$1/N$ expansion for stochastic fields in de Sitter spacetime

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Quantum fields in curved spacetime and inflation

- Inflation - quasi-de Sitter phase in the early universe
- Standard approach to obtain the scale invariant power-spectrum observed in the CMB is **perturbative**
- What about higher order corrections ?
- We follow a **semi-classical** approach

Nonperturbative dynamics of scalar fields in de Sitter

→ Explain the success of the standard approach

→ Use cosmological experiments to test fundamental quantum physics

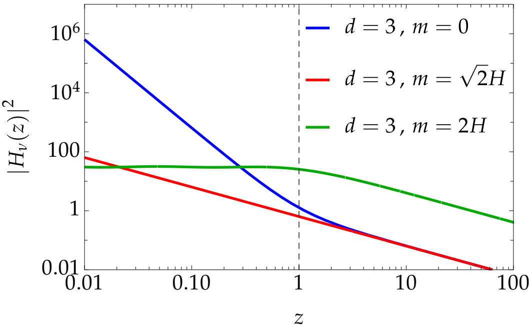
→ Explore unknown effects for spectator fields

Free scalar spectator in de Sitter

$$S = \int \mathrm{d}^D x \sqrt{-g} \left(\frac{1}{2} \phi \Box \phi - \frac{m^2}{2} \phi^2 \right)$$

Mode decomposition of ϕ, in the Bunch-Davies vacuum,

$$\phi(\eta, \vec{x}) \sim \int \frac{\mathrm{d}^d k}{(2\pi)^d} \left(\boxed{H_\nu \left(\frac{k}{a(\eta) H} \right)} e^{i\vec{k}\cdot\vec{x}} a_k + \mathrm{h.c.} \right), \quad \nu = \sqrt{\frac{d^2}{4} - \frac{m^2}{H^2}}$$



Infrared amplification can be interpreted as particle production, in analogy with the Schwinger effect for charged particles in a background electric field.

Nonperturbative treatments

The IR amplification leads to infrared and secular divergences

$$\sim \frac{H^4}{m^2}$$

N. C. Tsamis, R. P. Woodard '05 ; S. Weinberg '05 ; '06

Variety of nonperturbative treatments :

- △ Stochastic formalism
  A. A. Starobinsky '86 ; A. A. Starobinsky, J. Yokoyama '94
- △ Dynamical RG
  C. P. Burgess et al. '10
- △ Schwinger-Dyson equations
  B. Garbrecht, G. Rigopoulos '11 ; F. Gautier, J. Serreau '13 ; '15
- △ Functional renormalization group (FRG)
  J. Serreau '13 ; A. Kaya '13 ; M. Guilleux, J. Serreau '15
- △ Euclidean de Sitter
  A. Rajaraman '10 ; M. Beneke, P. Moch '13 ; D. López Nacir et al. '19
- △ $1/N$ expansion
  F. D. Mazzitelli, J. P. Paz '89 ; A. Riotto, M. Sloth '08 ; J. Serreau '11

Stochastic formalism

A. A. Starobinsky '86; A. A. Starobinsky, J. Yokoyama '94

Introduce the coarse grained field φ

$$\phi_a(t,\vec{x}) = \underbrace{\varphi_a(t,\vec{x})}_{\text{long}-\text{wavelength}} + \underbrace{\int \frac{\mathrm{d}^d k}{(2\pi)^d}\theta(k-\varepsilon a(t)H)\left(\varphi_{a,k}(t)e^{i\vec{k}\cdot\vec{x}}a_{\vec{k}} + \text{h.c.}\right)}_{\text{short}-\text{wavelength}}$$

In the Bunch-Davies vacuum, light mass limit and slow roll, φ behaves classically and verifies a stochastic equation (using rescaled fields and potential)

$$\partial_t \varphi_a + \partial_{\varphi_a} V = \xi_a \quad \text{with} \quad \langle \xi_a(t)\xi_b(t')\rangle = \delta_{ab}\delta(t-t')$$

$\xi \;\rightarrow$ UV modes exiting the horizon (separation in terms of physical scales)

Motivations

One-point functions, $\langle \varphi^n \rangle$, can be computed analytically from the equilibrium probability distribution

Unequal-time correlators :
Correlation of operators at different spacetime point (here on a single Hubble patch at different times)
Example : $\langle \varphi(t)\varphi(t') \rangle$, $\langle \varphi^2(t)\varphi^2(t') \rangle$, ...

→ Numerical resolution is possible

A. A. Starobinsky, J. Yokoyama '94 ; T. Markkanen et al. '19 , '20

→ They give information about

- spectral indices
- relaxation time to the stationary state
- decoherence properties

We compute these correlators analytically in a $1/N$ expansion

Eigenvalue problem

In terms of $P(\varphi_a, t)$ the probability distribution function (PDF),

$$\partial_t P = \partial_{\varphi_a}\left((\partial_{\varphi_a} V)P + \frac{1}{2}\partial_{\varphi_a} P\right) \underset{P=e^{-V}p}{\Rightarrow} \partial_t p = \left(-\frac{1}{2}\Delta_\varphi + W(\varphi_a)\right)p$$

where $W(\varphi_a) = \frac{1}{2}\left(V_{,aa} - V_{,a}^2\right)$

With eigenfunctions $\Psi_{n,\ell}$ and eigenvalues $\Lambda_{n,\ell}$,

$$\left(-\frac{1}{2}\Delta_\varphi + W(\varphi_a)\right)\Psi_{n,\ell} = \Lambda_{n,\ell}\Psi_{n,\ell}$$

The probability distribution functions reads

$$P(\varphi_a, t) = \underbrace{\frac{1}{\mathcal{N}_{\text{eq}}} e^{-2V(\varphi_a)}}_{\text{equilibrium distribution}} + e^{-V(\varphi_a)} \sum_{n\geq 1} \sum_{\ell=0}^{n} a_{n,\ell} \Psi_{n,\ell} e^{-\Lambda_{n,\ell} t}$$

In the non interacting case $\Lambda_n^{free} = nm^2$

Unequal time correlators

For a given operator $\mathcal{A}$, the correlator $\langle \mathcal{A}(t)\mathcal{A}(t')\rangle$ can be expressed using the spectral decomposition

$$\langle \mathcal{A}(t)\mathcal{A}(t')\rangle = \sum_{n\geq 0}\sum_{\ell=0}^{n} C^{\mathcal{A}}_{n,\ell} e^{-\Lambda_{n,\ell}|t-t'|}$$

with the coefficients

$$C^{\mathcal{A}}_{n,\ell} = \left[\int \mathrm{d}^N\varphi \Psi_{0,0}(\varphi_a)\mathcal{A}\Psi_{n,\ell}(\varphi_a)\right]^2$$

Remark : the spectral index $n_{\mathcal{A}}$ of the corresponding operator at long time depends on the lowest contributing eigenvalue $\Lambda_{\mathcal{A}}$

$$n_{\mathcal{A}} - 1 \equiv \frac{\log \mathcal{P}_{\mathcal{A}}(k)}{\log k} = \frac{2}{H}\Lambda_{\mathcal{A}}$$

T. Markkanen et al. ’19

$1/N$ expansion of the FP equation

$$V(\varphi_a) = \frac{m^2}{2}\varphi^2 + \frac{\lambda}{4N}\left(\varphi^2\right)^2$$

Using the generalized spherical harmonics, $Y_{\ell_i}(\theta_i)$,

$$\Psi_{n,\ell}(\varphi_a) = \mathcal{R}_{n,\ell}(x) Y_{\ell_i}(\theta_i) e^{-\Lambda_{n,\ell} t} \ ,$$

where $x = \sqrt{\varphi^2/N}$ and $|\ell_1| \le \ell_2 \le \cdots \le \ell_{N-1} \equiv \ell$, we find the radial equation

$$-\frac{\mathcal{R}''_{n,\ell}}{2N} - \frac{N-1}{2Nx}\mathcal{R}'_{n,\ell} + \left[\frac{\ell(\ell+N-2)}{2Nx^2} + W\right]\mathcal{R}_{n,\ell} = \Lambda_{n,\ell}\mathcal{R}_{n,\ell}$$

Write explicitely the exponential factor $\mathcal{R}_{n,\ell}(x) = e^{-Nv(x)} r_{n,\ell}(x)$, where $V(\varphi_a) = Nv(x)$ to get

$$-\frac{r''_{n,\ell}}{2N} - \left(\frac{N-1}{2Nx} - v'\right) r'_{n,\ell} + \frac{\ell(\ell+N-2)}{2Nx^2} r_{n,\ell} = \Lambda_{n,\ell} r_{n,\ell}$$

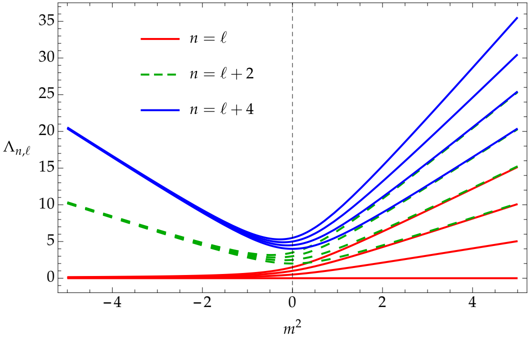
$1/N$ expansion from there gives analytical (and nontrivial) results

LO result

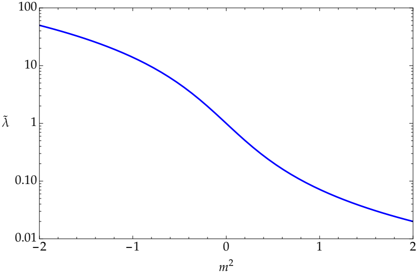
In particular, at LO, with $m_\pm^2 = \pm\frac{m^2}{2} + \sqrt{\frac{m^4}{4} + \frac{\lambda}{4}}$,

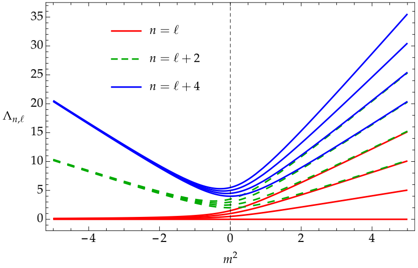
$$\Lambda_{n,\ell} = n m_+^2 + (n-\ell) m_-^2, \quad r_{n,\ell}(x) = a_0 x^\ell (1 - 2m_+^2 x^2)^{\frac{n-\ell}{2}} (1 - 2m_-^2 x^2)^{-\frac{n}{2}}$$

where $n - \ell$ is a positive even integer



Spectrum at LO



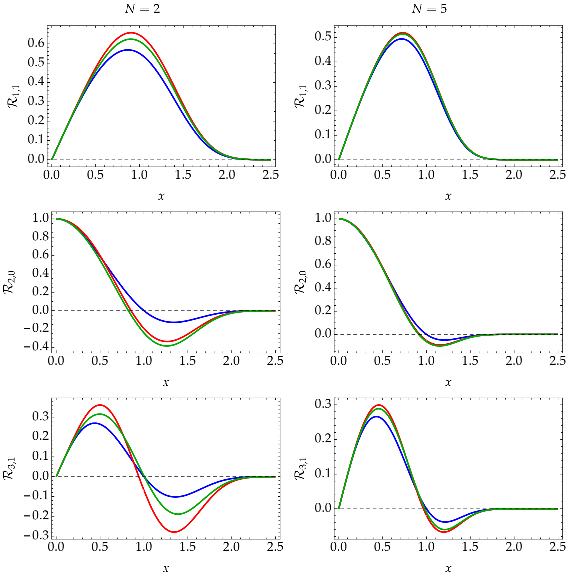


- Massless and deeply broken regime corresponds to highly nonperturbative behavior, $m_-^2 = -\tilde{\lambda} m_+^2$
- Gaussian spectrum (with different degeneracies) in the massless and deeply broken limit and fundamental frequency

$$\Lambda_{n,\ell} = (2n - \ell)\sqrt{\frac{\lambda}{4}}, \quad \Lambda_{n,\ell} = (n - \ell)\left|m^2\right|$$

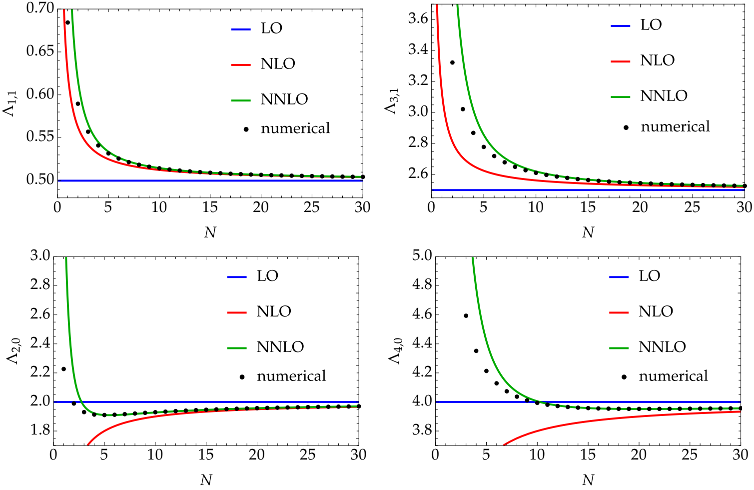
- In deeply broken limit, the scalar operators have a much smaller auto-correlation time in the deeply broken limit → **scalar sector does not couple to the Goldstone modes**

Comparison with finite N - massless case



Blue $\to$ LO
Red $\to$ NLO
Green $\to$ Numerical

Comparison with finite N - massless case



$$\frac{\Lambda_{n,\ell}}{\sqrt{\lambda}} = \frac{2n-\ell}{2}\left(1+\frac{3\ell-2}{4N}+\frac{5n^2-4\ell^2-\ell(5n-9)+2}{16N^2}+\mathcal{O}\left(\frac{1}{N^3}\right)\right)$$

Conclusion and perspectives

- The obtained results coincide with Lorentzian or Euclidean QFT comptations, showing the stochastic approach correctly captures such correlators
- We can probe the deeply nonperturbative regime of the interacting theory using a $1/N$ expansion and going to the massless or negative square mass limit
- We get analytical results for the autocorrelation time of any kind of two-point correlator
- The result in the $1/N$ expansion is qualitatively good at LO and quantitatively at NLO (or NNLO) for the obtained correlators down to low values of N

- ☐ A more complete numerical analysis at finite N remains to be done
- ☐ The auto-correlation times are directly related to decoherence timescales in the early universe
- ☐ Possible applications to cosmological models with spectator fields to constraint such models and possibly find new physical effects