

Thermal production and interaction rates in the Early Universe

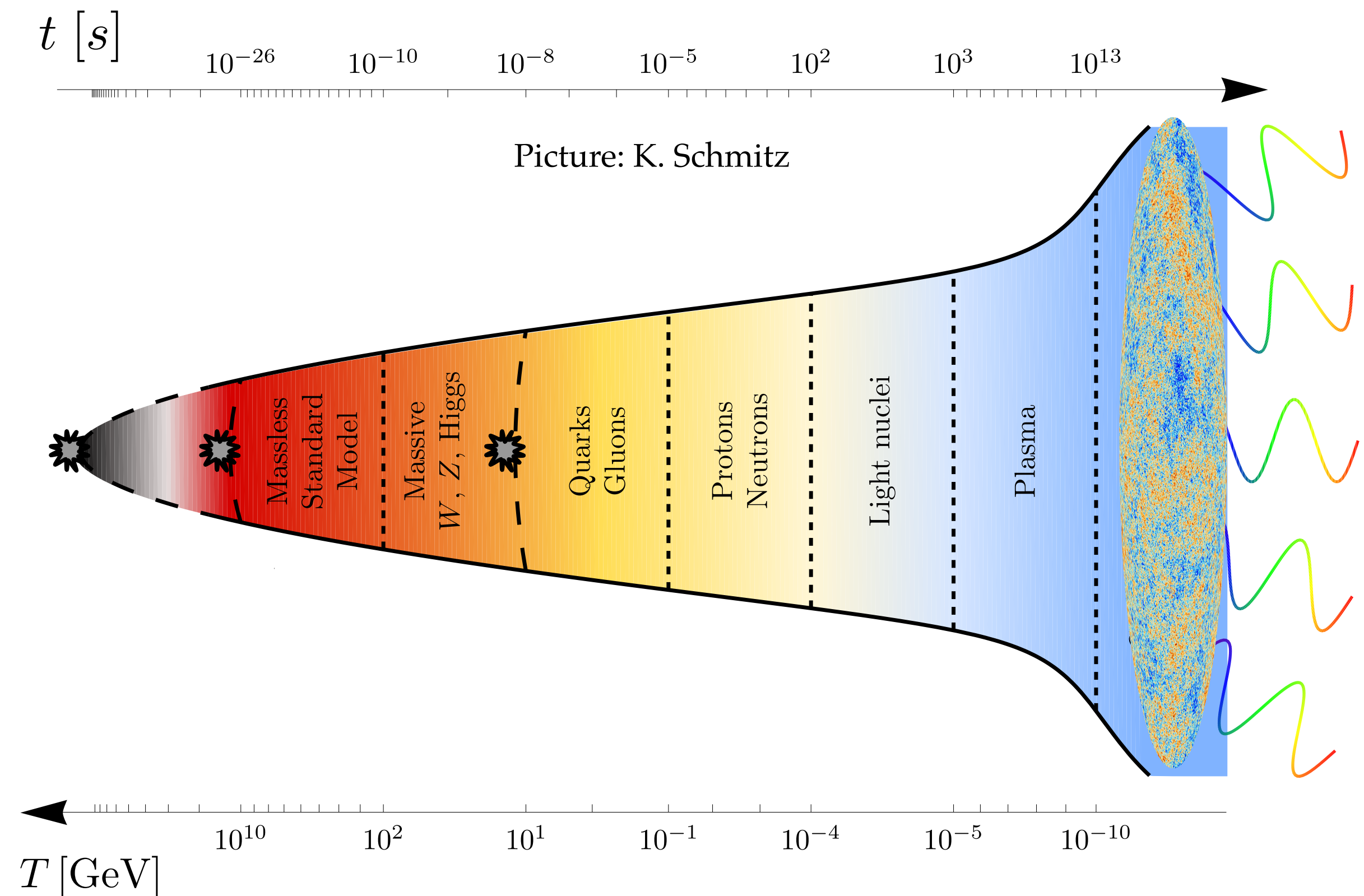


Jacopo Ghiglieri, SUBATECH, Nantes

PONT2020, Avignon, December 9 2020

Rates in the early universe

- Over the long thermal history, many phenomena enter and / or leave equilibrium
- DM candidates
- Mechanisms for the BAU
- Thermal relics
-
- governed by rates (production, equilibration, interaction) competing with H



In this talk

- Recent progress in defining and computing (some of) these rates using modern Thermal Field Theory (TFT) techniques (connection to previous approaches)
- Slowly-varying modes over a fast background
- Massless states: the example of gravitational waves
- Massive states: the example of right-handed neutrinos and NLO corrections
- Not shown:
 - Thermodynamics (phase transitions, the Hubble rate itself,...)

General approach

- Factor the system into “fast” and “slow” modes, and integrate out the former to obtain evolution eqs. for the latter



Production and equilibration

Production and equilibration

- A particle ϕ is weakly coupled (coupling h) to an equilibrated **bath** with its internal couplings g $\mathcal{L} = \mathcal{L}_\phi + h\phi J + \mathcal{L}_{\text{bath}}$
 J built of **bath fields**, one can prove to first order in h and **all orders** in g

$$\dot{f}_\phi(t, \mathbf{k}) = \Gamma(k) [f_{\text{eq}}(k^0) - f_\phi(t, \mathbf{k})] + \mathcal{O}(h^4)$$

$$\Gamma(k) = \frac{h^2}{2k^0} \int d^4 X e^{iK \cdot X} \langle [J(X), J(0)] \rangle$$

Bödeker Sangel Wörmann **PRD93** (2016)

- Single-particle phase-space distribution: $f(t, \mathbf{k})$, sensible only for sufficiently weakly interacting particles
- For conserved charges, equations for the density n can similarly be defined with no quasiparticle assumptions Bödeker Laine (2014)

Production and equilibration

$$\dot{f}_\phi(t, \mathbf{k}) = \Gamma(k) [f_{\text{eq}}(k^0) - f_\phi(t, \mathbf{k})] + \mathcal{O}(h^4) \quad \Gamma(k) = \frac{h^2}{2k^0} \int d^4 X e^{iK \cdot X} \langle [J(X), J(0)] \rangle$$

- The derivation is based (and relies) on a **separation of timescales** between production / equilibration and the plasma dynamics
- **All-order proof** of the equivalence of production and equilibration rates, $\Gamma_{\text{prod}} = \Gamma(k) f_{\text{eq}}(k^0)$. Goes beyond previous statements based on detailed balance in a leading-order Boltzmann approach.
- When doing perturbative expansions, Boltzmann expressions are recovered where applicable (LO). Higher orders are possible and natural in this form
- Easier to include non-perturbative input in this framework if needed
e.g. S. Biondini's talk yesterday

Production and equilibration

$$\dot{f}_\phi(t, \mathbf{k}) = \Gamma(k) [f_{\text{eq}}(k^0) - f_\phi(t, \mathbf{k})] + \mathcal{O}(h^4) \quad \Gamma(k) = \frac{h^2}{2k^0} \int d^4 X e^{iK \cdot X} \langle [J(X), J(0)] \rangle$$

- Applications of this TFT result to heavy ions (e.g. photon production, thermalisation) and cosmology. Not in this talk:
- Non-equilibrium Kadanoff-Baym equations yield similar results [Drewes \(2010\)](#) [Drewes Mendizabal Weniger \(2013\)](#) [Garny Hohenegger Kartavtsev \(2010-13\)](#)
- Cases where $f(t, \mathbf{k}) \gg 1$ (e.g. bosonic fields during reheating) and classical non-perturbative methods are used [Figueroa Florio Torrenti Valkenburg \(2020\)](#)

Production and equilibration

$$\dot{f}_\phi(t, \mathbf{k}) = \Gamma(k) [f_{\text{eq}}(k^0) - f_\phi(t, \mathbf{k})] + \mathcal{O}(h^4) \quad \Gamma(k) = \frac{h^2}{2k^0} \int d^4 X e^{iK \cdot X} \langle [J(X), J(0)] \rangle$$

- When using these equations in cosmology, the l.h.s is modified to include Hubble expansion

$$\dot{f}_\phi(t, \mathbf{k}) \rightarrow (\partial_t - H\mathbf{k} \cdot \nabla_{\mathbf{k}}) f_\phi(t, \mathbf{k})$$

and often (number, energy) densities are the quantity of interest, e.g. $n_\phi = \int_{\mathbf{k}} f_\phi$

$$\dot{n}_\phi + 3Hn_\phi = \int_{\mathbf{k}} \Gamma(k) [f_{\text{eq}}(k^0) - f_\phi(t, k)]$$

- If **scale separation** is present and $g \ll 1$, perturbative expansion of $\Gamma(k \gtrsim T)$ can reproduce standard Boltzmann. But **quasiparticle picture is not necessary!**
- Caution needed in extrapolating $\Gamma(k \gtrsim T)$ to $k \ll T$

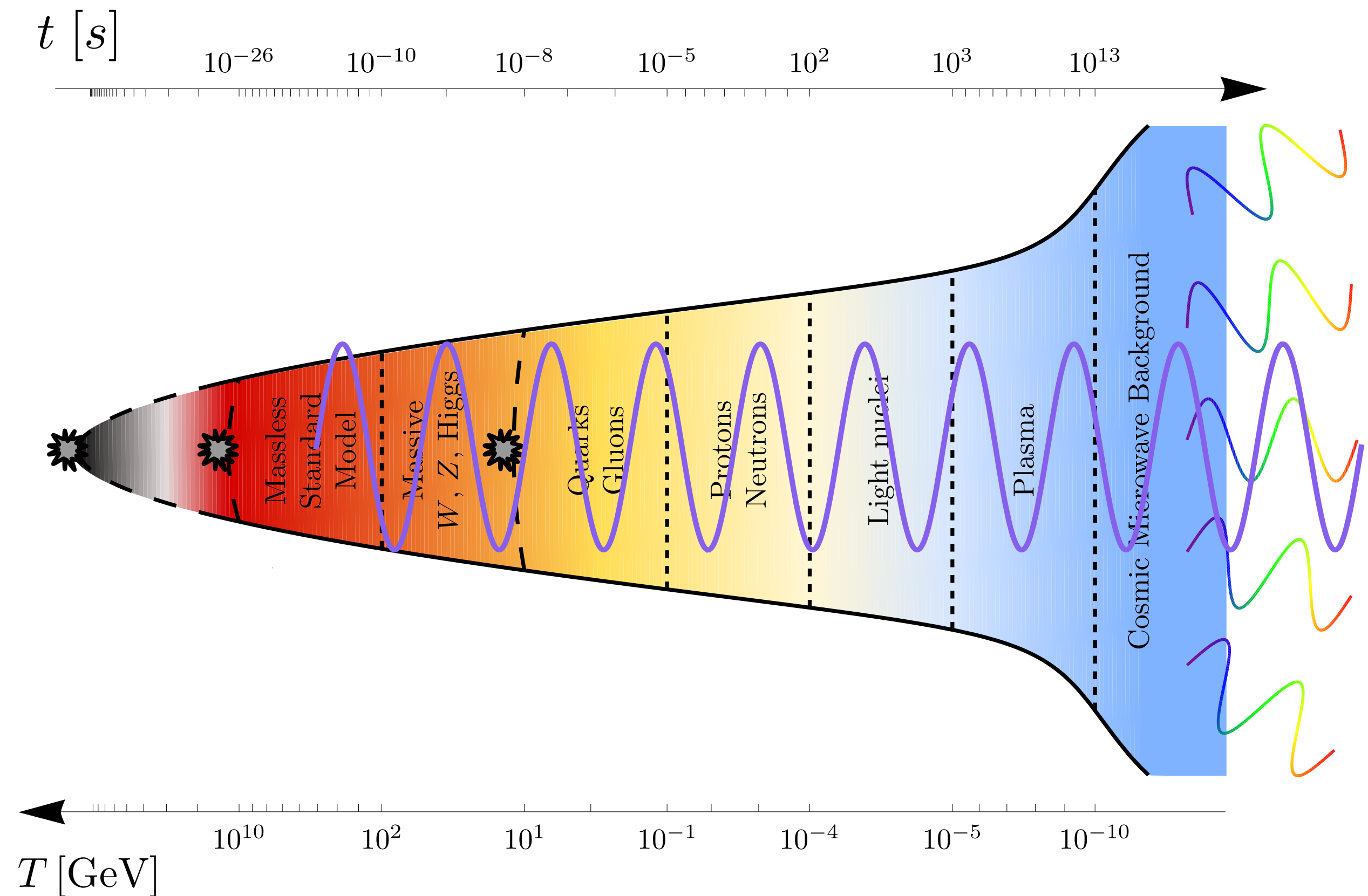
Massless particles: gravitational waves

JG Laine **JCAP1507** (2015) JG Jackson Laine Zhu **JHEP2007** (2020)
Ringwald Schütte-Engel Tamarit **2011.04731**

Gravitational waves in the early universe

Many potential sources of GWs

- Inflation
- Reheating
- Phase transitions
- ...



All model-dependent and / or speculative to a degree
Review: Caprini Figueroa *Class. Quant. Grav.* 35 (2018)

Gravitational waves from equilibrium

- GWs can be produced from eq. too. [Weinberg](#)
- Now $J \propto T^{\mu\nu} / m_{\text{Pl}}$, so as long as $T_{\text{max}} < m_{\text{Pl}}$ the GW-plasma coupling is indeed weak: [freeze-in](#) production over the history of the early universe?

- By the previous arguments: $\dot{f}_{\text{GW}}(t, \mathbf{k}) = \Gamma(k) [n_{\text{B}}(k) - f_{\text{GW}}(t, \mathbf{k})] + \mathcal{O}\left(\frac{1}{m_{\text{Pl}}^4}\right)$

$$\Gamma(k) = \frac{8\pi}{k m_{\text{Pl}}^2} \int d^4 X e^{ik(t-z)} \langle [T_{12}(X), T_{12}(0)] \rangle$$

- $\Gamma(k)$ also determines the absorption rate of previously emitted GWs from other sources

[Baym Patil Pethick PRD96 \(2017\)](#) [Flauger Weinberg PRD99 \(2019\)](#)

[JG Laine JCAP1507 \(2015\)](#)

Leading order for $k \sim T$

$$\Gamma(k) = \frac{8\pi}{k m_{\text{Pl}}^2} \int d^4 X e^{ik(t-z)} \langle [T_{12}(X), T_{12}(0)] \rangle$$

- What it means: cut two-point function with thermal propagators. A naive example: at LO T_{12} is bilinear in the fields of the QFT, so

$$= \left| \begin{array}{c} \text{Triangle Diagram} \end{array} \right|^2 \sim \int d^4 P \, n(p^0) n(p^0 + k^0) |\mathcal{M}|^2 \delta(P^2) \delta((P+K)^2)$$

thermal distribution functions x matrix element x on-shell kinematics

- Kinematically forbidden: need extra scatterings
- A complete LO calculation for $k \sim T$ requires all $2 \leftrightarrow 2$ scatterings between SM particles yielding a graviton

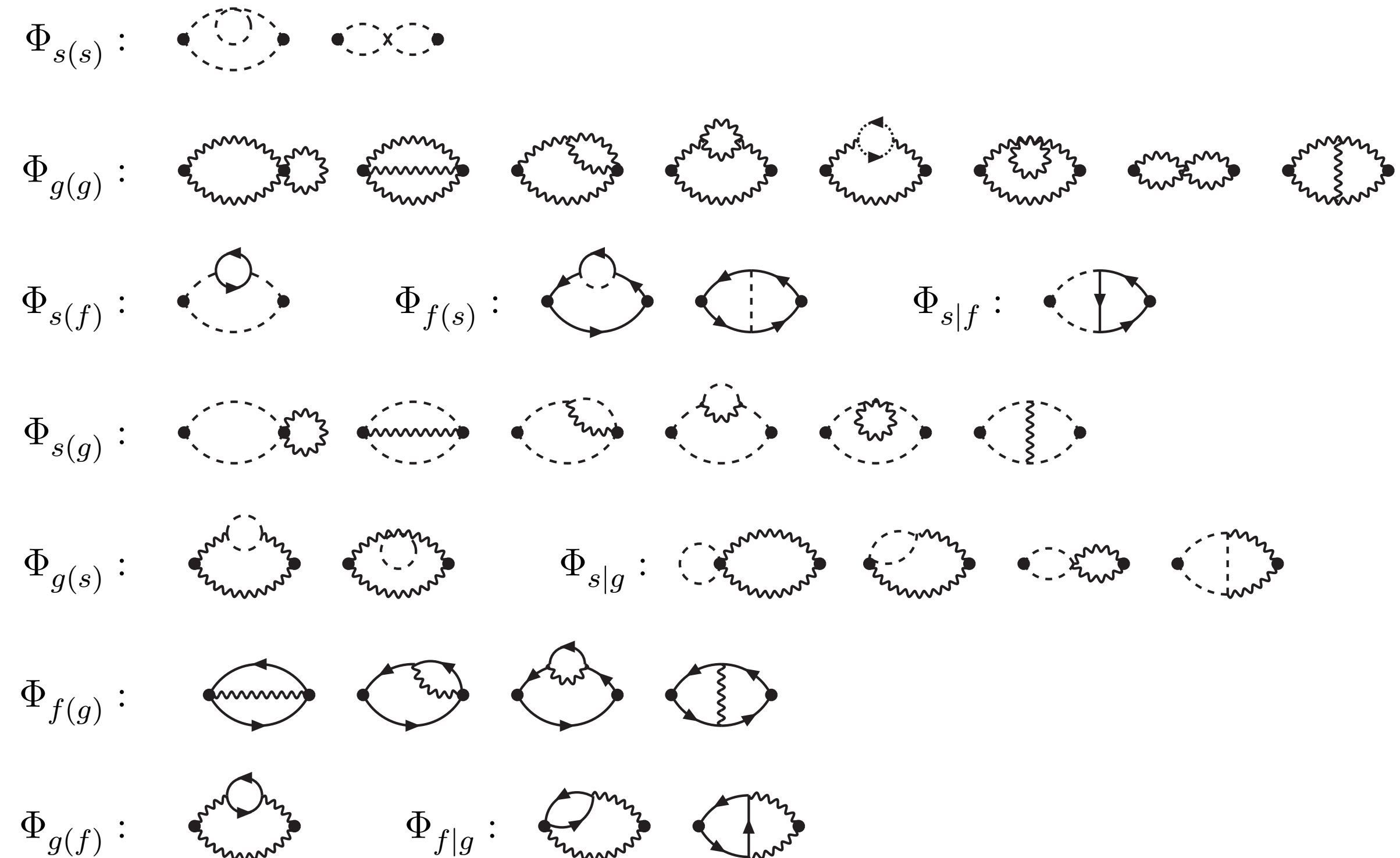
Leading order for $k \sim T$

$$\Gamma(k) = \frac{8\pi}{k m_{\text{Pl}}^2} \int d^4 X e^{ik(t-z)} \langle [T_{12}(X), T_{12}(0)] \rangle$$

- Work from this def, compute all two-loop graphs in the SM for the TT correlator and take the cuts

- Powerful method to get thermal spectral functions at thermal frequencies and nonzero virtualities too

Laine Zhu Jackson *et al*
(2010-20)

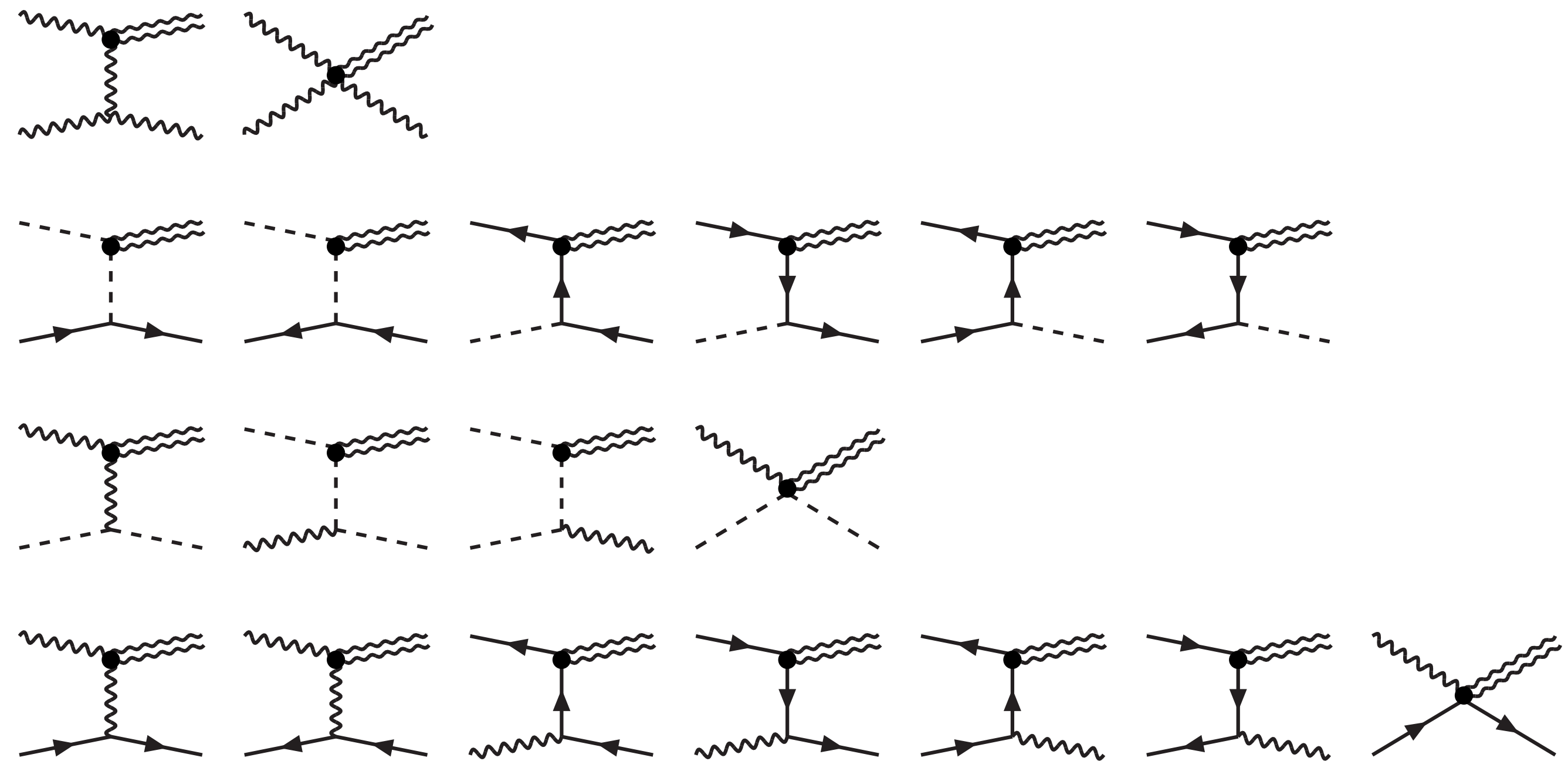


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- Work from this def, compute all two-loop graphs in the SM for the TT correlator and take the cut

- Cutting the two-loop diagrams gives rise to the squares of these diagrammatic structures (crossings not shown)

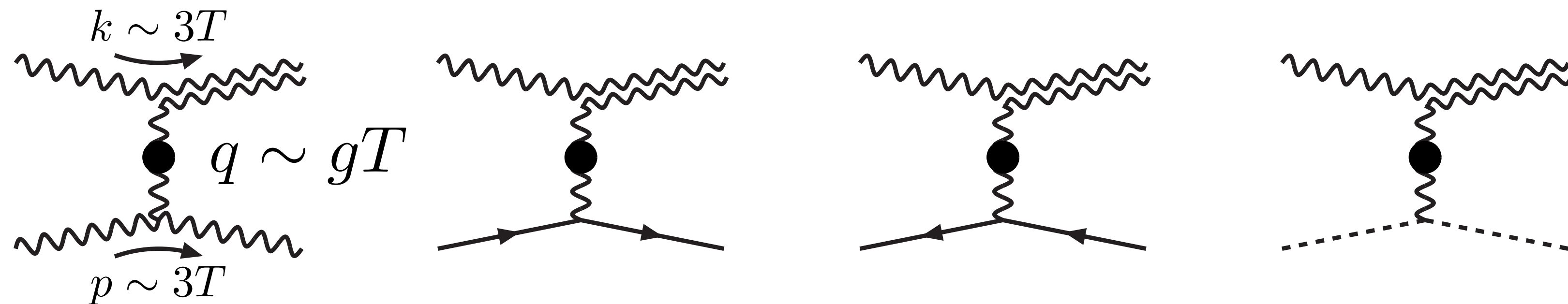


Leading order for $k \sim T$

- Hence, at LO for $k \sim T$, equivalence with kinetic theory

$$\dot{f}_{\text{GW}}(t, \mathbf{k}) = \Gamma(k) n_{\text{B}}(k) = \frac{1}{8k} \int d\Omega_{2 \rightarrow 2} \sum_{abc} \left| \mathcal{M}_{cG}^{ab}(\mathbf{p}_1, \mathbf{p}_2; \mathbf{k}_1, \mathbf{k}) \right|^2 f_a(p_1) f_b(p_2) [1 \pm f_c(k_1)]$$

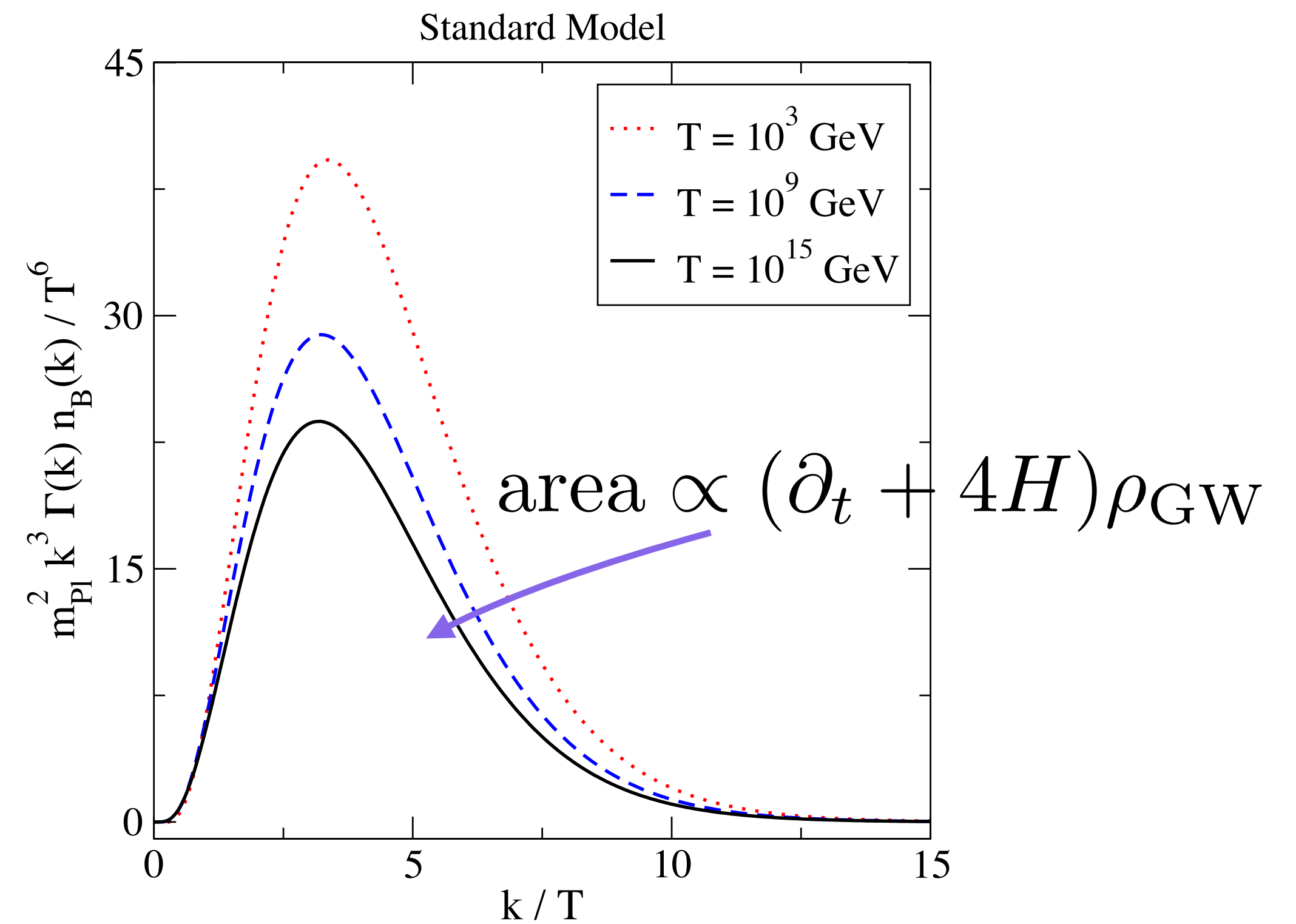
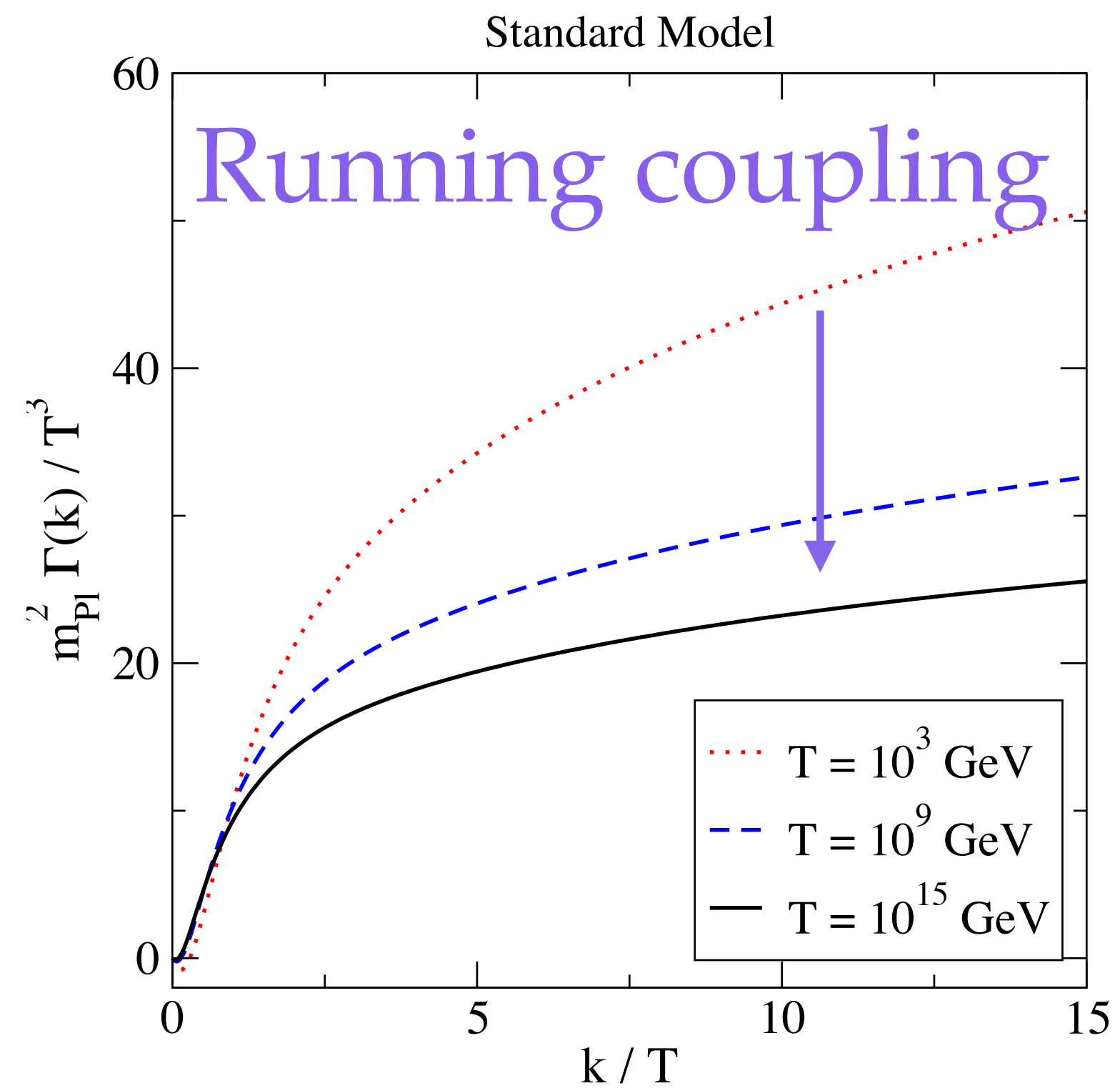
- The **phase space integration** runs over log-IR divergent soft gauge boson exchanges

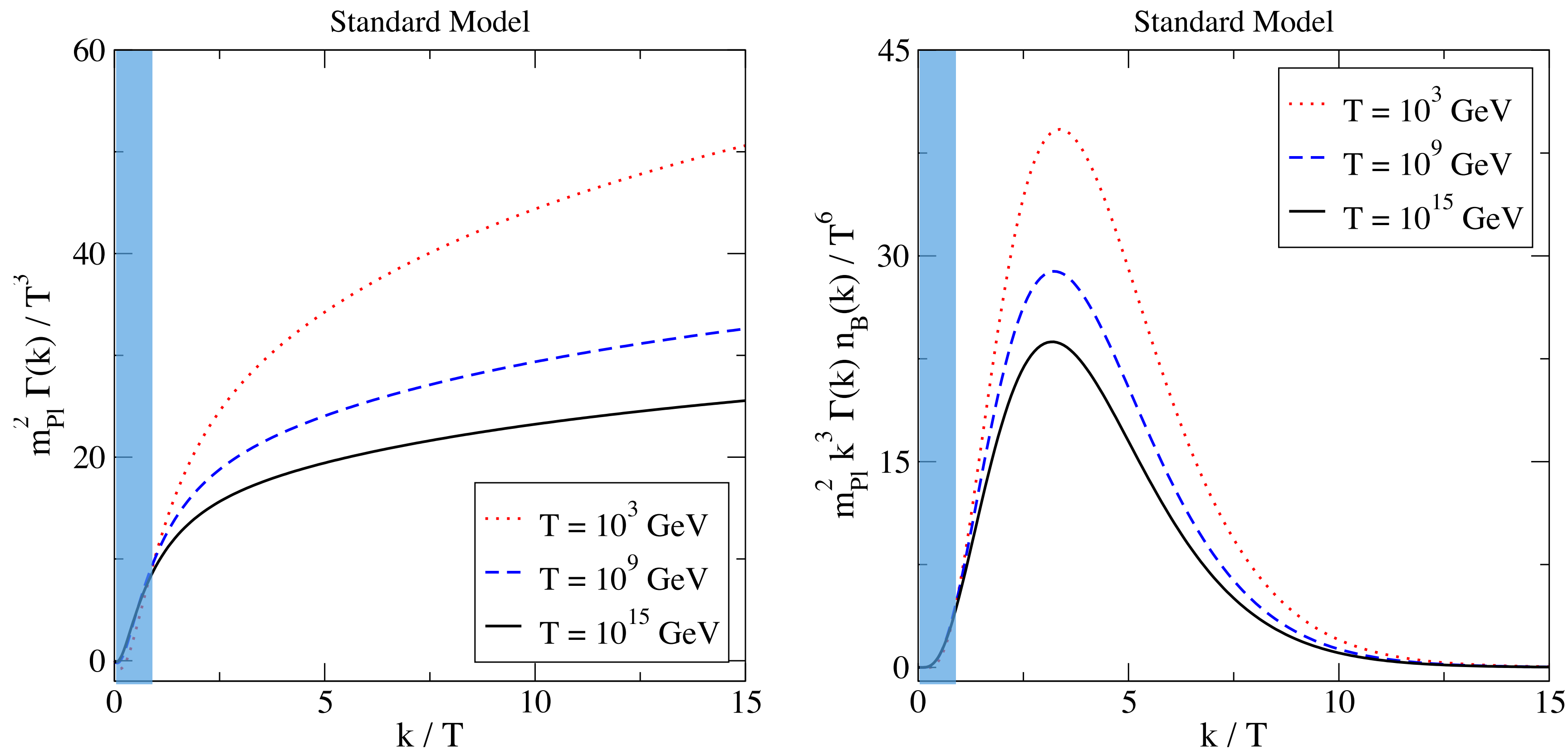


- Sensitivity to **collectivity**: screening, plasma oscillations and Landau damping. Treated by **Hard Thermal Loop resummation**: based on recent developments in TFT we implement a well-behaved subtraction and replacement with the HTL resummed evaluation [JG Laine \(2015-16\)](#)

Leading order for $k \sim T$

$$\frac{d \rho_{\text{GW}}}{dt dk} = \frac{k^3 \Gamma(k) n_B(k)}{\pi^2} = \frac{k^3 T n_B(k)}{\pi^2 m_{\text{Pl}}^2} \left\{ \sum_{i=1}^3 d_i m_{\text{D}i}^2 \ln \left(1 + \frac{4k^2}{m_{\text{D}i}^2} \right) + g^2 T^2 \chi \left(\frac{k}{T} \right) \right\}$$

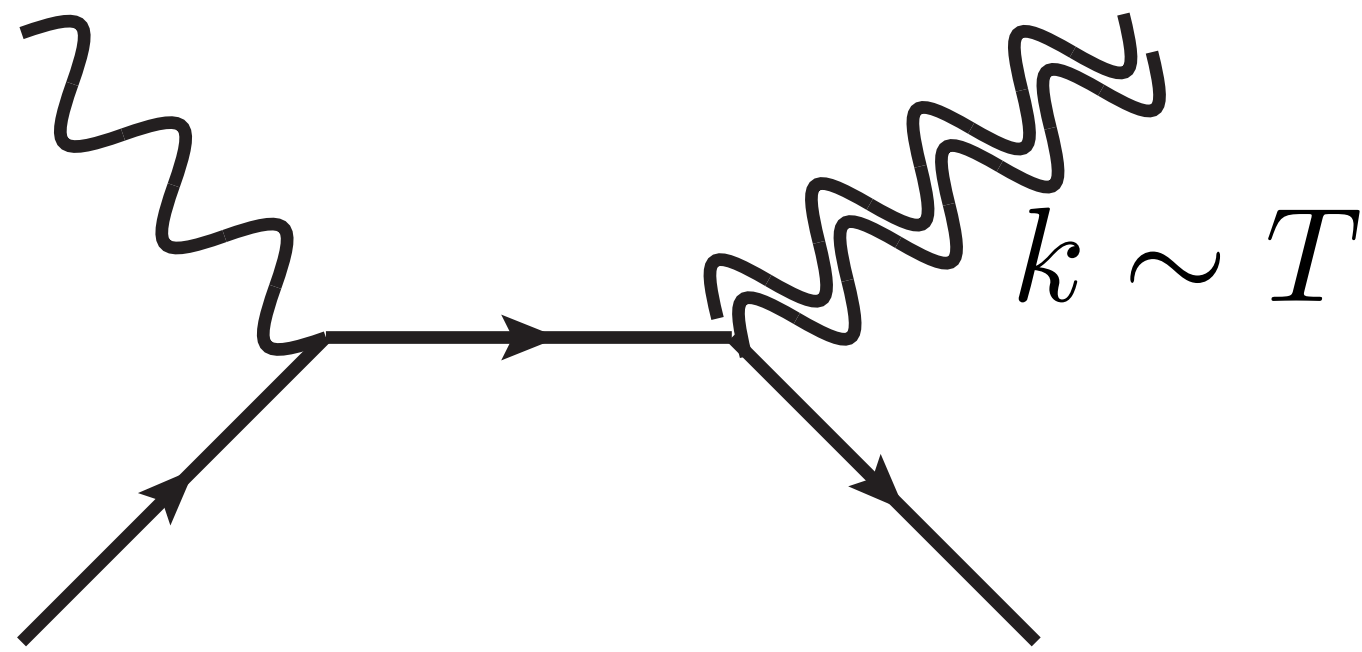




JG Jackson Laine Zhu **JHEP2007** (2020)

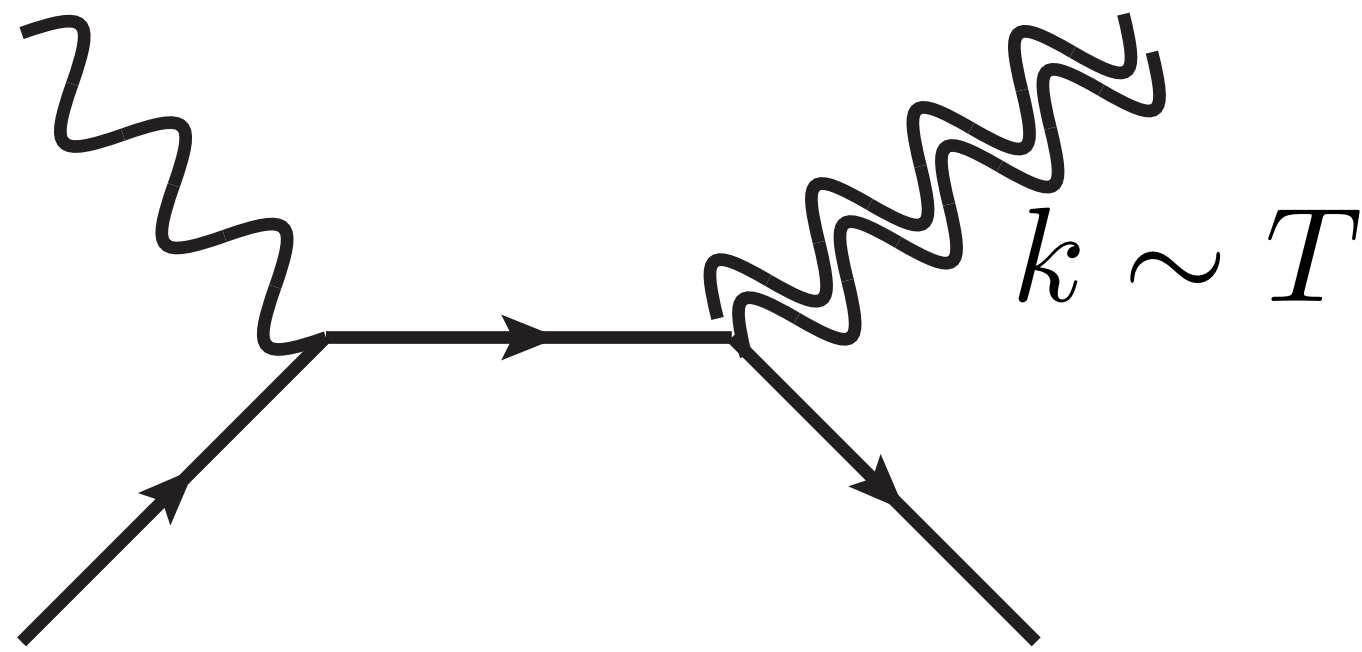
- The rate is valid for $k \gtrsim T$. At smaller k our rate is not LO correct, but extrapolates to $k=0$ better than what was happening in similar calculations for gravitino and axion production
(e.g. Pradler Steffen **PRD75** (2007), Rychkov Strumia **PRD75** (2007))

Going to the IR

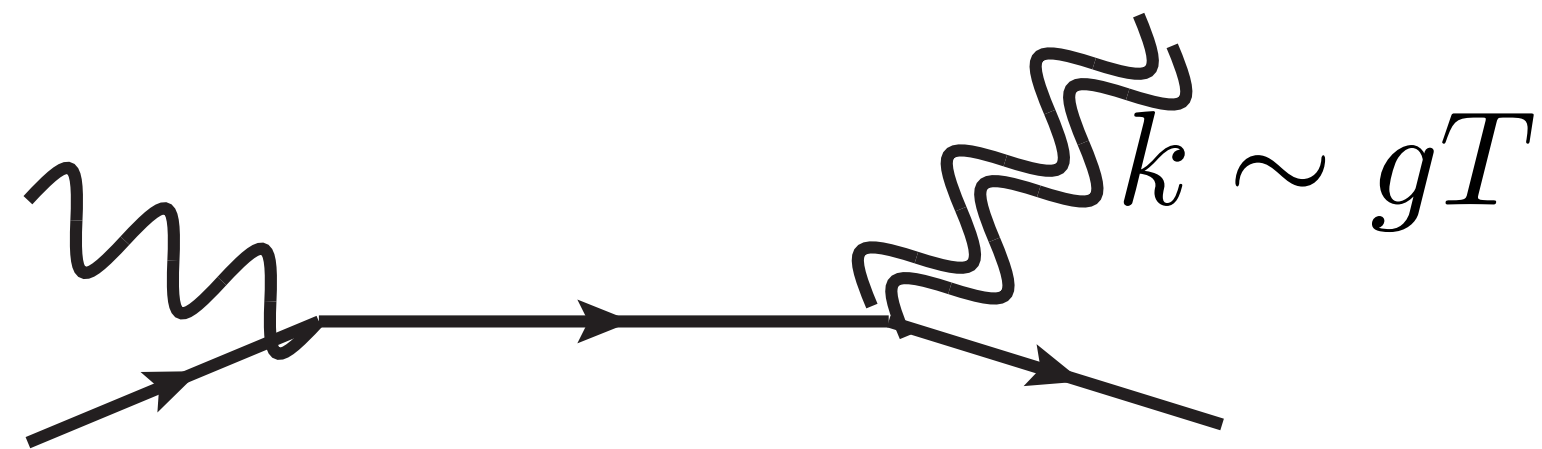


Well-defined vacuum-like particle external states, at most HTL internally

Going to the IR

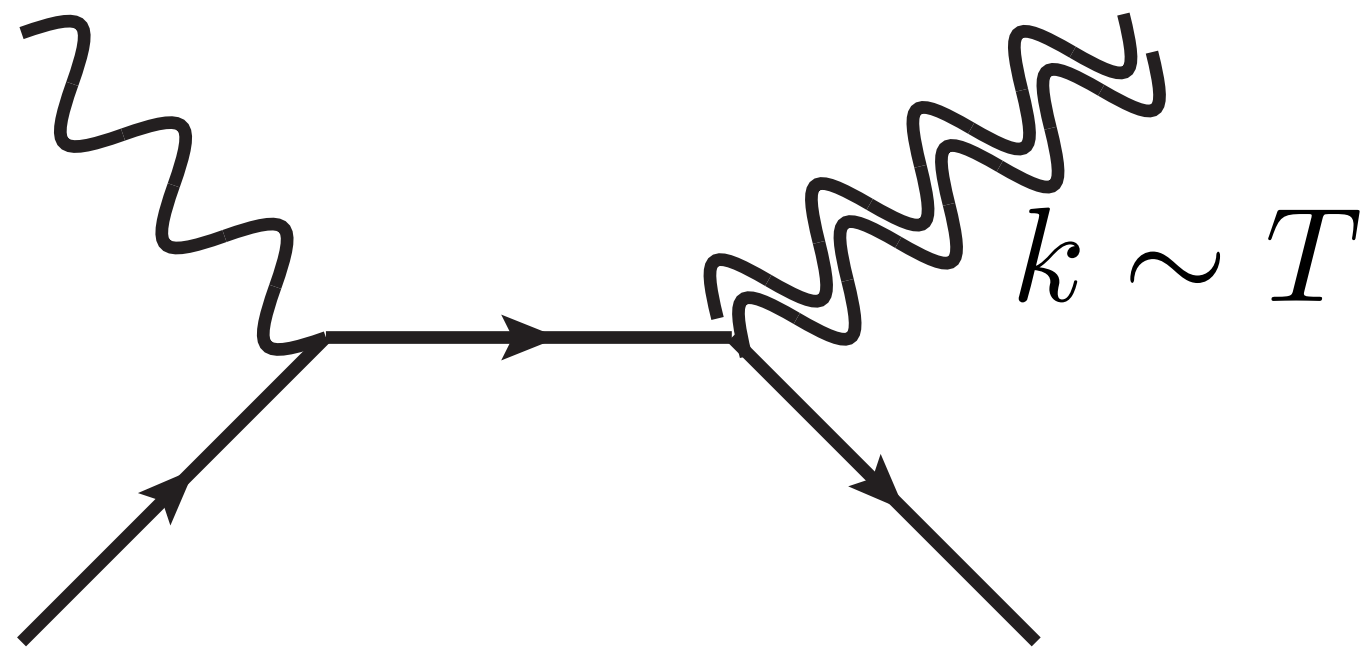


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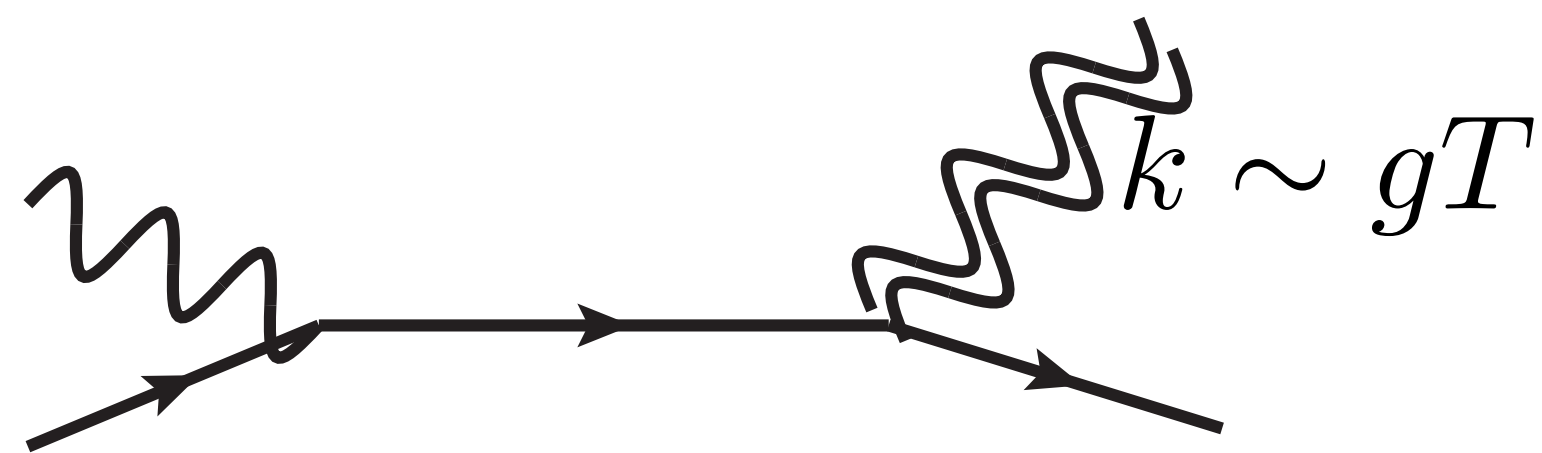


🤔 Longer-lived intermediate states, collinear and soft kinematics. Changes to simple particle picture

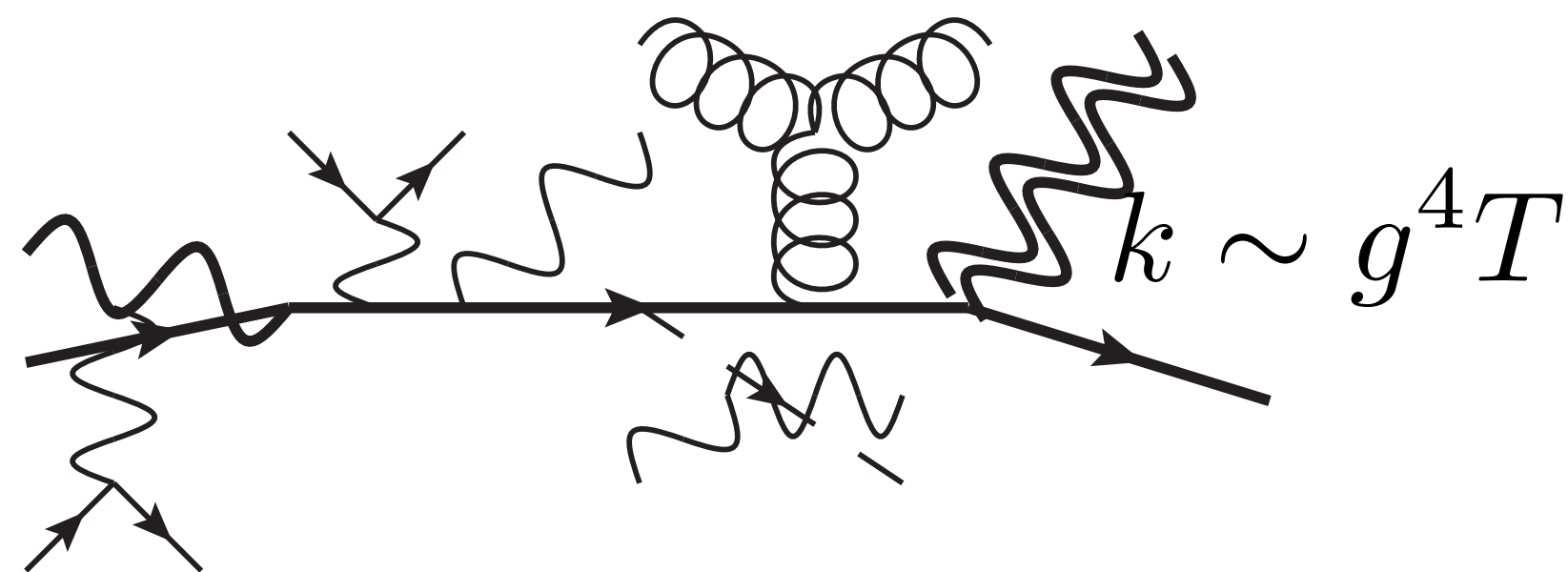
Going to the IR



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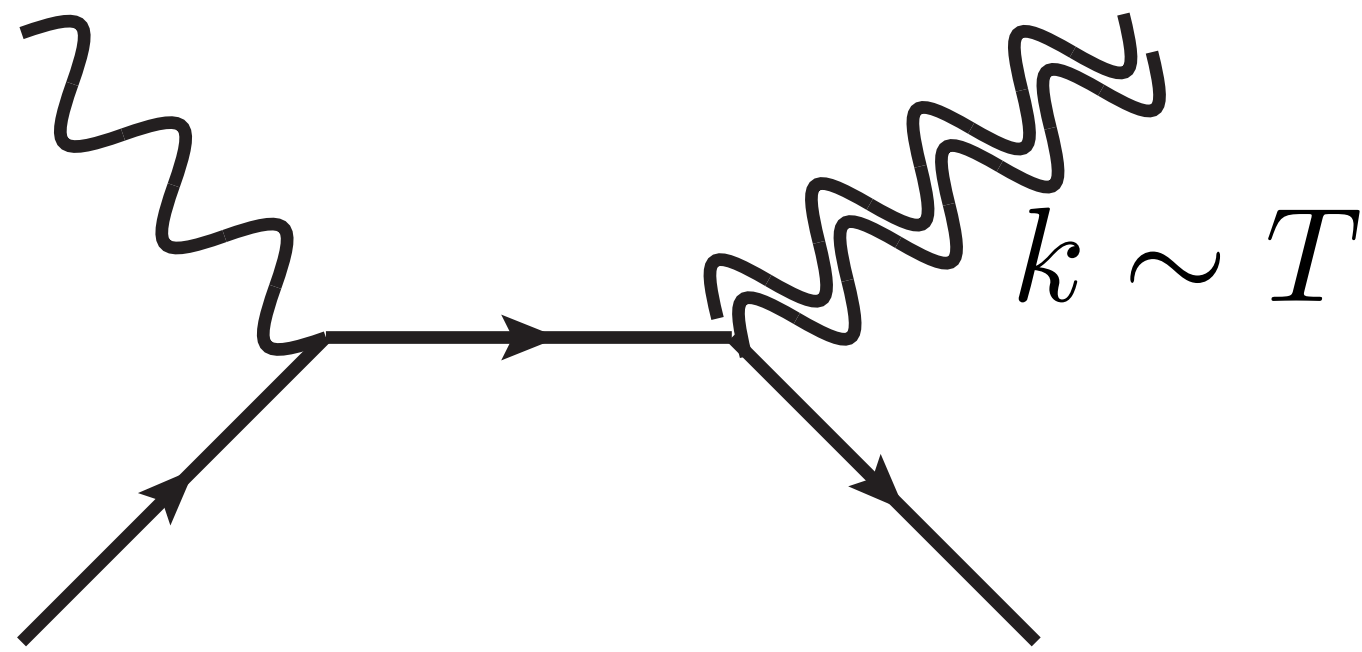


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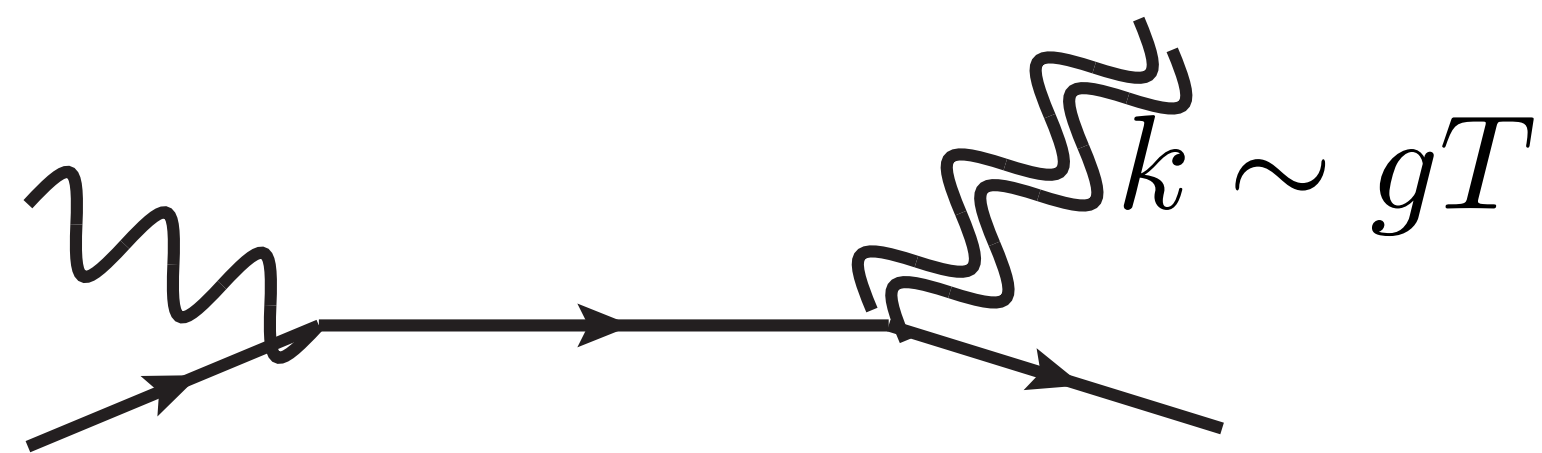


😲 Duration of order mean free time: scattering picture completely breaks down, GW does not resolve the microscopic scale

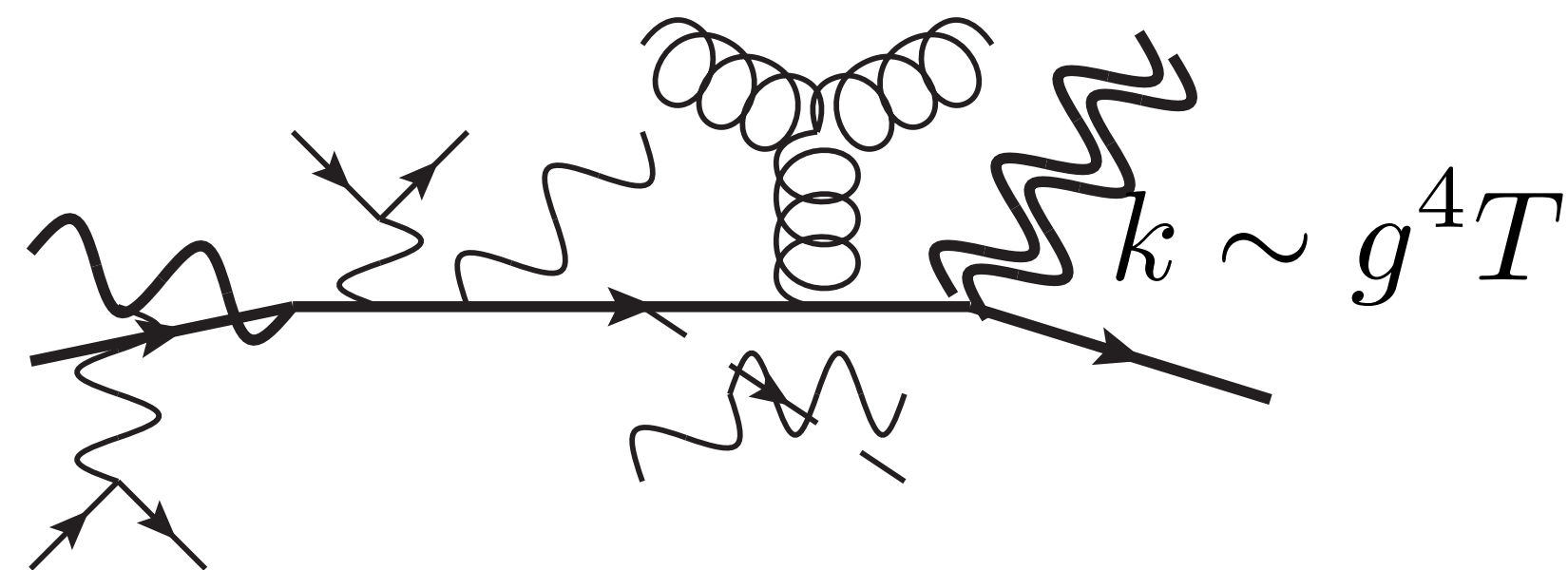
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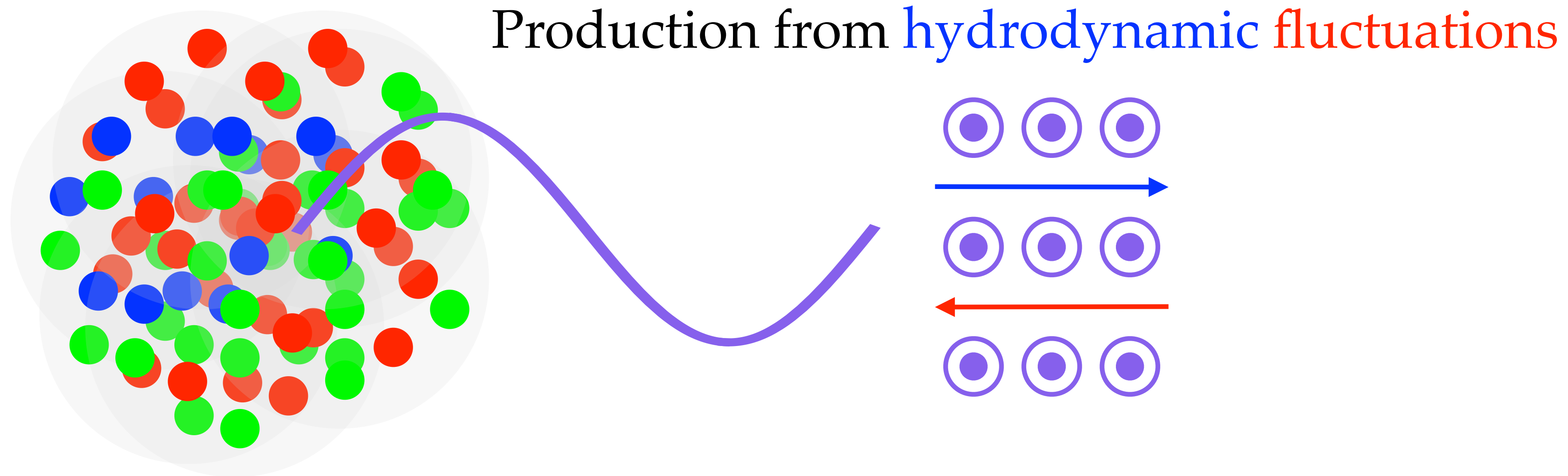
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😲 Duration of order mean free time: scattering picture completely breaks down, GW does not resolve the microscopic scale

- Nothing here specific to GW

Going to the IR



- TFT formalism shows that the IR rate is proportional to the *shear viscosity* of the plasma

$$\Gamma(k) = \frac{8\pi}{k m_{\text{P}1}^2} \int d^4 X e^{ik(t-z)} \langle [T_{12}(X), T_{12}(0)] \rangle \quad n_{\text{B}}(k) \Gamma(k) \stackrel{k \lesssim g^4 T}{=} \frac{16\pi T \eta}{k m_{\text{P}1}^2}$$

Going to the IR

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- For the SM at $T > 160$ GeV η is dominated by the slowest processes in eq., those involving right-handed leptons only

$$\eta \simeq \frac{16T^3}{g_1^4 \ln(5T/m_{\text{D1}})} \quad \rightarrow \quad \eta \simeq 400 T^3$$

g_1 hypercharge coupling with screening mass $m_{\text{D1}} = \sqrt{11/6} g_1 T$

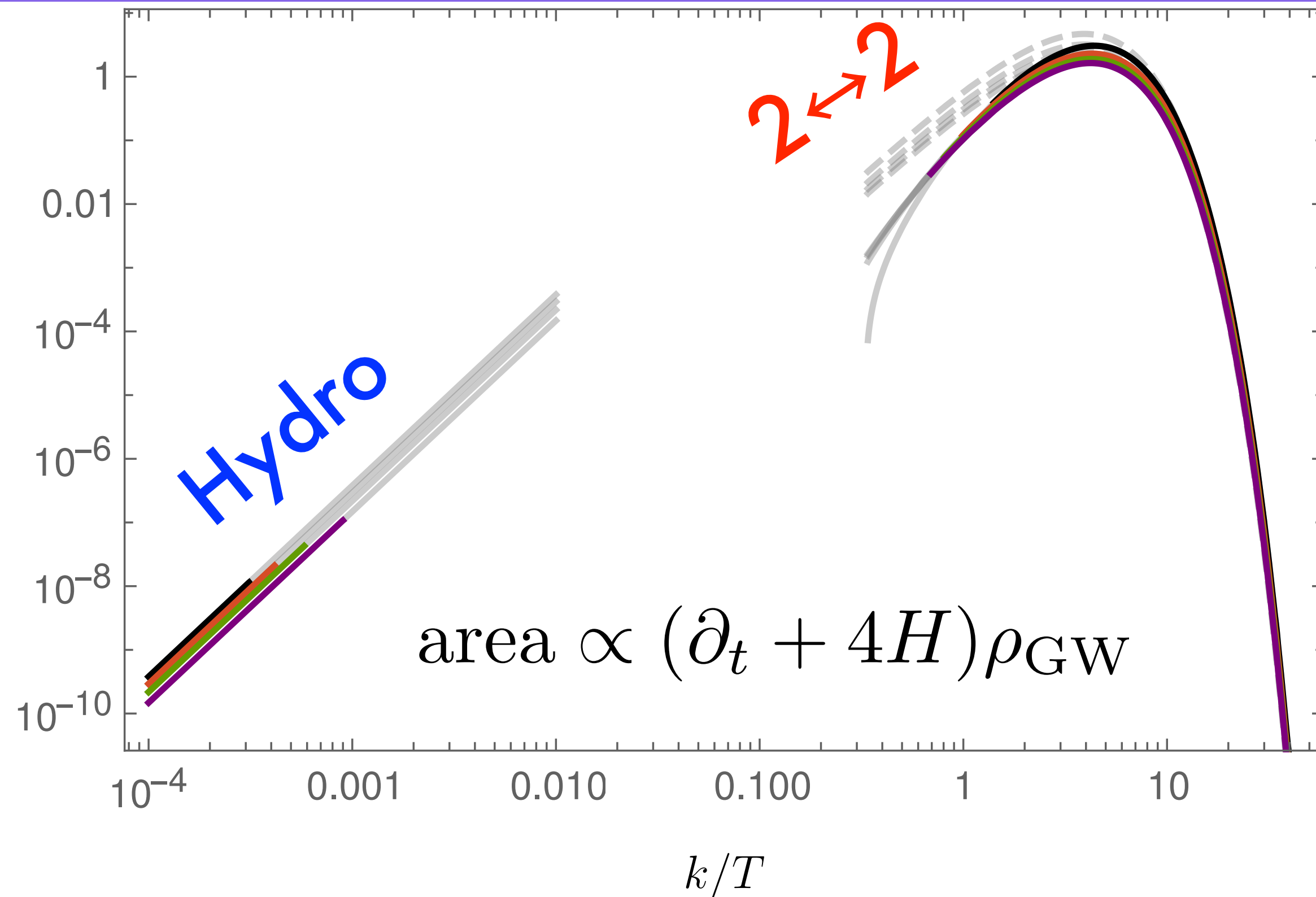
Only a leading-log estimate, no complete LO for $T > 160$ GeV

Arnold Moore Yaffe (2000-2003)

JG Laine JCAP1507 (2015)

Cosmological implications

$$\frac{k^4 m_{\text{Pl}}^2 n_{\text{B}}(k)}{16\pi T^7} \Gamma(k)$$



- Peak: frequency at $k \approx 4T$. Redshifts at decoupling to $k_{\text{dec}} \approx 4T_{\text{dec}} (3.9 / 106.75)^{1/3} \sim T_{\text{dec}}$. Today $f \approx 100$ GHz. Amplitude determined by T_{max} .

Cosmological implications

- Direct detection challenging in the medium term
- Thermal production stores energy in GWs. BBN and CMB observations constrain the energy density stored in radiation at those epochs: GW contribution to N_{eff}
Smith Pierpaoli Kamionkowski **PRL97** (2006) Henrot-Versille *et al* *Class. Quant. Grav.* **32** (2015) Caprini Figueroa *Class. Quant. Grav.* **35** (2018)
- The SM predictions have 10^{-3} uncertainty, the experimental accuracy 10^{-1} , expected to increase with next-generation detectors **CMB-S4**
- Requiring $\Delta N_{\text{eff}} = 10^{-3}$ yields $T_{\text{max}} < 2 \cdot 10^{17}$ GeV for a SM universe, 2x more than that for a MSSM scenario (the extra GW production from the larger number of thermal d.o.f.s is more than compensated by the extra dilution)

JG Jackson Laine Zhu **JCHEP2007** (2020) Ringwald Schütte-Engel Tamarit **2011.04731**

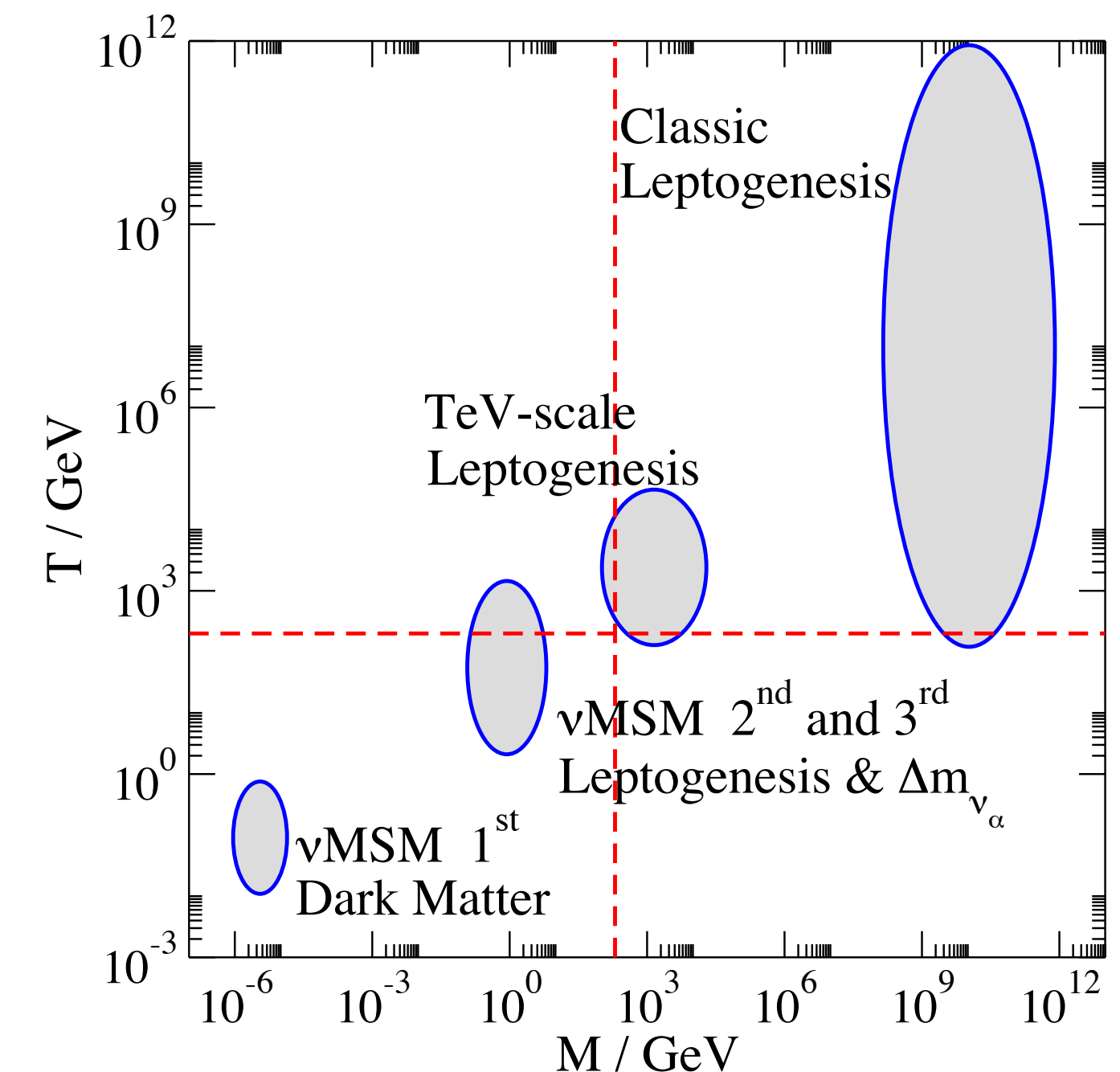
Massive particles

Massive particles: right-handed neutrinos

- n sterile (SM gauge singlet), Majorana neutrinos coupling to the three active lepton flavours and the (conjugate) Higgs field

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{2} \sum_I \bar{N}_I (i\gamma^\mu \partial_\mu - M_I) N_I - \sum_{I,a} (\bar{N}_I h_{Ia} \tilde{\phi}^\dagger a_L l_a + \bar{l}_a a_R \tilde{\phi} h_{Ia}^* N_I)$$

- Can address active neutrino masses (seesaw) and baryon asymmetry (leptogenesis) over a wide range of parameters
Fukugita Yanagida [PLB174](#) (1986)
- A specific realisation (νMSM) can also provide a keV-scale DM right-handed neutrino
Asaka Blanchet Shaposhnikov [PLB620](#), [PLB631](#) (2005)
- Asymmetry generation and RHN production require rates from $T \gg M_I$ to $T \ll M_I$

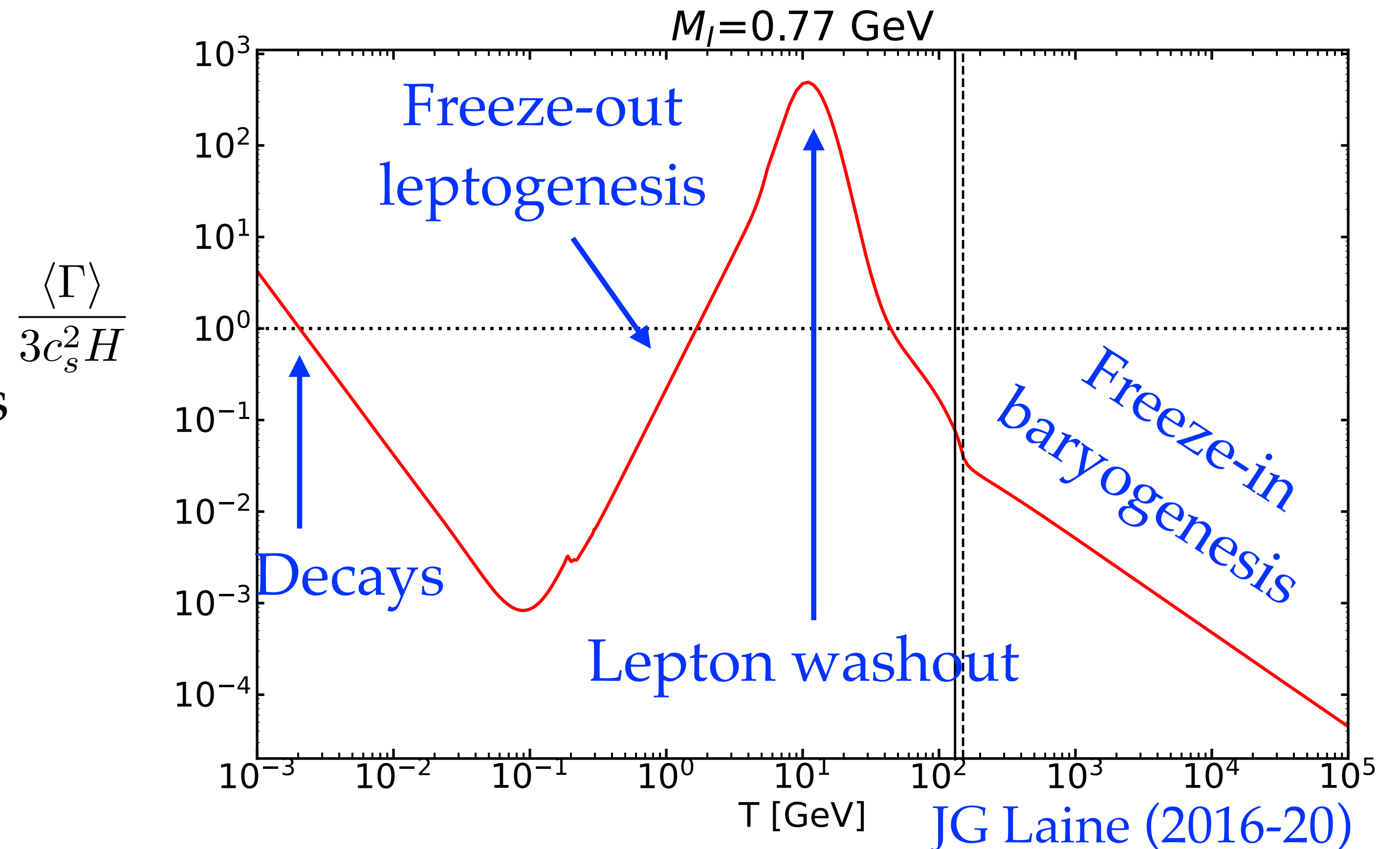


Plot by Mikko Laine

Massive particles: right-handed neutrinos

$$(\partial_t - H\mathbf{k} \cdot \nabla_{\mathbf{k}})f_N(t, \mathbf{k}) = \Gamma(k)[n_F(k^0) - f_N(t, \mathbf{k})] \quad \Gamma(k) = \sum_a \frac{|h_{Ia}|^2}{2k^0} \int d^4X e^{iK \cdot X} \langle [\tilde{\phi}^\dagger a_L l(X), \bar{l} a_R \tilde{\phi}(0)] \rangle$$

- Production, equilibration, freeze-out and decay rates from the formalism over many decades



Massive particles: right-handed neutrinos

- Symmetric phase
$$\Gamma(k) = \sum_a \frac{|h_{Ia}|^2}{2k^0} \int d^4X e^{iK \cdot X} \langle [\tilde{\phi}^\dagger a_L l(X), \bar{l} a_R \tilde{\phi}(0)] \rangle$$
- $T \ll M_I$ [Salvio Lodone Strumia \(2011\)](#), [Laine Schröder \(2012\)](#), [Biondini Brambilla Escobedo Vairo \(2012\)](#)
- $T \sim M_I$ [Laine \(2013\)](#)
- $T \gg M_I$ [Anisimov Besak Bödeker \(2010-12\)](#), [Garbrecht Glowna Herranen \(2013\)](#), [Ghisoiu Laine \(2014\)](#)
- Broken phase
 - $M_I \sim \text{GeV}$ [JG Laine \(2016-20\)](#), [Jackson Laine \(2019\)](#)
 - $M_I \sim \text{keV}$ [Asaka Laine Shaposhnikov \(2006\)](#), [JG Laine \(2015-20\)](#) [Bödeker Klaus \(2020\)](#)
- These calculations provide a pattern for models with many regimes to be followed

Massive particles: the ultrarelativistic regime

$$T \gg M_I$$

$$\Gamma(k) = \sum_a \frac{|h_{Ia}|^2}{2k^0} \int d^4X e^{iK \cdot X} \langle [\tilde{\phi}^\dagger a_L l(X), \bar{l} a_R \tilde{\phi}(0)] \rangle$$

- In a first approximation mass seems negligible
- Just $2 \leftrightarrow 2$ processes (with fermion HTL included)?

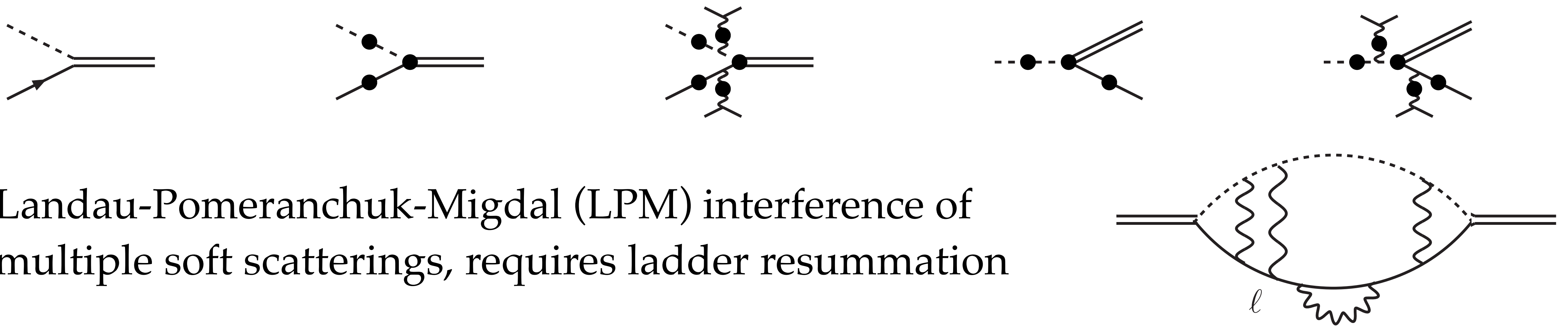


Massive particles: the ultrarelativistic regime

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- Effective $1 \leftrightarrow 2$ processes



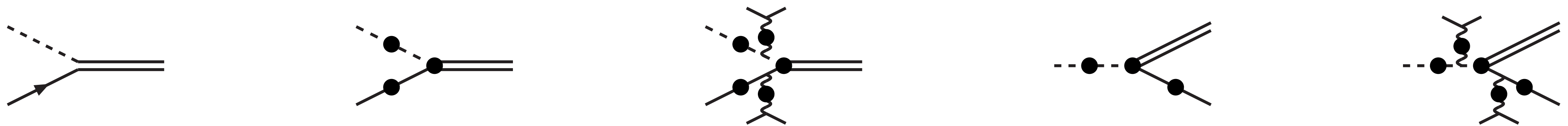
- Landau-Pomeranchuk-Migdal (LPM) interference of multiple soft scatterings, requires ladder resummation
- Borrow techniques from hot QCD to deal with LPM resummation
Baier Dokshitzer Mueller Peigné Schiff (1995-97) Zakharov (1996-97) Arnold Moore Yaffe (2001-2003)

Massive particles: the ultrarelativistic regime

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- Effective $1 \leftrightarrow 2$ processes



- Absent from GW production calculation at LO (suppression in derivative coupling) and similar production calculations (gravitino, axion, etc...)

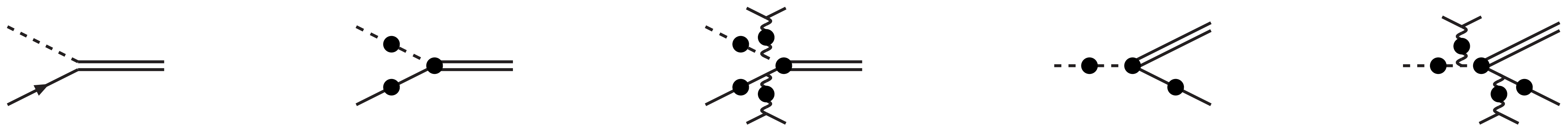
Salvio Strumia Xue (2013)

Massive particles: the ultrarelativistic regime

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- Effective $1 \leftrightarrow 2$ processes



- Very important beyond sterile neutrinos
- Thermalisation during reheating (number-nonconserving and efficient energy equilibration) Davidson Sarkar (2001) Harigaya Mukaida (2014) Mukaida Yamada (2015)
Large body of literature on QCD thermalisation, review in Berges Heller Mazeliauskas Venugopalan (2020)

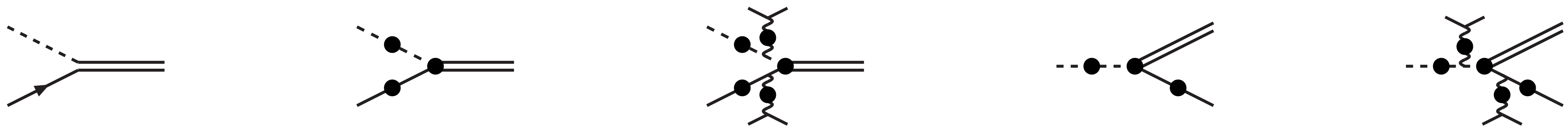
Anisimov Besak Bödeker (2010-12) Ghisoiu Laine (2014)

Massive particles: the ultrarelativistic regime

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- Effective $1 \leftrightarrow 2$ processes



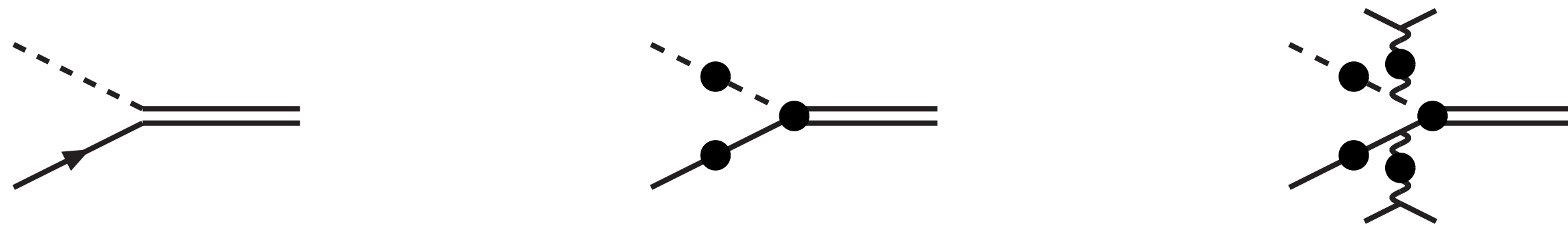
- Very important beyond sterile neutrinos
- Equilibration of the Yukawa interactions of right-handed electrons
Bödeker Schröder (2019)

Massive particles: the ultrarelativistic regime

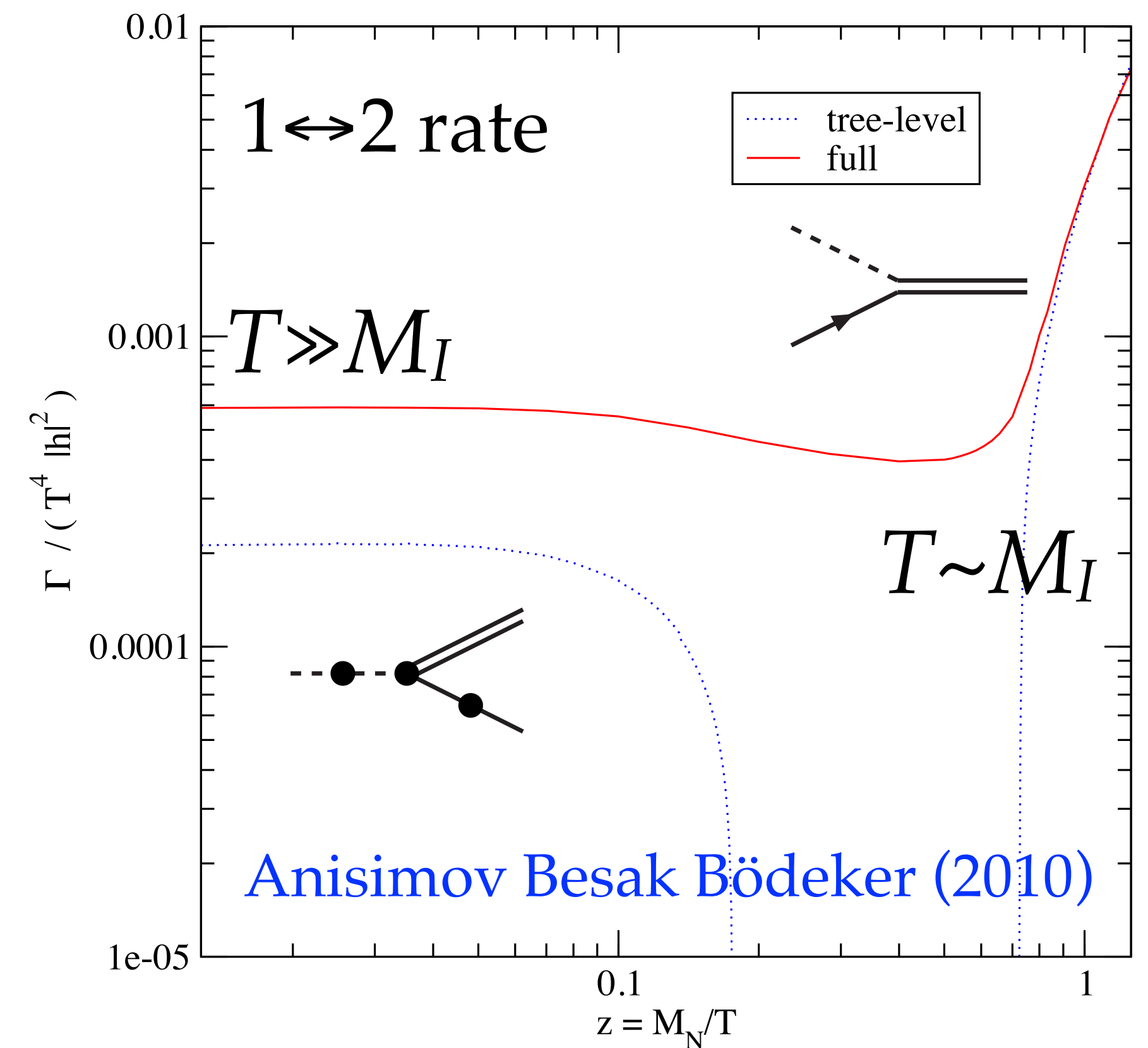
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- Effective $1 \leftrightarrow 2$ processes



- Large enhancement in the high- T regime

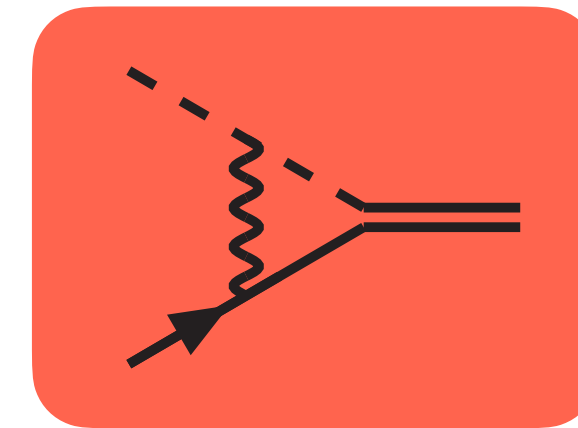
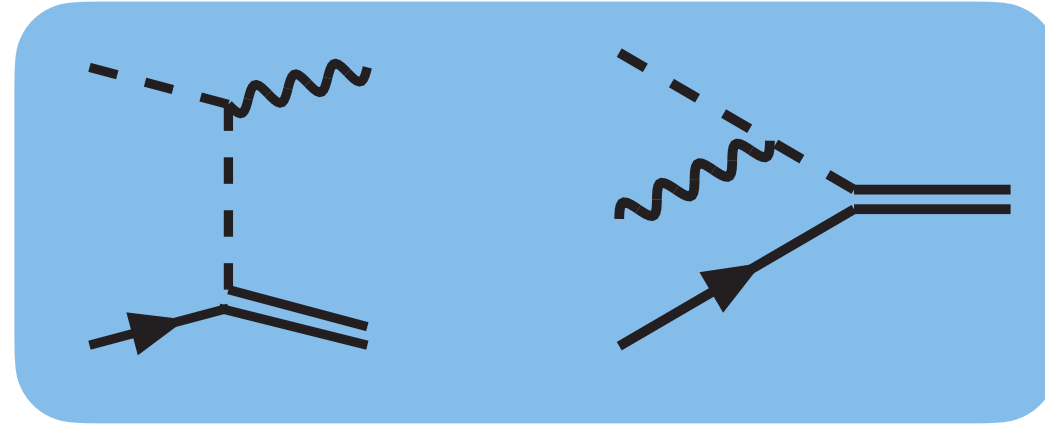
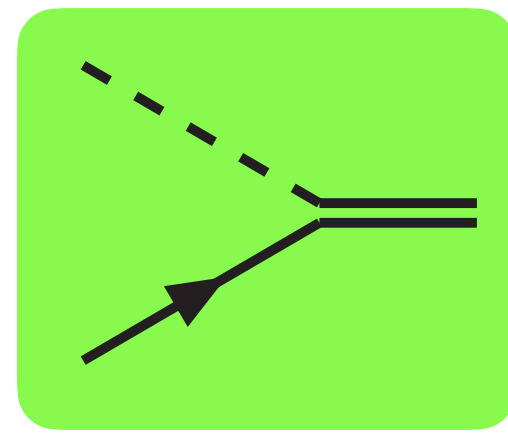


Massive particles: the relativistic regime

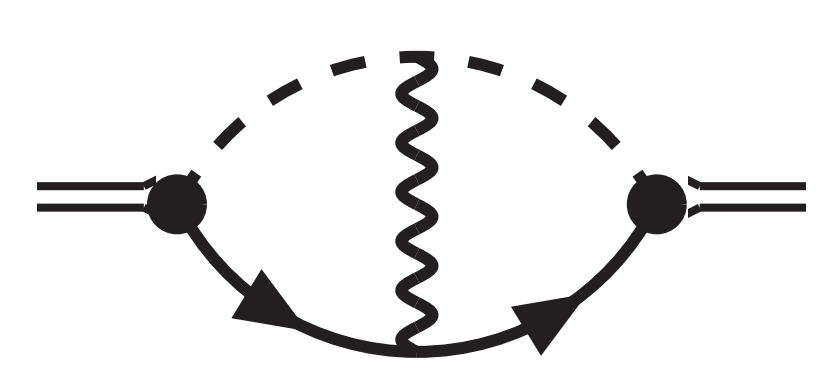
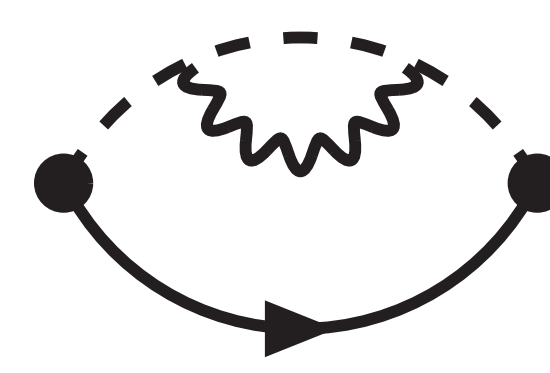
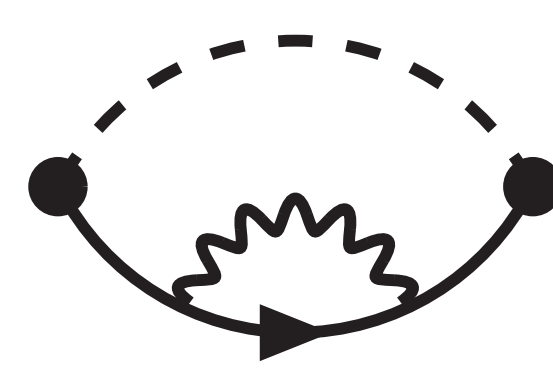
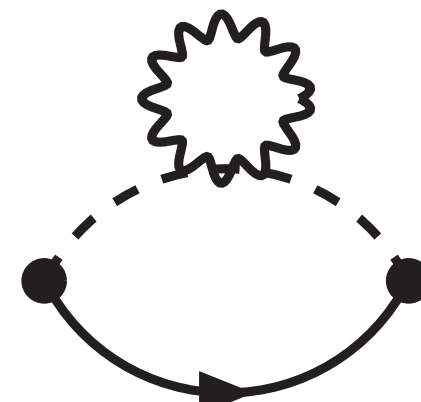
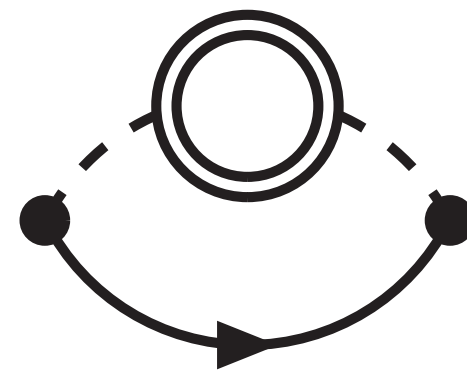
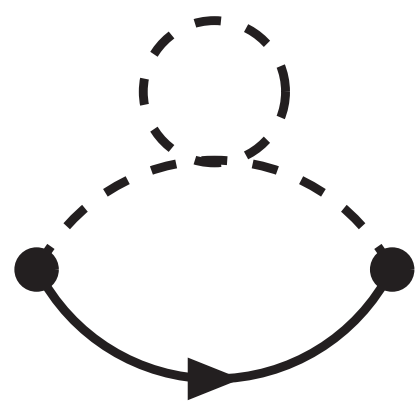
$$T \sim M_I$$

$$\Gamma(k) = \sum_a \frac{|h_{Ia}|^2}{2k^0} \int d^4X e^{iK \cdot X} \langle [\tilde{\phi}^\dagger a_L l(X), \bar{l} a_R \tilde{\phi}(0)] \rangle$$

- A **leading-order 2→1 process** receiving **real (2→2, 3→1)** and **virtual (2→1) NLO** corrections that have been computed and merged with the $T \gg M_I$ range



- Real and virtual corrections are **individually IR divergent**, only **sum is physical**



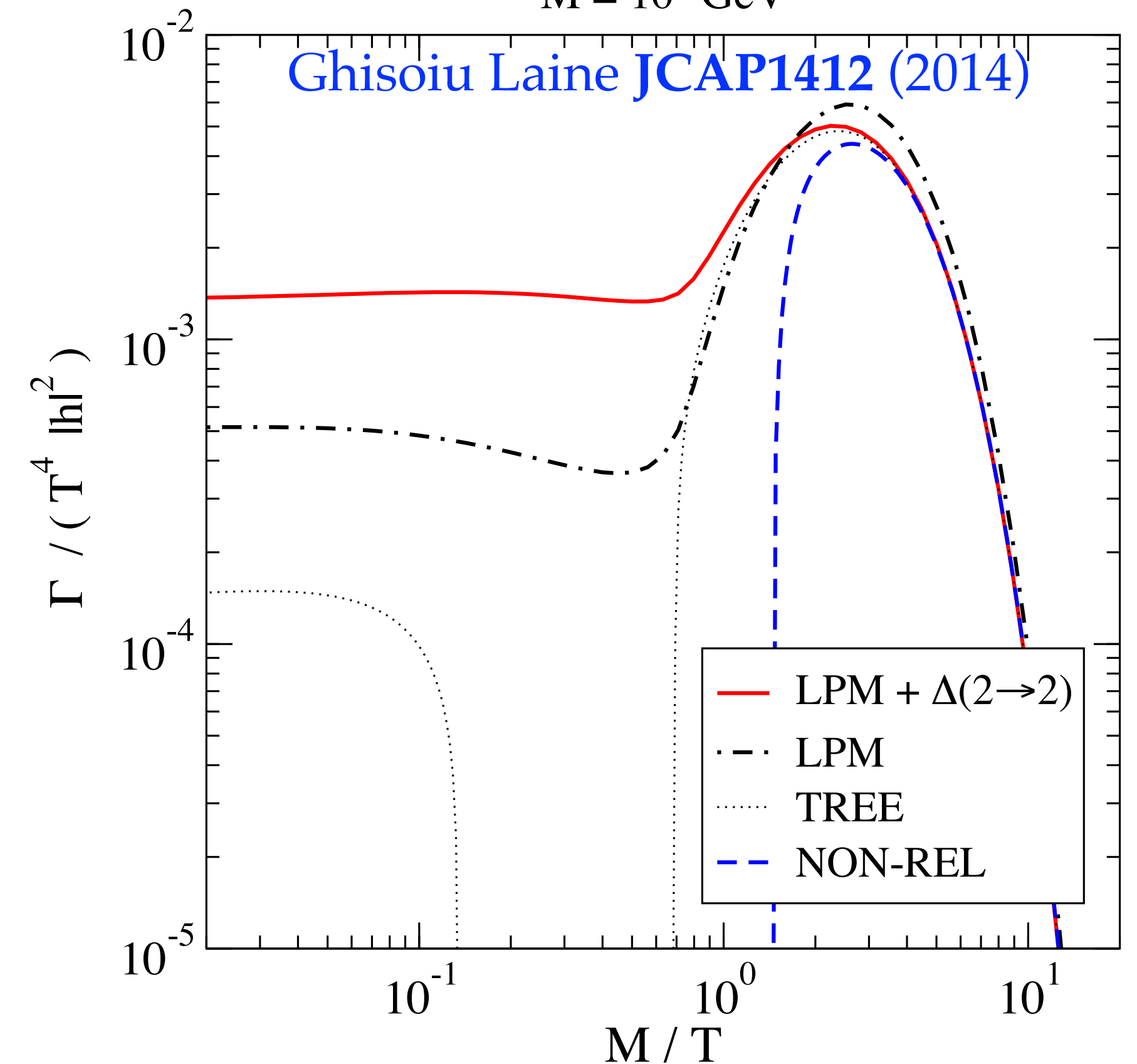
Massive particles: the relativistic regime

$$T \sim M_I$$

$$\Gamma(k) = \sum_a \frac{|h_{Ia}|^2}{2k^0} \int d^4X e^{iK \cdot X} \langle [\tilde{\phi}^\dagger a_L l(X), \bar{l} a_R \tilde{\phi}(0)] \rangle$$

$M = 10^7 \text{ GeV}$

- A leading-order $2 \rightarrow 1$ process receiving real ($2 \rightarrow 2$, $3 \rightarrow 1$) and virtual ($2 \rightarrow 1$) **NLO** corrections that have been computed and **merged** with the $T \gg M_I$ range
- Agreement with OPE / EFT based calculations in the **non-relativistic** regime
 Salvio Lodone Strumia **JHEP1108 (2011)**
 Laine Schröder **JHEP1202 (2012)** Biondini Brambilla
 Escobedo Vairo **JHEP1312 (2013)**



Laine **JHEP1305 JHEP1308 (2013)** Ghisoiu Laine **JCAP1412 (2014)**

Conclusions

- TFT formalism for thermal rates
- Does not require quasi-particles, though it reproduces quasi-particle Boltzmann results where they apply
- Relies on timescale separation
- Thermal production of **gravitational waves**: guaranteed to be there, contributes to N_{eff} . No stringent bounds for SM-like universes. Methods applicable to light/massless states non-renormalizably coupled to plasma
- Thermal production of massive particles: the case of **heavy neutral leptons / sterile neutrinos**. Many regimes to be examined, great progress with *interdisciplinary connections* to hot QCD and NLO available in some regimes

Extra slides

Leading order for $k \sim T$

$$\Gamma(k) = \frac{8\pi}{k m_{\text{Pl}}^2} \int d^4 X e^{ik(t-z)} \langle [T_{12}(X), T_{12}(0)] \rangle$$

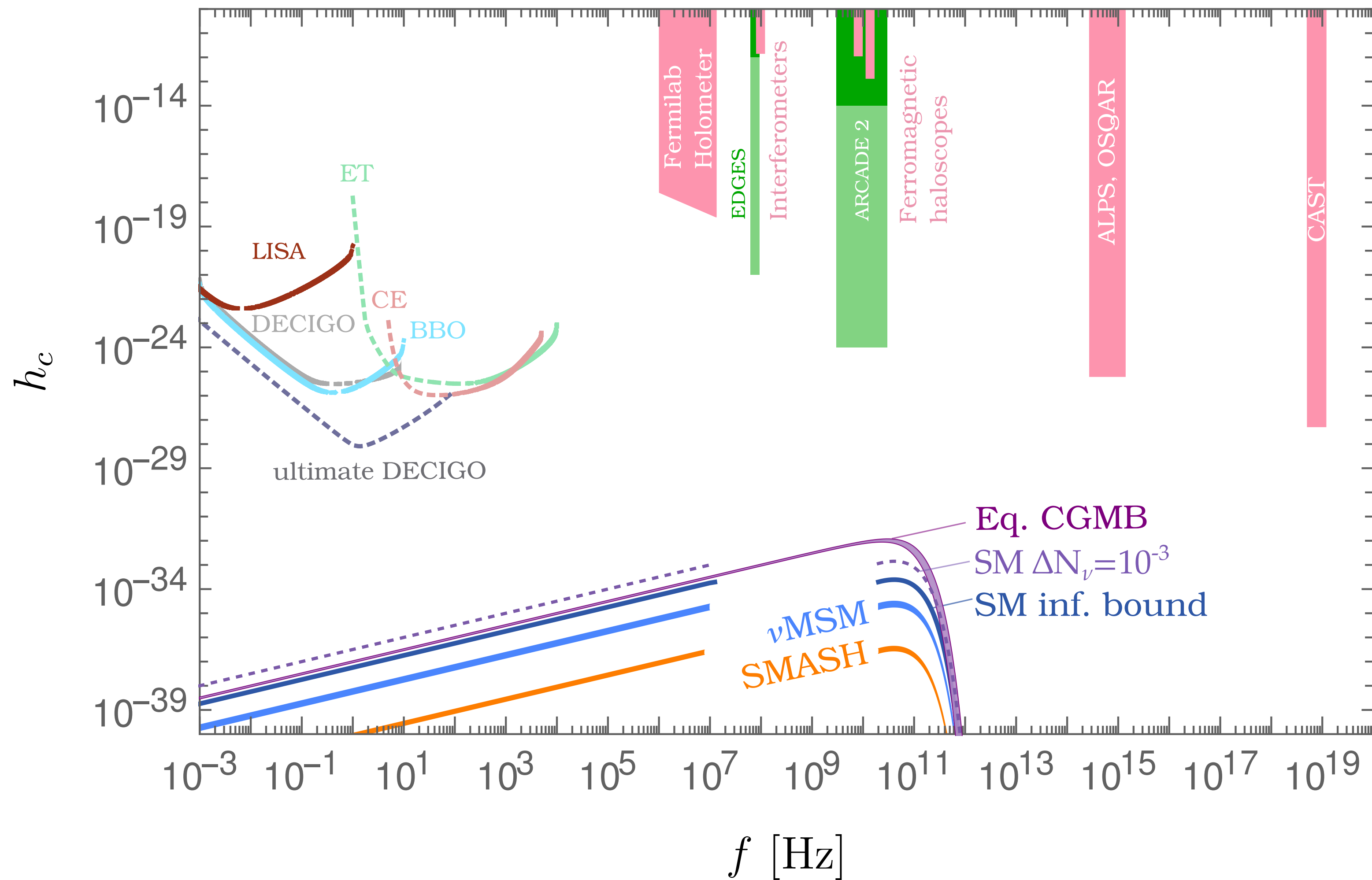
- Hence, at LO for $k \sim T$, equivalence with kinetic theory*

$$\dot{f}_{\text{GW}}(t, \mathbf{k}) = \Gamma(k) n_{\text{B}}(k) = \frac{1}{8k} \int d\Omega_{2 \rightarrow 2} \sum_{abc} \left| \mathcal{M}_{cG}^{ab}(\mathbf{p}_1, \mathbf{p}_2; \mathbf{k}_1, \mathbf{k}) \right|^2 f_a(p_1) f_b(p_2) [1 \pm f_c(k_1)]$$

- Use **automation** to cross-check: FeynRules ([Alloul Christensen Degrande Duhr Fuks 1310.1921](#)) to generate the Feynman rules, FeynArts/FormCalc ([Hahn Paßehr Schappacher hep-ph/0012260 1604.04661](#)) to generate and square all LO diagrams. Tensor boson polarisation sum implemented by us

*Up to subtlety discussed later

Cosmological implications



Leading order for $k \sim T$

- Both methods give, to LO in the gauge couplings and top Yukawa

$$\begin{aligned} \Gamma(k) n_B(k) = & \frac{1}{8k} \frac{32\pi}{m_{\text{Pl}}^2} \int d\Omega_{2 \rightarrow 2} \left\{ \right. \\ & + n_B(p_1) n_B(p_2) [1 + n_B(k_1)] (g_1^2 + 15g_2^2 + 48g_3^2) \left(\frac{st}{u} + \frac{su}{t} + \frac{tu}{s} \right) \\ & - n_F(p_1) n_B(p_2) [1 - n_F(k_1)] \left[6|h_t|^2 t + (10g_1^2 + 18g_2^2 + 48g_3^2) \frac{s^2 + u^2}{t} \right] \\ & - n_B(p_1) n_F(p_2) [1 - n_F(k_1)] \left[6|h_t|^2 u + (10g_1^2 + 18g_2^2 + 48g_3^2) \frac{s^2 + t^2}{u} \right] \\ & \left. + n_F(p_1) n_F(p_2) [1 + n_B(k_1)] \left[6|h_t|^2 s + (10g_1^2 + 18g_2^2 + 48g_3^2) \frac{t^2 + u^2}{s} \right] \right\} \end{aligned}$$

- Full gauge group structure available from method 1, i.e.

$$\left| \mathcal{M}_{gG}^{gg}(\mathbf{p}_1, \mathbf{p}_2; \mathbf{k}_1, \mathbf{k}) \right|^2 = \frac{32\pi}{m_{\text{Pl}}^2} 2(N_c^2 - 1) N_c g_3^2 \left(\frac{st}{u} + \frac{su}{t} + \frac{tu}{s} \right)$$

- This is consistent with expectations from factorisation of $e\gamma \rightarrow eG$ in Compton amplitude and kinematic factors

Bjerrum-Bohr Holstein Planté Vanhove **PRD91** (2015)