

DARK MATTER BOUND STATES IN THE EARLY UNIVERSE: WHERE DO WE STAND?

Simone Biondini

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Progress on Old and New Themes in cosmology

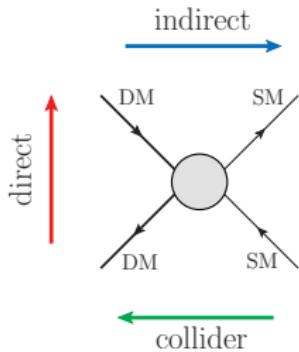
December 8th, 2020

in collaboration with Mikko Laine, and Stefan Vogl

OUTLINE

- 1 MOTIVATION AND INTRODUCTION
- 2 SPECTRAL FUNCTION AND NREFTS
- 3 APPLICATIONS TO DM MODELS
- 4 CONCLUSIONS

PARTICLE INTERPRETATION OF DM AND FREEZE-OUT



- Evidence for DM from many compelling (gravitational) observations
 - ◊ from CMB anisotropies with Λ CDM *Planck Collab. Results 2018*

- DM as a particle: many candidates (e.g. review G. Bertone 2018)
- Any model has to comply with

$$\Omega_{\text{DM}} h^2(M_{\text{DM}}, M_{\text{DM}'}, \alpha_{\text{DM}}, \alpha_{\text{SM}}) = 0.1200 \pm 0.0012$$

THERMAL FREEZE-OUT

- Boltzmann equation for DM (χ)

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle\sigma v\rangle(n_\chi^2 - n_{\chi,\text{eq}}^2)$$

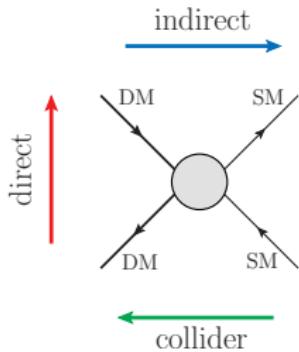
- relevant processes $\chi\chi \leftrightarrow \text{SM SM}$

- $\langle\sigma v\rangle$: input from particle physics with $v \sim \sqrt{T/M} < 1$

$$\langle\sigma v\rangle \approx \langle a + bv^2 + \dots \rangle \Rightarrow \boxed{\langle\sigma v\rangle^{(0)} \approx \frac{\alpha^2}{M^2}}$$

K. Griest and D. Seckel (1991), P. Gondolo and G. Gelmini (1991)

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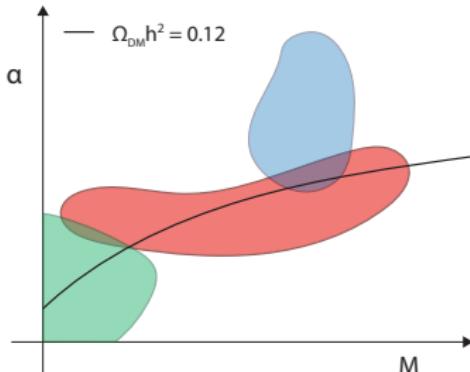
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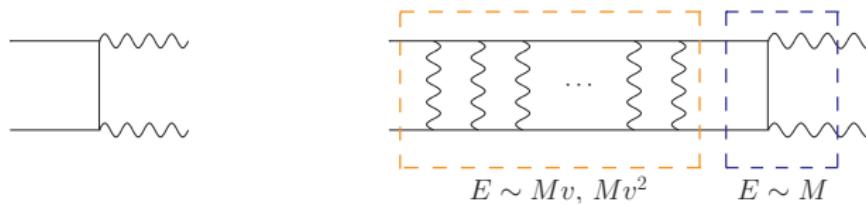
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GOING TOWARDS A REALISTIC PICTURE

- unitarity and gauge invariance → DM interacts with gauge bosons and scalars

F. Kahlhoefer, K. Schmidt-Hoberg, T. Schwetz and S. Vogl (2016)

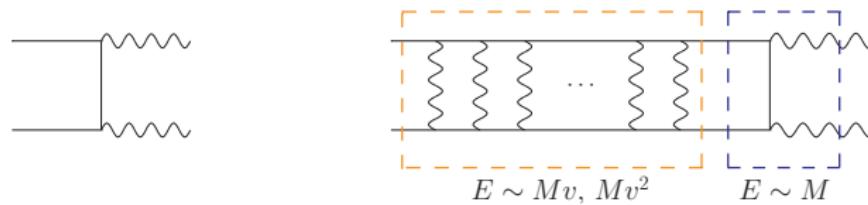


- repeated soft interactions: Sommerfeld enhancement $\langle\sigma v\rangle \approx S \langle\sigma v\rangle^{(0)}$ and....

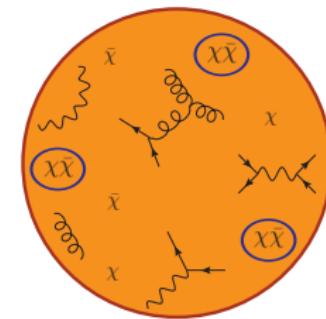
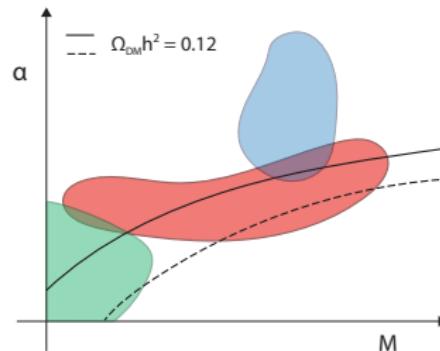
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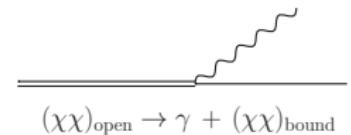
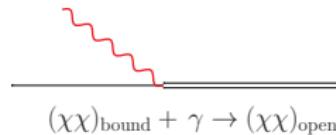
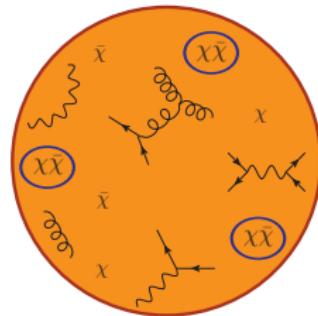
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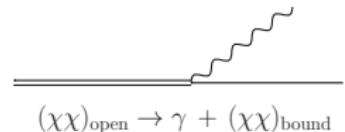
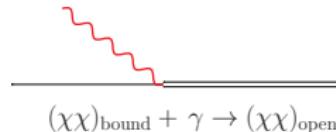
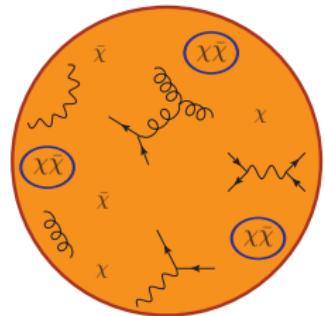
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DARK MATTER BOUND STATES



DARK MATTER BOUND STATES



- bound states as additional d.o.f. in the Boltzmann equation B. von Harling and K. Petraki 2014

$$\frac{dY_\chi}{dz} = -\frac{c_1 \bar{S}_{\text{ann}}}{z^2} (Y_\chi^2 - Y_{\chi, \text{eq}}^2) - \frac{c_1 \bar{S}_{\text{BSF}}}{z^2} Y_\chi^2 + z c_2 f_{\text{ion}}(z) Y_b$$

$$\frac{dY_b}{dz} = \frac{c_1 \text{bar}S_{\text{BSF}}}{4z^2} Y_\chi^2 - z c_2 [1 + f_{\text{ion}}(z)] Y_b$$

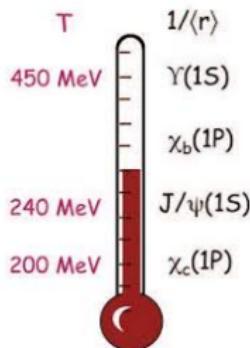
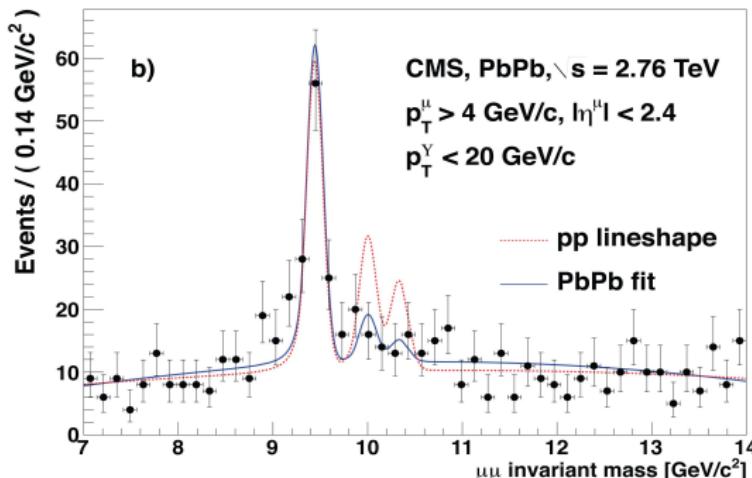
U(1) QED-like dark matter model

M. Cirelli, P. Panci, K. Petraki, F. Sala, M. Taoso (2016), S.P. Liew and F. Luo (2016); P. Asadi, M. Baumgart, P. J. Fitzpatrick, E. Krupczak and T. R. Slatyer (2016), M. Beneke, A. Bharucha, F. Dighera, C. Hellmann, A. Hryczuk, S. Recksiegel and P. Ruiz-Femenia (2016), A. Mitridate, M. Redi, J. Smirnov and A. Strumia (2017), E. Braaten, E. Johnson and H. Zhang (2017), J. L. Feng, M. Kaplinghat, H. Tu and H. B. Yu (2019)...

PROBLEMS AND ISSUES

- many bound states may appear in the spectrum
- their existence (formation/dissociation) depends on the temperature
- bound-states calculations can be performed in pNREFTs

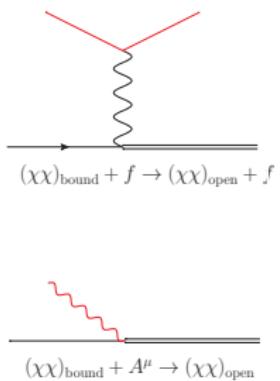
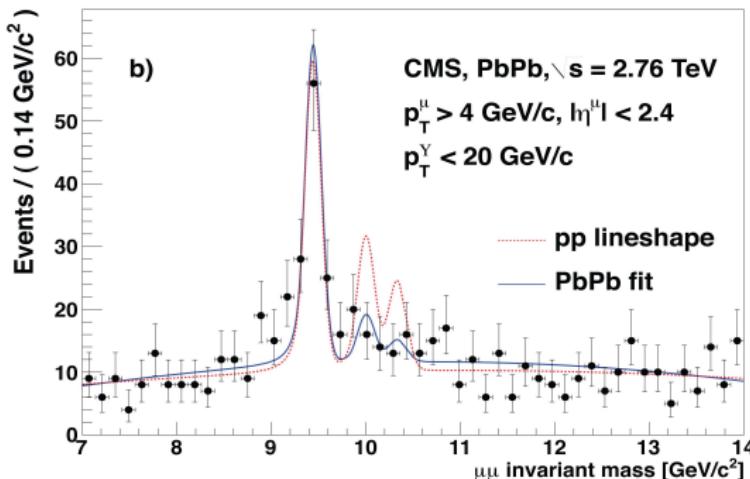
T. Matsui and H. Satz (1986); M. Laine, O. Philipsen, P. Romatschke and M. Tassler hep-ph/0611300, N. Brambilla, J. Ghiglieri, P. Petreczky, A. Vairo 0804.0993; M. Escobedo and J. Soto 0804.0691;



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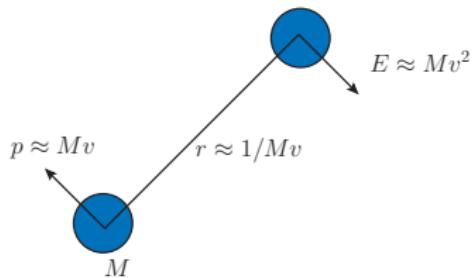
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NREFTs AND PNREFTs FOR DARK MATTER

APPLY AN EFT APPROACH

- Non-relativistic scales: $M \gg Mv \gg Mv^2$ (Coulomb potential $v \sim \alpha$)
- Thermal scales: πT and $m_D \approx \alpha^{1/2} T$, if weakly-coupled plasma $\pi T \gg m_D$



$$\mathcal{L}_{\text{RT}} = \frac{1}{2} \bar{\chi} (i \not{D} - M) \chi$$

$$\mathcal{L}_{\text{NREFT}} = \psi^\dagger \left(i D^0 - \frac{D^2}{2M} + \dots \right) \psi + \frac{c}{M^2} \psi^\dagger \psi^\dagger \psi \psi$$

$$\mathcal{L}_{\text{pNREFT}} = \int d^3r \text{Tr} \left\{ \phi^\dagger [i \partial_0 - V_\phi - \delta M_\phi] \phi \right\} + \dots$$

- 1) work with the natural d.o.f. at a given scale

$E \sim Mv$ non-relativistic heavy particles

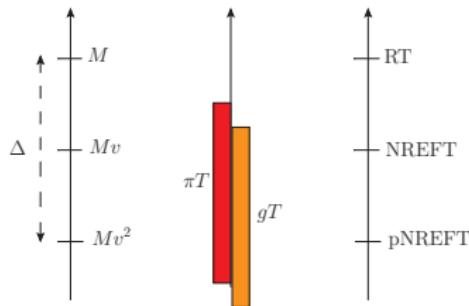
$E \sim Mv^2$ non-relativistic pairs $\psi\psi \rightarrow \phi_s + \phi_b \Rightarrow V_\phi = V(r, \textcolor{red}{T}, \textcolor{orange}{m}_D) + i\Gamma(r, \textcolor{red}{T}, \textcolor{orange}{m}_D)$

- 2) systematic expansions and power counting $\Rightarrow \langle \sigma v \rangle \approx \frac{c}{M^2} \langle \phi^\dagger \phi \rangle_T$

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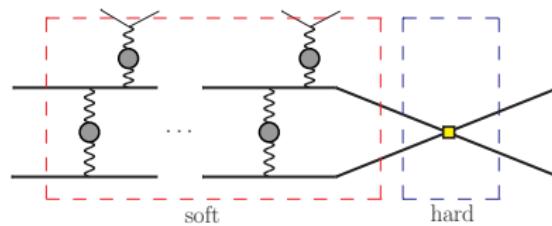
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SPECTRAL FUNCTION AND NREFTs II

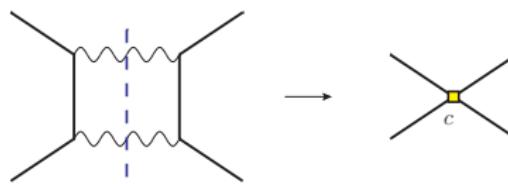


- $M \gg T, m_D, Mv, Mv^2$

- Annihilation of a heavy pair: DM-DM, with energies $\sim 2M$

$$\mathcal{O} = i \frac{c}{M^2} \psi^\dagger \psi^\dagger \psi \psi, \quad c \approx \alpha^2 \quad (\text{inclusive s-wave annihilation})$$

G. T. Bodwin, E. Braaten and G. P. Lepage hep-ph/9407339



- $M \gg T \Rightarrow \Delta x \sim \frac{1}{k} \sim \frac{1}{M} \ll \frac{1}{T}$ local and insensitive to thermal scales

BEYOND THE FREE CASE: THE SPECTRAL FUNCTION

$$(\partial_t + 3H)n = -\langle \sigma v \rangle (n^2 - n_{\text{eq}}^2), \quad \langle \sigma v \rangle = \frac{4}{n_{\text{eq}}^2} \frac{c}{M^2} \gamma \quad \text{where } \gamma = \langle \psi^\dagger \psi^\dagger \psi \psi \rangle_T$$

D. Bodeker and M. Laine 1205.4987; S. Kim and M. Laine 1602.08105; S. Kim and M. Laine 1609.00474

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$$\gamma = \int_{2M-\Lambda}^{\infty} \frac{d\omega}{\pi} e^{-\frac{\omega}{T}} \int_{\mathbf{k}} \rho(\omega, \mathbf{k}) + \mathcal{O}(e^{-4M/T}), \quad \alpha^2 M \ll \Lambda \sim M$$

- non-relativistic dynamics:

$$E_m \equiv \omega = E' + 2M + \frac{k^2}{4M} \text{ and } H = -\frac{\nabla^2}{M} + \mathbf{V}(\mathbf{r}, T)$$

- the spectral function $\rho(E')$

$$[H - i\Gamma(\mathbf{r}, T) - E'] G(E'; \mathbf{r}, \mathbf{r}') = N \delta^3(\mathbf{r} - \mathbf{r}') \quad \lim_{\mathbf{r}, \mathbf{r}' \rightarrow 0} \text{Im} G(E'; \mathbf{r}, \mathbf{r}') = \rho(E')$$

- $\Gamma(\mathbf{r}, T)$ is related to gauge-boson and $2 \rightarrow 2$ inelastic scatterings dissociation processes (and formation)

FROM ρ TO A SCHRÖDINGER EQUATION

- non-relativistic dynamics:

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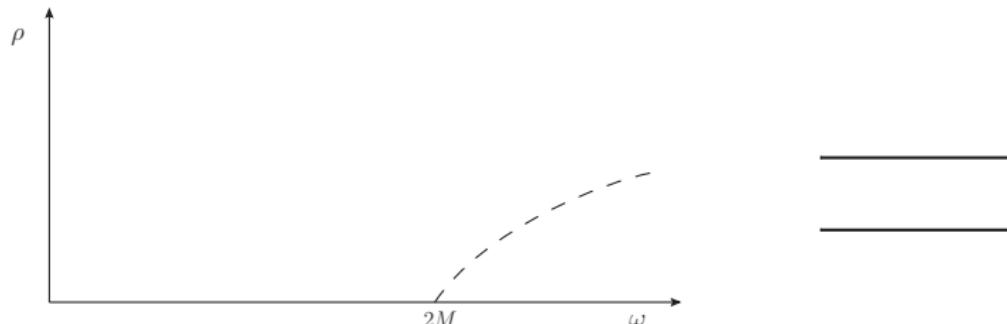
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$$\gamma \approx \left(\frac{MT}{2\pi} \right)^{\frac{3}{2}} e^{-\frac{2M}{T}} \int_{-\Lambda}^{\infty} \frac{dE'}{\pi} e^{-\frac{E'}{T}} \rho_{\text{free}}(E') \Rightarrow \langle \sigma v \rangle = \frac{c}{M^2}$$



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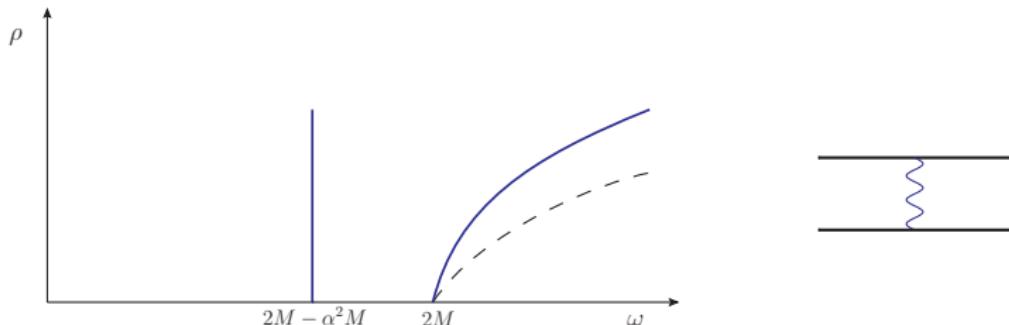
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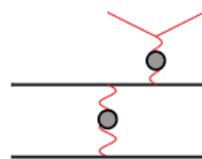
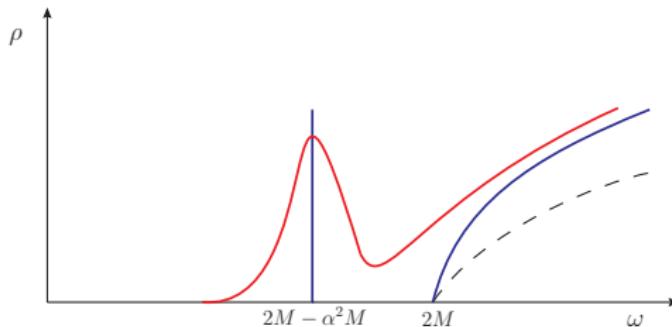
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INERT DOUBLET MODEL

- Supplement SM with χ SU(2) doublet, no coupling with fermions, unbroken vacuum
- We focus on the high-mass regime of the model: $M \gtrsim 530$ GeV

T. Hambye, F.-S. Ling, L. Lopez Honorez and J. Rocher, 0903.4010

$$\begin{aligned}\mathcal{L}_\chi &= (D^\mu \chi)^\dagger (D_\mu \chi) - M^2 \chi^\dagger \chi \\ &- \left\{ \lambda_2 (\chi^\dagger \chi)^2 + \lambda_3 \phi^\dagger \phi \chi^\dagger \chi + \lambda_4 \phi^\dagger \chi \chi^\dagger \phi + \left[\frac{\lambda_5}{2} (\phi^\dagger \chi)^2 + h.c. \right] \right\}\end{aligned}$$

ELECTROWEAK THERMAL POTENTIALS

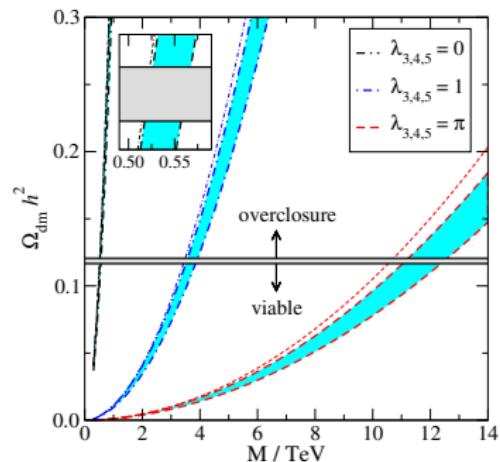
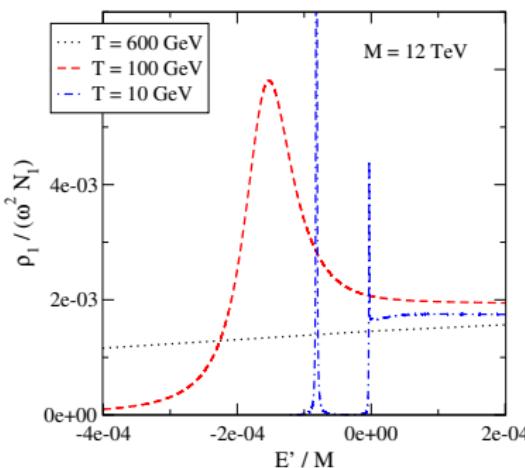
$$\mathcal{V}_W(r) \equiv \frac{g_2^2}{4} \int_{\mathbf{k}} e^{i\mathbf{k} \cdot \mathbf{r}} i \langle W_0^+ W_0^- \rangle_T(0, k) \quad \text{similar for } B^\mu \text{ and } W^3$$

$$i \langle W_0^+ W_0^- \rangle_T = \frac{1}{\mathbf{k}^2 + m_{\widetilde{W}}^2} - \frac{i\pi T}{k} \frac{m_{E2}^2}{(\mathbf{k}^2 + m_{\widetilde{W}}^2)^2}$$

BOUND STATES WITH ELECTROWEAK GAUGE BOSONS

- the thermally modified Sommerfeld factors are defined as

$$\bar{S}_i = \frac{\int_{-\Lambda}^{\infty} \frac{dE'}{\pi} \rho(E') e^{-E'/T}}{\int_{-\Lambda}^{\infty} \frac{dE'}{\pi} \rho_{\text{free}}(E') e^{-E'/T}} = \left(\frac{4\pi}{MT}\right)^{\frac{3}{2}} \int_{-\Lambda}^{\infty} \frac{dE'}{\pi} e^{[\text{Re}\mathcal{V}_i(\infty) - E']/T} \frac{\rho_i(E')}{N_i}$$



- at small enough T bound states start to form and contribute to the annihilation cross section, up to 20% effect for large λ 's

S.B. and M. Laine (2017)

COLOURED MEDIATORS: SIMPLIFIED MODELS

MAJORANA FERMION DM + SCALAR COLOURED MEDIATOR

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_\chi + \mathcal{L}_\eta + \mathcal{L}_{\text{int}}$$

$$\mathcal{L}_\chi = \frac{1}{2}\bar{\chi}i\cancel{D}\chi - \frac{M}{2}\bar{\chi}\chi, \quad \mathcal{L}_\eta = (D^\mu\eta)^\dagger(D_\mu\eta) - M_\eta^2\eta^\dagger\eta - \lambda_2(\eta^\dagger\eta)^2$$

$$\mathcal{L}_{\text{int}} = -y\eta^\dagger\bar{\chi}P_Rq - y^*\bar{q}P_L\chi\eta - \lambda_3\eta^\dagger\eta H^\dagger H$$

M. Garny, A. Ibarra and S. Vogl 1503.01500

SCALAR DM + FERMION COLOURED

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_\chi + \mathcal{L}_\eta + \mathcal{L}_{\text{int}}$$

$$\mathcal{L}_S = \frac{1}{2}\partial_\mu S\partial^\mu S - \frac{M_S^2}{2}S^2 - \frac{\lambda_2}{4!}S^4 - \frac{\lambda_3}{2}S^2H^\dagger H, \quad \mathcal{L}_F = \bar{F}(i\cancel{D} - M_F)F$$

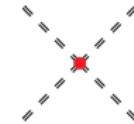
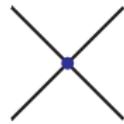
$$\mathcal{L}_{\text{int}} = -yS\bar{F}P_Rq - y^*S\bar{q}P_LF,$$

M. Garny, A. Ibarra and S. Vogl 1503.01500

- we look at the **coannihilation scenario**: $(M_\eta - M_\chi)/M_\chi, (M_F - M_S)/M_S \lesssim 0.2$

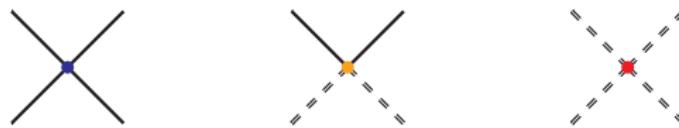
QCD-CHARGED COLORED MEDIATOR $\chi - \eta$ MODEL

- Non-relativistic fields $\eta = \frac{1}{\sqrt{2M}} \left(\phi e^{-iMt} + \varphi^\dagger e^{iMt} \right)$ and $\chi = (\psi e^{-iMt}, -i\sigma_2 \psi^* e^{iMt})$



QCD-CHARGED COLORED MEDIATOR $\chi - \eta$ MODEL

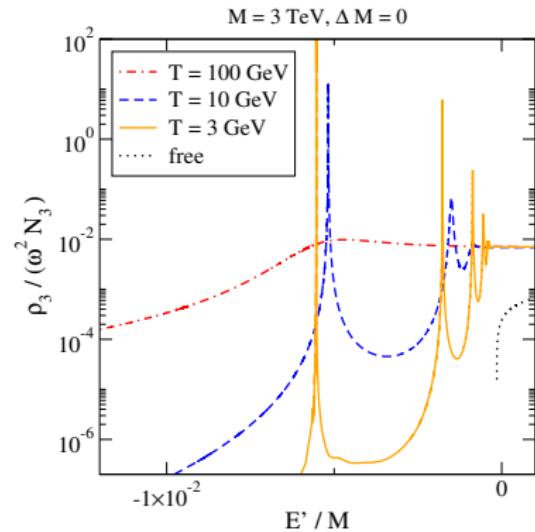
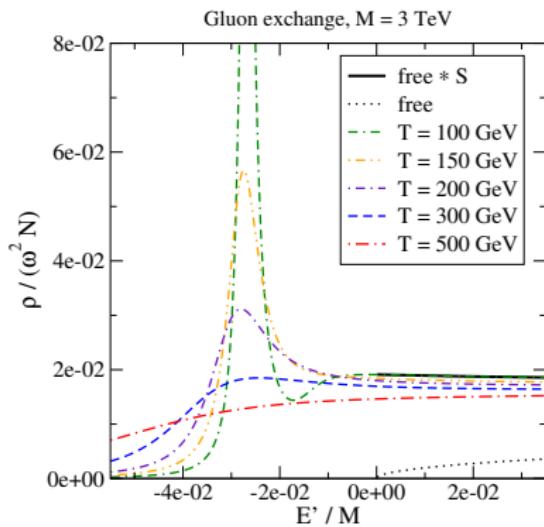
- Non-relativistic fields $\eta = \frac{1}{\sqrt{2M}} (\phi e^{-iMt} + \varphi^\dagger e^{iMt})$ and $\chi = (\psi e^{-iMt}, -i\sigma_2 \psi^* e^{iMt})$



$$\langle \sigma_{\text{eff}} v \rangle = \frac{2c_1 + 4c_2 N_c e^{-\Delta M_T/T} + [c_3 \bar{S}_3 + c_4 \bar{S}_4 C_F + 2c_5 \bar{S}_5 (N_c + 1)] N_c e^{-2\Delta M_T/T}}{(1 + N_c e^{-\Delta M_T/T})^2}$$

- \bar{S}_i from gluon exchange/gluo-dissociation/inelastic-parton scattering
- $\Delta M_T \equiv \Delta M + \frac{(g_s^2 C_F + \lambda_3) T^2}{12M} - \frac{g_s^2 C_F m_D}{8\pi}$, $\Delta M = M_\eta - M$
- ΔM plays an important role on
 - the importance of bound state effects
 - conversion rate $\chi \leftrightarrow \eta$

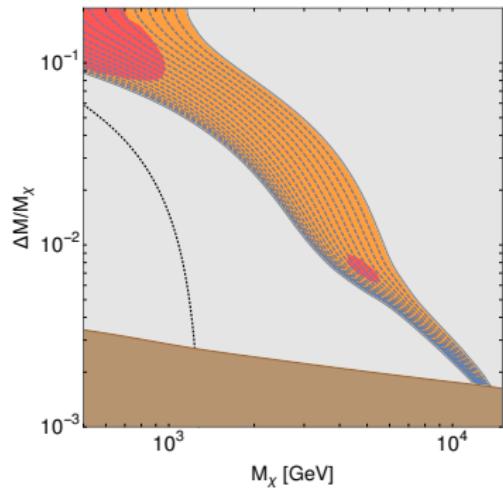
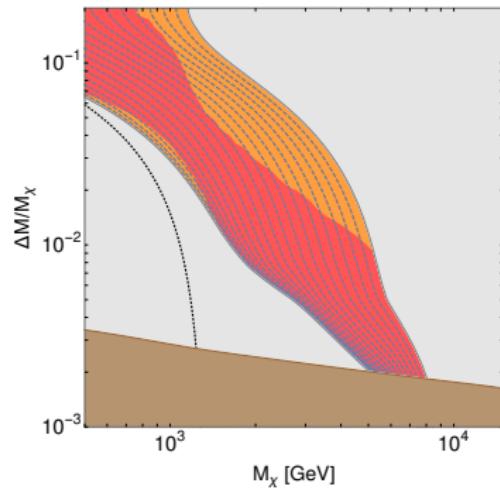
SPECTRAL FUNCTION, BOUND STATES AND MELTING



- bound states start to form at $z \sim 20$, $z \equiv M_\chi / T$
- at high temperatures: reduced Sommerfeld effect with respect to a massless gluon

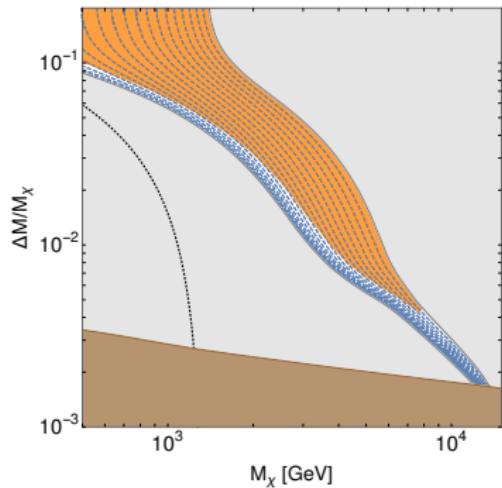
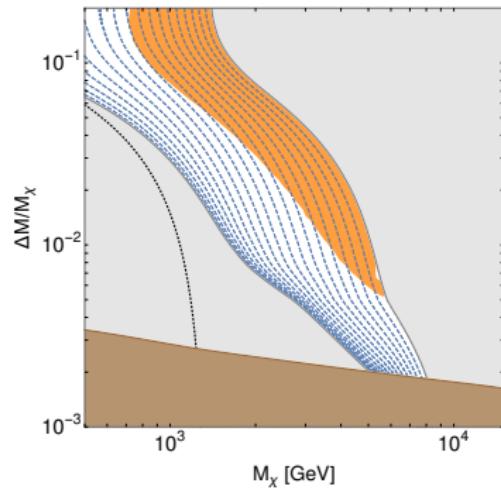
SB and M. Laine (2018)

PARAMETER SPACE FOR $\text{DM}_\chi + \eta$ MODEL S.B. AND S. VOGL (2018)



- $2\Delta M > |E_1|$, left $\lambda_3 = 0$, right $\lambda_3 = 1.5$, $y \in [0.1, 2]$, dotted-black $y = 0.3$ (free)
 - Xenon1T sensitivity and DARWIN-like detector sensitivity see talk by José Matias-Lopes
 - larger values λ_3 boost the annihilations because $c_3 \bar{S}_3 \approx (g_s^4 + \lambda_3^2) \bar{S}_3$
- $M_\chi \simeq 1.2 \text{ TeV} \rightarrow M_\chi \simeq 2.0 (3.1) \text{ TeV}$ for $\Delta M / M_\chi = 10^{-2}$ and $\lambda_3 = 0.0 (1.5)$

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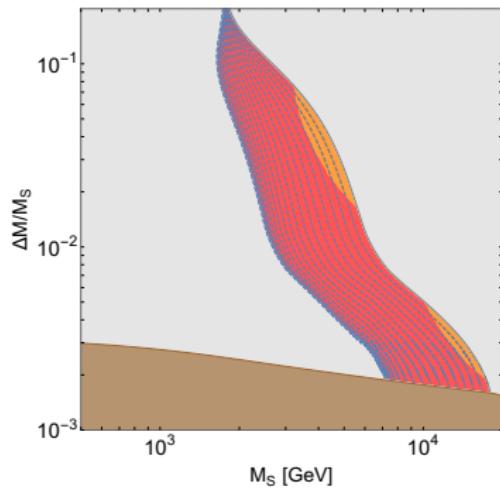
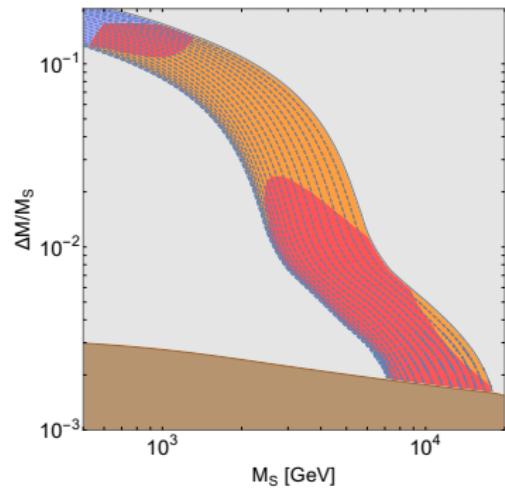


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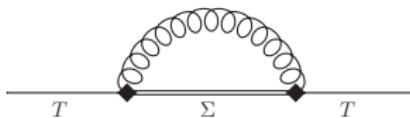
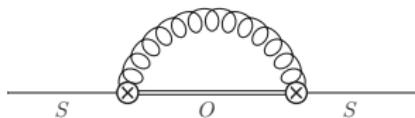
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PARAMETER SPACE FOR $DM_S + F$ MODEL

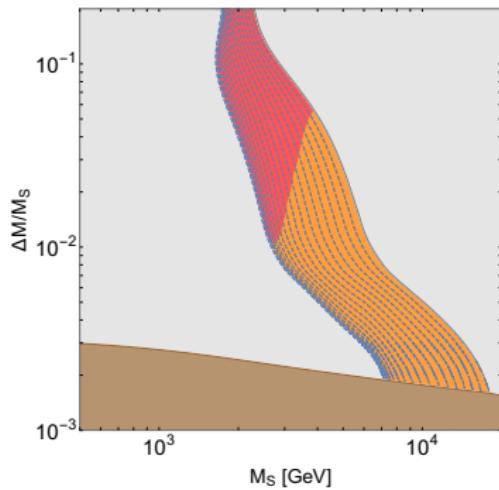
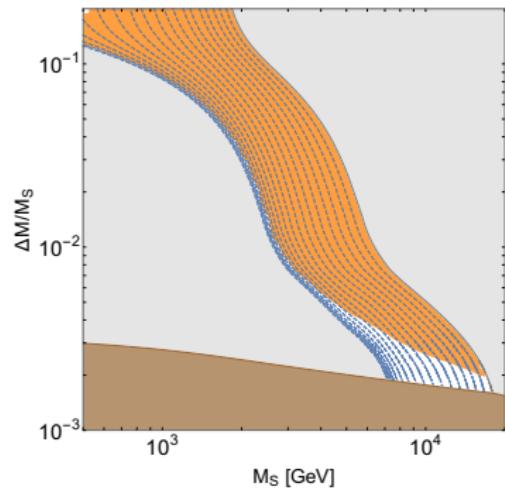
S.B. AND S. VOGL (2019)



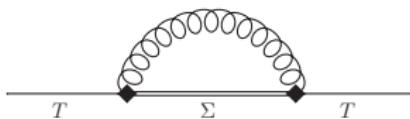
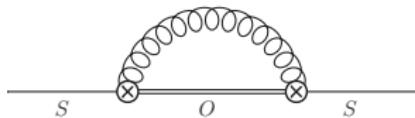
- **throat-like behaviour** due to $SS \rightarrow H^\dagger H$; for $\lambda_3 = 1.5$, $(m_t/M_S)^2$ suppression is mild
- there is an attractive potential and bound states from antitriplet (FF): $3 \otimes 3 = \bar{3} \oplus 6$



PARAMETER SPACE FOR $DM_S + F$ MODEL S.B. AND S. VOGL (2019)



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CONCLUSIONS AND OUTLOOK

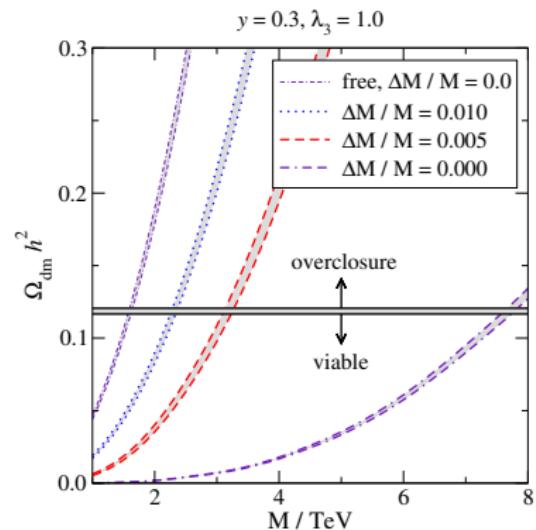
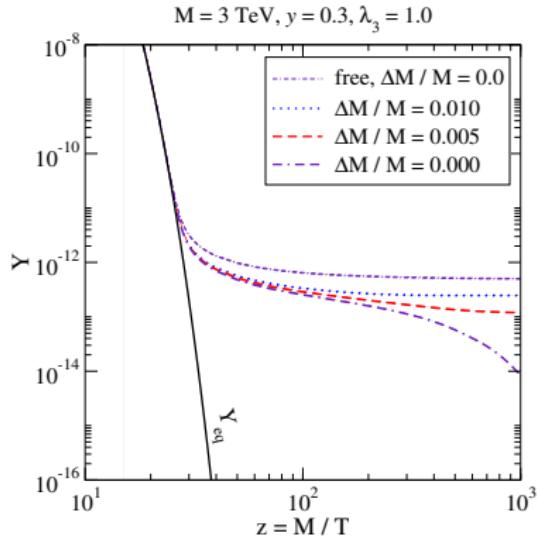
- the freeze-out calculation is factorized into $\langle \sigma v \rangle \approx c_i \langle \mathcal{O}_i \rangle_T$
- $E \sim M$: matching coefficients at $T = 0 \rightarrow$ NREFTs
- **existence of bound states** from a thermally modified Schrödinger equation for $\rho(E)$

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-
- Modest effect for the Inert Doublet Model: 1% to 20% (electroweak interactions)
 - Simplified models with mediator charged under QCD: **much larger effect**
 - ⇒ parameter space compatible with relic density **change substantially**: up to 18-20 TeV
 - ⇒ experimental prospects have to be adjusted accordingly
 - **data files available \bar{S}_3 at <http://www.laine.itp.unibe.ch/sommerfeld/>**
-
- Provide new updated benchmarks for cosmologically viable parameter space:
self-interactions in the hidden sector, SM charged coannihilators, scalar force carriers ...
 - Derive evolution equations for DM pairs (bound state and scattering states) from out-of-equilibrium field theory T. Binder, L. Covi and K. Mukaida (2018); S.B. and M. Laine (2019)

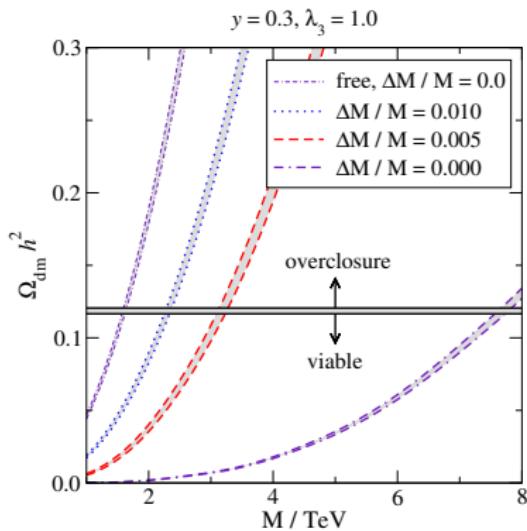
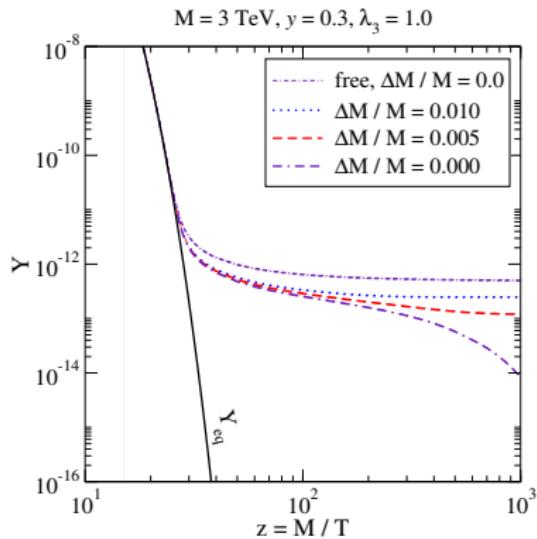
BACK-UP SLIDES

ISSUE WITH BOLTZMANN AND SPECTRAL FUNCTION



- a blind $\Delta M = 0$ brings to very large masses M , **however the splitting cannot be arbitrary small!**
- if $2\Delta M - |E_1| < 0$ the lightest two-particle states are $(\eta^\dagger \eta)$
 $\Rightarrow (\chi\chi)$ rapidly convert into $(\eta^\dagger \eta)$ that are short lived and promptly decay

ISSUE WITH BOLTZMANN AND SPECTRAL FUNCTION



- What if we deal with a simpler DM model without coannihilator?
- Then exponential growth of \bar{S}_s drives the DM to very small values

A DIFFERENT RATE EQUATION?

- Recently an alternative form of the BE has been suggested T. Binder, L. Covi and K. Mukaida (2018)

$$\dot{n} + 3Hn = -\langle \sigma v \rangle (e^{2\beta\mu(n)} - 1)n_{\text{eq}}^2$$

- μ couples to the total number of dark sector particles
- number density operator, total number of particles and eq. number density

$$\hat{N} = \int_x \hat{n}(x), \quad n_{\text{eq}} = \lim_{\mu \rightarrow 0} \langle \hat{n} \rangle$$

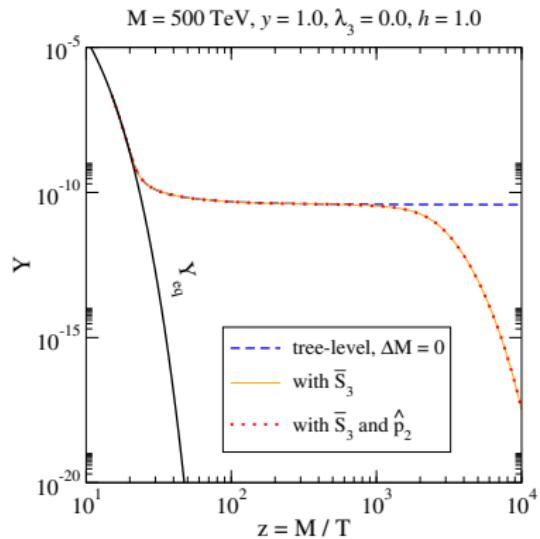
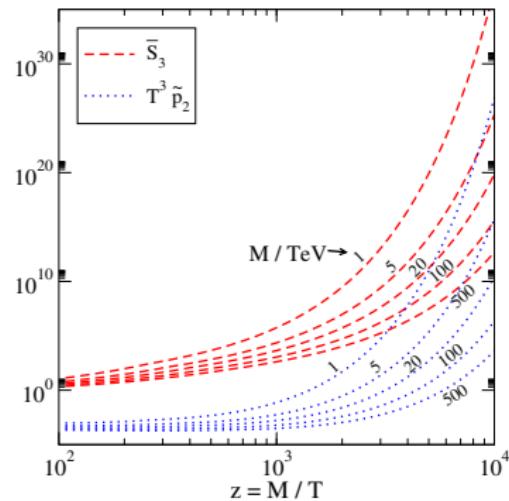
- the density matrix has the form

$$\hat{\rho} = \frac{\exp[-\beta(\hat{H} - \mu\hat{N})]}{Z}, \quad Z = e^{\mu\beta V}, \quad n(\mu) = \frac{\partial p}{\partial \mu}$$

- expand pressure in the fugacity expansion $p = p_0 + p_1 e^{\beta\mu} + p_2 e^{2\beta\mu} + \dots$, and obtain n

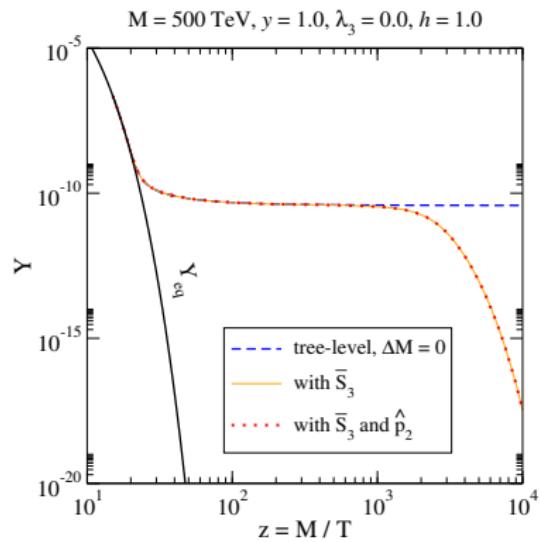
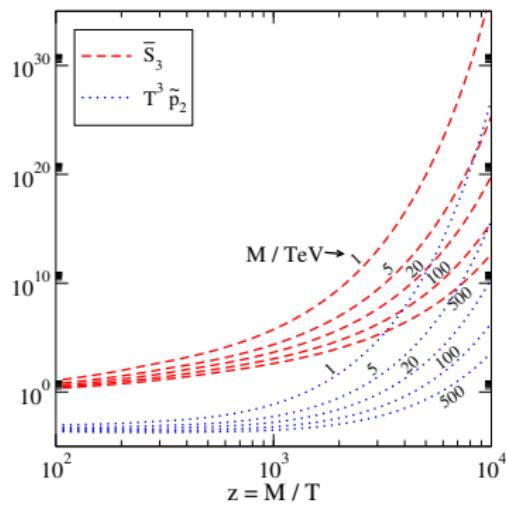
$$nT = p_1 e^{\beta\mu} + 2p_2 e^{2\beta\mu} + \dots, \quad e^{\beta\mu} n_{\text{eq}} \approx \frac{2n}{\sqrt{1 + 8\hat{p}_2 n}}$$

AGAIN THE SAME PROBLEM



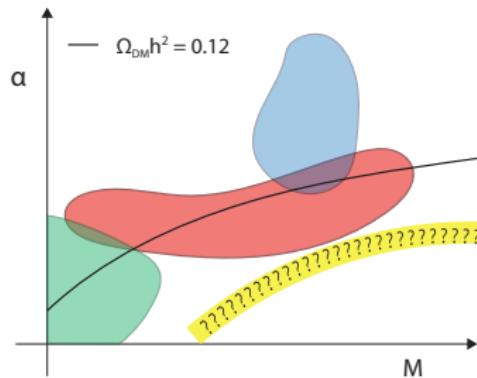
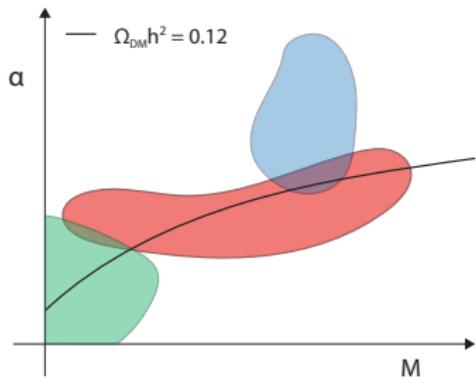
$$\hat{p}_2 \simeq \frac{N_c^2}{(N_c + e^{\beta \Delta M_T})^2} \tilde{p}_2, \quad T^3 \tilde{p}_2 \equiv \frac{2}{N_c^2} \left(\frac{\pi T}{M} \right)^{3/2} (e^{\Delta E \beta} - 1), \quad \bar{S}_3 \approx \left(\frac{4\pi}{MT} \right)^{3/2} \frac{e^{\Delta E \beta}}{\pi a^3}$$

AGAIN THE SAME PROBLEM

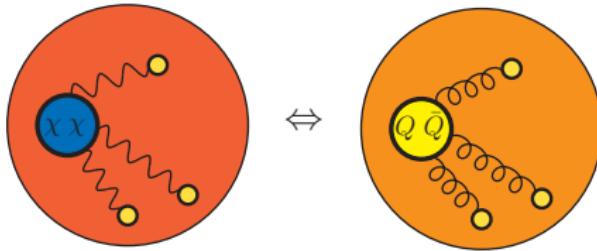


$$e^{\beta\mu} n_{\text{eq}} \approx \frac{2n}{\sqrt{1 + 8\hat{p}_2 n}}, \quad 8\hat{p}_2 n = 8T^3 \hat{p}_2 \frac{s}{T^3} Y, \quad \Omega_{\text{dm}} h^2 = \frac{Y(z_f)M}{[3.645 \times 10^{-12} \text{TeV}]} \approx 0.12$$

...OUTLOOK



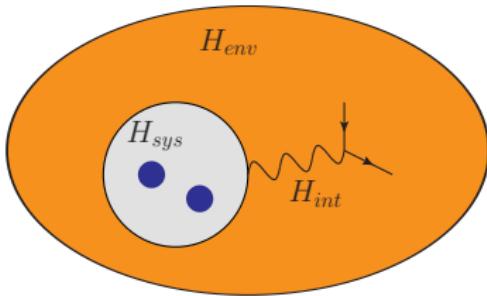
- generic DM model with $\alpha_{\text{DM}}, \dots$, sit down and write the pNREFT from the beginning



OUTLOOK...

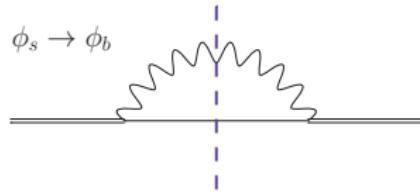
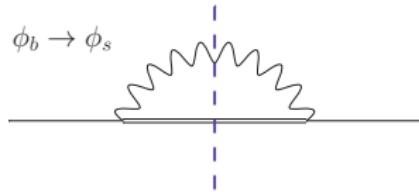
APPLY AN OPEN QUANTUM SYSTEM APPROACH

- heavy particle as a probe of the thermal environment
- hard probe can gain or dissipate energy with the environment



- formation, dissociation and regeneration
→ **real-time dynamics/non-equilibrium**
- scattering and bound states mix: $\rho \approx \rho_s + \rho_b$

$$\frac{d\rho}{dt} = -i[H_{sys}, \rho] + \sum_n \left(C_n \rho C_n^\dagger - \frac{1}{2} \{ C_n^\dagger C_n, \rho \} \right)$$



FROM ρ TO A SCHRÖDINGER EQUATION

- non-relativistic dynamics:

$$E_m \equiv \omega = E' + 2M + \frac{k^2}{4M} \text{ and } H = -\frac{\nabla^2}{M} + V(r, T)$$

- the spectral function $\rho(E')$ is obtained from

$$[H - i\Gamma(r, T) - E'] G(E'; \mathbf{r}, \mathbf{r}') = N\delta^3(\mathbf{r} - \mathbf{r}') \quad \lim_{r, r' \rightarrow 0} \text{Im}G(E'; \mathbf{r}, \mathbf{r}') = \rho(E')$$

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- form inhomogeneous to homogeneous equation M.J. Strassler and M.E. Peskin (1991)

$$x \equiv \alpha Mr \quad V \equiv \alpha^2 M \tilde{V}, \quad \Gamma \equiv \alpha^2 M \tilde{\Gamma}, \quad E' \equiv \alpha^2 M \tilde{E}'$$

- solve for the solution which is regular at the origin, $u_\ell(x) \sim x^{\ell+1}$ for $x \ll 1$

$$\left[-\frac{d^2}{dx^2} + \frac{\ell(\ell+1)}{x^2} + \tilde{V} - i\tilde{\Gamma} - \tilde{E}' \right] u_\ell(x) = 0$$

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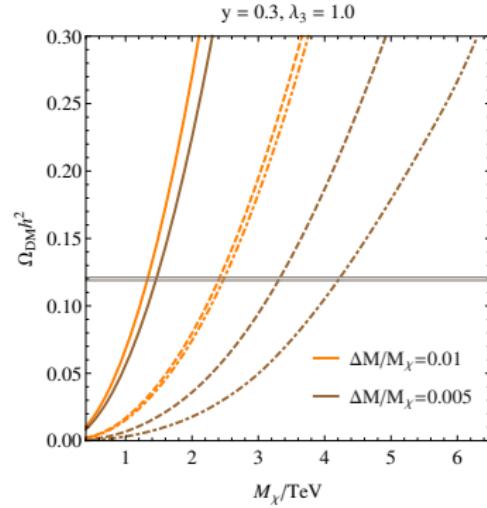
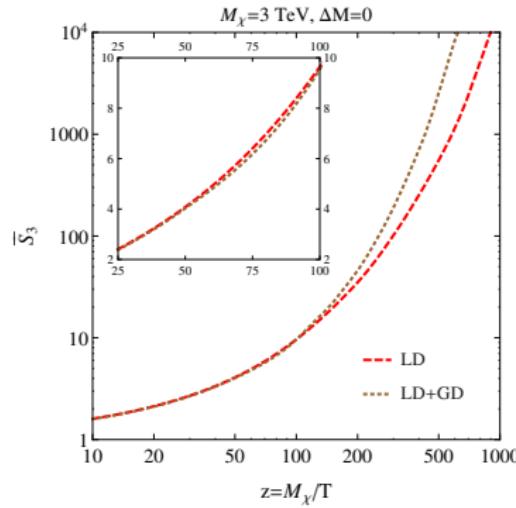
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- the s-wave ($\ell = 0$) spectral function is obtained from

$$\rho(E') = \frac{\alpha NM^2}{4\pi} \int_0^\infty dx \text{Im} \left[\frac{1}{u_0^2(x)} \right]$$

GLUO-DISSOCIATION

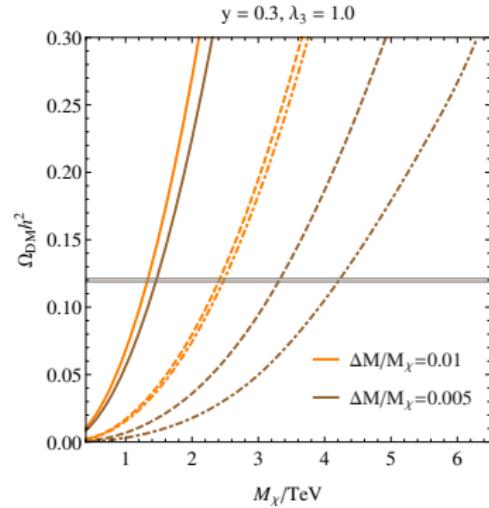
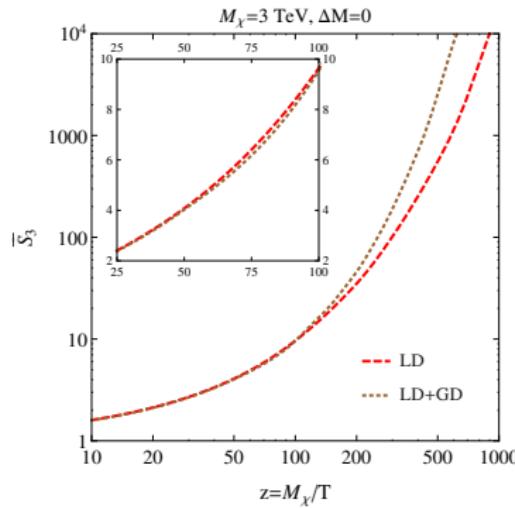


- borrow the result from heavy quarkonium $M \gg Mv \gg T \gg \Delta V$

$$\delta V_{\text{GD}} = \frac{4}{3} C_F \frac{\alpha_s}{\pi} r^2 T^2 \Delta V f(\Delta V/T), \quad \Gamma_{\text{GD}} = \frac{2}{3} C_F \alpha_s r^2 (\Delta V)^3 n_B(\Delta V),$$

N. Brambilla, J. Ghiglieri, A. Vairo and P. Petreczky 0804.0993

GLUO-DISSOCIATION



- this is not a rigorous way of implementing it:
other terms are missing beyond the static limit that are of the same order in the thermal width

N. Brambilla, M. A. Escobedo, J. Ghiglieri, J. Soto and A. Vairo (2010)

SECOND MODEL AND PNREFT

$$\mathcal{L}_{\text{pNRQCD}} = \mathcal{L}_{\text{light}} + \mathcal{L}_{\text{pNRQCD}}^{F\bar{F}} + \mathcal{L}_{\text{pNRQCD}}^{FF},$$

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$$\begin{aligned}\mathcal{L}_{\text{pNRQCD}}^{F\bar{F}} &= \int d^3r \text{Tr} \left\{ S^\dagger [i\partial_0 - \mathcal{V}_s - \delta M_s] S + O^\dagger [iD_0 - \mathcal{V}_o - \delta M_o] O \right\} \\ &+ \int d^3r \left(\text{Tr} \left\{ O^\dagger \mathbf{r} \cdot g \boldsymbol{\mathcal{E}} S + S^\dagger \mathbf{r} \cdot g \boldsymbol{\mathcal{E}} O \right\} + \frac{1}{2} \text{Tr} \left\{ O^\dagger \mathbf{r} \cdot g \boldsymbol{\mathcal{E}} O + O^\dagger \mathbf{r} \cdot g \boldsymbol{\mathcal{E}} O \right\} \right) + \dots\end{aligned}$$

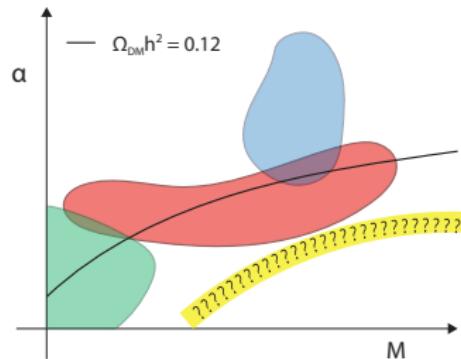
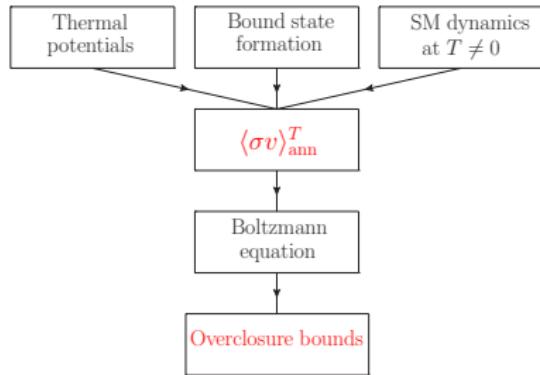
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SUMMARY OF THE THEORETICAL FRAMEWORK

RELIC DENSITY CAN BE FACTORIZED IN SOME STEPS

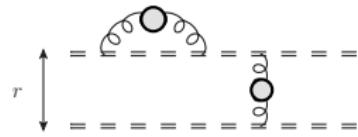
M. LAINE AND S. KIM 1609.00474

- Calculate the matching coefficients from the hard annihilation process, $E \sim 2M$
- Compute the static potentials and thermal widths
- Extract the spectral function \Rightarrow annihilation rate
- Solve the Boltzmann equation with the **thermal** cross section



THERMAL POTENTIALS

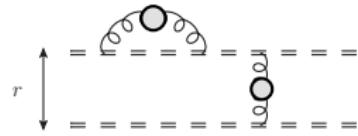
- the potential that plays a role involves QCD gluons



$$V \equiv \frac{g_s^2}{2} \int_{\mathbf{k}} e^{i\mathbf{k} \cdot \mathbf{r}} \left[\frac{1}{\mathbf{k}^2 + m_D^2} - i \frac{\pi T}{k} \frac{m_D^2}{(\mathbf{k}^2 + m_D^2)^2} \right]$$
$$m_D = g_s \sqrt{\frac{N_c}{3} + \frac{N_f}{6}} T$$

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$$m_D = g_s \sqrt{\frac{N_c}{3} + \frac{N_f}{6}} T$$

$$\langle \sigma_{\text{eff}} v \rangle = \frac{2c_1 + 4c_2 N_c e^{-\Delta M_T/T} + [c_3 \bar{S}_3 + c_4 \bar{S}_4 C_F + 2c_5 \bar{S}_5 (N_c + 1)] N_c e^{-2\Delta M_T/T}}{(1 + N_c e^{-\Delta M_T/T})^2}$$

MATCHING COEFFICIENTS

$$\begin{aligned}\mathcal{L}_{\text{NREFT}} &= i \left\{ c_1 \psi_p^\dagger \psi_q^\dagger \psi_q \psi_p + c_2 (\psi_p^\dagger \phi_\alpha^\dagger \psi_p \phi_\alpha + \psi_p^\dagger \varphi_\alpha^\dagger \psi_p \varphi_\alpha) \right. \\ &+ \left. c_3 \phi_\alpha^\dagger \varphi_\alpha^\dagger \varphi_\beta \phi_\beta + c_4 \phi_\alpha^\dagger \varphi_\beta^\dagger \varphi_\gamma \phi_\delta T_{\alpha\beta}^a T_{\gamma\delta}^a + c_5 (\phi_\alpha^\dagger \phi_\beta^\dagger \phi_\beta \phi_\alpha + \varphi_\alpha^\dagger \varphi_\beta^\dagger \varphi_\beta \varphi_\alpha) \right\}\end{aligned}$$

$$c_1 = 0, \quad c_2 = \frac{|y|^2(|h|^2 + g_s^2 C_F)}{128\pi M^2},$$

$$c_3 = \frac{1}{32\pi M^2} \left(\lambda_3^2 + \frac{g_s^4 C_F}{N_c} \right), \quad c_4 = \frac{g_s^4 (N_c^2 - 4)}{64\pi M^2 N_c}, \quad c_5 = \frac{|y|^4}{128\pi M^2}.$$

CONVERSION RATES: DM_S+F MODEL



...

$$\begin{aligned}\Gamma_{2 \rightarrow 1} &= \frac{|y|^2 N_c M_S}{4\pi} \left(\frac{\Delta M}{M_S} \right)^2 n_F(\Delta M) \\ \Gamma_{2 \rightarrow 2} &= \frac{|y|^2 N_c}{8M_S} \int_p \frac{\pi m_q^2 n_F \left(\Delta M + \frac{p^2}{2M_S} \right)}{p(p^2 + m_q^2)}\end{aligned}$$

