# Coupling to matter in degenerate scalar-tensor theories

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Based on **2004.11619** with C. Deffayet

# Outline and results

- Degenerate higher-order scalar-tensor theories (DHOST)
  - Review

    see also N. Frusciante's talk
  - Matter coupling problem
- Pathological matter fields
  - Loss of degeneracy
  - Non-commutation of constraints
- Spinor fields in DHOST
  - Not pathological (so far)

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- This motivates the study of **generalized scalar-tensor theories**

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Horndeski (1974)

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- These models are potentially pathological

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  - Second-order eqs. of motion

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    **Horndeski theory, Galileon** — Nicolis et al. (2008), Deffayet et al. (2009)

  - Higher-order eqs. of motion + degeneracy

    **Beyond Horndeski, DHOST** — Gleyzes, Langlois, Piazza, Vernizzi (2015), Crisostomi, Koyama, Tasinato (2016), Ben Achour, Langlois, Noui (2016)

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    $g_{\mu\nu}$ , $\phi$ , $\nabla_\mu\phi$ $+$ conjugate momenta

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  - Result

    $\mathbf{2} + \mathbf{1}$ **dynamical degrees of freedom**

# Matter coupling

- Key observation is that matter fields may spoil constraints

de Rham & Matas (2016)

| | |
|---|---|
| DHOST | 3 DoF |
| matter | $N$ DoF |
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- We will say that the coupling to a given matter sector is **inconsistent** if the total degree of freedom count is

$$> 3 + N$$

- If the matter sector is consistent within GR, but inconsistent within DHOST, then we will conclude that the **ghost has reappeared**

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  Isenberg & Nester (1977)

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Our framework

- Minimal coupling to the metric (but in a general frame)

- Quadratic DHOST $\mathcal{L} \supset \nabla^2\phi\,,\ (\nabla^2\phi)^2$

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**Example:** non-Maxwell vector field

$$\mathcal{L}_m = \sqrt{-g}\,\nabla^\mu B^\nu \nabla_\mu B_\nu$$

$$\text{rank}\,\mathcal{H}^{(\text{DHOST})} + \text{rank}\,\mathcal{H}^{(\text{matter})} < \text{rank}\,\mathcal{H}$$

because $\mathcal{H}$ is not block-diagonal

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(II) Some constraint in the matter sector fails to Poisson-commute with the DHOST constraint

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**Example:** cubic Galileon/KGB

$$\mathcal{L}_m = \sqrt{-g} \left[ -\frac{1}{2}(\nabla\pi)^2 + \kappa(\nabla\pi)^2 \Box\pi \right]$$

Hessian constraint ✓

Poisson bracket constraint $X$

# Spinor fields

Spinor fields in DHOST are *potentially* problematic

- they couple to the spin connection

  $\rightarrow$ Hessian matrix not block-diagonal

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  $\rightarrow$ must Poisson-commute with DHOST constraint

$$\mathcal{L}_m = -\frac{1}{2}\sqrt{-g}\,\bar{\lambda}\gamma^\mu\left(\partial_\mu + \Omega_\mu\right)\lambda$$

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We have shown that a large class of spinor models is actually **consistent**

- any quadratic action with arbitrarily many Majorana and/or Dirac spin-1/2 fields
- most general self-interacting Majorana spin-1/2 field, linear in $\nabla\lambda$

## Open questions

- Beyond minimal coupling
  - curvature couplings
  - scalar-matter couplings $\phi T^{\mu}{}_{\mu} \quad , \quad \partial_\mu\phi\partial_\nu\phi T^{\mu\nu}$
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**Thank you**