Marco Drewes, Université catholique de Louvain

A new Benchmark Value for N_{eff} in the SM

09. 10. 2020

PONT 2020

(unfortunately not in the Palais des Papes in Avignon)

In collaboration with Jack J. Bennett, Gilles Buldgen, Pablo F. de Salas, Stefano Gariazzo, Sergio Pastor, and Yvonne Y. Y. Wong

arXiv:2012.02726 [hep-ph],
JCAP 2003 (2020) 003, arXiv:1911.04504 [hep-ph],
JCAP 07 (2019) 014, arXiv:1905.11290 [astro-ph.CO].

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The Ultimate Value of N_{eff} in the SM?

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$$N_{\text{eff}}^{\text{SM}} = 3.0433 \pm 0.0002$$

arXiv:2012.02726 [hep-ph]

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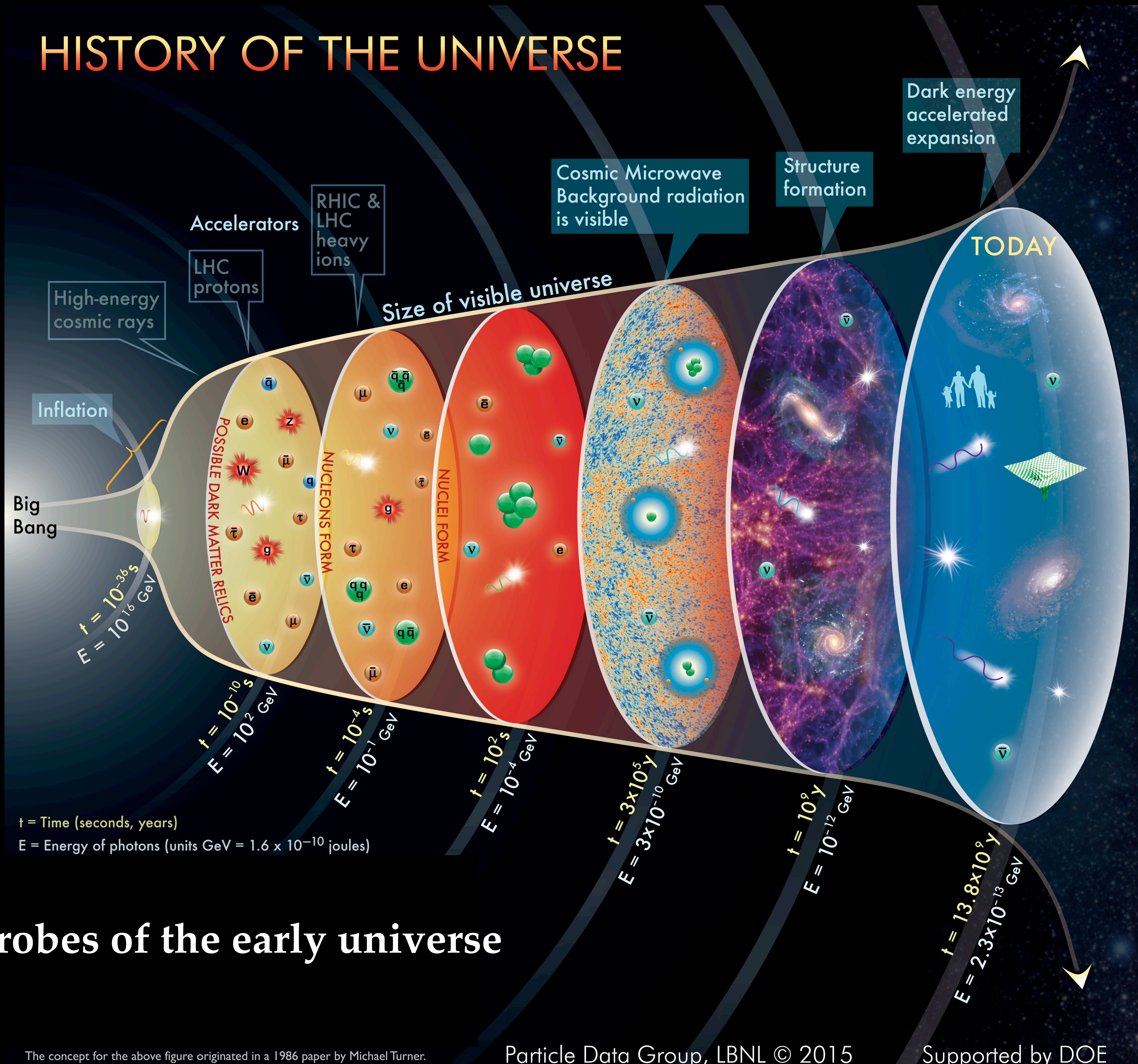
arXiv:2012.02726 [hep-ph]

Disclaimer:

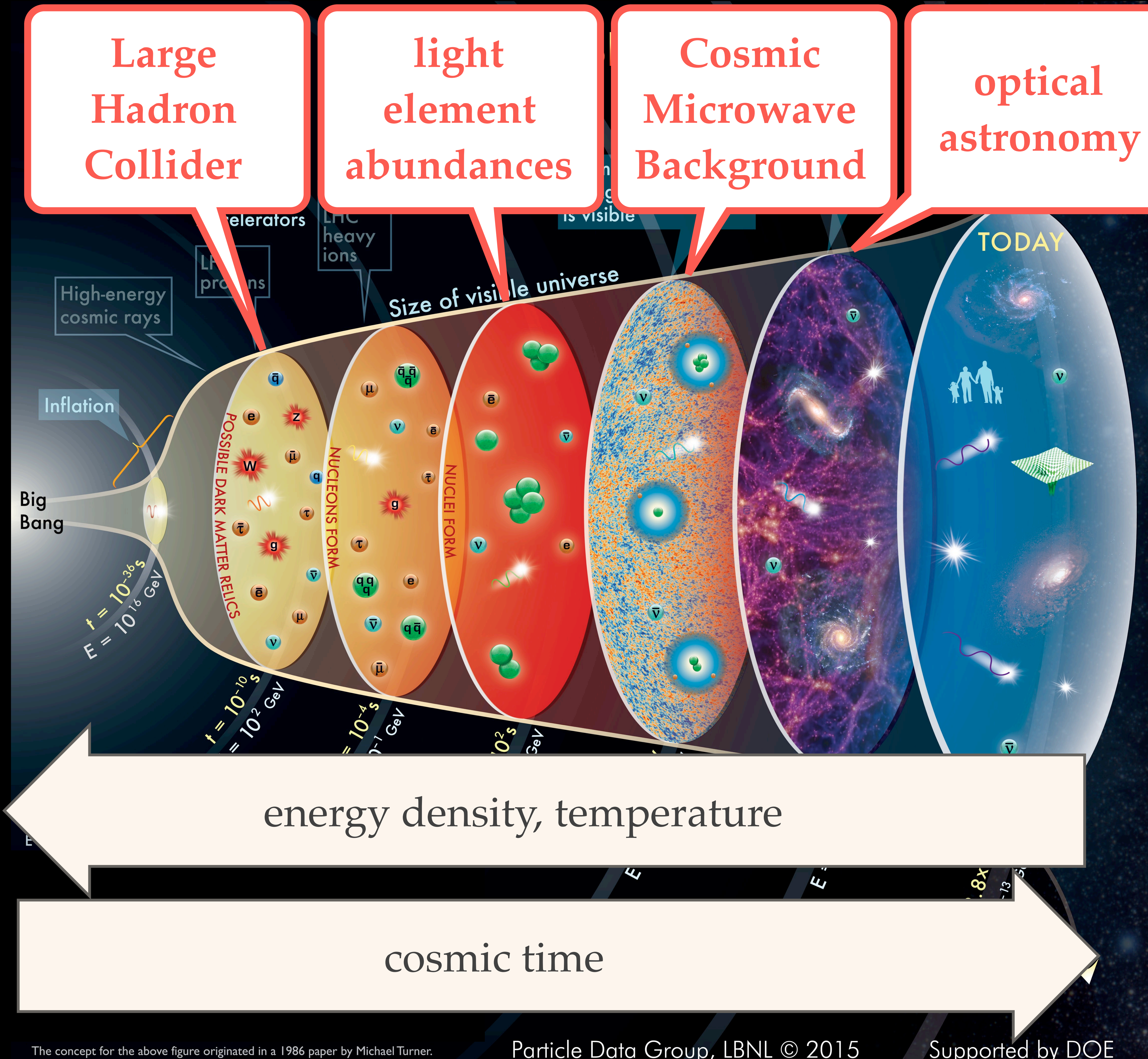
This talk is strongly focussed on arXiv:2012.02726 [hep-ph].

Other notable recent contributions include:

- Escudero 2001.04466 [hep-ph]
- Akita / Yamaguchi 2005.07047 [hep-ph]
- Froustey / Pitrou / Volpe 2008.01074 [hep-ph]
- [I apologise if I missed your work]



Probes of the early universe


Large Hadron Collider
light element abundances
Cosmic Microwave Background
optical astronomy
Size of visible universe
TODAY
High-energy cosmic rays
Inflation
Big Bang
t = 10^-36 s
E = 10^16 GeV
POSSIBLE DARK MATTER RELICS
NUCLEONS FORM
NUCLEI FORM
heavy ions
t = 10^-10 s
energy density, temperature
cosmic time
The concept for the above figure originated in a 1986 paper by Michael Turner.
Particle Data Group, LBNL © 2015
Supported by DOE

N_{eff} as a Probe of Cosmology and Particle Physics

Expansion of the universe is controlled by energy density,

$$H^2 = \frac{8\pi}{3} G\rho,$$

The Standard Model assumptions are:

- particle content of the Standard Model of particle physics (here: including neutrino masses)
- gravity described by General Relativity
- All species in thermal equilibrium at T >> 1 MeV

Under these assumptions one can define:

$$\left.\frac{\rho_\nu}{\rho_\gamma}\right|_{T/m_e \to 0} \equiv \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\rm eff}^{\rm SM}$$

N_{eff} as a Probe of Cosmology and Particle Physics

Expansion of the universe is controlled by energy density,

$$H^2 = \frac{8\pi}{3} G\rho,$$

Deviations from the Standard Model and/or General Relativity can be parameterised as:

$$\rho_\gamma + \rho_{\text{neutrinos}} + [\text{new physics effects}] \equiv \rho_\gamma + N_{\text{eff}}\,\rho_\nu = \frac{\pi^2}{15} T_\gamma^4 \left[1 + N_{\text{eff}} \frac{7}{8} \left(\frac{4}{11}\right)^{4/3}\right]$$

This can include:

- presence of extra particle species
- deviations from equilibrium
- deviations from the Hubble Law / General Relativity
- …

***N_{eff}* probes both, particle physics and cosmology!**

N_{eff} in the SM

N_{eff} = 3 under the assumptions

- primordial plasma is an ideal gas
- neutrino decoupling happens instantaneous
- neutrino decoupling happens at $T >> m_e$

All three assumptions are violated in reality!

A new Benchmark Value for N_{eff} in the SM

$$N_{\rm eff}^{\rm SM} = 3.0433 \pm 0.0002$$

arXiv:2012.02726 [hep-ph]

$$\left.\begin{aligned} N_{\rm eff} &= 2.96^{+0.34}_{-0.33}, \\ \sum m_\nu &< 0.12\ \text{eV}, \end{aligned}\right\}$$ 95 %, *Planck* TT,TE,EE+lowE +lensing+BAO.

arXiv:1807.06209 [astro-ph.CO]

Motivations for a Precision Computation

Upcoming experiment:

- CMB S4 will decrease observational error by a order of magnitude
- Theory error should be much smaller, so that the theory prediction can be treated as an "exact number"

Controversy in the literature:

$$N_{\text{eff}} = 3.044$$ arXiv:1905.11290 [hep-ph]

$$N_{\text{eff}} = 3.052$$ arXiv:1512.02205 [astro-ph.CO]

So what's new?

New ingredients in arXiv:2012.02726 [hep-ph]:

- Use new `FortEPiaNO` code
- Include thermal correction to the QED equation of state
- complete numerical evaluation of the neutrino–neutrino collision integral in the presence of neutrino flavour oscillations
- assessment of the uncertainties:

(i) numerical convergence.
(ii) the approximate modelling of the weak collision integrals in the presence of flavour oscillations,
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  arXiv:1905.11290 [astro-ph.CO] https://bitbucket.org/ahep_cosmo/fortepiano_public

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QED Equation of State

QED equation of state can be computed from partition function Z

In practice lnZ is expanded in powers of e

$$\ln Z = \ln Z^{(0)} + \ln Z^{(2)} + \ln Z^{(3)} + \; \cdots$$

From this, contributions to energy, pressure and entropy care computed

$$P^{(n)} = \frac{T}{V} \ln Z^{(n)},$$

$$\rho^{(n)} = \frac{T^2}{V} \frac{\partial \ln Z^{(n)}}{\partial T} = -P^{(n)} + T \frac{\partial P^{(n)}}{\partial T},$$

$$s^{(n)} = \frac{1}{V} \frac{\partial \left[T \ln Z^{(n)} \right]}{\partial T} = \frac{\rho^{(n)} + P^{(n)}}{T},$$

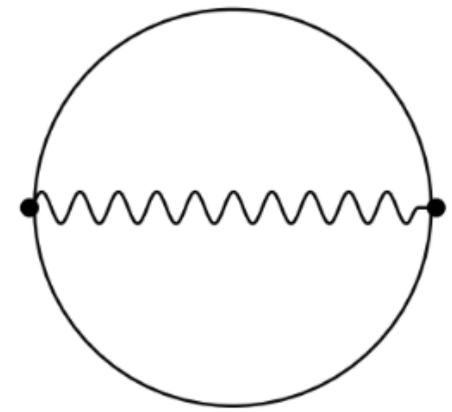
QED Equation of State

At zeroth order one finds ideal gas

$$P^{(0)} = \frac{T}{\pi^2} \int_0^\infty \mathrm{d}p \, p^2 \; \ln \left[\frac{(1 + \mathrm{e}^{-E_e/T})^2}{(1 - \mathrm{e}^{-E_\gamma/T})} \right],$$

$$\rho^{(0)} = \frac{1}{\pi^2} \int_0^\infty \mathrm{d}p \, p^2 \left[\frac{2E_e}{e^{E_e/T} + 1} + \frac{E_\gamma}{e^{E_\gamma/T} - 1} \right],$$

QED Equation of State O(e²)

The first correction comes from $\ln Z^{(2)} = -\frac{1}{2}$ 

$$P^{(2)} = \frac{T}{V} \ln Z^{(2)} = -\frac{e^2 T^2}{12\pi^2} \int_0^\infty \mathrm{d}p \, \frac{p^2}{E_p} n_D - \frac{e^2}{8\pi^4} \left(\int_0^\infty \mathrm{d}p \, \frac{p^2}{E_p} n_D \right)^2 + \frac{e^2 m_e^2}{16\pi^4} \iint_0^\infty \mathrm{d}p \, \mathrm{d}\tilde{p} \, \frac{p\tilde{p}}{E_p E_{\tilde{p}}} \ln \left| \frac{p+\tilde{p}}{p-\tilde{p}} \right| n_D \tilde{n}_D,$$

Kapusta / Gale, Finite-temperature field theory: Principles and applications.

Usually the log-dependent term is neglected

QED Equation of State O(e²)[no log]

Neglecting the log term yields

$$P^{(2)} = \frac{T}{V}\ln Z^{(2)} = -\frac{e^2T^2}{12\pi^2}\int_0^\infty \mathrm{d}p\,\frac{p^2}{E_p}n_D - \frac{e^2}{8\pi^4}\left(\int_0^\infty \mathrm{d}p\,\frac{p^2}{E_p}n_D\right)^2$$

$$\rho^{(2)\not{\ln}} = -\frac{e^2T^2}{12\pi^2}\int_0^\infty \mathrm{d}p\frac{p^2}{E_p}\left(n_D + T\partial_T n_D\right) + \frac{e^2}{8\pi^4}\left(\int_0^\infty \mathrm{d}p\frac{p^2}{E_p}n_D\right)^2 - \frac{e^2}{4\pi^4}\left(\int_0^\infty \mathrm{d}p\frac{p^2}{E_p}\,n_D\right)\left(\int_0^\infty \mathrm{d}p\frac{p^2}{E_p}\,T\partial_T n_D\right),$$

QED Equation of State O(e²)

Adding the log term yields

$$P^{(2)} = \frac{T}{V} \ln Z^{(2)} = -\frac{e^2T^2}{12\pi^2} \int_0^\infty \mathrm{d}p \, \frac{p^2}{E_p} n_D - \frac{e^2}{8\pi^4} \left(\int_0^\infty \mathrm{d}p \, \frac{p^2}{E_p} n_D \right)^2 + \frac{e^2 m_e^2}{16\pi^4} \iint_0^\infty \mathrm{d}p \, \mathrm{d}\tilde{p} \, \frac{p\tilde{p}}{E_p E_{\tilde{p}}} \ln \left| \frac{p+\tilde{p}}{p-\tilde{p}} \right| n_D \tilde{n}_D,$$

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$$\rho^{(2)\,\ln} = \frac{e^2 m_e^2}{16\pi^4} \iint_0^\infty \mathrm{d}p \, \mathrm{d}\tilde{p} \, \frac{p\tilde{p}}{E_p E_{\tilde{p}}} \ln \left| \frac{p+\tilde{p}}{p-\tilde{p}} \right| \, n_D \left(2T\partial_T \tilde{n}_D - \tilde{n}_D \right)$$

Avoiding Mistakes

- The previous formulae look a bit complicated, and one may be tempted to simplify them…

- It is well known that some properties of a system of interaction particles can be described in the **quasiparticle picture**, i.e., by absorbing the effects of the interactions into modified dispersion relations and couplings of effective quasiparticles [Landau, Weldon, Klimov…]

- So maybe one can simply insert modified dispersion relations into the ideal gas energy and pressure?

$$E_\gamma^2(p) \to E_\gamma^2(p,T) = p^2 + \delta m_\gamma^2(T),$$
$$E_e^2(p) \to E_e^2(p,T) = p^2 + m_e^2 + \delta m_e^2(p,T)$$

Avoiding Mistakes

However: While the quasiparticle picture works reasonably well in transport equations [e.g. Arnold/Yaffe/Moore hep-ph/0209353], it in general does not describe bulk properties well.

- In the present context it only works well to order e^2

- The quasiparticle approximation describes interaction rates well, but "double counts" the effect of the interactions when being used to compute bulk properties

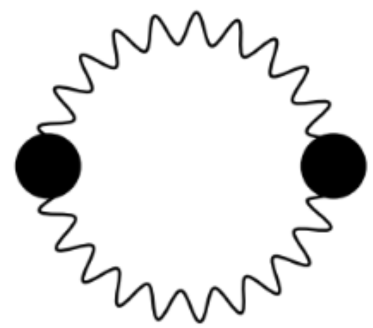
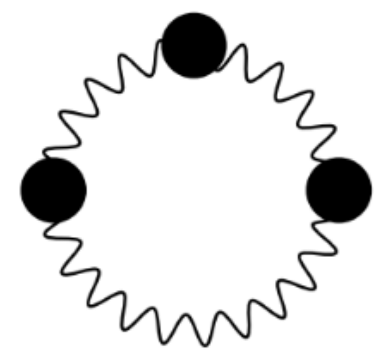
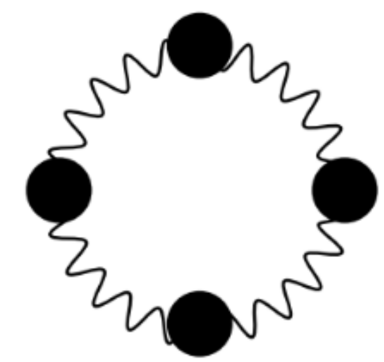
  $\Rightarrow$ need to add factor 1/2 in front of corrections!

  This factor 1/2 was missed in [1512.02205], which explains the huge value $N_{\rm eff} = 3.052$

  Discrepancy in te literature solved!

QED Equation of State O(e^3)

The next correction comes from

$\ln Z^{(3)} = \frac{1}{2}\Big[\ \frac{1}{2}$  $-\ \frac{1}{3}$  $+\ \frac{1}{4}$  $+\ \cdots\ \Big]$

Kapusta / Gale, Finite-temperature field theory: Principles and applications.

It reads

$$P^{(3)} = \frac{T}{V}\ln Z^{(3)} = \frac{e^3 T}{12\pi^4} I^{3/2}(T) \qquad \rho^{(3)} = \frac{e^3 T^2}{8\pi^4} I^{1/2} \partial_T I$$

with

$$I(T) = \int_0^\infty \mathrm{d}p \left(\frac{p^2 + E_p^2}{E_p} \right) n_D$$

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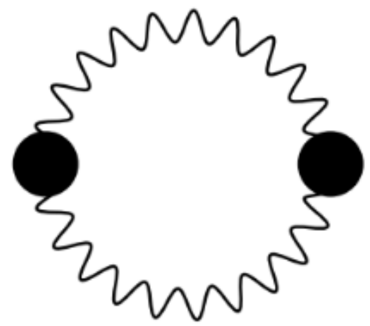
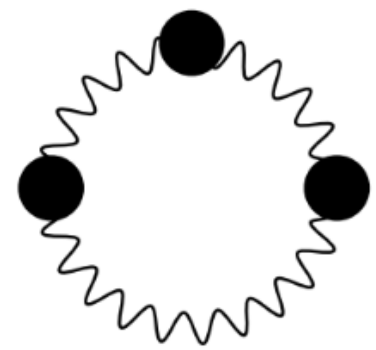
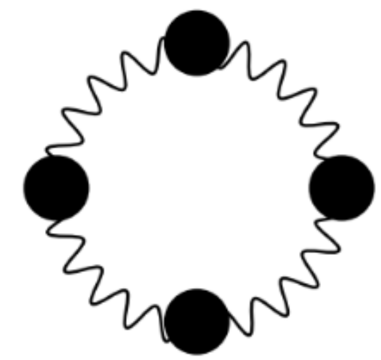
$\ln Z^{(3)} = \frac{1}{2}\Big[\frac{1}{2}$  $-\frac{1}{3}$  $+\frac{1}{4}$  $+\ \cdots\Big]$

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with

$$I(T) = \int_0^\infty \mathrm{d}p \left(\frac{p^2 + E_p^2}{E_p}\right) n_D$$

- This correction was previously neglected
- It turns out to be important

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Quantum Kinetic Equations

N_{eff} is computed by solving the continuity equation

$$(\mathrm{d}/\mathrm{d}t)\rho_{\rm tot} + 3H(\rho_{\rm tot} + P_{\rm tot}) = 0,$$

and a density matrix equations for the neutrinos

$$\partial_t \varrho - pH\partial_p = -\mathrm{i}[\mathbb{H}, \varrho] + \mathcal{I}[\varrho]$$

effective Hamiltonian including neutrino refraction (“matter potential” or “thermal mass”)

collision integral

Quantum Kinetic Equations

Using $\quad x \equiv a m_e, \; y \equiv a p, \;\; z \equiv a T_\gamma \quad$ this reads

$$\frac{\mathrm{d}}{\mathrm{d}x}\bar{\rho}_{\mathrm{tot}}(x, z(x)) = \frac{1}{x}\left[\bar{\rho}_{\mathrm{tot}}(x, z(x)) - 3\bar{P}_{\mathrm{tot}}(x, z(x))\right]$$

and

$$\frac{\mathrm{d}\varrho(x,y)}{\mathrm{d}x} = \frac{1}{m_e}\frac{m_e^4}{x^4}\sqrt{\frac{3m_{\mathrm{Pl}}^2}{8\pi\bar{\rho}_{\mathrm{tot}}}}\left(-\mathrm{i}\Big[\mathbb{H}(x,y,z(x)), \varrho(x,y)\Big] + \mathcal{I}[\varrho(x,y)]\right)$$

with

$$\mathbb{H}(x,y,z(x)) = \frac{x^6}{m_e^6}\frac{U\mathbb{M}U^\dagger}{2y} - 2\sqrt{2}G_F y\left(\frac{\mathbb{E}_\ell(z(x)) + \mathbb{P}_\ell(z(x))}{m_W^2} + \frac{4}{3}\frac{\mathbb{E}_\nu(x, z(x))}{m_W^2\cos^2\theta_W}\right)$$

Quantum Kinetic Equations

Using $x \equiv a m_e,\ y \equiv a p,\ \ z \equiv a T_\gamma$ this reads

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and

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vacuum mass M and mixing matrix *U*

“matter potentials”

Collision Integrals

Neutrino-electron scattering and annihilation:

$$\mathcal{I}_{\nu e}[\varrho(x,y)] = \frac{G_F^2}{(2\pi)^3 y^2}\left\{I_{\nu e}^{\rm sc}[\varrho(x,y)] + I_{\nu e}^{\rm ann}[\varrho(x,y)]\right\} ,$$

$$\begin{aligned} I_{\nu e}^{\rm sc} = & \int {\rm d}y_2 {\rm d}y_3 \frac{y_2}{\epsilon_2}\frac{y_4}{\epsilon_4} \\ & \times \Big\{ (\Pi_{2a}^{\rm sc}(y,y_2) + \Pi_{2b}^{\rm sc}(y,y_4)) \left[F_{\rm sc}^{LL}\left(\varrho^{(1)}, f_e^{(2)}, \varrho^{(3)}, f_e^{(4)}\right) + F_{\rm sc}^{RR}\left(\varrho^{(1)}, f_e^{(2)}, \varrho^{(3)}, f_e^{(4)}\right)\right] \\ & \quad -2x^2 \Pi_1^{\rm sc}(y,y_3)\left[F_{\rm sc}^{RL}\left(\varrho^{(1)}, f_e^{(2)}, \varrho^{(3)}, f_e^{(4)}\right) + F_{\rm sc}^{LR}\left(\varrho^{(1)}, f_e^{(2)}, \varrho^{(3)}, f_e^{(4)}\right)\right]\Big\} , \end{aligned}$$

$$\begin{aligned} I_{\nu e}^{\rm ann} = & \int {\rm d}y_2 {\rm d}y_4 \frac{y_3}{\epsilon_3}\frac{y_4}{\epsilon_4} \\ & \times \Big\{ \Pi_{2b}^{\rm ann}(y,y_4) F_{\rm ann}^{LL}\left(\varrho^{(1)}, \varrho^{(2)}, f_e^{(3)}, f_e^{(4)}\right) + \Pi_{2a}^{\rm ann}(y,y_3) F_{\rm ann}^{RR}\left(\varrho^{(1)}, \varrho^{(2)}, f_e^{(3)}, f_e^{(4)}\right) \\ & \quad +x^2 \Pi_1^{\rm ann}(y,y_2)\left[F_{\rm ann}^{RL}\left(\varrho^{(1)}, \varrho^{(2)}, f_e^{(3)}, f_e^{(4)}\right) + F_{\rm ann}^{LR}\left(\varrho^{(1)}, \varrho^{(2)}, f_e^{(3)}, f_e^{(4)}\right)\right]\Big\} , \end{aligned}$$

$$\begin{aligned} D_1(a,b,c,d) &= \frac{16}{\pi}\int_0^\infty \frac{{\rm d}\lambda}{\lambda^2} \prod_{i=a,b,c,d} \sin(\lambda i) , \\ D_2(a,b,c,d) &= -\frac{16}{\pi}\int_0^\infty \frac{{\rm d}\lambda}{\lambda^4} \prod_{i=a,b} [\lambda i \cos(\lambda i) - \sin(\lambda i)] \prod_{j=c,d} \sin(\lambda j) , \\ D_3(a,b,c,d) &= \frac{16}{\pi}\int_0^\infty \frac{{\rm d}\lambda}{\lambda^6} \prod_{i=a,b,c,d} [\lambda i \cos(\lambda i) - \sin(\lambda i)] . \end{aligned}$$

$$\begin{aligned} \Pi_1^{\rm sc}(y,y_3) &= y\, y_3\, D_1 + D_2(y,y_3,y_2,y_4), \\ \Pi_1^{\rm ann}(y,y_2) &= y\, y_2\, D_1 - D_2(y,y_2,y_3,y_4), \\ \Pi_{2a}^{\rm sc}(y,y_2)/2 &= y\,\epsilon_2\, y_3\,\epsilon_4\, D_1 + D_3 - y\,\epsilon_2 D_2(y_3,y_4,y,y_2) - y_3\,\epsilon_4 D_2(y,y_2,y_3,y_4), \\ \Pi_{2b}^{\rm sc}(y,y_4)/2 &= y\,\epsilon_2\, y_3\,\epsilon_4\, D_1 + D_3 + \epsilon_2\, y_3 D_2(y,y_4,y_2,y_3) + y\,\epsilon_4 D_2(y_2,y_3,y,y_4), \\ \Pi_{2a}^{\rm ann}(y,y_3)/2 &= y\, y_2\,\epsilon_3\,\epsilon_4\, D_1 + D_3 + y\,\epsilon_3 D_2(y_2,y_4,y,y_3) + y_2\,\epsilon_4 D_2(y,y_3,y_2,y_4), \\ \Pi_{2b}^{\rm ann}(y,y_4)/2 &= y\, y_2\,\epsilon_3\,\epsilon_4\, D_1 + D_3 + y_2\,\epsilon_3 D_2(y,y_4,y_2,y_3) + y\,\epsilon_4 D_2(y_2,y_3,y,y_4), \end{aligned}$$

$$\begin{aligned} F_{\rm sc}^{ab} &\equiv f_e^{(4)}(\mathbb{1} - f_e^{(2)}) G^a \varrho^{(3)} G^b (\mathbb{1} - \varrho^{(1)}) - f_e^{(2)}(\mathbb{1} - f_e^{(4)}) \varrho^{(1)} G^b (\mathbb{1} - \varrho^{(3)}) G^a + {\rm h.c.} \\ F_{\rm ann}^{ab} &\equiv f_e^{(3)} f_e^{(4)} G^a (\mathbb{1} - \varrho^{(2)}) G^b (\mathbb{1} - \varrho^{(1)}) - (\mathbb{1} - f_e^{(3)})(\mathbb{1} - f_e^{(4)}) G^a \varrho^{(2)} G^b \varrho^{(1)} + {\rm h.c.}, \end{aligned}$$

$$\begin{aligned} G^L &= {\rm diag}(g_L, \tilde{g}_L, \tilde{g}_L), \\ G^R &= {\rm diag}(g_R, g_R, g_R) \end{aligned}$$

Collision Integrals

Neutrino-neutrino scattering:

$$\mathcal{I}_{\nu\nu}[\varrho(x,y)] = \frac{G_F^2}{(2\pi)^3 y^2}\left\{I_{\nu\nu}^{\rm sc}[\varrho(x,y)] + I_{\nu\nu}^{\rm pair}[\varrho(x,y)]\right\} \ ,$$

$$I_{\nu\nu}^{\rm sc} = \frac{1}{4}\int \mathrm{d}y_2 \mathrm{d}y_3\, \Pi_{2a}^{\rm sc}(y,y_2)\, F_{\rm sc}^{\nu\nu}\left(\varrho^{(1)},\varrho^{(2)},\varrho^{(3)},\varrho^{(4)}\right),$$

$$I_{\nu\nu}^{\rm pair} = \frac{1}{4}\int \mathrm{d}y_2 \mathrm{d}y_3\, \Pi_{2b}^{\rm sc}(y,y_4)\, F_{\rm pair}^{\nu\nu}\left(\varrho^{(1)},\varrho^{(2)},\varrho^{(3)},\varrho^{(4)}\right).$$

$$F_{\rm sc}^{\nu\nu} \equiv (\mathbb{1}-\varrho^{(1)})\varrho^{(3)}\left[(\mathbb{1}-\varrho^{(2)})\varrho^{(4)} + \mathrm{tr}(\cdots)\right] - \varrho^{(1)}(\mathbb{1}-\varrho^{(3)})\left[\varrho^{(2)}(\mathbb{1}-\varrho^{(4)}) + \mathrm{tr}(\cdots)\right] + \mathrm{h.c.},$$

$$\begin{aligned} F_{\rm pair}^{\nu\nu} \equiv\, & (\mathbb{1}-\varrho^{(1)})(\mathbb{1}-\varrho^{(2)})\left[\varrho^{(4)}\varrho^{(3)} + \mathrm{tr}(\cdots)\right] - \varrho^{(1)}\varrho^{(2)}\left[(\mathbb{1}-\varrho^{(4)})(\mathbb{1}-\varrho^{(3)}) + \mathrm{tr}(\cdots)\right] \\ & + (\mathbb{1}-\varrho^{(1)})\varrho^{(3)}\left[\varrho^{(4)}(\mathbb{1}-\varrho^{(2)}) + \mathrm{tr}(\cdots)\right] - \varrho^{(1)}(\mathbb{1}-\varrho^{(3)})\left[(\mathbb{1}-\varrho^{(4)})\varrho^{(2)} + \mathrm{tr}(\cdots)\right] + \mathrm{h.c.}, \end{aligned}$$

$$\begin{aligned} \Pi_1^{\rm sc}(y,y_3) &= y\, y_3\, D_1 + D_2(y,y_3,y_2,y_4), \\ \Pi_1^{\rm ann}(y,y_2) &= y\, y_2\, D_1 - D_2(y,y_2,y_3,y_4), \\ \Pi_{2a}^{\rm sc}(y,y_2)/2 &= y\,\epsilon_2\, y_3\,\epsilon_4\, D_1 + D_3 - y\,\epsilon_2 D_2(y_3,y_4,y,y_2) - y_3\,\epsilon_4 D_2(y,y_2,y_3,y_4), \\ \Pi_{2b}^{\rm sc}(y,y_4)/2 &= y\,\epsilon_2\, y_3\,\epsilon_4\, D_1 + D_3 + \epsilon_2\, y_3 D_2(y,y_4,y_2,y_3) + y\,\epsilon_4 D_2(y_2,y_3,y,y_4), \\ \Pi_{2a}^{\rm ann}(y,y_3)/2 &= y\, y_2\,\epsilon_3\,\epsilon_4\, D_1 + D_3 + y\,\epsilon_3 D_2(y_2,y_4,y,y_3) + y_2\,\epsilon_4 D_2(y,y_3,y_2,y_4), \\ \Pi_{2b}^{\rm ann}(y,y_4)/2 &= y\, y_2\,\epsilon_3\,\epsilon_4\, D_1 + D_3 + y_2\,\epsilon_3 D_2(y,y_4,y_2,y_3) + y\,\epsilon_4 D_2(y_2,y_3,y,y_4), \end{aligned}$$

includes terms to fourth oder in ϱ… highly non-linear!

Collision Integrals

- Most of the complication lies in the collision integrals.
- It is very common to approximate the momentum dependence in different ways, use linear response, assume kinetic equilibrium etc…
- In this work we use the full momentum dependence
- We test different approximations:

  1. diagonal $\{\mathcal{I}_{\nu\nu}[\varrho]\}_{\alpha\alpha} = 0$
  2. Solve $\{\mathcal{I}_{\nu\nu}[\varrho]\}_{\alpha\alpha}$ assuming diagonal ϱ
  3. keep full dependence

So what's new?

New ingredients in arXiv:2012.02726 [hep-ph]:

- Use new `FortEPiaNO` code

- Include thermal correction to the QED equation of state

- complete numerical evaluation of the neutrino–neutrino collision integral in the presence of neutrino flavour oscillations

- assessment of the uncertainties:

(i) numerical convergence.
(ii) the approximate modelling of the weak collision integrals in the presence of flavour oscillations,
(iii) measurement errors in the physical parameters of the weak sector.

Results!!!

Conservation Law Tests

We use four tests of conservation laws to assess the numerical error

1. Switch off collision integral
   coming neutrino number and energy density must be conserved

2. Neutrino-Neutrino collisions only
   coming neutrino number and energy density must be conserved

3. Neutrino-electron elastic scattering only
   coming neutrino number density must be conserved

4. No electron-positron annihilations into neutrinos
   coming neutrino number density must be conserved

Conservation Law Tests

N_y grid	$\mathcal{I}[\varrho]$	N_y	Number [%]	Energy [%]
GL	No collisions	30	0.0000	0.0000
		40	0.0000	0.0000
		50	0.0000	0.0000
	Neutrino–neutrino only	30	0.0001	0.0001
		40	0.0000	0.0000
		50	0.0000	0.0000
	Neutrino-electron elastic only	30	0.0021	–
		40	0.0025	–
		50	0.0026	–
	No e^+e^--annihilation	30	0.0087	–
		40	0.0073	–
		50	0.0064	–
NC	No collisions	60	0.0000	0.0000
		80	0.0000	0.0000
		100	0.0000	0.0000
	Neutrino–neutrino only	60	0.0000	0.0000
		80	0.0000	0.0000
		100	0.0000	0.0000
	Neutrino–electron elastic only	60	0.0029	–
		80	0.0029	–
		100	0.0029	–
	No e^+e^--annihilation	60	0.0029	–
		80	0.0029	–
		100	0.0029	–

Gauss-Laguerre (GL) quadrature

Newton–Cotes (NC) quadrature

- **NC has better convergence**
- **accuracy of conservation law can be used to estimate the error on N_{eff}**

QED Equation of State

	$N_{\rm eff}^{\rm SM}$
Benchmark A — $\{\mathcal{I}_{\nu\nu}[\varrho]\}_{\alpha\alpha} = 0$	
Assuming: • Normal ordering • $(2)\cancel{\ln} + (2)\ln + (3) +$ type (a) weak rates • Damping for both $\{\mathcal{I}_{\nu e}[\varrho]\}_{\alpha\beta}$ and $\{\mathcal{I}_{\nu\nu}[\varrho]\}_{\alpha\beta}$ • $N_y = 35$, GL spacing of y_i nodes	**3.0429**
Alternative estimates	
Flavour oscillations	
Inverted ordering	3.0429
No oscillations	3.0420
Finite-temperature QED corrections	
$(2)\cancel{\ln}$	3.04392
$(2)\cancel{\ln} + (2)\ln$	3.04389
$(2)\cancel{\ln} + (3)$	3.04296
$(2)\cancel{\ln} + (2)\ln + (3)$	3.04292

Adding the O(e³) term is equally important as adding neutrino oscillations!

Dependence on Particle Physics Parameters

	Parameter [Units]	Value $\pm$ 1σ uncertainty
QED	$\alpha/10^{-3}$	$7.2973525693 \pm 0.0000000011$
	m_e [MeV]	$0.51099895000 \pm 0.00000000015$
Weak	$\sin^2\theta_W$	0.23871 ± 0.00009
	G_F [$10^{-5}\mathrm{GeV}^{-2}$]	1.1663787 ± 0.0000006
	m_W [GeV]	80.379 ± 0.012
Neutrino	$\sin^2\theta_{12}/10^{-1}$	$3.20^{+0.20}_{-0.16}$
	$\sin^2\theta_{13}/10^{-2}$	$2.160^{+0.083}_{-0.069}$
	$\sin^2\theta_{23}/10^{-1}$	$5.47^{+0.20}_{-0.30}$
	Δm^2_{21} [$10^{-5}\mathrm{eV}^2$]	$7.55^{+0.20}_{-0.16}$
	Δm^2_{31} [$10^{-3}\mathrm{eV}^2$] (NO)	2.50 ± 0.03
	Δm^2_{31} [$10^{-3}\mathrm{eV}^2$] (IO)	$-2.42^{+0.03}_{-0.04}$

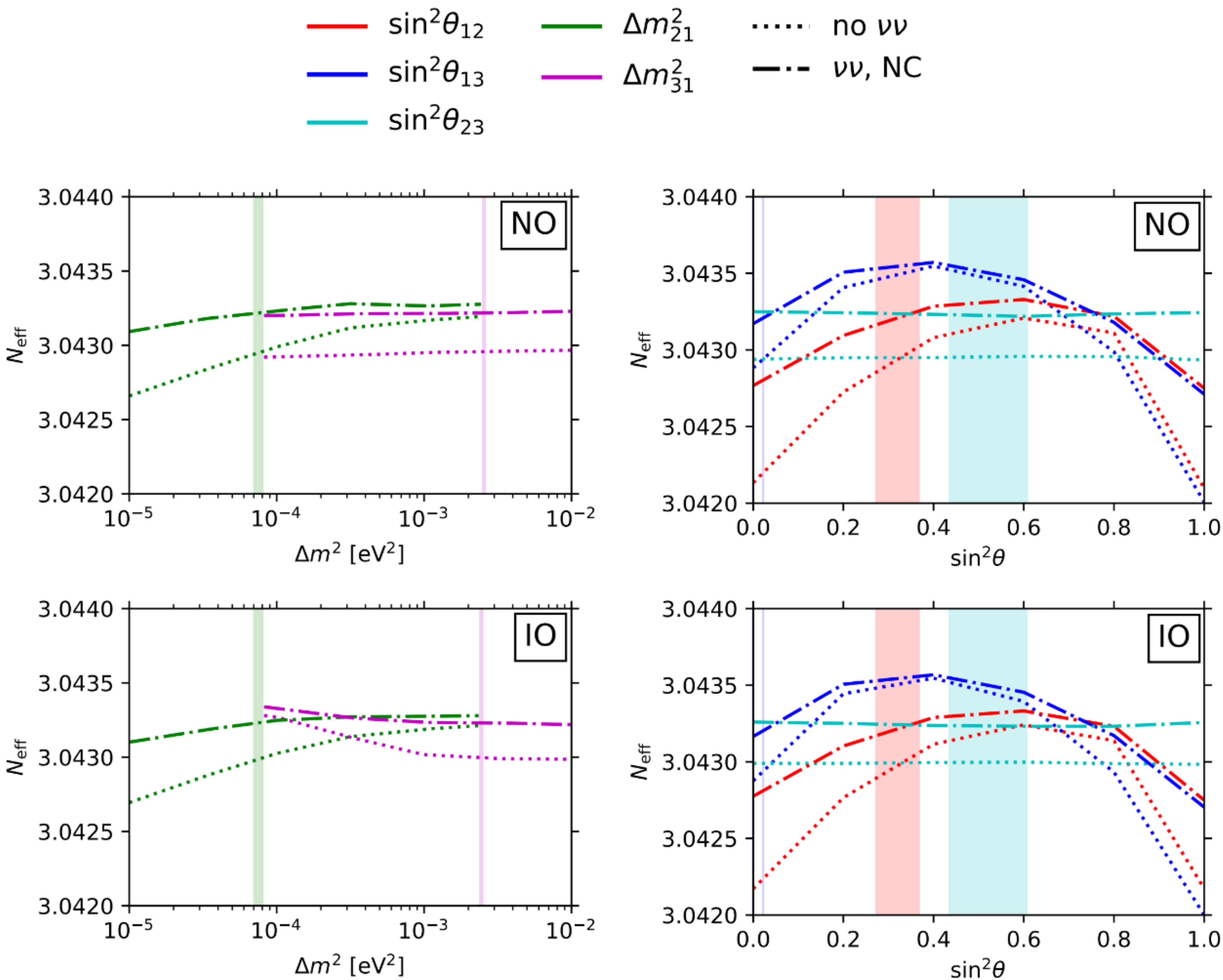
Dependence on Particle Physics Parameters

- errors on QED parameters are totally negligible

- effect of errors on weak interaction parameters is smaller than numerical uncertainty

- errors on neutrino mass splitting have no significant effect (neutrinos oscillate many times in one Hubble time, the averaged oscillation time only depends on the mixing angles)

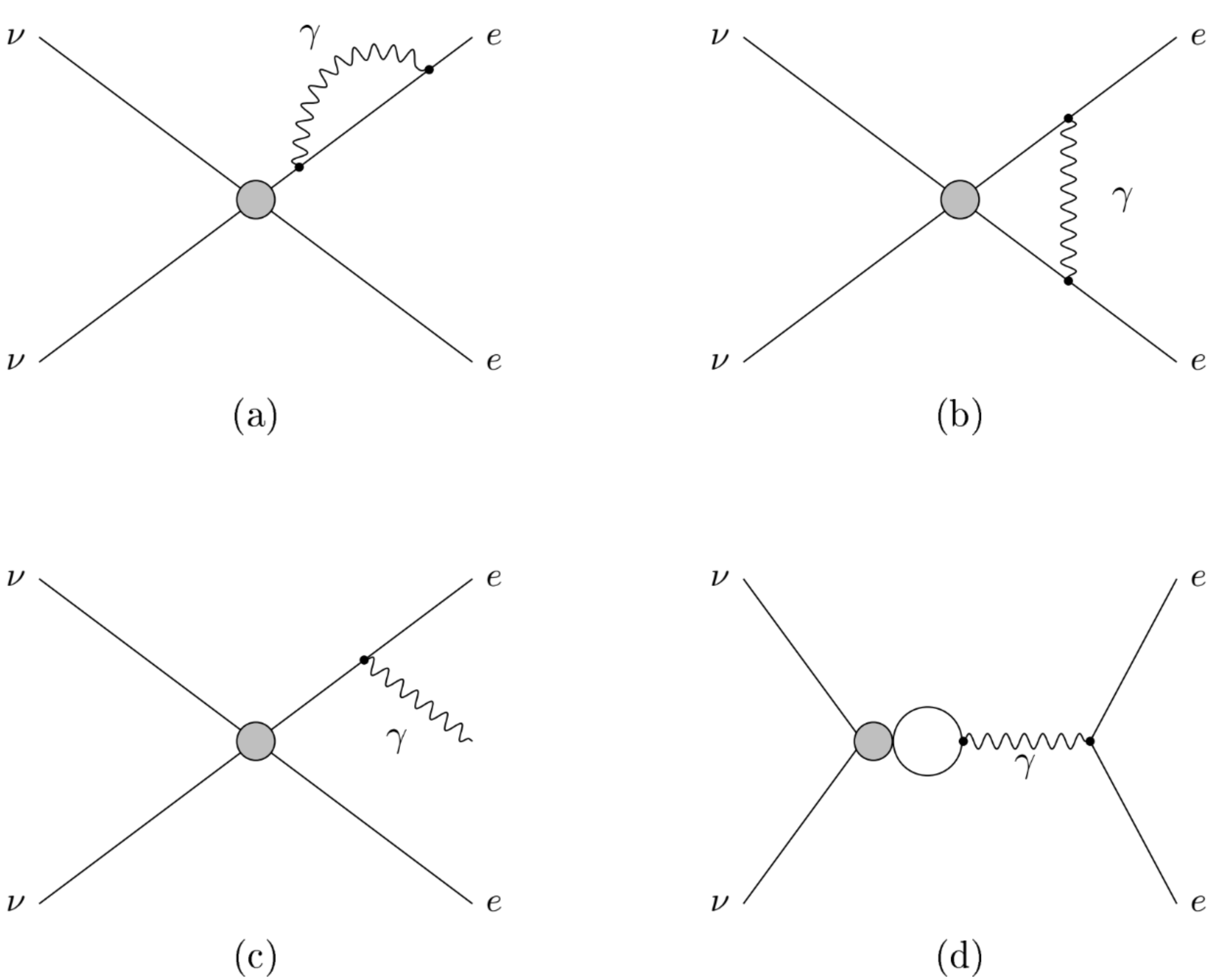
- error on $\sin^2 \theta_{23}$ also has no significant effect (it simply converts muon-neutrinos into tauon-neutrinos, both of which have the same effect on N_{eff})



$\sin^2\theta_{12}$
$\sin^2\theta_{13}$
$\sin^2\theta_{23}$
Δm^2_{21}
Δm^2_{31}
no $\nu\nu$
$\nu\nu$, NC
NO
IO
$N_{\rm eff}$
Δm^2 [eV^2]
$\sin^2\theta$
3.0420
3.0425
3.0430
3.0435
3.0440
10^{-5}
10^{-4}
10^{-3}
10^{-2}
0.0
0.2
0.4
0.6
0.8
1.0

What was not accounted for?

- We did not incorporate thermal QED corrections to collision integrals



- Estimate in [2001.04466] suggests $\delta N_{eff} \sim -0.0007$

- To be investigated in more detail…

Conclusion

- The updated calculation of N_{eff} improves on several aspects:
  - Include thermal correction to the QED equation of state
  - complete numerical evaluation of the neutrino–neutrino collision integral in the presence of neutrino flavour oscillations
  - assessment of the uncertainties:
- We believe that this exhausts all possible effects within the SM that would change N_{eff} appreciably.
- In that sense, this may be **the ultimate computation of N_{eff} in the SM**

Please complain if you disagree with this,
and let us know what we may have missed!

Backup Slides

Dependence on Momentum Discretisation

N_y grid	$\{\mathcal{I}_{\nu\nu}[\varrho]\}_{\alpha\alpha}$	N_y	$N_{\rm eff}^{\rm SM}$ (no osc)	$N_{\rm eff}^{\rm SM}$ (NO)
GL	$\{\mathcal{I}_{\nu\nu}[\varrho]\}_{\alpha\alpha} = 0$	30	3.0420	3.0430
		40	3.0420	3.0430
		50	3.0420	3.0430
	Diagonal ϱ	30	3.0488	3.0490
		40	3.0464	3.0468
		50	3.0452	3.0456
	Full	30	3.0488	3.0490
		40	3.0464	3.0468
		50	3.0452	3.0456
NC	$\{\mathcal{I}_{\nu\nu}[\varrho]\}_{\alpha\alpha} = 0$	60	3.0420	3.0430
		80	3.0419	3.0429
		100	3.0419	3.0429
	Diagonal ϱ	60	3.0427	3.0432
		80	3.0427	3.0432
		100	3.0427	3.0432
	Full	60	3.0427	3.0433
		80	3.0427	3.0433
		100	3.0427	3.0433

Gauss-Laguerre (GL) quadrature

Newton–Cotes (NC) quadrature

Dependence on Momentum Discretisation

N_y grid	$\{\mathcal{I}_{\nu\nu}[\varrho]\}_{\alpha\alpha}$	N_y	$N_{\text{eff}}^{\text{SM}}$ (no osc)	$N_{\text{eff}}^{\text{SM}}$ (NO)
GL	$\{\mathcal{I}_{\nu\nu}[\varrho]\}_{\alpha\alpha} = 0$	30	3.0420	3.0430
		40	3.0420	3.0430
		50	3.0420	3.0430
	Diagonal ϱ	30	3.0488	3.0490
		40	3.0464	3.0468
		50	3.0452	3.0456
	Full	30	3.0488	3.0490
		40	3.0464	3.0468
		50	3.0452	3.0456
NC	$\{\mathcal{I}_{\nu\nu}[\varrho]\}_{\alpha\alpha} = 0$	60	3.0420	3.0430
		80	3.0419	3.0429
		100	3.0419	3.0429
	Diagonal ϱ	60	3.0427	3.0432
		80	3.0427	3.0432
		100	3.0427	3.0432
	Full	60	3.0427	3.0433
		80	3.0427	3.0433
		100	3.0427	3.0433

Gauss-Laguerre (GL) quadrature

NC has better convergence

Newton–Cotes (NC) quadrature

Dependence on Initialisation Time