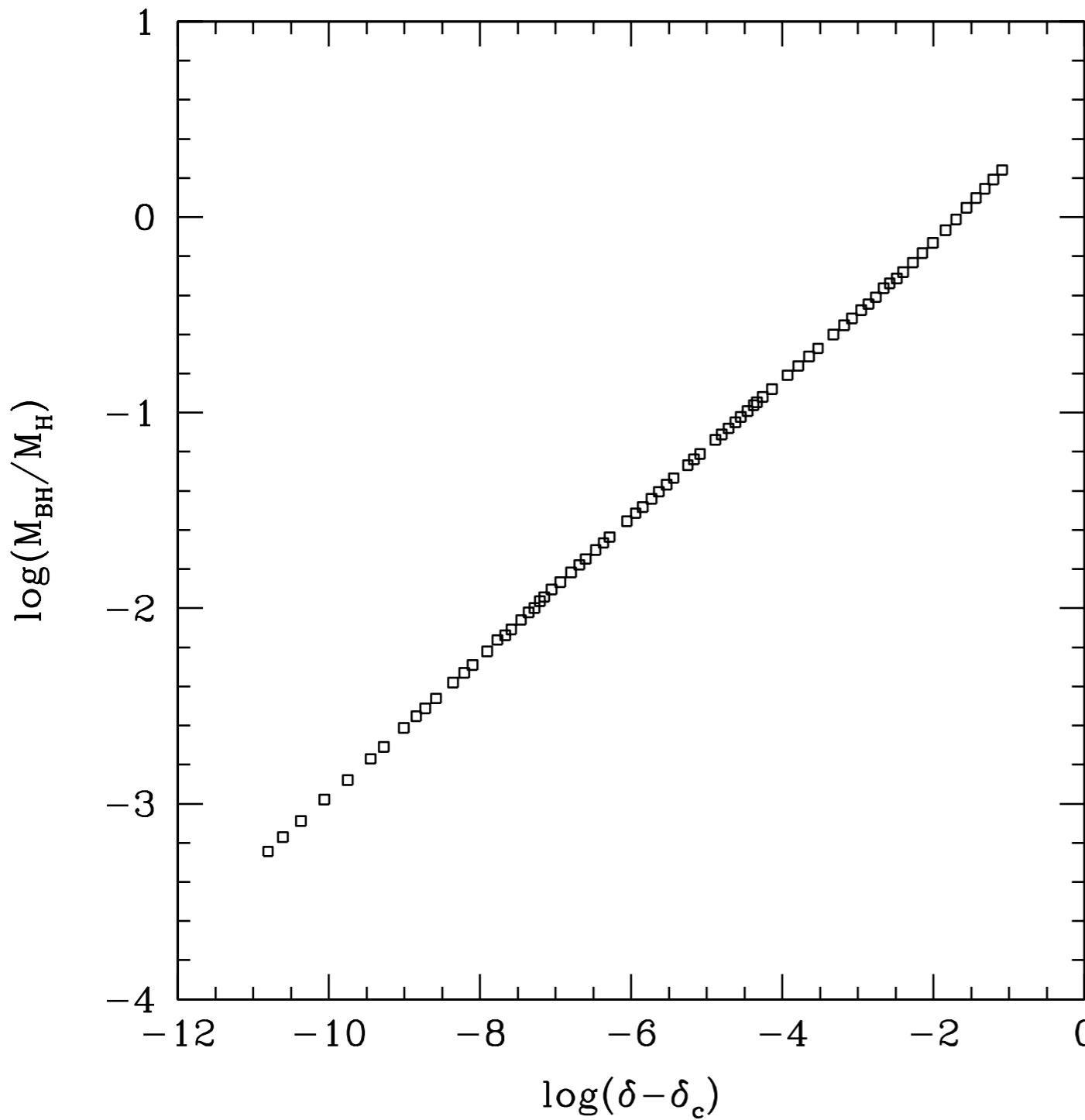
- PBHs can in principle span a large wide range of masses and if not evaporated ( $M > 10^{15} g$ ) are interesting candidates for dark matter, intermediate mass black holes and the seeds of supermassive black holes.
- Numerical hydrodynamical simulations in spherical symmetry of a cosmological perturbation, characterised by an amplitude  $\delta$ , have shown:
  - $\delta > \delta_c \Rightarrow$  PBH formation
  - $\delta < \delta_c \Rightarrow$  perturbation bounce
  - $\delta_c = 0.4 - 2/3$  depending on the perturbation shape

# Numerical Results: PBH formation/bounce

$$R(r, t) = 2M(r, t)$$



# Numerical Results: Scaling Law



$$M_{PBH} = \mathcal{K}(\delta - \delta_c)^\gamma M_H$$

$\mathcal{K}, \delta_c$  – shape dependent

$$\gamma \simeq 0.36$$

$M_H$  – cosmological horizon mass

*IM, Miller, Polnarev - CQG (2009,2013)*

# Curvature profile

- The asymptotic metric ( $t \rightarrow \theta$ ), describing super-horizon cosmological perturbations in the comoving synchronous gauge can be written as:

$$ds^2 \simeq -dt^2 + a^2(t) \left[ \frac{1}{1 - K(r)r^2} dr^2 + r^2 d\Omega^2 \right]$$

$$ds^2 \simeq -dt^2 + a^2(t)e^{2\zeta(\tilde{r})} [d\tilde{r}^2 + \tilde{r}^2 d\Omega^2]$$

- In the “linear regime” of cosmological perturbations, adiabatic perturbations on super horizon scales can be described by a time independent curvature profile using the quasi-homogeneous / gradient expansion approach.

$$K(r)r^2 = -\tilde{r}\zeta'(\tilde{r}) [2 + \tilde{r}\zeta'(\tilde{r})]$$

$$r = \tilde{r}e^{\zeta(\tilde{r})}$$

- The zero-order perturbation in the curvature is related to the first non zero order perturbation in the metric/hydro variable.

# Perturbation amplitude / profile

$$\frac{\delta\rho}{\rho_b} = \left(\frac{1}{aH}\right)^2 \frac{2}{3} \left[ K(r) + \frac{r}{3} K'(r) \right]$$

$$\frac{\delta\rho}{\rho_b} = - \left(\frac{1}{aH}\right)^2 \frac{4}{9} \left[ \nabla^2 \zeta(\tilde{r}) + \frac{1}{2} (\nabla \zeta(\tilde{r}))^2 \right] e^{-2\zeta(\tilde{r})}$$

$$\mathcal{C} := \frac{2[M(r, t) - M_b(r, t)]}{R(r, t)} = \frac{2}{3} K(r) r^2 = -\frac{4}{3} \tilde{r} \zeta'(\tilde{r}) \left[ 1 + \frac{1}{2} \tilde{r} \zeta'(\tilde{r}) \right]$$

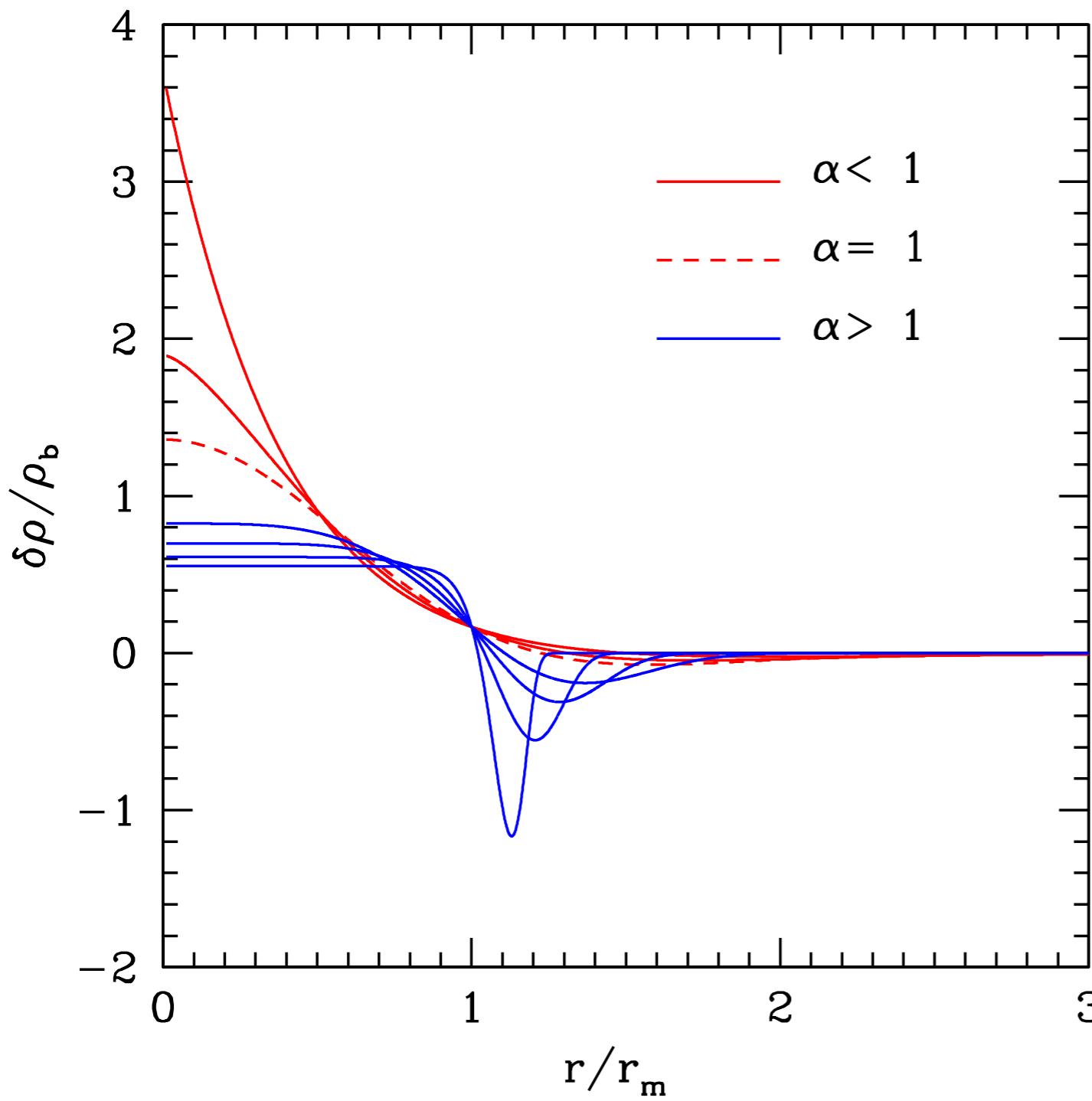
- The typical scale of the perturbation is identified as the location where the perturbation reaches its maximum compactness.

$$r_m : C'(r_m) = 0 \quad \delta(r_m, t_H) := \frac{1}{V_b} \int_0^{r_m} 4\pi \frac{\delta\rho}{\rho_b} r^2 dr = \mathcal{C}(r_m)$$

- The perturbation amplitude can measured as the mass excess within a characteristic scale at “horizon crossing time”, independent from the curvature profile.

$$\delta(r_m, t_H) = 3 \frac{\delta\rho}{\rho_b}(r_m, t_H)$$

$$K(r) = \mathcal{A} \exp \left[ -\frac{1}{\alpha} \left( \frac{r}{r_m} \right)^{2\alpha} \right] \Rightarrow \frac{\delta\rho}{\rho_b} = \frac{\delta\rho_0}{\rho_b} \left[ 1 - \frac{2}{3} \left( \frac{r}{r_m} \right)^{2\alpha} \right] \left[ -\frac{1}{\alpha} \left( \frac{r}{r_m} \right)^{2\alpha} \right]$$



$$\frac{\delta\rho_0}{\rho_b} = e^{1/\alpha} \delta_m = \frac{2}{3} \mathcal{A} r_m^2$$

### Shape parameter

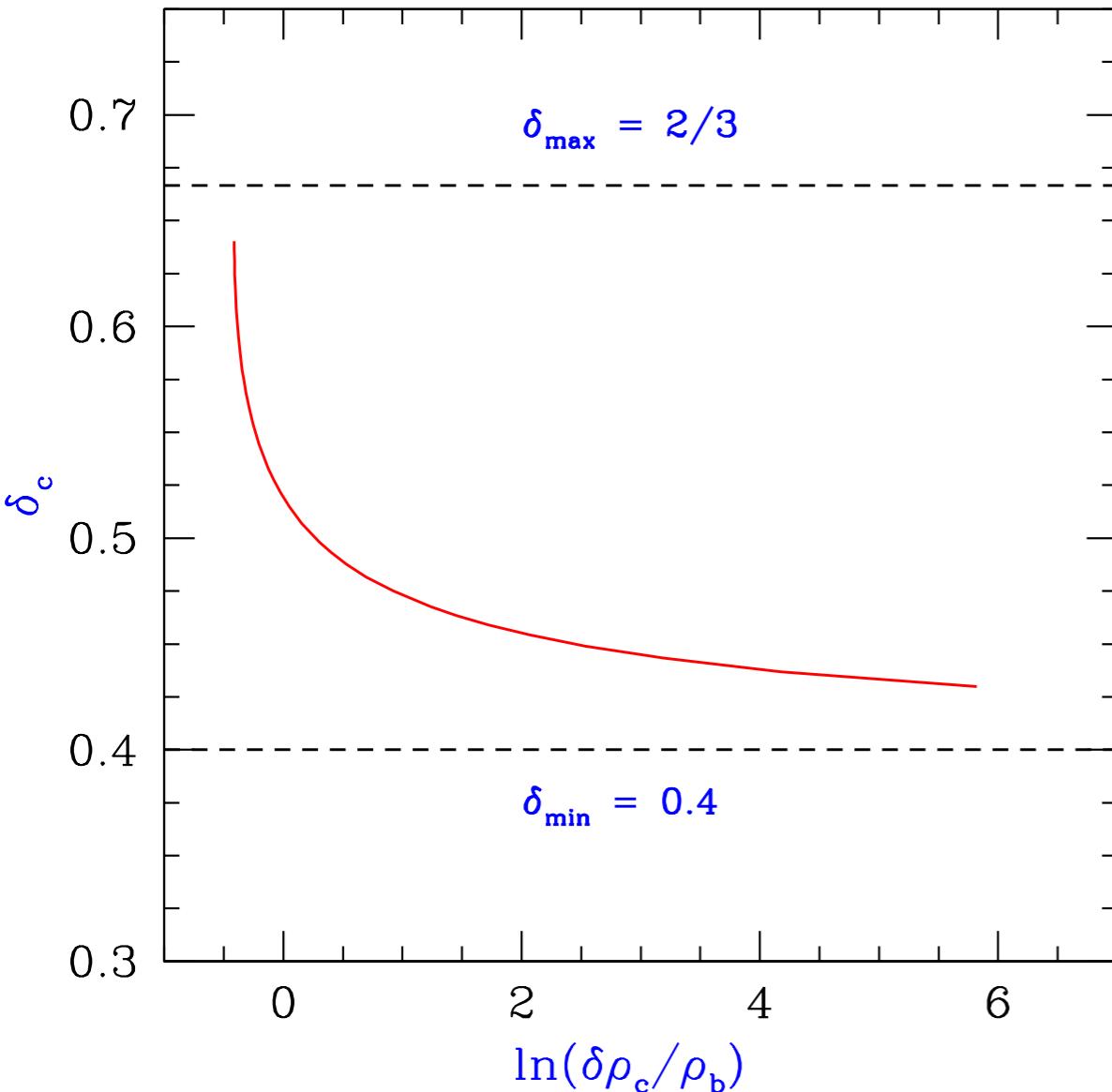
$$\alpha = -\frac{\mathcal{C}''(r_m) r_m^2}{4\mathcal{C}(r_m)}$$

$$\alpha = -\frac{\Phi_m'' \tilde{r}_m^2}{4\Phi_m \left(1 - \frac{1}{2}\Phi_m\right) (1 - \Phi_m)}$$

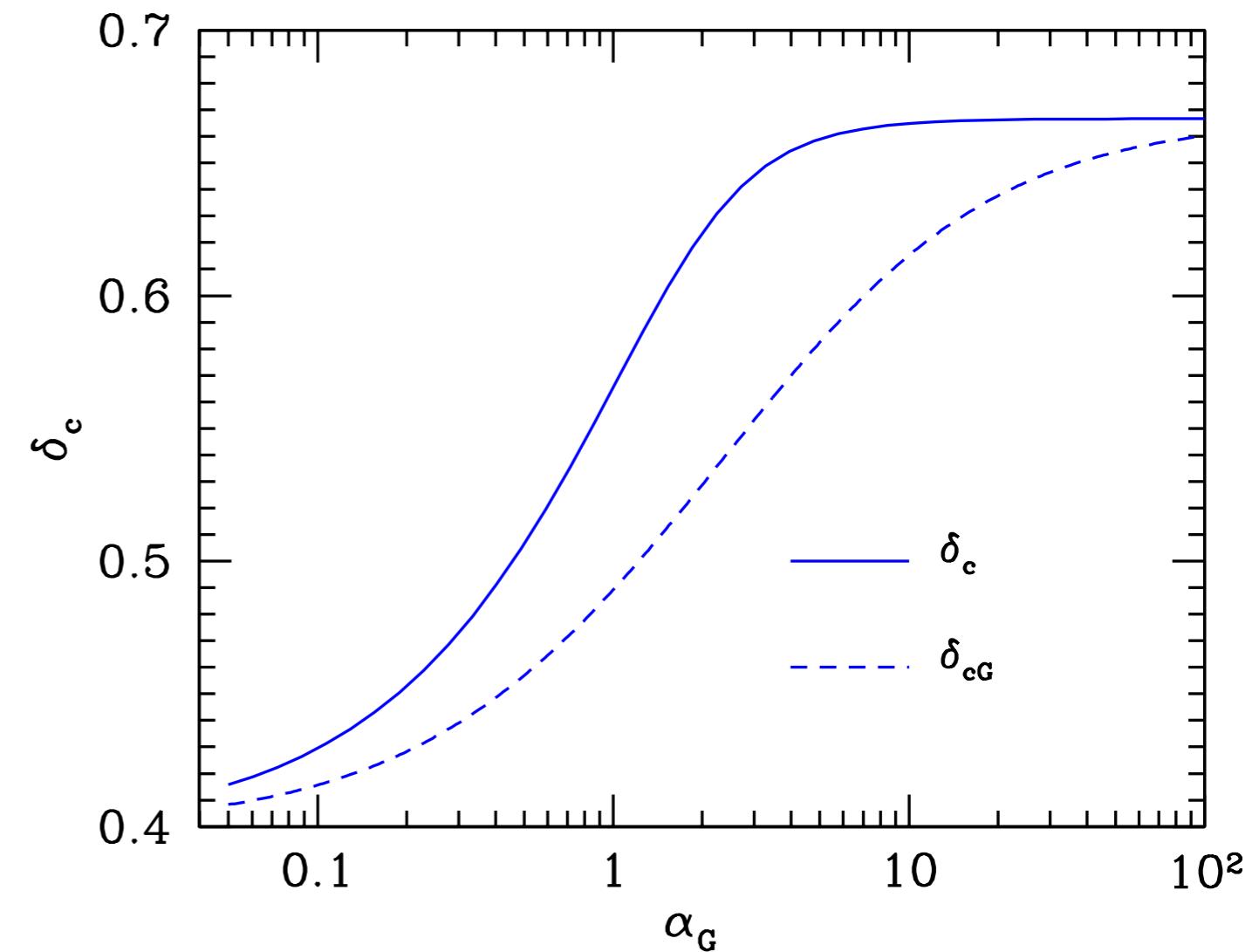
$$\Phi_m = -\tilde{r}_m \zeta'(\tilde{r}_m)$$

# PBH threshold

$$\delta_c \simeq \frac{4}{15} e^{-\frac{1}{\alpha}} \frac{\alpha^{1-5/2\alpha}}{\Gamma\left(\frac{5}{2\alpha}\right) - \Gamma\left(\frac{5}{2\alpha}, \frac{1}{\alpha}\right)}$$



$$0.4 \leq \delta_c \leq \frac{2}{3} \quad \frac{\delta\rho_0}{\rho_b} \geq \frac{2}{3}$$



$$\alpha = -\frac{\alpha_G}{\left(1 - \frac{1}{2}\Phi_m\right)\left(1 - \Phi_m\right)}$$

# PBH threshold prescription

Curvature power spectrum  $\mathcal{P}_\zeta$



Characteristic overdensity scale  $k_* \hat{r}_m$



Characteristic shape parameter  $\alpha$



Threshold  $\delta_c$

1. **The power spectrum of the curvature perturbation:** take the primordial power spectrum  $\mathcal{P}_\zeta$  of the Gaussian curvature perturbation and compute, on superhorizon scales, its convolution with the transfer function  $T(k, \eta)$

$$P_\zeta(k, \eta) = \frac{2\pi^2}{k^3} \mathcal{P}_\zeta(k) T^2(k, \eta).$$

2. **The comoving length scale  $\hat{r}_m$**  of the perturbation is related to the characteristic scale  $k_*$  of the power spectrum  $P_\zeta$ . Compute the value of  $k_* \hat{r}_m$  by solving the following integral equation

$$\int dk k^2 \left[ (k^2 \hat{r}_m^2 - 1) \frac{\sin(k \hat{r}_m)}{k \hat{r}_m} + \cos(k \hat{r}_m) \right] P_\zeta(k, \eta) = 0.$$

3. **The shape parameter:** compute the corresponding shape parameter  $\alpha$  of the collapsing perturbation, including the correction from the non linear effects, by solving the following equation

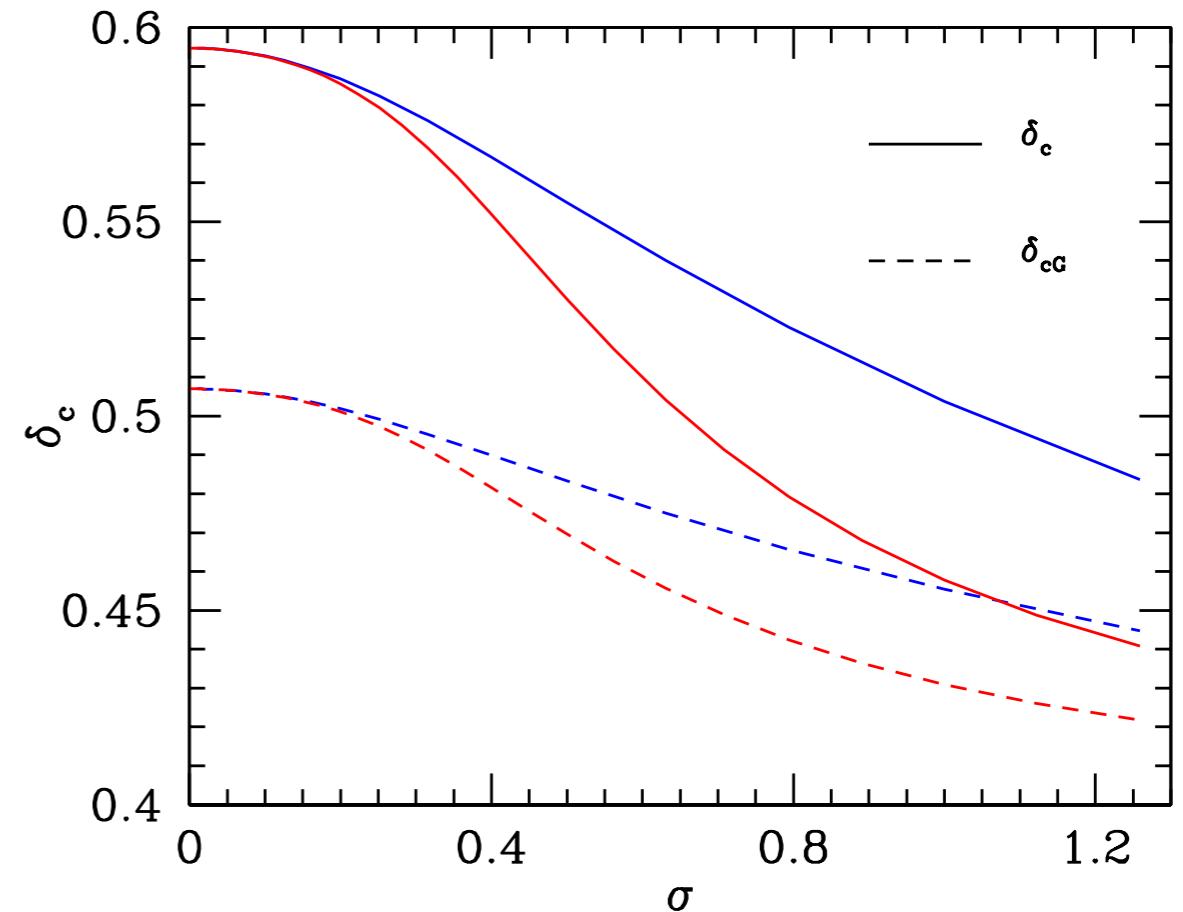
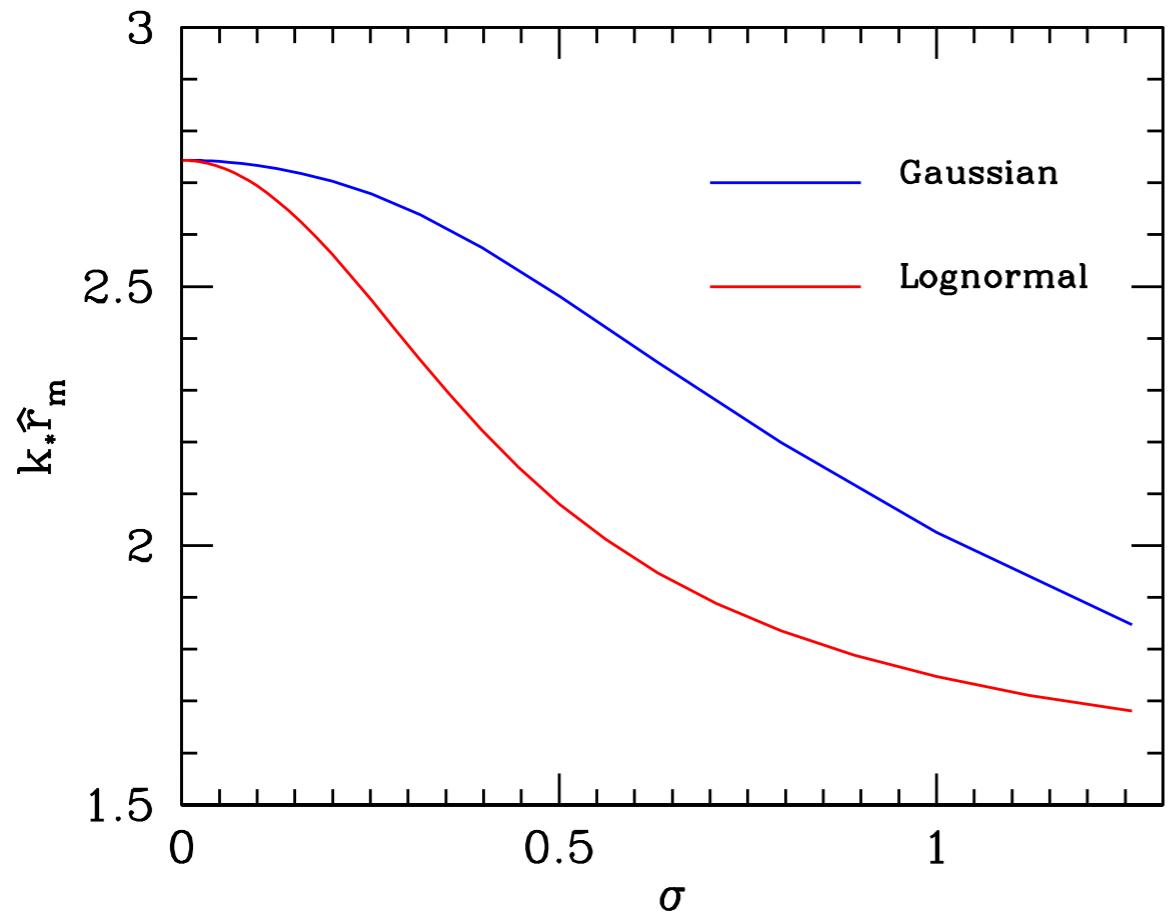
$$F(\alpha) [1 + F(\alpha)] \alpha = -\frac{1}{2} \left[ 1 + \hat{r}_m \frac{\int dk k^4 \cos(k \hat{r}_m) P_\zeta(k, \eta)}{\int dk k^3 \sin(k \hat{r}_m) P_\zeta(k, \eta)} \right]$$

$$F(\alpha) = \sqrt{1 - \frac{2}{5} e^{-1/\alpha}} \frac{\alpha^{1-5/2\alpha}}{\Gamma\left(\frac{5}{2\alpha}\right) - \Gamma\left(\frac{5}{2\alpha}, \frac{1}{\alpha}\right)}.$$

4. **The threshold  $\delta_c$ :** compute the threshold at cosmological horizon crossing as function of  $\alpha$ . This expression takes into account also the non linear effects of the horizon crossing studied with numerical simulations

$$\delta_c \simeq \begin{cases} \alpha^{0.125} - 0.05 & 0.1 \lesssim \alpha \lesssim 3 \\ \alpha^{0.06} + 0.025 & 3 \lesssim \alpha \lesssim 8 \\ 1.15 & \alpha \gtrsim 8 \end{cases}$$

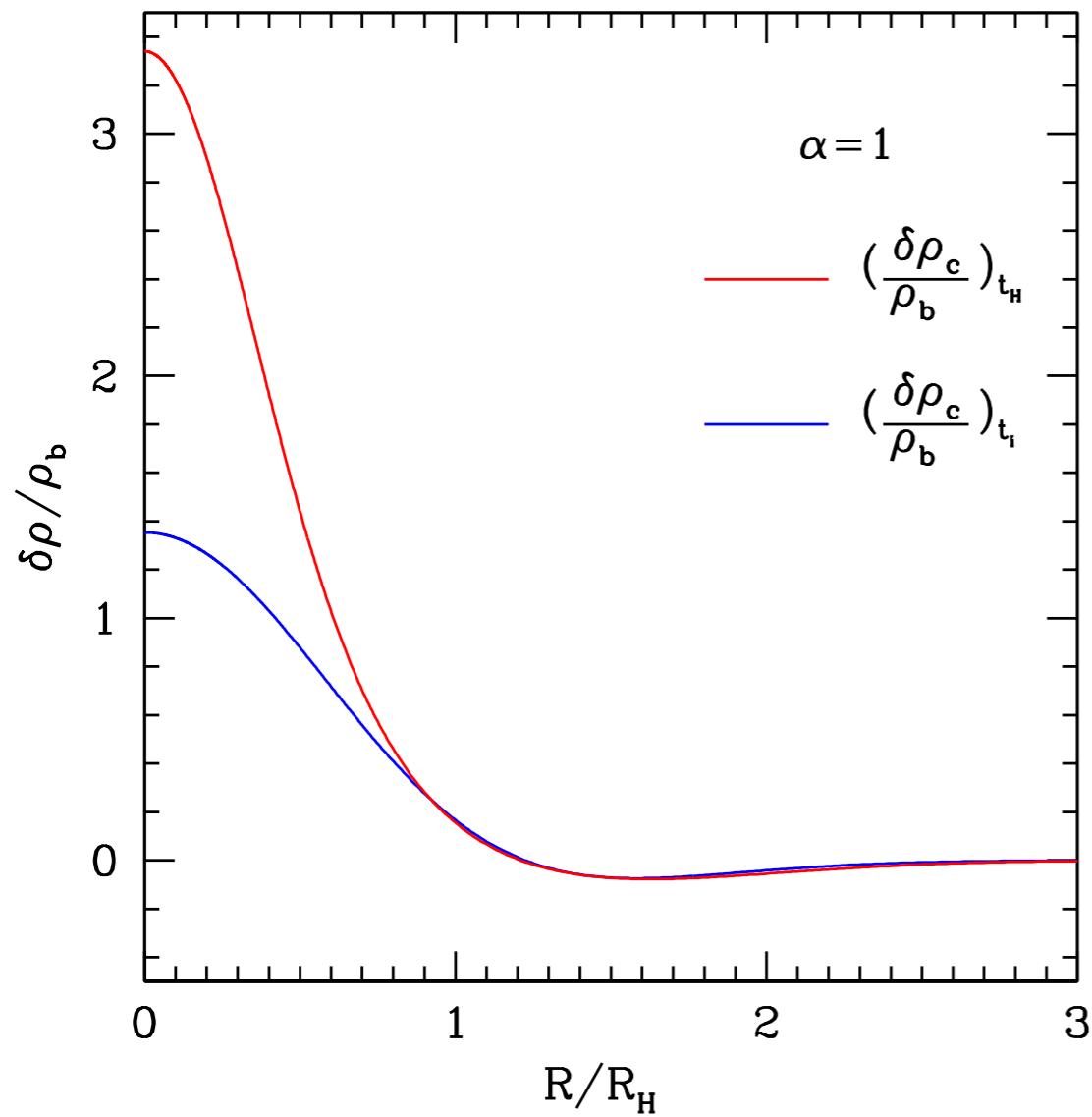
# Power Spectrum



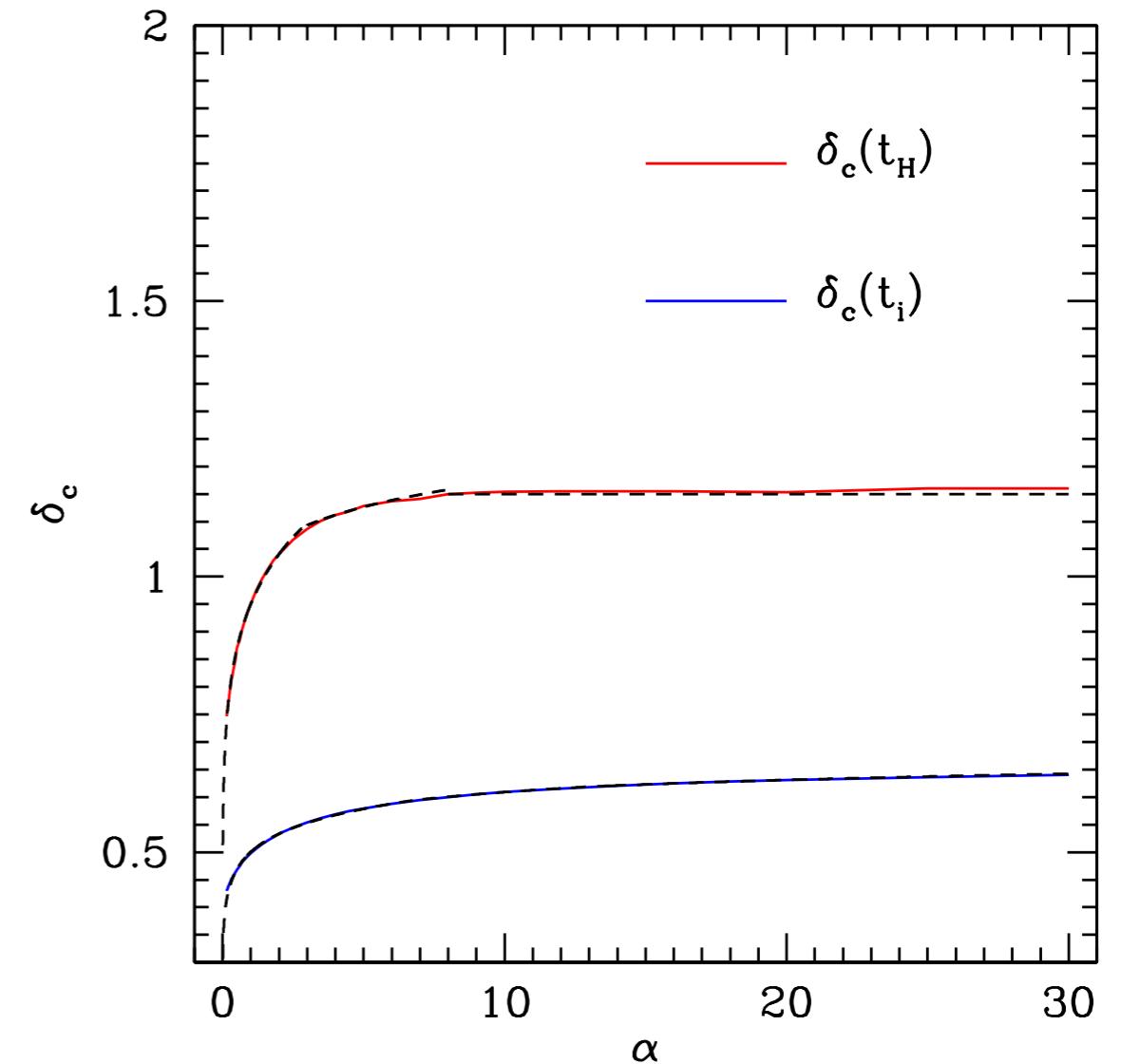
$$\text{Gaussian: } \mathcal{P}_\zeta(k) = \mathcal{P}_0 \exp \left[ -(k - k_*)^2 / 2\sigma^2 \right]$$

$$\text{Lognormal: } \mathcal{P}_\zeta(k) = \mathcal{P}_0 \exp \left[ -\ln^2(k/k_*) / 2\sigma^2 \right]$$

# Non linear horizon crossing



$$\delta_c(t_i) \simeq \begin{cases} \alpha^{0.047} - 0.50 & 0.1 \lesssim \alpha \lesssim 7 \\ \alpha^{0.035} - 0.475 & 7 \lesssim \alpha \lesssim 13 \\ \alpha^{0.026} - 0.45 & 13 \lesssim \alpha \lesssim 30 \end{cases}$$



$$\delta_c(t_H) \simeq \begin{cases} \alpha^{0.125} - 0.05 & 0.1 \lesssim \alpha \lesssim 3 \\ \alpha^{0.06} + 0.025 & 3 \lesssim \alpha \lesssim 8 \\ 1.15 & \alpha \gtrsim 8 \end{cases}$$

$0.4 \leq \delta_c(t_i) < 0.6$	$0.7 \lesssim \delta_c(t_H) \lesssim 1.15$
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# PBH Abundance

- PBH Abundance in Peak Theory (Gaussian approximation):

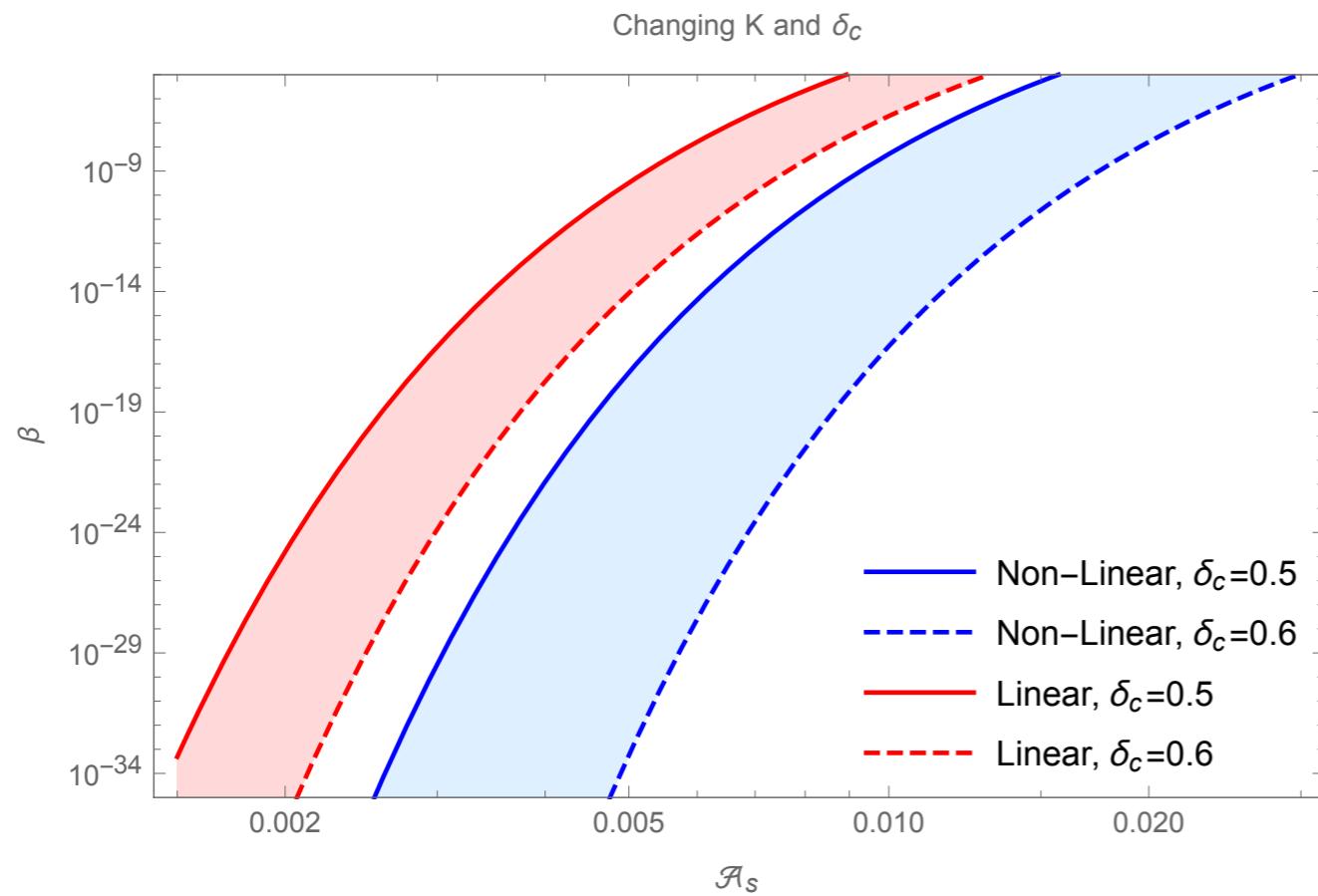
$$\beta_f \simeq \sqrt{\frac{2}{\pi}} \mathcal{K} \left( \frac{k_*}{a_m H_m} \right)^3 \sigma^\gamma \nu_c^{1-\gamma} \gamma^{\gamma+1/2} e^{-\frac{\nu_c^2}{2}}$$
$$\nu_c \equiv \frac{\delta_c}{\sigma}$$
$$\sigma^2 = \left( \frac{2}{3} r_m \right)^4 \int dk k^3 \mathcal{P}_\zeta(k) T^2(k, \eta)$$

- If  $M_{PBH} \sim 10^{16} g$  are Dark Matter  $\Rightarrow \beta_f \simeq 10^{-8} \sqrt{\frac{M_{PBH}}{M_\odot}} \simeq 10^{-16}$

- Narrow peak:  $\frac{k_*}{\sigma} \gg 1 \Rightarrow \nu_c \simeq 0.22 \sqrt{\frac{k_*}{\sigma \mathcal{P}_0}} \Rightarrow \mathcal{P}_0 \sim 7 \times 10^{-4} \frac{k_*}{\sigma} \gg 10^{-3}$

- Broad peak:  $\frac{k_*}{\sigma} \ll 1 \Rightarrow \nu_c \simeq 0.46 (\mathcal{P}_0)^{-1/2} \Rightarrow \mathcal{P}_0 \sim 3 \times 10^{-3}$

# Non linear effects: power spectrum constraints



Delta function power spectrum  
**S.Young, IM, C.Byrnes JCAP (2019)**

- In order to generate the same number of PBHs when taking the non-linear (NL) relation into account, compared to the linear relation, the power spectrum amplitude needs to increase

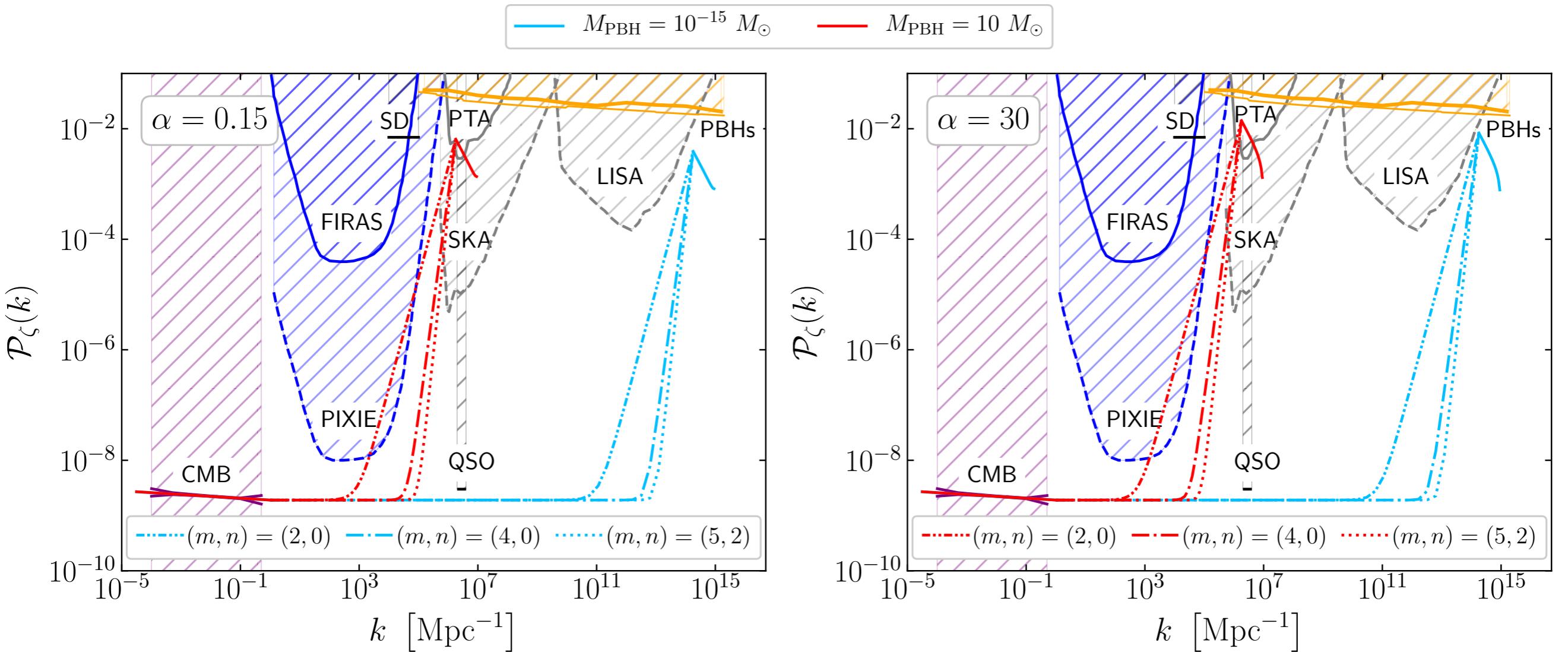
$$\delta_m = \frac{4}{3} \Phi_m \left( 1 - \frac{1}{2} \Phi_m \right)$$

$$\Phi_m = -\tilde{r}_m \zeta'(\tilde{r}_m)$$

$$1.5 \lesssim \frac{\mathcal{P}_{0_{NL}}}{\mathcal{P}_{0_L}} = \frac{16 \left( 1 - \sqrt{1 - \frac{3}{2} \delta_c} \right)^2}{9 \delta_c^2} \lesssim 4$$

- For the typical value of  $\delta_c \sim 0.5$ , power spectrum constraints are weakened by a factor of 2

# PBHs Constraints - reconstructing the power spectrum



*Alba Kalaja, Nicola Bellomo, Nicola Bartolo, Daniele Bertacca, Sabino Matarrese,  
IM, Alvise Racanelli, Licia Verde - JCAP (2019)*

# Conclusions

- There is no universal threshold for PBH formation. The curvature profile (pressure gradients) play a key role determining the particular value of the threshold. This can be related to the morphology of the power spectrum of cosmological perturbations.
- PBH formation is characterised by non liner curvature profile. The linear approximation does not give accurate results.
- The threshold, including also the non linear effects of the horizon crossing, could be computed with Gaussian statistics plus an algebraic correction accounting for the non linear effects. - *IM, De Luca, Franciolini, Riotto - arXiv (2020)*
- The abundance of PBHs is exponentially sensitive to the threshold. The shape of the peak of the power spectrum is very important.
- A large enough feature of the power spectrum on small scales (large  $k$ ) could account for an important component of dark matter in PBHs.