Primordial gravitational waves with a blue-tilted spectrum

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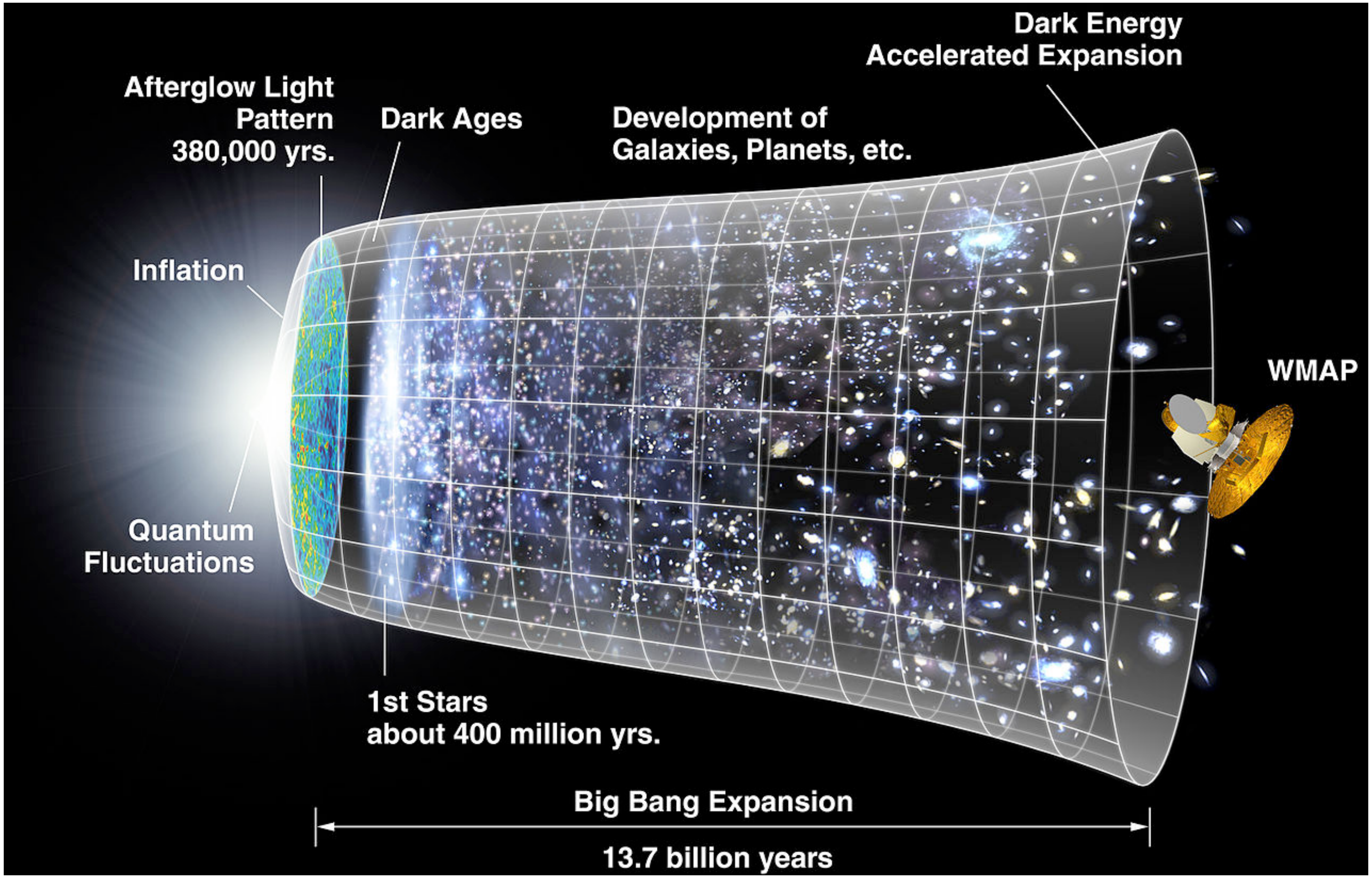
PONT 2020

References:
G. Calcagni, S. Kuroyanagi, arXiv: 2012.00170
S. Kuroyanagi, T. Takahashi, S. Yokoyama, arXiv: 2011.03323

Inflation

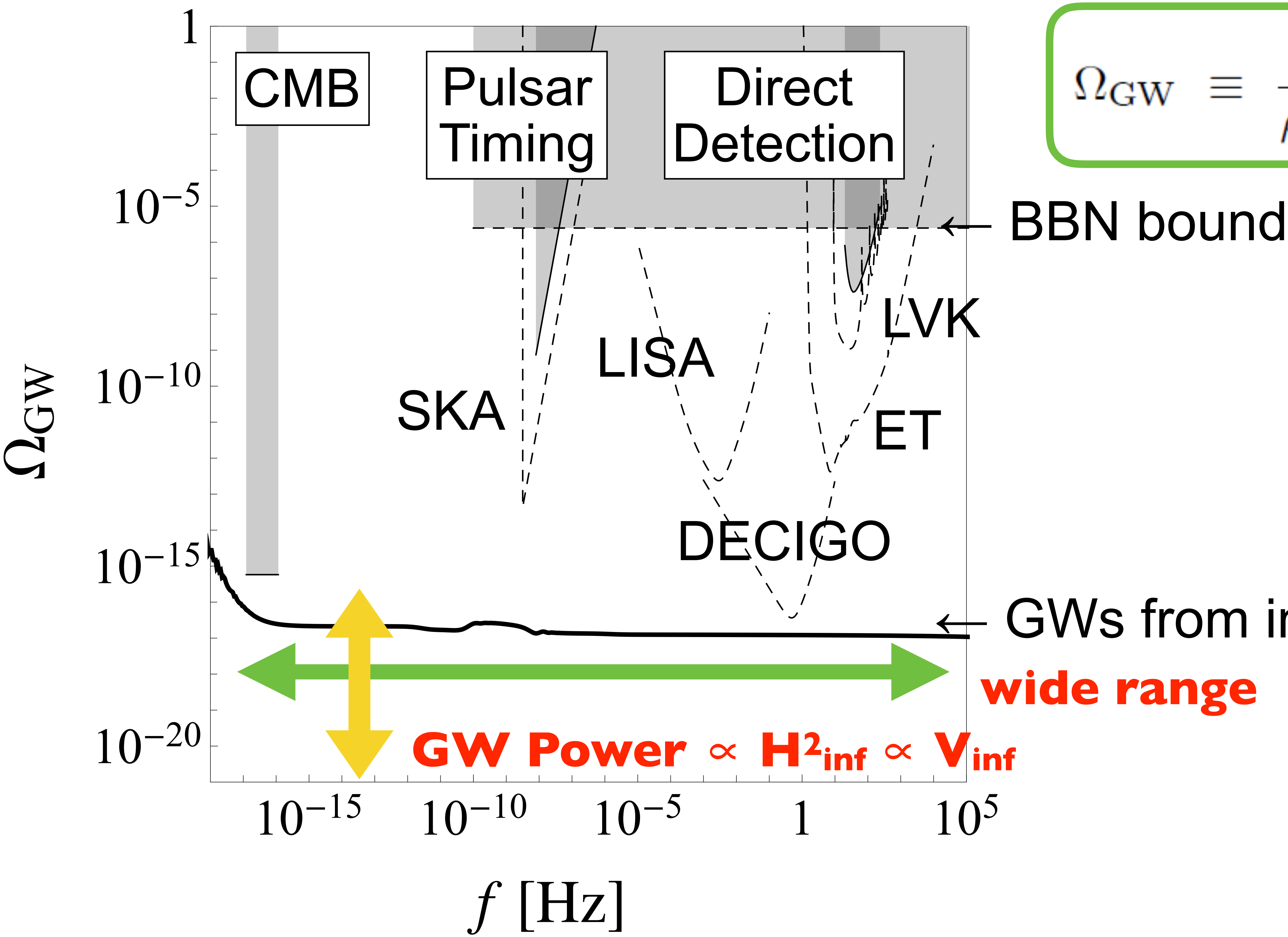
Accelerated expansion in the early Universe

Solves horizon/flatness/monopole problem

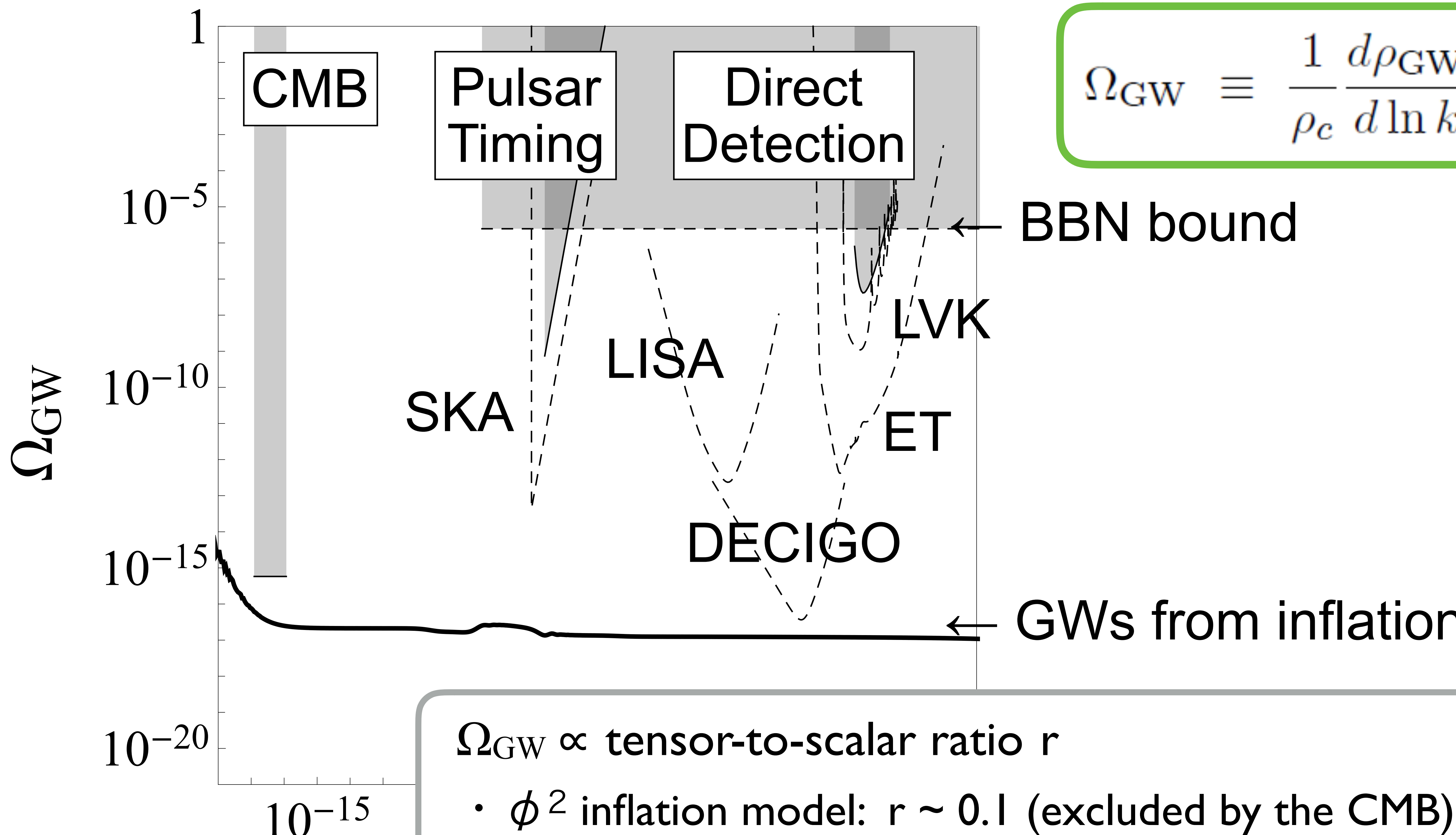


Gravitational waves - Smoking gun of inflation

$$\Omega_{GW} \equiv \frac{1}{\rho_c}\frac{d\rho_{GW}}{d\ln k}$$



Gravitational waves - Smoking gun of inflation



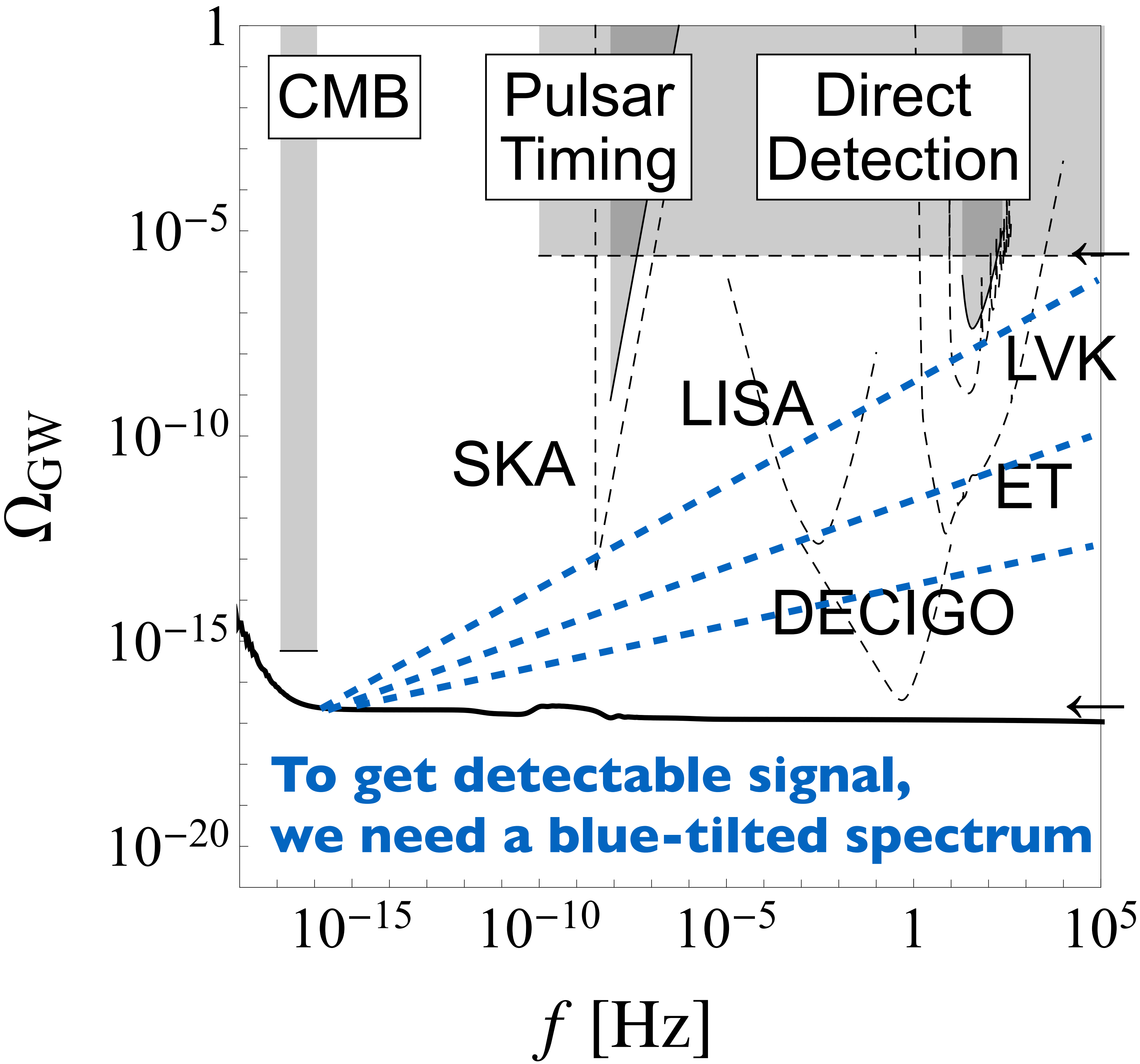
$$\Omega_{\rm GW} \equiv \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\ln k}$$

$\Omega_{\rm GW} \propto$ tensor-to-scalar ratio r

- ϕ^2 inflation model: r ~ 0.1 (excluded by the CMB)
- Starobinsky inflation model: r ~ 10^{-3}

→ not detectable by interferometers

Gravitational waves - Smoking gun of inflation



$$\Omega_{\rm GW} \equiv \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\ln k}$$

→ cannot be realized by standard slow-roll inflation

Theories beyond GR at inflationary energy scale can realize a blue-tilted spectrum

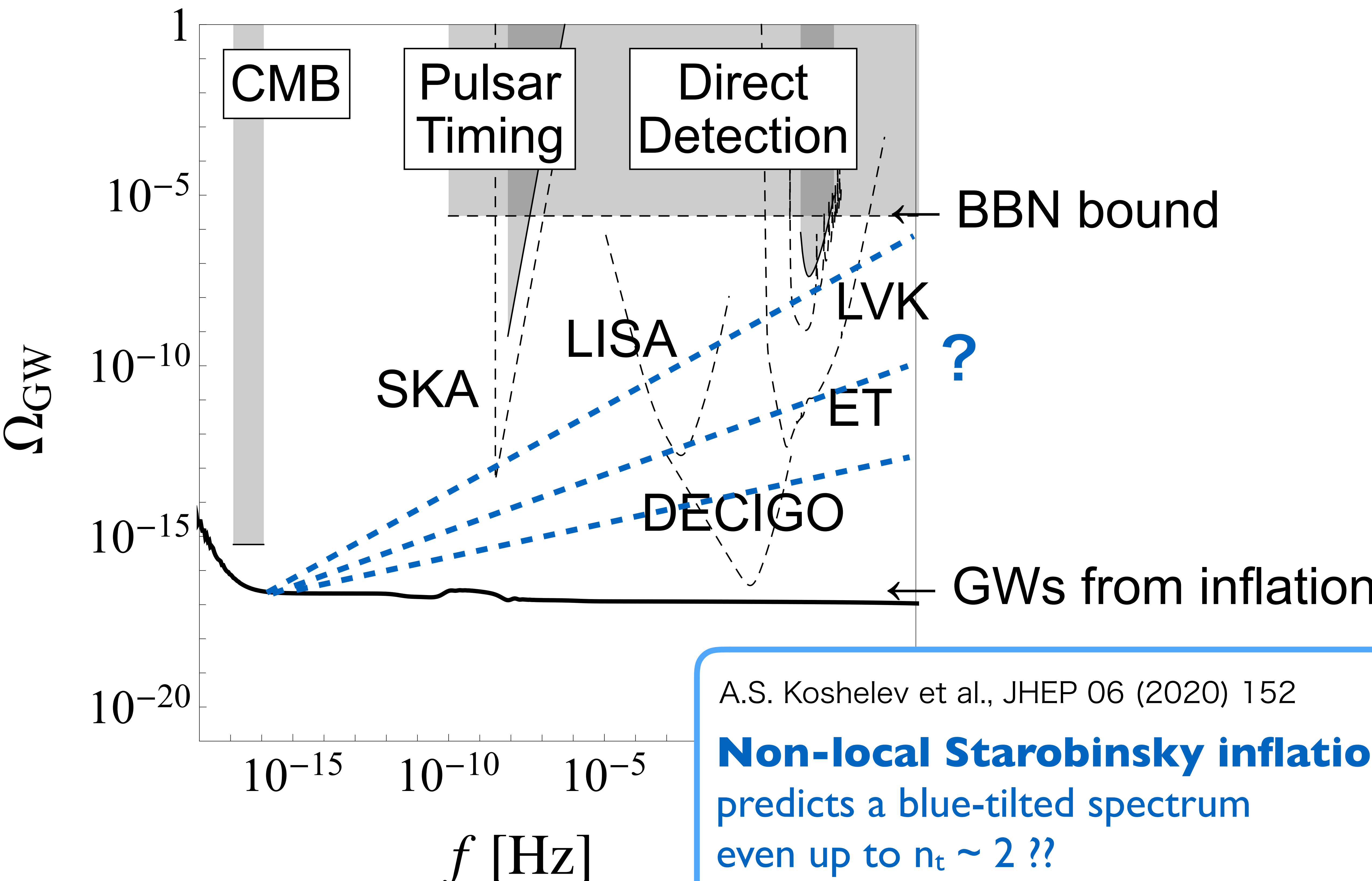
→ Any models motivated by quantum gravity?

G. Calcagni, S. Kuroyanagi, arXiv: 2012.00170

We investigate…

- **Non-local Starobinsky inflation**
- Brandenberger–Ho non-commutative inflation
- Multi-fractional spacetimes
- String-gas cosmology
- New ekpyrotic scenario

Non-local Starobinsky inflation



A.S. Koshelev et al., JHEP 06 (2020) 152

Non-local Starobinsky inflation
predicts a blue-tilted spectrum
even up to $n_t \sim 2$??

Starobinsky inflation

Action

A. A. Starobinsky, PLB 91 (1980) 99–102

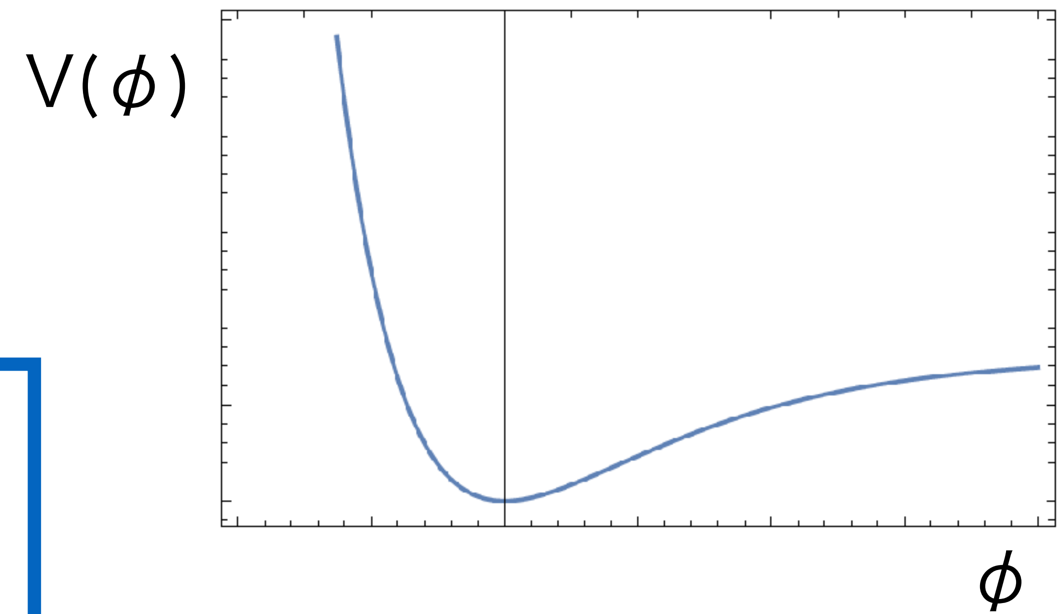
$$S = \frac{1}{2\kappa^2}\int d^4x\,\sqrt{|g|}\left[R + \frac{R^2}{6M_*^2}\right]$$

curvature-squared correction to the Einstein–Hilbert action

Einstein frame

$$S = \int d^4x\sqrt{|g|}\left[\frac{\hat{R}}{2\kappa^2} - \frac{1}{2}\hat{\partial}_\mu\phi\hat{\partial}^\mu\phi - V(\phi)\right]$$

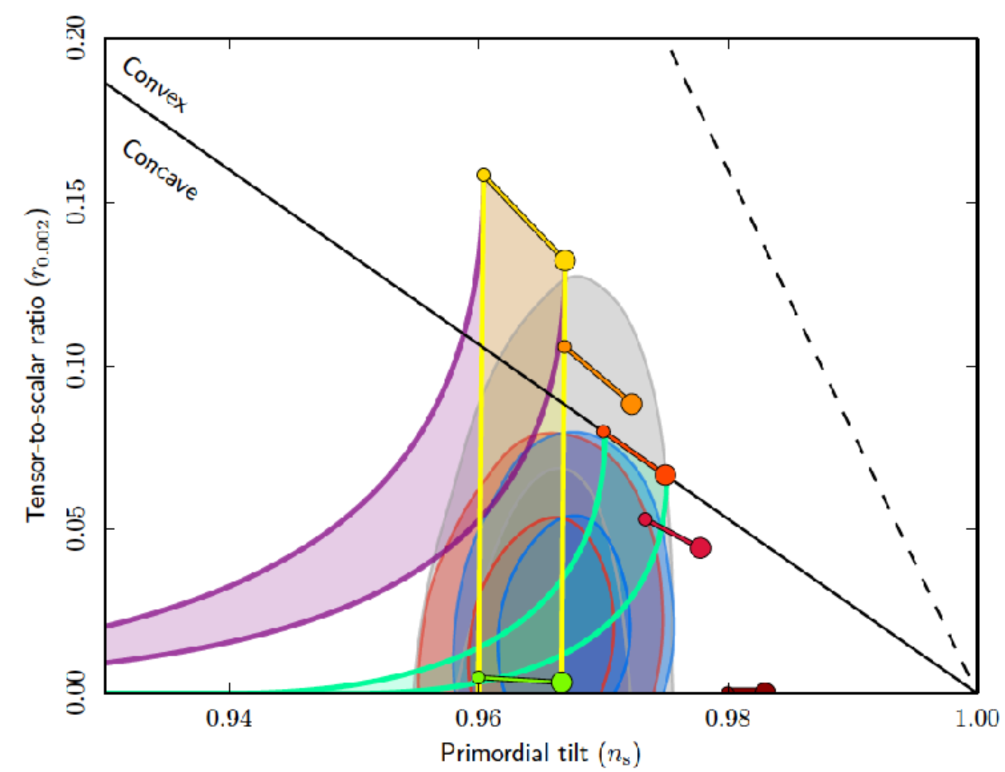
$$V(\phi) = \frac{3M_*^2}{4\kappa^2}\left(1 - e^{-\sqrt{\frac{2}{3}}\kappa\phi}\right)^2$$



$n_s \simeq 0.967$

$r \simeq 3 \times 10^{-3}$

→ Good agreement with CMB observations



Non-local Starobinsky inflation

Action

A.S. Koshelev et al., JHEP 11 (2016) 067

$$S = \frac{1}{2\kappa^2}\int d^4x\,\sqrt{|g|}\,[R + R\,\underline{\gamma_S(\Box)}R + C_{\mu\nu\rho\sigma}\underline{\gamma_C(\Box)}C^{\mu\nu\rho\sigma}]$$

vanish in a FLRW background

Embedded in quantum gravity

- Weyl tensor term is introduced to make the theory renormalizable
- Form factors are introduced to preserve unitarity (ghost freedom) and improve renormalizability

Form factors

Action

$$S = \frac{1}{2\kappa^2}\int d^4x\,\sqrt{|g|}\,[R + R\,\gamma_{\rm S}(\Box)R + C_{\mu\nu\rho\sigma}\gamma_{\rm C}(\Box)C^{\mu\nu\rho\sigma}]$$

chosen in such a way that the theory be
free of ghosts on a given background

$$\gamma_{\rm S}(z) = -\frac{1}{6M_*^2(z+2z_*)}\left[e^{\mathrm{H}_0(z+2z_*)}\left(1-\frac{M_*^2}{m^2}z\right)-\left(1+\frac{2M_*^2}{m^2}z_*\right)\right]$$

$$\gamma_{\rm C}(z) = \left(\frac{z_*}{m^2}+\frac{1}{2M_*^2}\right)\frac{e^{\mathrm{H}_2(z-4z_*)}-1}{z-4z_*} \qquad z := \frac{\Box}{M_*^2} \qquad z_* := \frac{R}{6M_*^2}$$

infinitely many derivatives in the form factor
→improve renormalizability

EoM for GWs (in Jordan Frame)

$$\left(\bar{\Box} - \frac{\bar{R}}{6}\right) e^{\mathrm{H}_2(z-4z_*)} h^{ij} = 0$$

$$\mathcal{P}_\mathrm{t} \propto (1-3\epsilon)\, e^{-\tilde{\mathrm{H}}_2(z_*)} \qquad \tilde{\mathrm{H}}_2(z) := \mathrm{H}_2(z-4z_*)$$

tensor-to-scalar ratio $\quad r \simeq \dfrac{12}{\mathcal{N}^2} e^{-\tilde{\mathrm{H}}_2(z_*)} \qquad z_* := \dfrac{R}{6M_*^2}$

spectral index $\quad n_\mathrm{t} \simeq -\dfrac{3}{2\mathcal{N}^2} + \dfrac{1}{\mathcal{N}} z_* \tilde{\mathrm{H}}_2'(z_*)$

spectral running $\quad \alpha_\mathrm{t} \simeq -\dfrac{3}{\mathcal{N}^3} - \dfrac{1}{12\mathcal{N}^3} z_* \tilde{\mathrm{H}}_2'(z_*) - \dfrac{1}{\mathcal{N}^2} z_*^2 \tilde{\mathrm{H}}_2''(z_*)$

$\mathcal{N} := \ln \dfrac{a(t_\mathrm{e})}{a(t)}$: e-folding number

EoM for GWs (in Jordan frame)

$$\left(\bar{\Box} - \frac{\bar{R}}{6}\right) e^{\mathrm{H}_2(z-4z_*)} h^{ij} = 0$$

$R \propto H^2$
slowly changing variable
$\frac{1}{H}\dot{z}_* \simeq -2\epsilon z_* \simeq -\frac{z_*}{\mathcal{N}}$

$$\mathcal{P}_\mathrm{t} \propto (1 - 3\epsilon)\, e^{-\tilde{\mathrm{H}}_2(z_*)}$$

tensor-to-scalar ratio $r \simeq \frac{12}{\mathcal{N}^2} e^{-\tilde{\mathrm{H}}_2(z_*)}$ $\quad z_* := \frac{R}{6M_*^2}$

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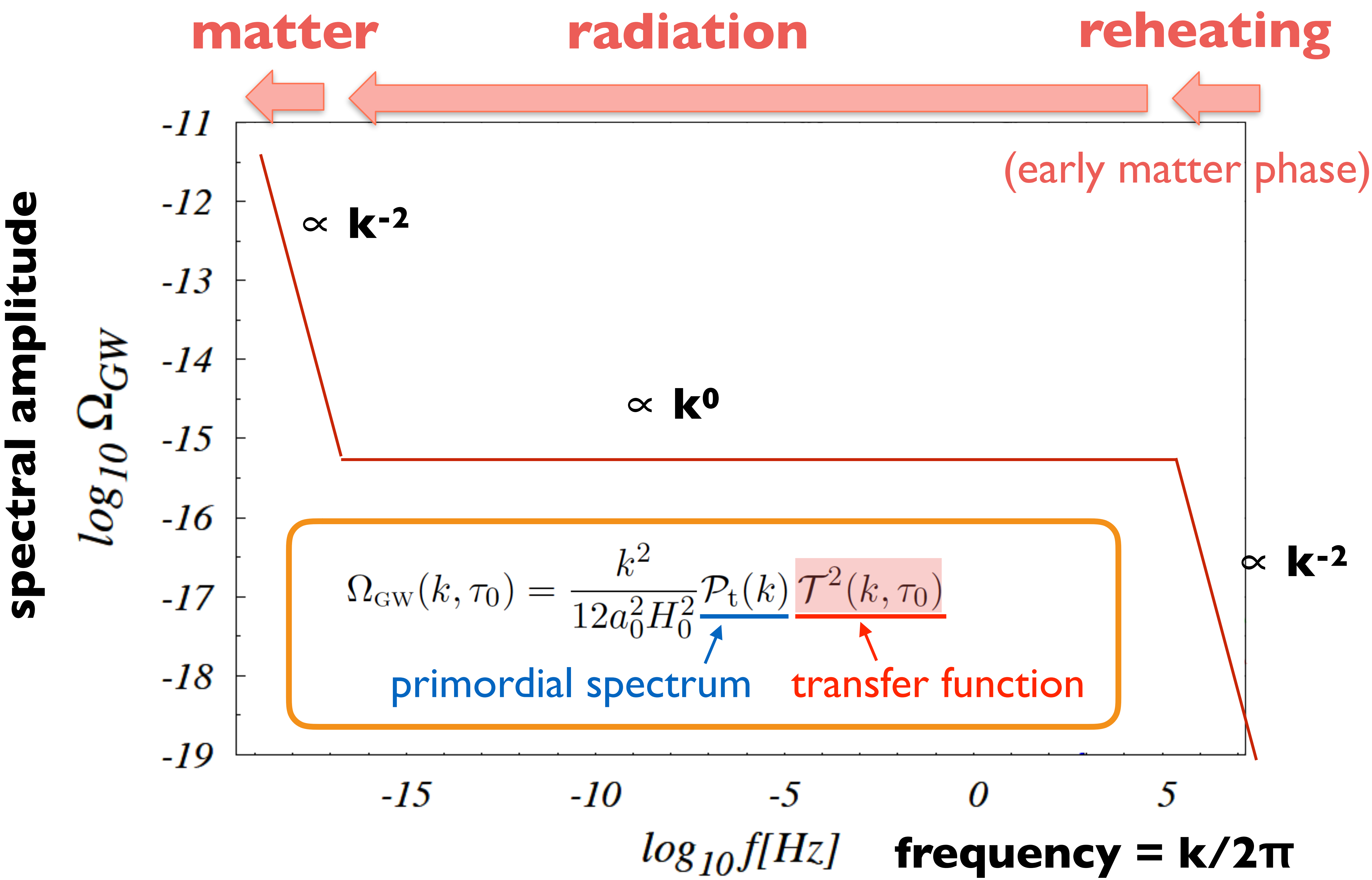
standard slow-roll terms
$\epsilon \simeq \frac{1}{2\mathcal{N}}$: suppressed at higher orders

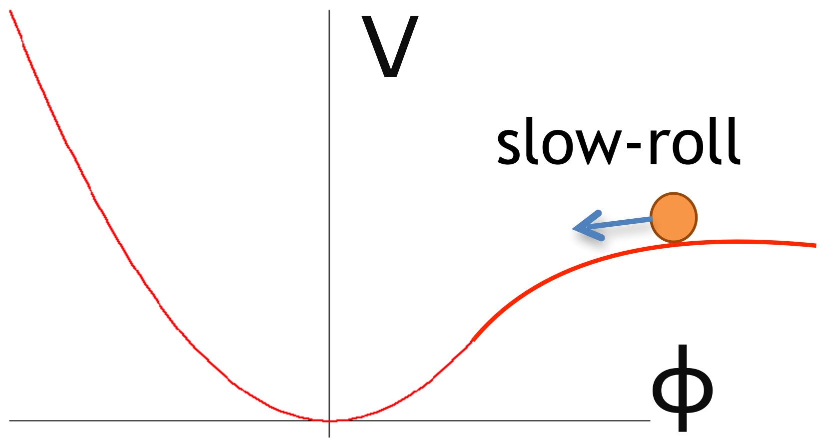
$\mathcal{N} := \ln \frac{a(t_\mathrm{e})}{a(t)}$: e-folding number

Spectral shape

Hubble expansion history affects the spectral shape



Spectral shape



primordial spectrum

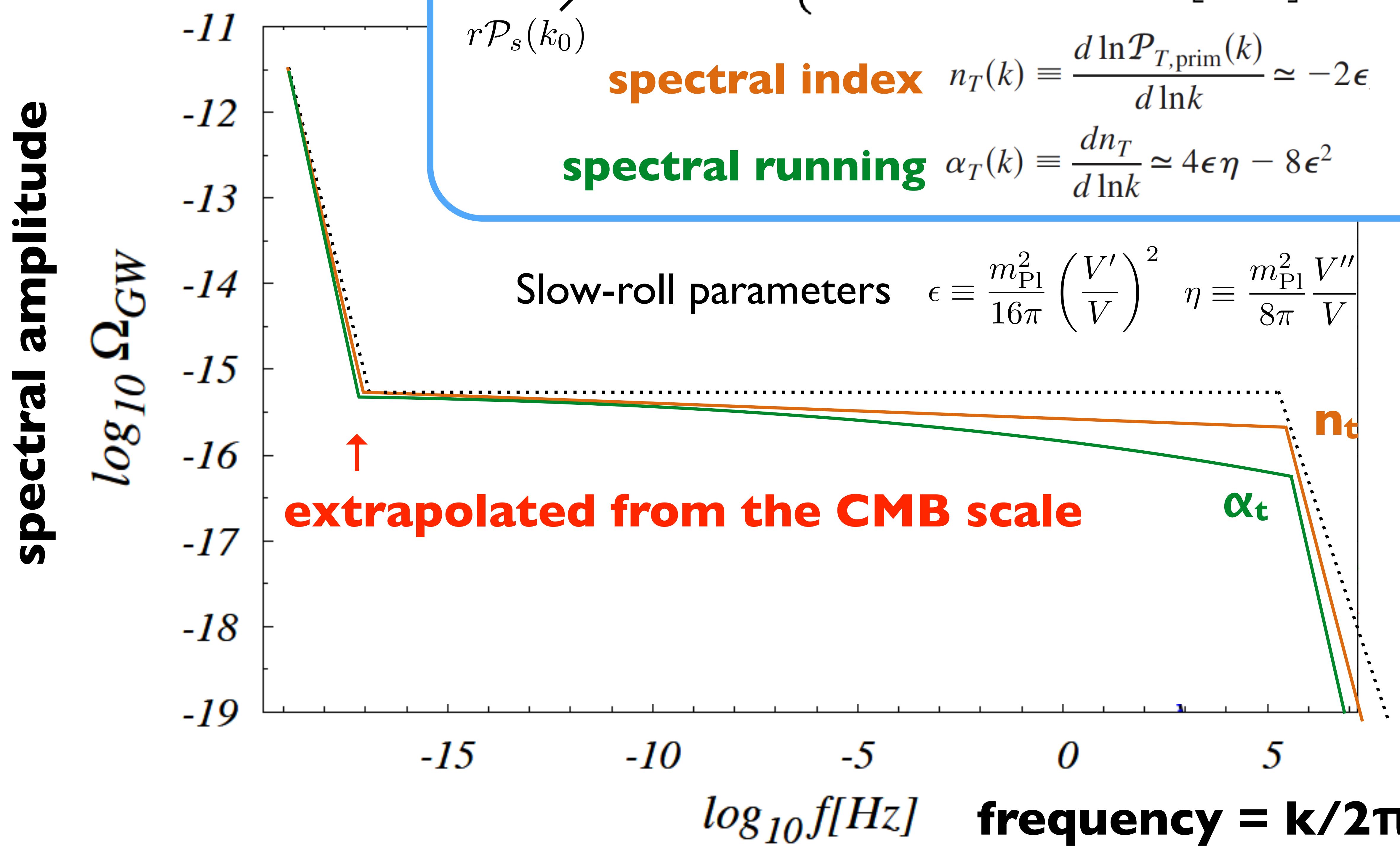
$$\mathcal{P}_{\rm t}(k) = \mathcal{P}_{\rm t}(k_0)\, \exp\left\{ n_{\rm t}(k_0) \ln \frac{k}{k_0} + \frac{\alpha_{\rm t}(k_0)}{2} \left[\ln \frac{k}{k_0} \right]^2 + \cdots \right\}$$

$r\mathcal{P}_s(k_0)$

spectral index $n_T(k) \equiv \dfrac{d \ln \mathcal{P}_{T,\rm prim}(k)}{d \ln k} \simeq -2\epsilon$

spectral running $\alpha_T(k) \equiv \dfrac{d n_T}{d \ln k} \simeq 4\epsilon\eta - 8\epsilon^2$

Slow-roll parameters $\epsilon \equiv \dfrac{m_{\rm Pl}^2}{16\pi} \left(\dfrac{V'}{V} \right)^2 \quad \eta \equiv \dfrac{m_{\rm Pl}^2}{8\pi} \dfrac{V''}{V}$



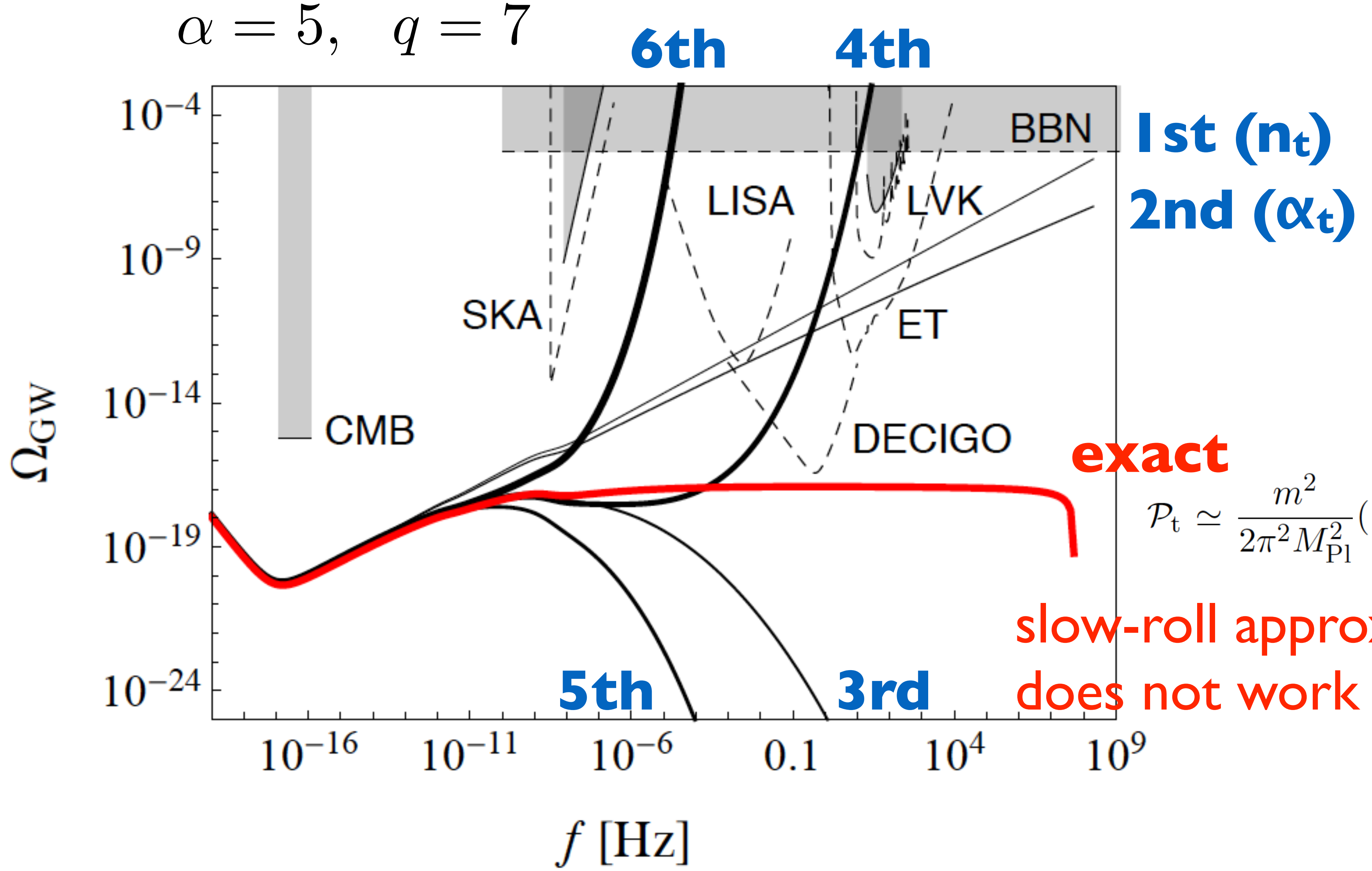
Result

Kuz'min form factor

Yu. V. Kuz'min, Sov. J. Nucl. Phys. 50, 1011 (1989)

$$\mathrm{H}_{\mathrm{Kuz}}(z) := \alpha \left\{ \ln p(z) + \Gamma[0, p(z)] + \gamma_{\mathrm{E}} \right\} \qquad p^2(z) = z^q$$

$\alpha = 5, \quad q = 7$



$$\mathcal{P}_{\mathrm{t}} \simeq \frac{m^2}{2\pi^2 M_{\mathrm{Pl}}^2}(1-3\epsilon)\, e^{-\tilde{\mathrm{H}}_2(z_*)}$$

slow-roll approximation does not work

Reason

tensor-to-scalar ratio $r \simeq \frac{12}{\mathcal{N}^2} e^{-\tilde{\mathrm{H}}_2(z_*)}$ $\qquad z_* := \frac{R}{6M_*^2}$

spectral index $n_\mathrm{t} \simeq -\frac{3}{2\mathcal{N}^2} + \frac{1}{\mathcal{N}} z_* \tilde{\mathrm{H}}_2'(z_*)$

spectral running $\alpha_\mathrm{t} \simeq -\frac{3}{\mathcal{N}^3} - \frac{1}{12\mathcal{N}^3} z_* \tilde{\mathrm{H}}_2'(z_*) - \frac{1}{\mathcal{N}^2} z_*^2 \tilde{\mathrm{H}}_2''(z_*)$

standard slow-roll terms

$\epsilon \simeq \frac{1}{2\mathcal{N}}$: suppressed at higher orders

non-local corrections

$$\mathcal{O}_l := \frac{d^l \ln \mathcal{P}_\mathrm{t}}{(d \ln k)^l} \simeq -\left(-\frac{z_*}{\mathcal{N}}\right)^l \tilde{\mathrm{H}}_2^{(l)}(z_*) + O\left(\frac{1}{\mathcal{N}^{l+1}}\right)$$

diverges when $z_* \gg 1$, a regime where the non-local effect is large

Result

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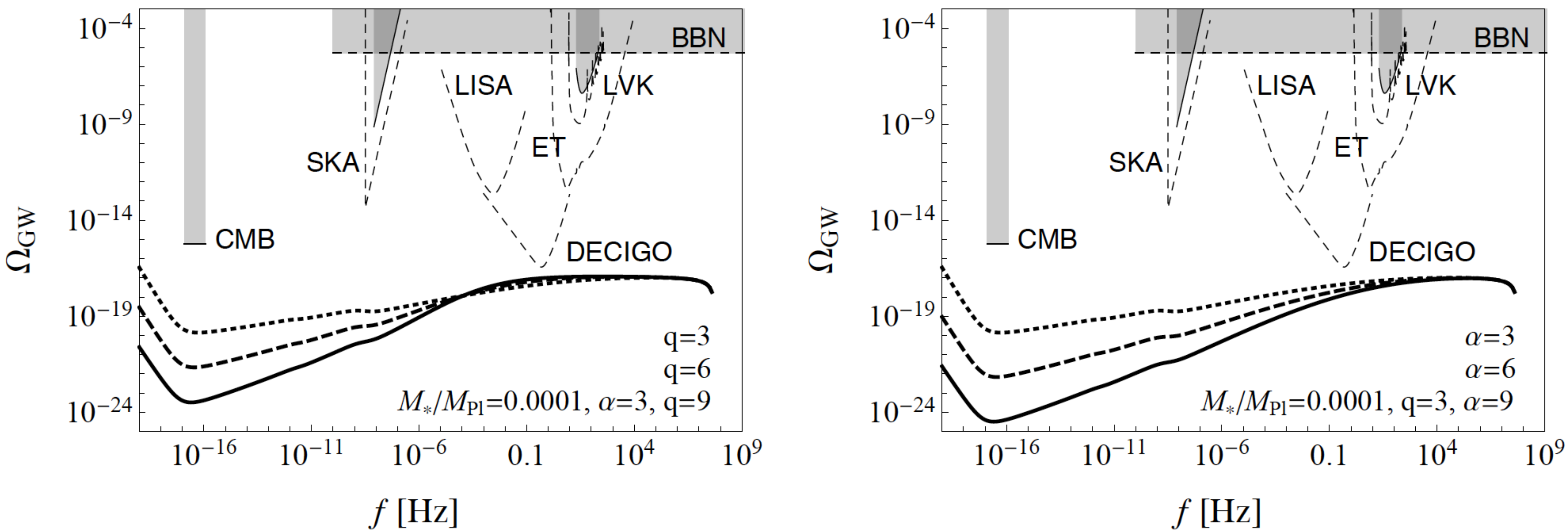
Yu. V. Kuz'min, Sov. J. Nucl. Phys. 50, 1011 (1989)

$$\mathrm{H}_{\mathrm{Kuz}}(z) := \alpha\left\{\ln p(z) + \Gamma[0, p(z)] + \gamma_{\mathrm{E}}\right\} \qquad p^2(z) = z^q$$

exact formula $\mathcal{P}_{\mathrm{t}} \simeq \frac{m^2}{2\pi^2 M_{\mathrm{Pl}}^2}(1-3\epsilon)\, e^{-\tilde{\mathrm{H}}_2(z_*)}$



Instant reheating is assumed

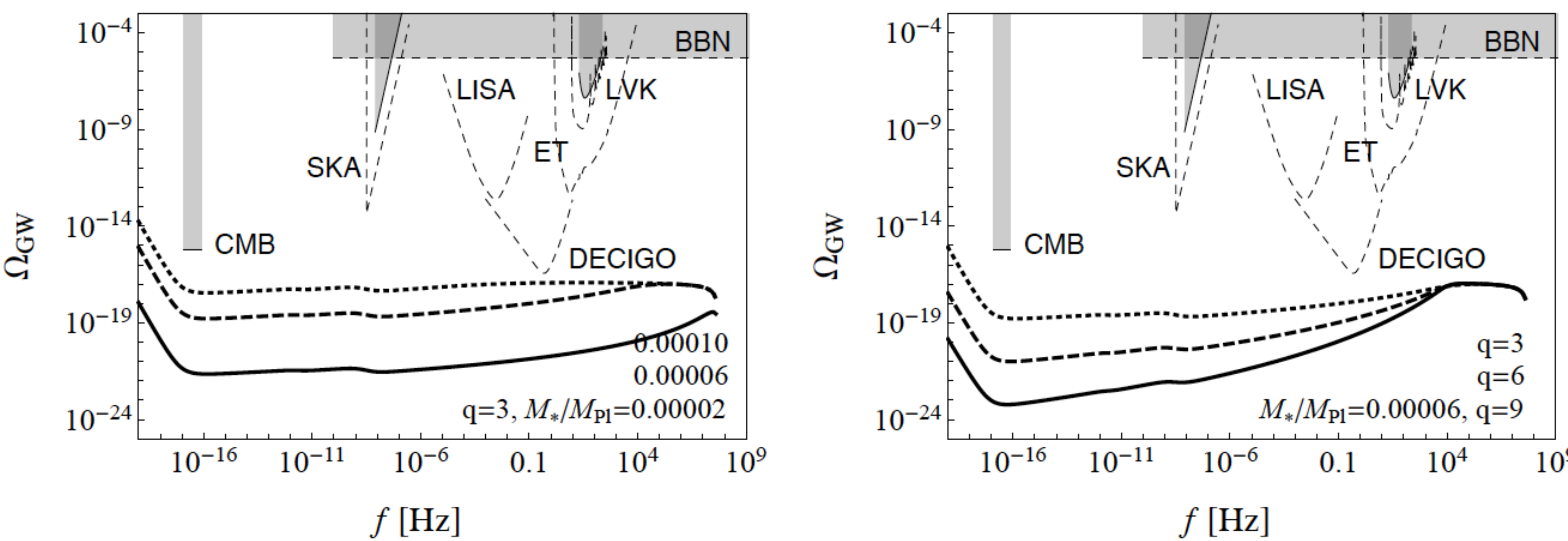
Result

Tomboulis form factor

E.T. Tomboulis, hep-th/9702146
L. Modesto, PRD 86, 044005 (2012)

$$\mathrm{H_{Tom}}(z) := \frac{1}{2}\left\{\ln p^2(z) + \Gamma[0, p^2(z)] + \gamma_{\mathrm{E}}\right\} \qquad p^2(z) = z^q$$

exact formula $\mathcal{P}_{\mathrm{t}} \simeq \frac{m^2}{2\pi^2 M_{\mathrm{Pl}}^2}(1-3\epsilon)\, e^{-\tilde{\mathrm{H}}_2(z_*)}$



Instant reheating is assumed

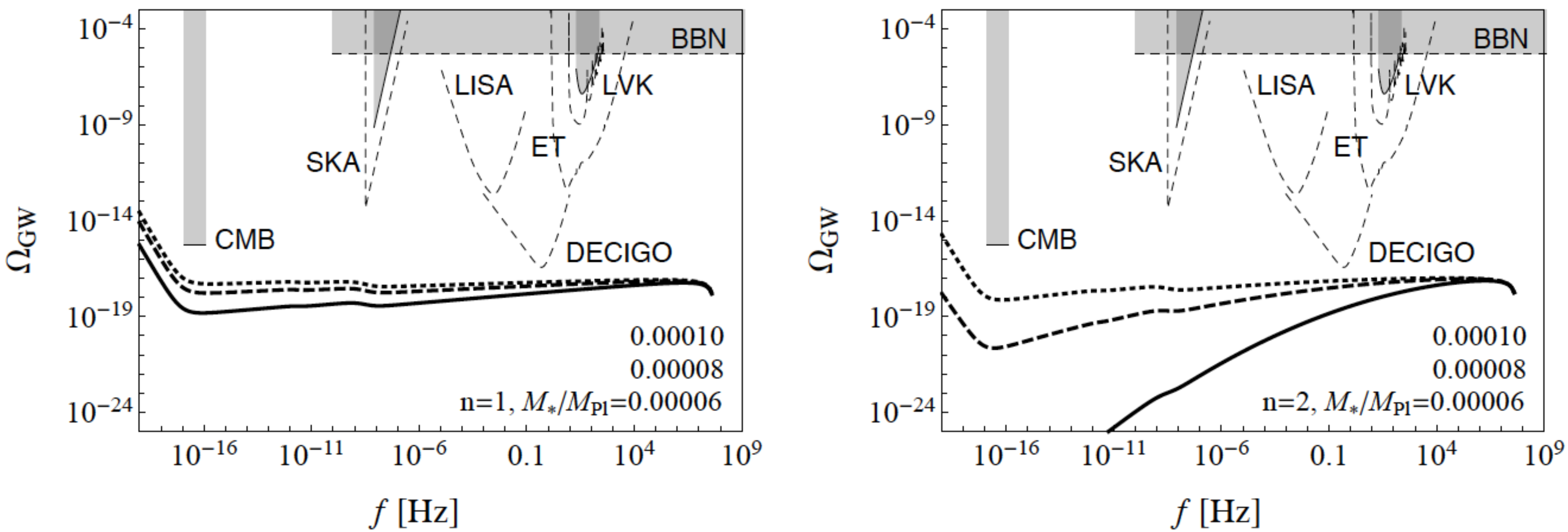
Result

Monomial form factor

G. Wataghin, Zeitschrift Physik 88, 92 (1934)
N.V. Krasnikov, Theor. Math. Phys. 73, 1184 (1987)

$$\mathrm{H}_{\mathrm{mon}}(z) := (-z)^n$$

exact formula $$\mathcal{P}_{\mathrm{t}} \simeq \frac{m^2}{2\pi^2 M_{\mathrm{Pl}}^2}(1-3\epsilon)\, e^{-\tilde{\mathrm{H}}_2(z_*)}$$



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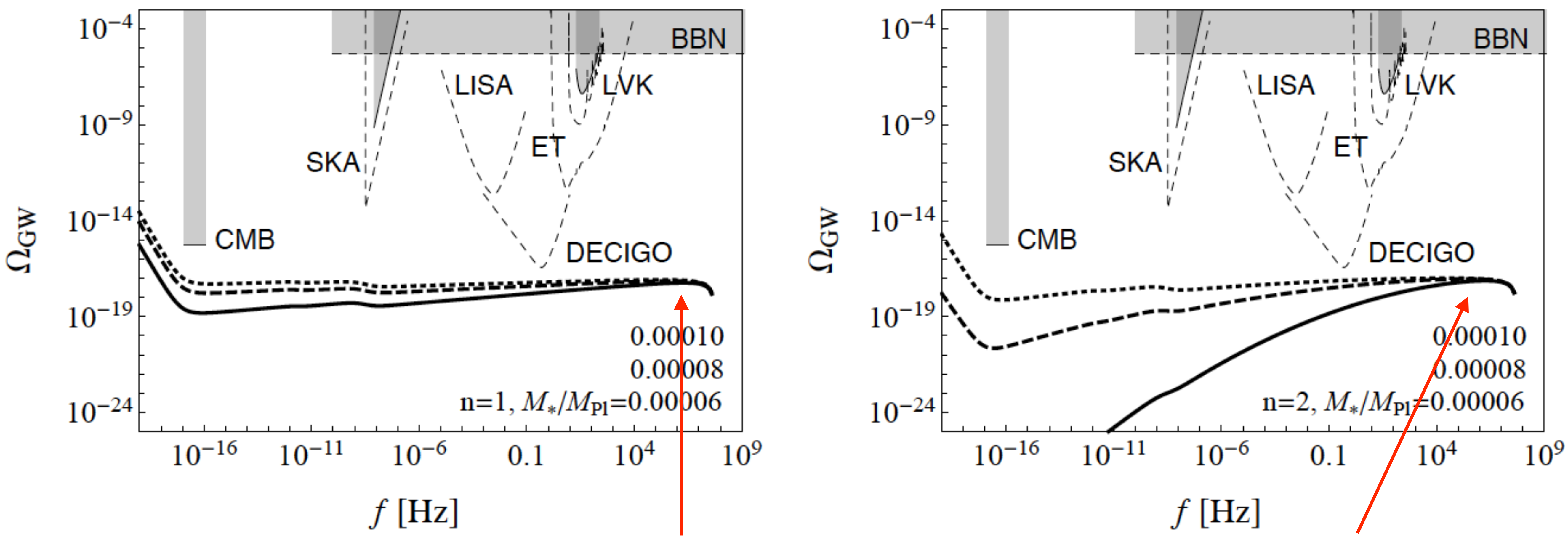
N.V. Krasnikov, Theor. Math. Phys. 73 1184 (1987)

$$\mathrm{H}_{\mathrm{mon}}(z) := (-z)^n$$

exact formula $$\mathcal{P}_{\mathrm{t}} \simeq \frac{m^2}{2\pi^2 M_{\mathrm{Pl}}^2}(1-3\epsilon)\boxed{e^{-\tilde{\mathrm{H}}_2(z_*)}} \rightarrow 1 \quad \text{for} \quad z_* \ll 1$$



converges to standard Starobinsky inflation at high frequency

Result

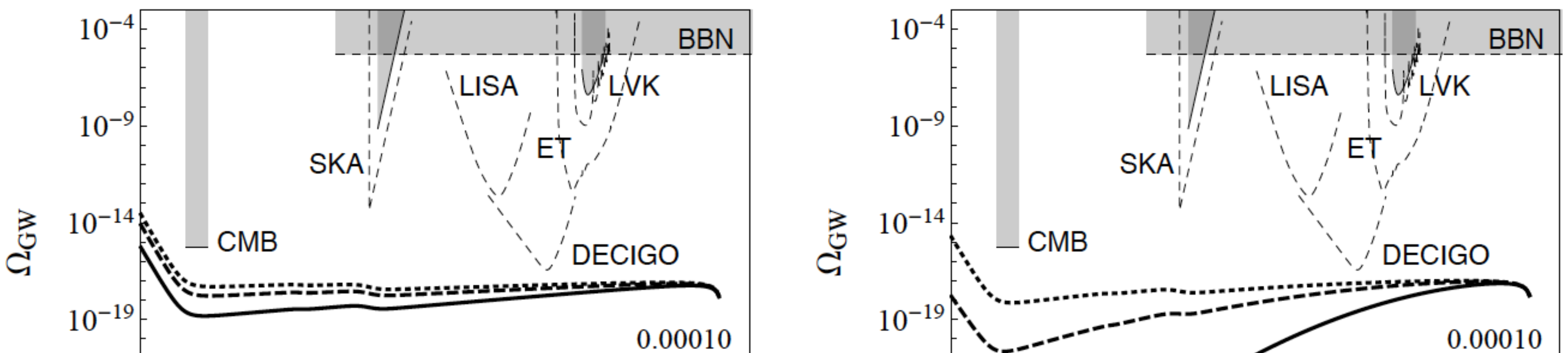
Monomial form factor

G. Wataghin, Zeitschrift Physik 88, 92 (1934)
N.V. Krasnikov, Theor. Math. Phys. 73, 1184 (1987)

$$\mathrm{H}_{\mathrm{mon}}(z) := (-z)^n$$

exact formula

$$\mathcal{P}_{\mathrm{t}} \simeq \frac{m^2}{2\pi^2 M_{\mathrm{Pl}}^2}(1-3\epsilon)\, e^{-\tilde{\mathrm{H}}_2(z_*)}$$

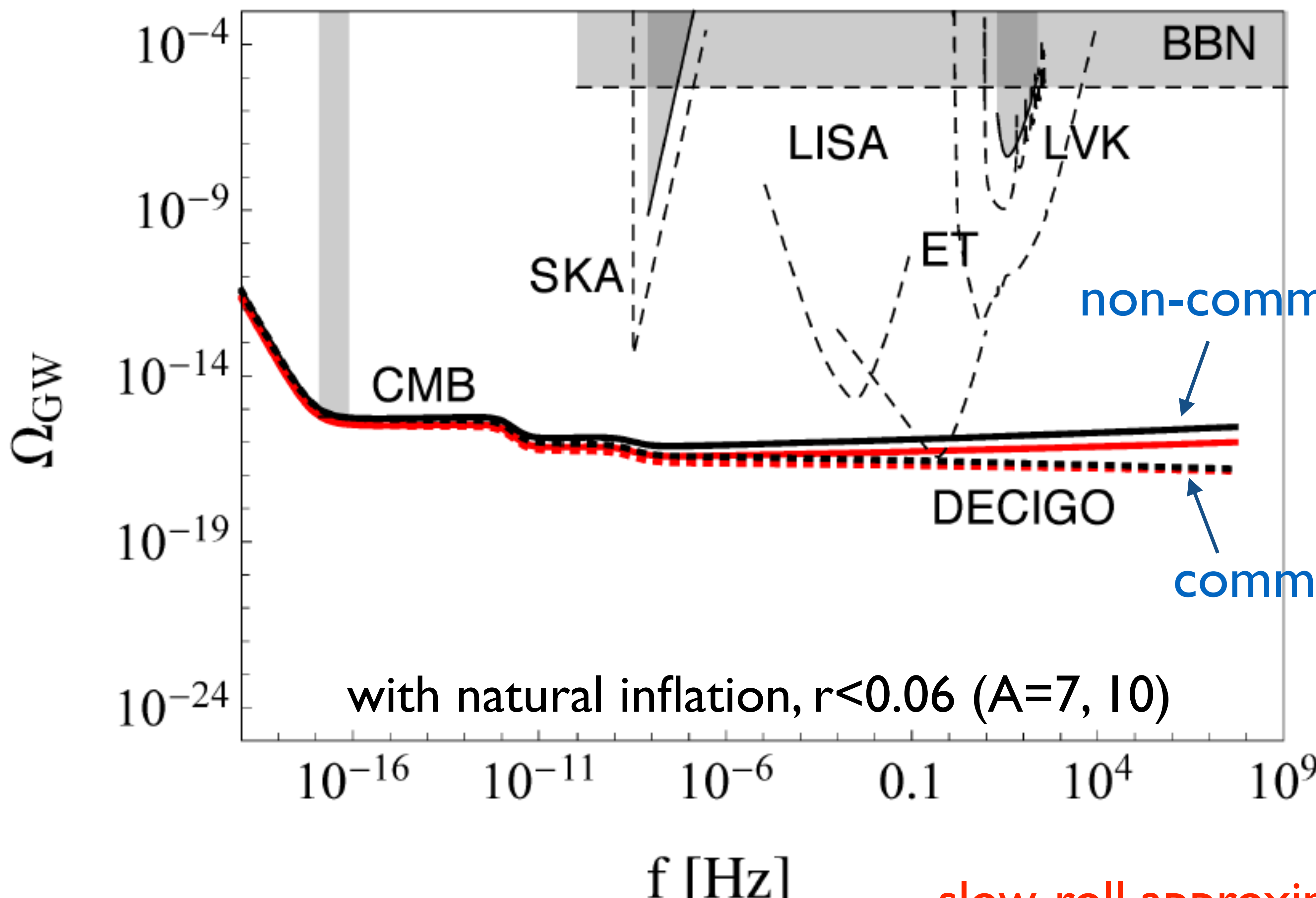


Main message:
A blue-tilt at the CMB scale does not always mean a detectable signal at interferometer scales

Brandenberger–Ho non-commutative inflation

R. Brandenberger and P. M. Ho, Phys. Rev. D 66, 023517 (2002)

Time and space coordinates do not commute: $[\tilde{\tau}, x] = i/M^2$



non-commutative case

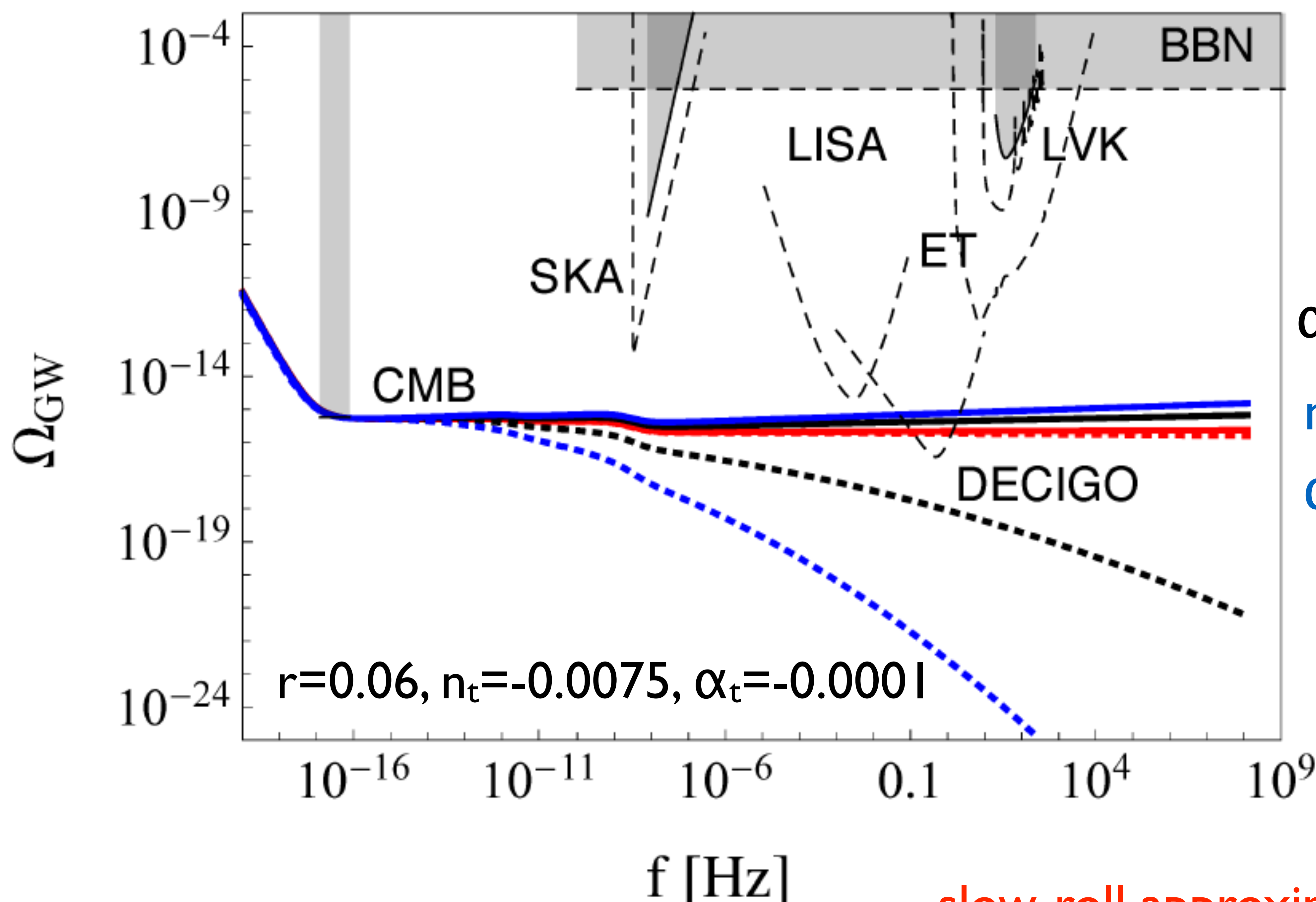
commutative case

slow-roll approximation is valid

Multi-fractional spacetimes

G. Calcagni, Phys. Rev. Lett. 104, 251301 (2010)

The dimension of spacetime changes with the probed scale



α=-0.5, -3, -5

$n_t \to \alpha n_T$

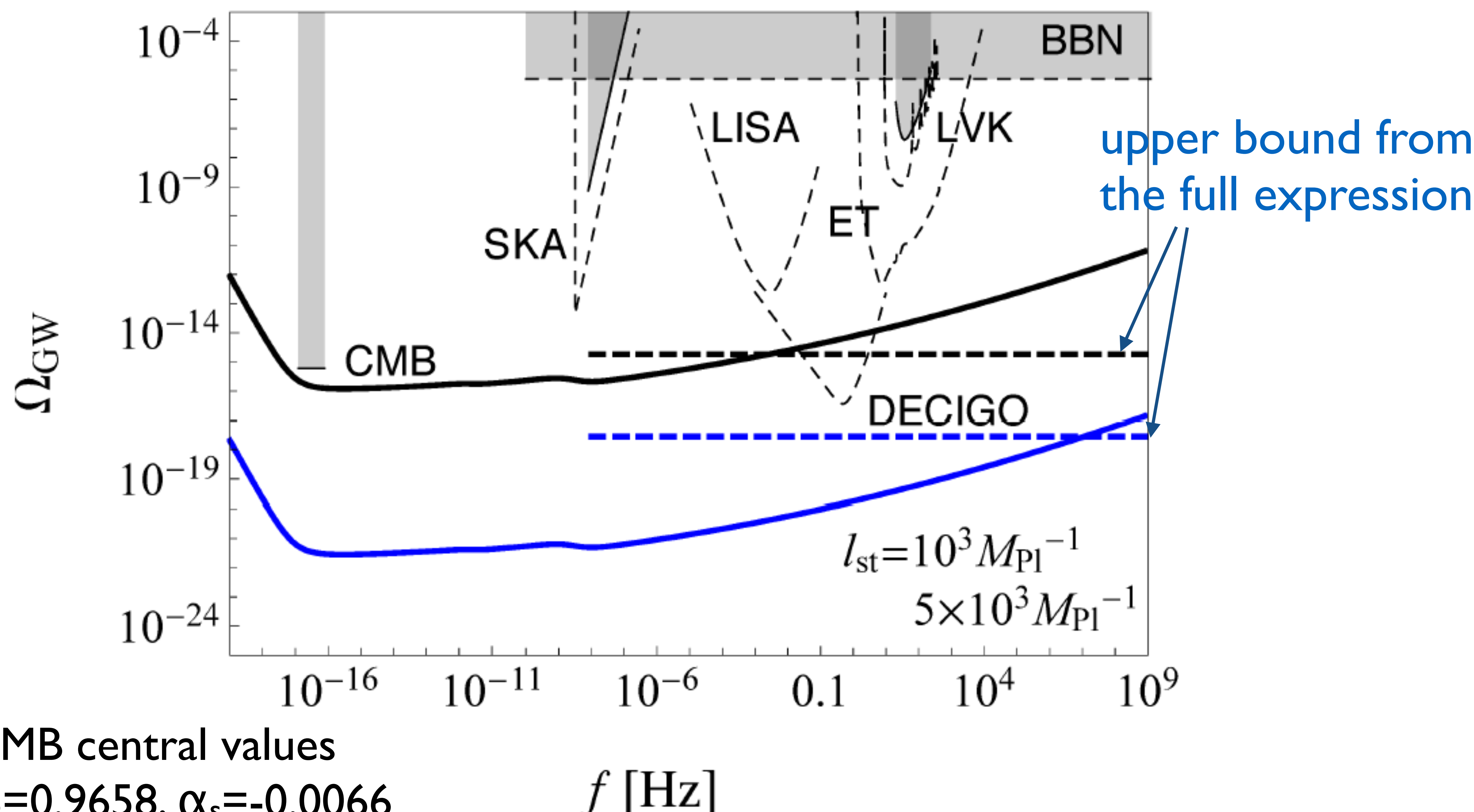
$\alpha_t \to \alpha^2 \alpha_T$

slow-roll approximation is valid

String-gas cosmology

R.H. Brandenberger et al., Phys. Rev. Lett. 98, 231302 (2007)

produces both scalar and tensor primordial spectra
via a thermal mechanism alternative to inflation

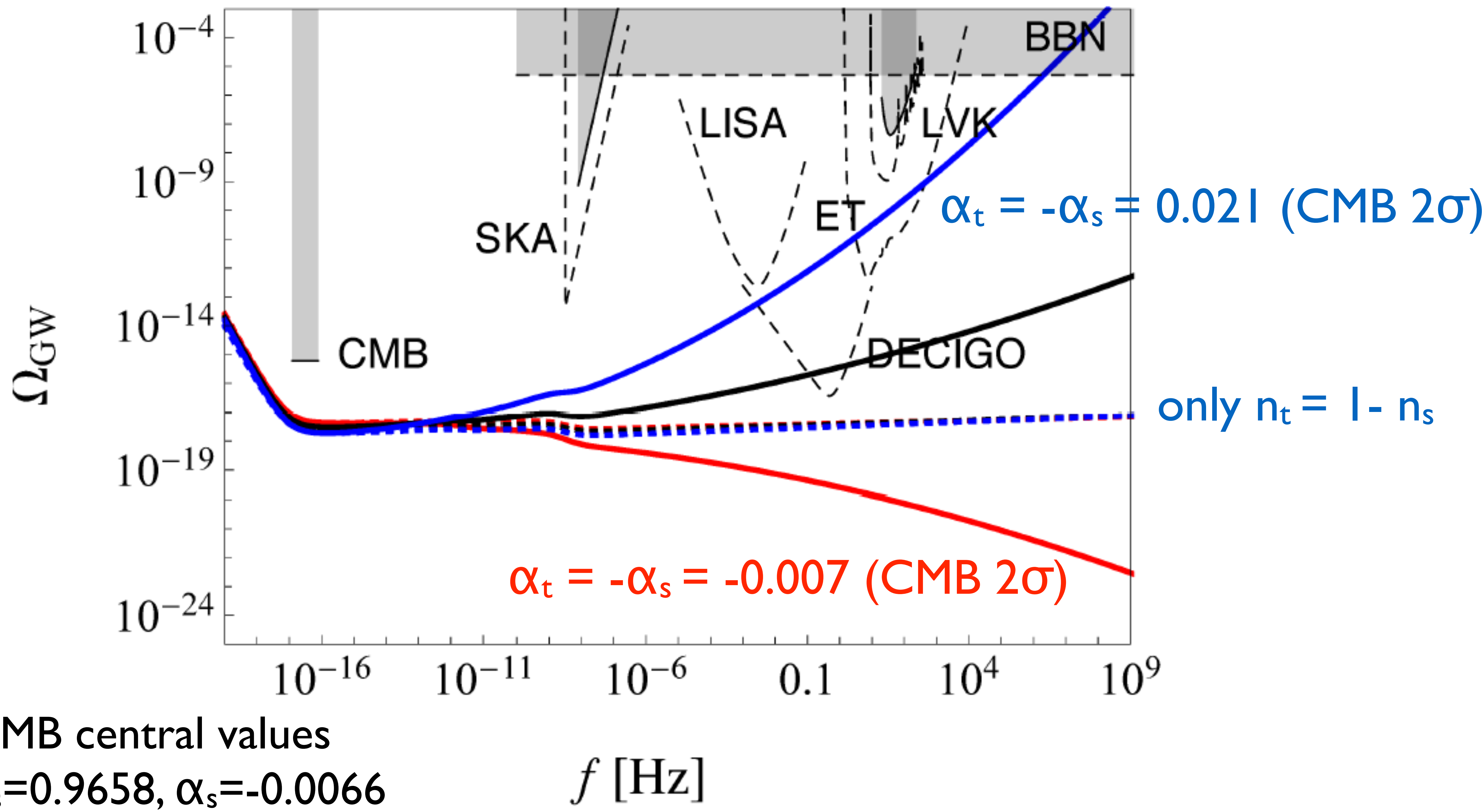


CMB central values
n_s=0.9658, α_s=-0.0066

New ekpyrotic scenario

R. Brandenberger and Z. Wang, Phys. Rev. D 101, 063522 (2020)

contraction (ekpyrosis) + bounce + expansion
perturbations generated by brane collision (+ S-brane)



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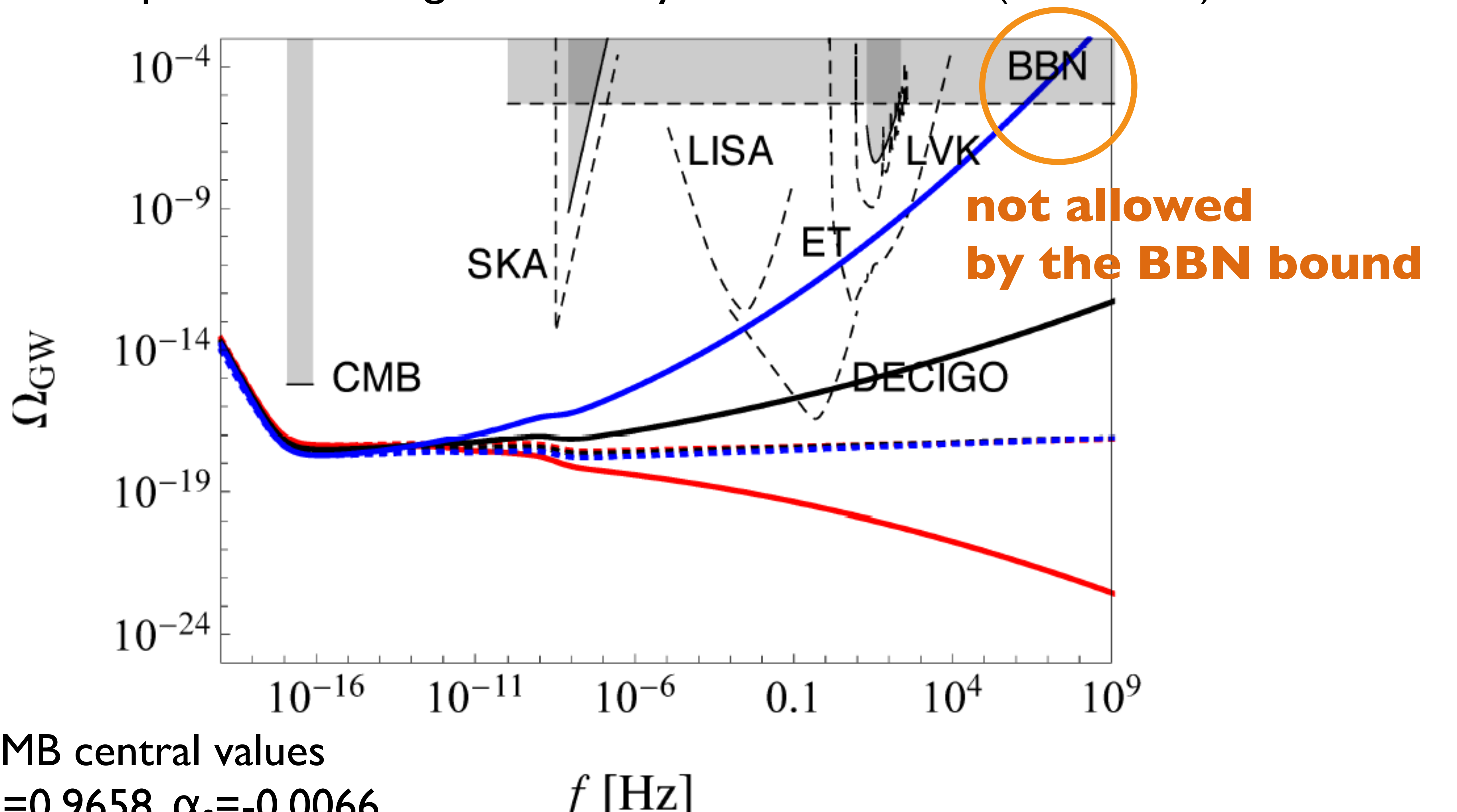

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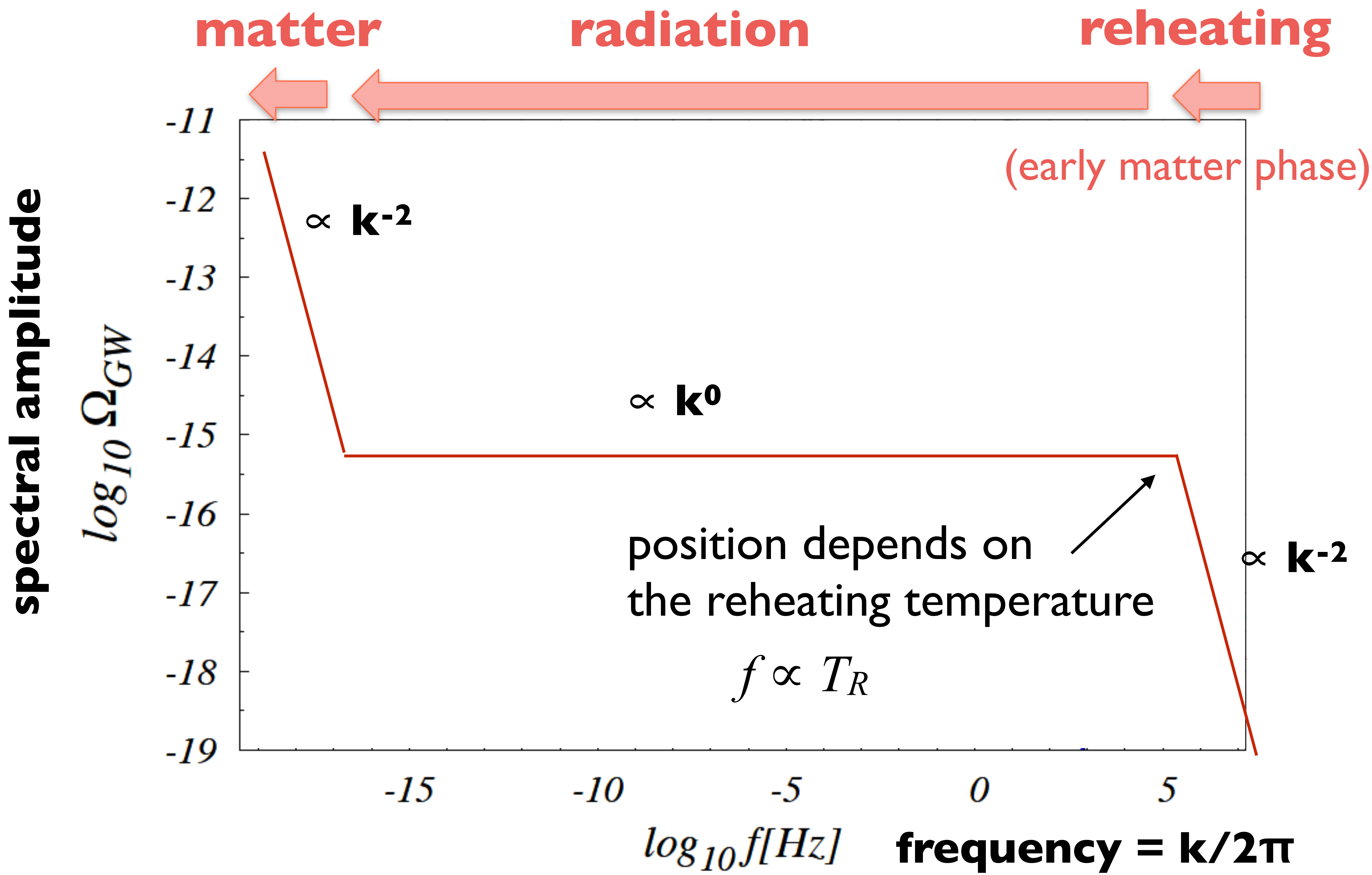
n_s=0.9658, α_s=-0.0066

Early matter phase helps to evade the upper bounds



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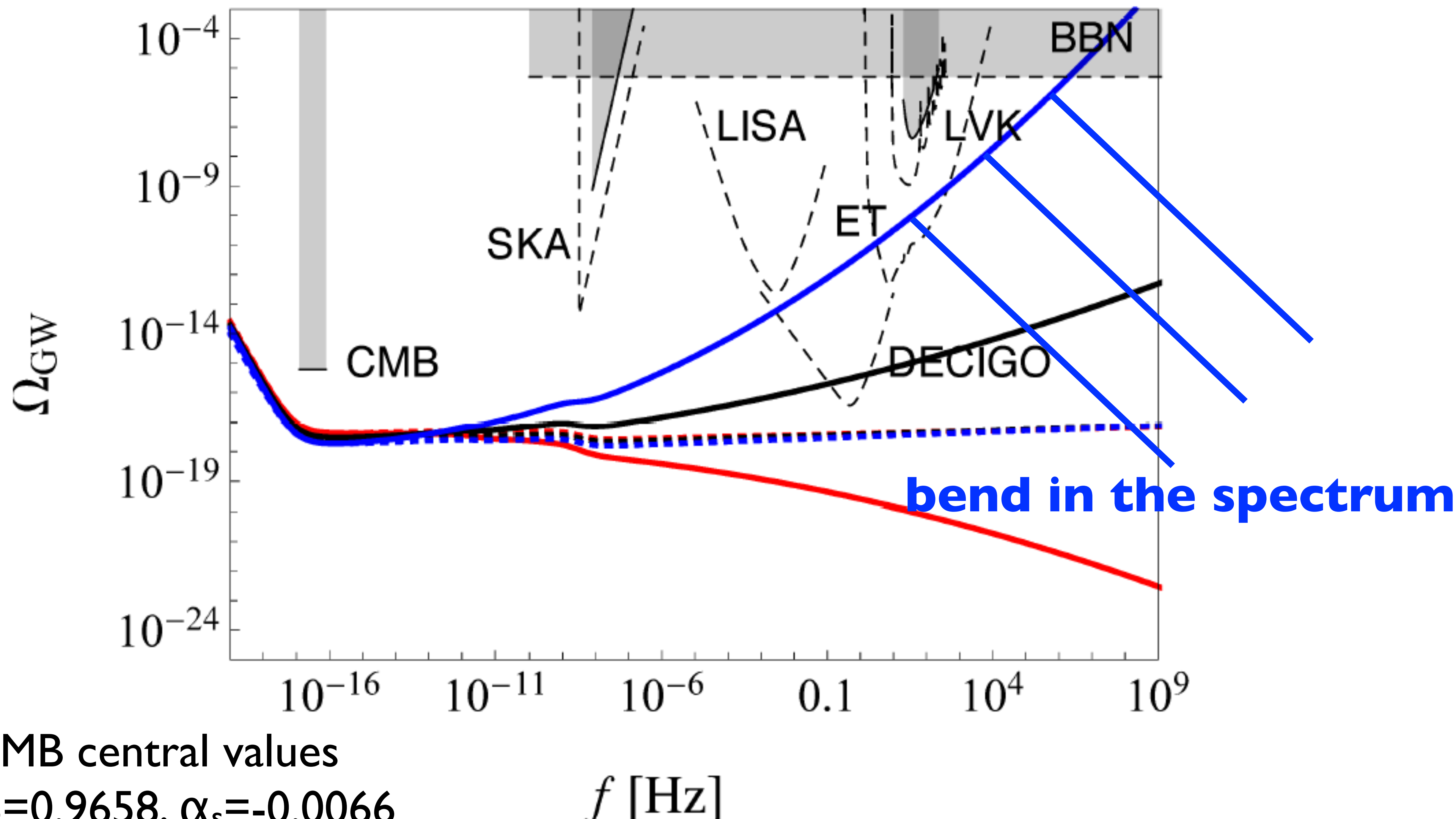
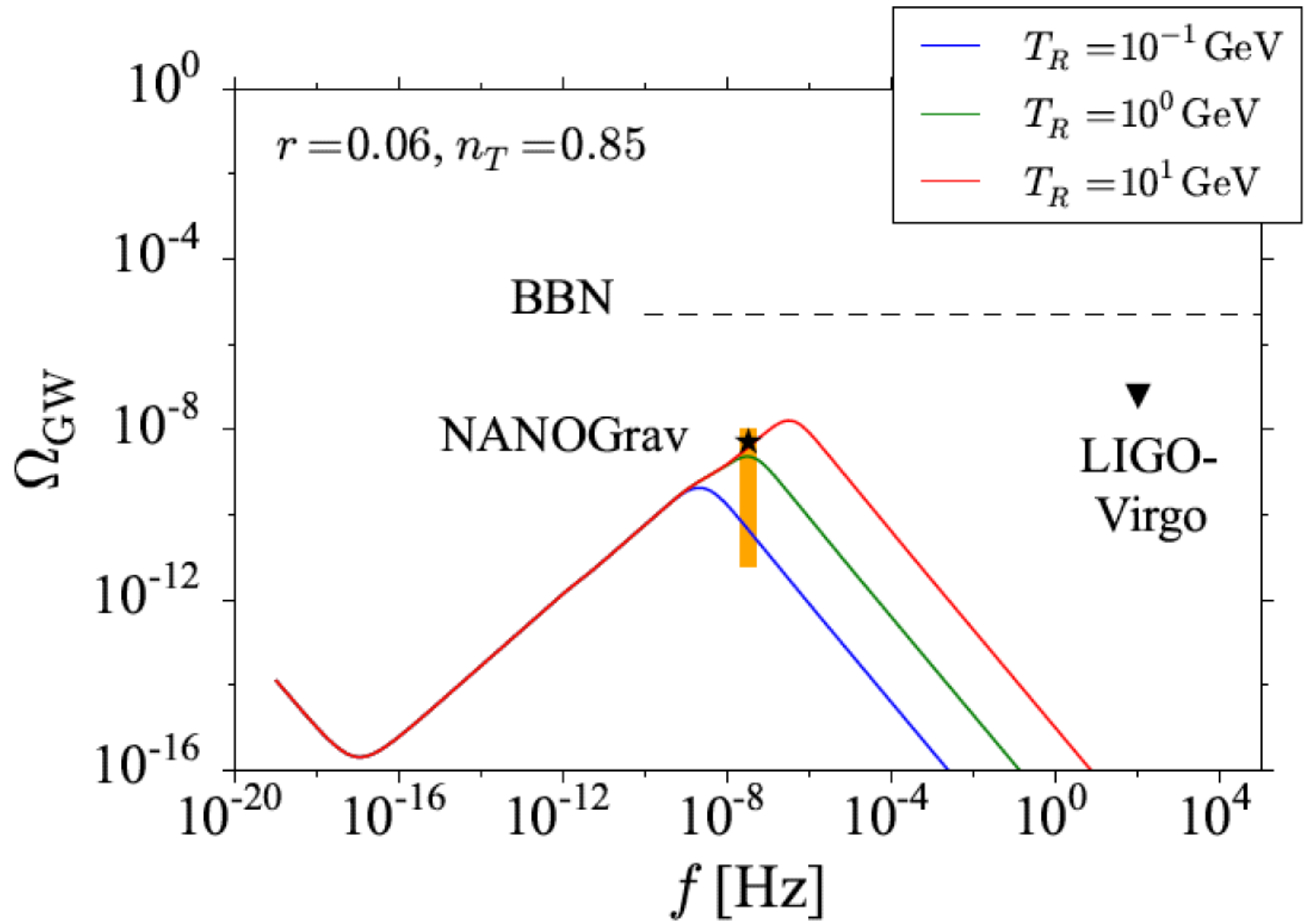


CMB central values
n_s=0.9658, α_s=-0.0066

Extra topic: explaining the NANOGrav signal by a blue-tilted spectrum?

S. Kuroyanagi, T. Takahashi, S. Yokoyama, arXiv: 2011.03323

We need a strongly blue-tilted spectrum: nt ~ 0.8 - 0.9

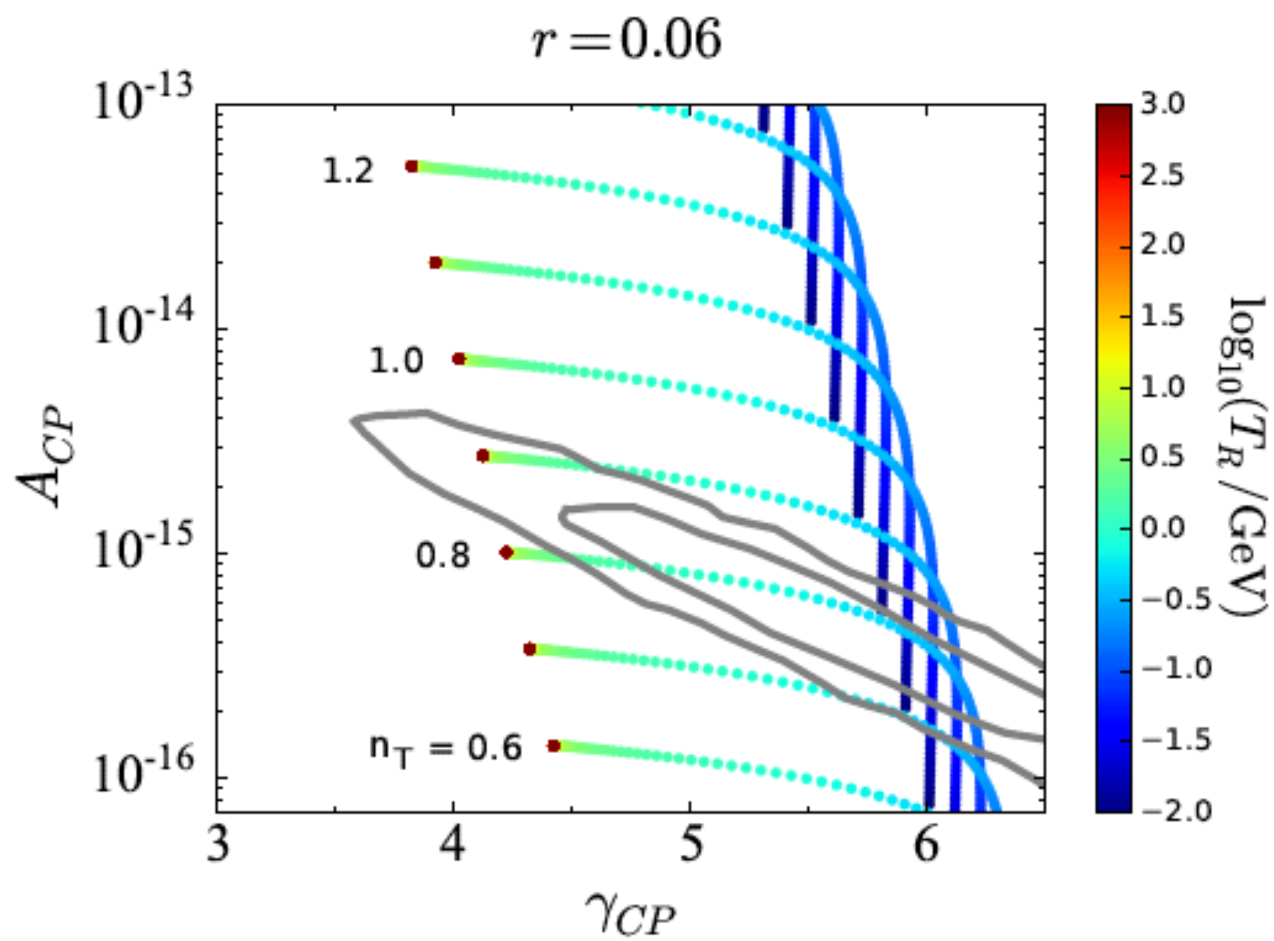


Long matter phase (low reheating temperature) is necessary to avoid the BBN and LIGO bound

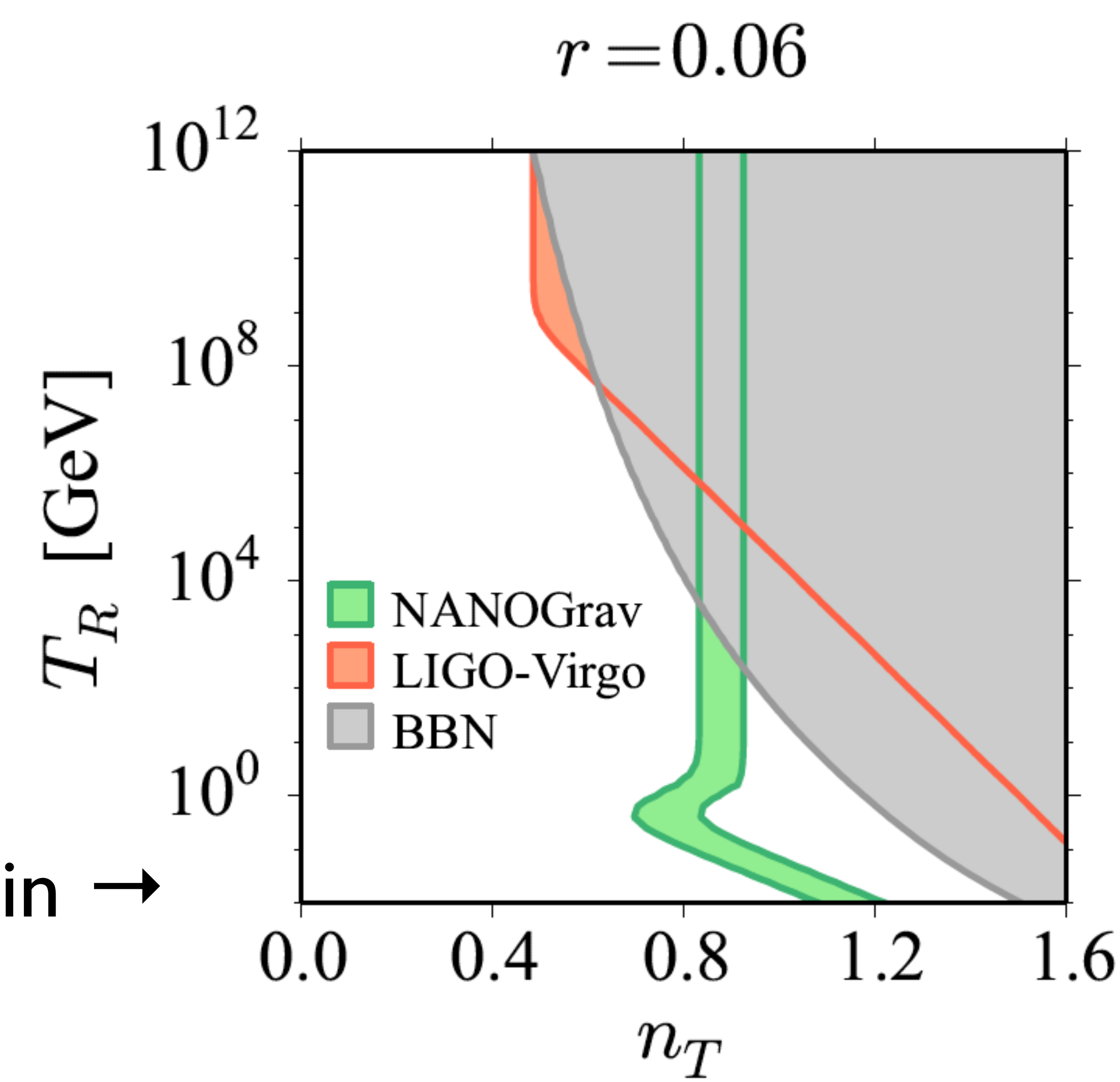
Extra topic: explaining the NANOGrav signal by a blue-tilted spectrum?

S. Kuroyanagi, T. Takahashi, S. Yokoyama, arXiv: 2011.03323

spectral amplitude



spectral tilt



Inflationary GWs can explain → the signal if $T_R < 10^3$GeV

Summary

GWs can become a powerful probe of the very early Universe

- GWs from inflation are promising, but the amplitude is typically small for standard slow-roll inflation
- Non-local Starobinsky inflation predicts a blue-tilted GW spectrum at CMB scales. However, we found that it does not give a detectable GW signal at interferometer scales.
- Other models of quantum gravity predict a signal detectable by DECIGO, but likely not by LISA.
- Early matter dominated phase (e.g. reheating) enables to evade the BBN constraint, and helps to give detectable signal by ET.