Probing pre-BBN era with Primordial Gravitational Waves: Modified Gravity and Non-Standard Cosmology

Istituto Nazionale di Fisica Nucleare

Anish Ghoshal

Istituto Nazionale di Fisica Nucleare
Tor Vergata, Rome, Italy

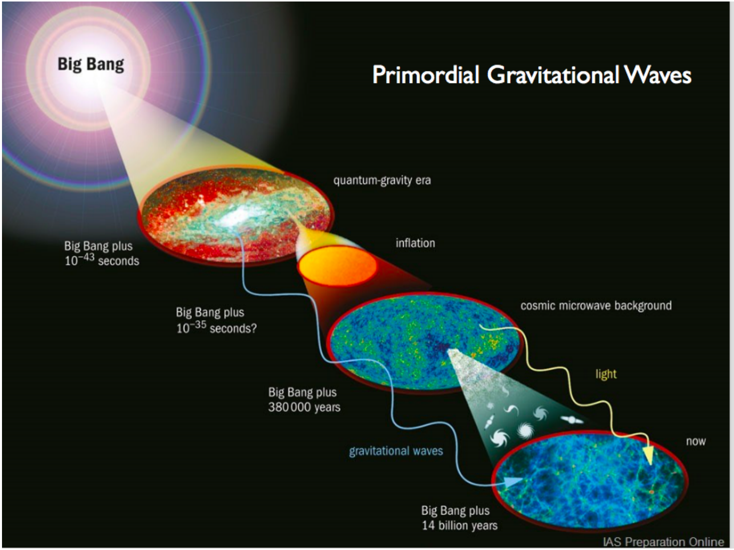
anishghoshal1@protonmail.ch

December 2020 Avignon, France (PONT)

Outline of talk:

- UV completion in particle particle and gravity.
- The unknown pre-BBN era.
- Various GW sources of cosmic origin.
- Primordial Gravitational Waves.
  - Propagation of Gravitational Waves in Early Universe.
  - Modified GW Propagation in non-standard cosmology.
  - Modified GW Propagation in modified gravity.
  - Signal-building on the GW sensitivity map.
- Gravitational Waves as *test-of-gravity* experiment.
- Recent NanoGrav GW detection.
- Conclusion

History of the Universe



Gravitational Waves

- Gravitational Waves (GW) first detected in 2016.
- New Window into the Early Universe.
- Sources of GW from cosmic origin & corresponding spectrum:
  - Inflation: Primordial GW.
  - Inflation: Secondary GW.
  - First-order Phase Transition.
  - Re-heating.
  - Graviton Radiation.
  - Topological Defects.
  - Oscillon.
  - Primordial BH.
- Peccei-Quinn Phase Transition.

Gravitational Waves - - a Primer

perturbations of the background metric: $ds^2 = a^2(\tau)(\eta_{\mu\nu} + h_{\mu\nu}(\mathbf{x},\tau))dx^\mu dx^\nu$

scale factor: cosmological expansion   background metric   GW

governed by linearized Einstein equation $(\tilde{h}_{ij} = a h_{ij}, \text{ TT - gauge})$

$$\tilde{h}''_{ij}(\boldsymbol{k},\tau) + \underbrace{\left(k^2 - \frac{a''}{a}\right)}_{\sim a^2 H^2} \tilde{h}_{ij}(\boldsymbol{k},\tau) = \underbrace{16\pi G\, a\, \Pi_{ij}(\boldsymbol{k},\tau)}_{\text{source term from } \delta T_{\mu\nu}}$$

source: anisotropic stress-energy tensor

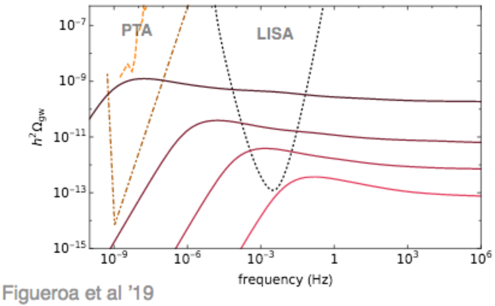
$k \gg aH:\ h_{ij} \sim \cos(\omega\tau)/a\,,\quad k \ll aH:\ h_{ij} \sim \text{const.}$

a useful plane wave expansion: $h_{ij}(\boldsymbol{x},\tau) = \sum_{P=+,\times} \int_{-\infty}^{+\infty} \frac{dk}{2\pi} \int d^2\hat{\boldsymbol{k}}\, h_P(\boldsymbol{k})\, \underbrace{T_k(\tau)}_{\sim a(\tau_i)/a(\tau)}\, e^P_{ij}(\hat{k})\, e^{-ik(\tau - \hat{\boldsymbol{k}}\boldsymbol{x})}$

transfer function , expansion coefficients , polarization tensor $P = +, \times$

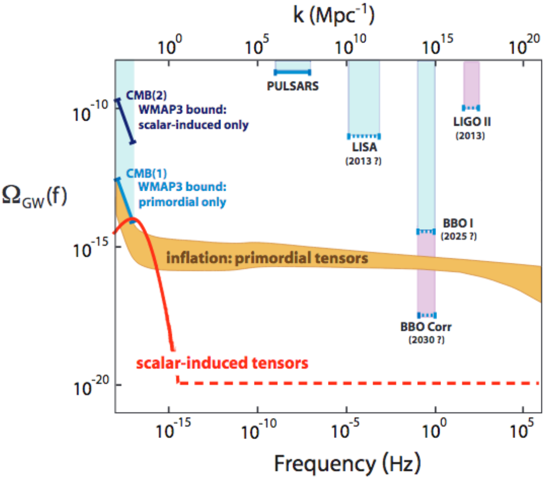
GW - - Cosmic String

Topological defects like cosmic strings give rise to scale invariant GW spectrum.



GW - - Scalar Induced Secondary GW

Secondary Tensor Spectrum induced by first-order scalar perturbation via mixing.
Can be tuned to generate high amplitude in high frequency regions.



Baumann (2007)

GW - - PBH-induced GW

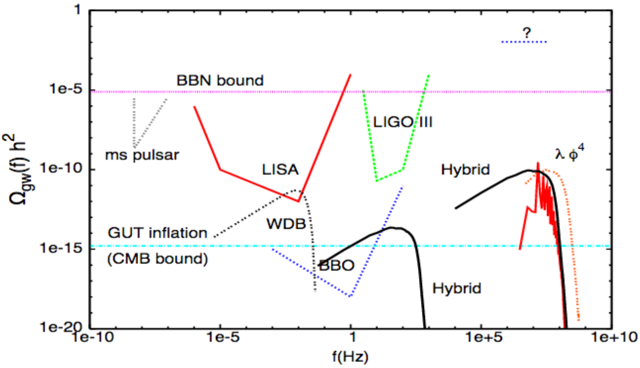
PBH-induced GW.

$$\frac{\Omega_{\rm GW}(k)}{\Omega_r} = \frac{c_g}{72}\int_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} \mathrm{d}d \int_{\frac{1}{\sqrt{3}}}^{\infty} \mathrm{d}s \left[\frac{(s^2-\frac{1}{3})(d^2-\frac{1}{3})}{s^2+d^2}\right]^2 \times P_\zeta(kx)P_\zeta(ky)\left(I_c^2+I_s^2\right) \ , \quad (13)$$

Baumann (2007)

GW - - (P)-reheating

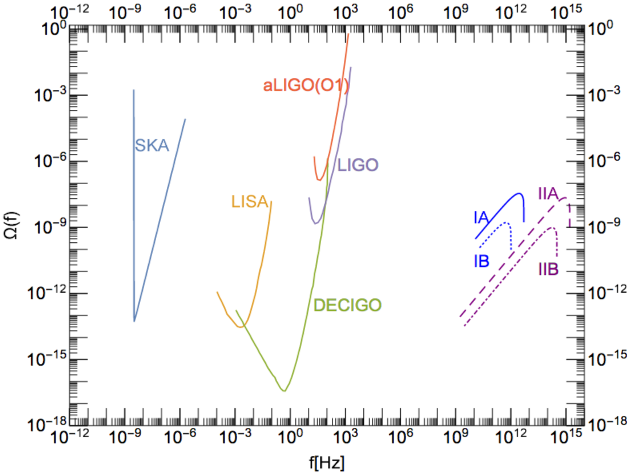
Production during inflaton oscillating in FRW background.



Figuera (2007)

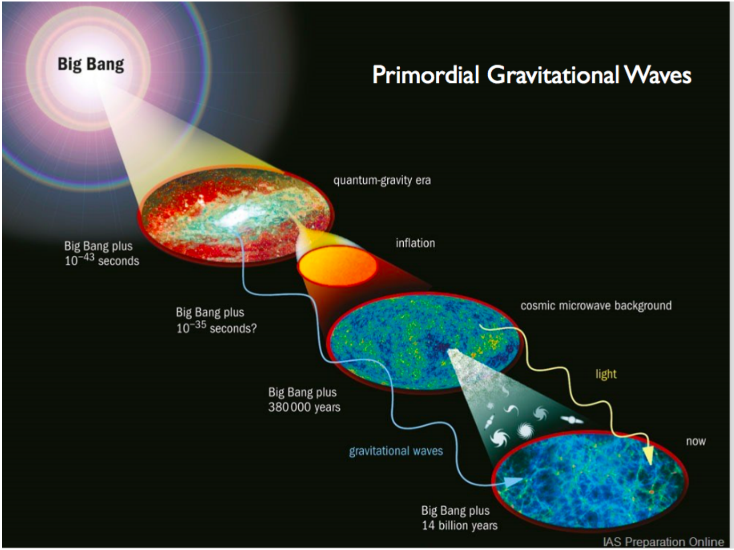
GW - - Graviton Bremmstrahlung

Inflaton radiating away gravitons forming Stochastic GW background.



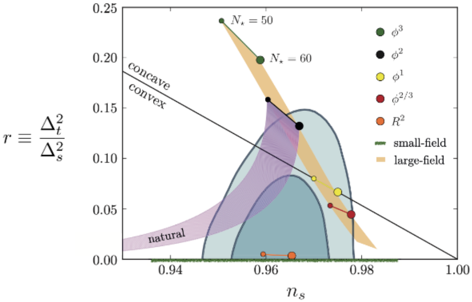
Nakayama (2018)

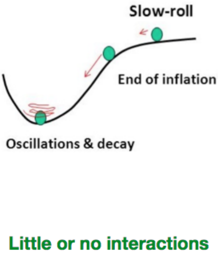
History of the Universe - - Again



Inflation - - UV completion

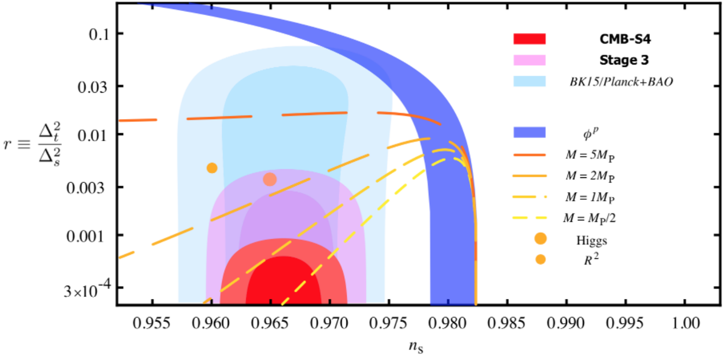
Well known story about the inflation scalar field inflaton rolling down a potential with little or no interactions:





Inflation - - UV completion

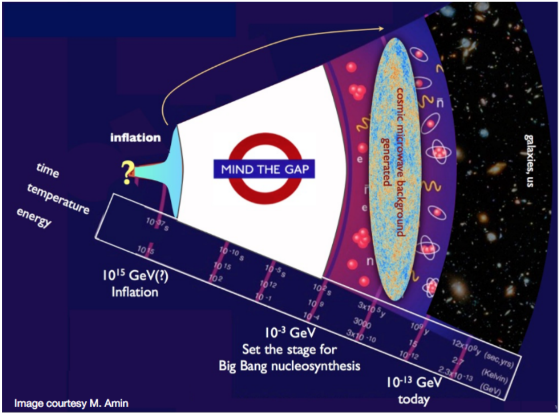
Going to be more well-known in very near future:



Non-Gaussianity parameters:
$f_{NL}^{\rm local} = -0.9 \pm 5.1$, $f_{NL}^{\rm equilibrium} = -26 \pm 47$, $f_{NL}^{\rm orthogonal} = -38 \pm 24$.
Detection of CMB BB-modes will tell us the scale of inflation.

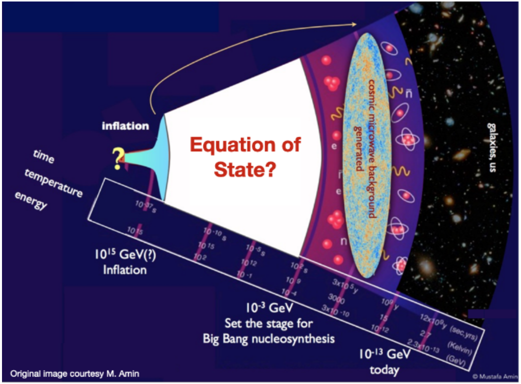
Inflation - - UV completion

However huge gap in our knowledge in between the inflation ending & the beginning of radiation-domination era:



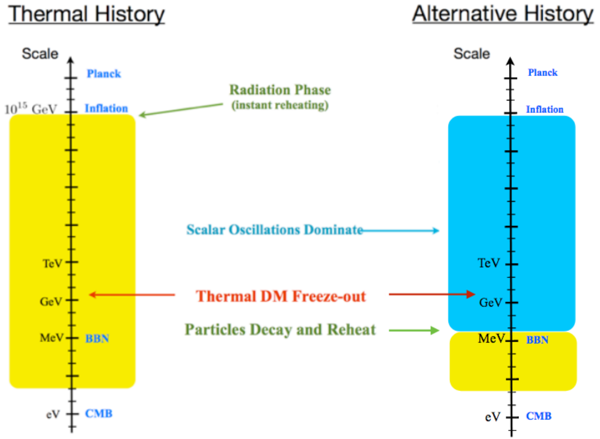
Inflation - - UV completion

What dominated this era ? New gravity ? New Matter ? Predictions from UV-completions.



Inflation - - UV completion

Non-thermal History of the Universe during this era, motivated from string theory, UV-completion of gravity (See for instance Sinha (2015)):



Probe of this era via PGW.
Watson slides PHENO (2020)

The idea: Naively

Propagation of Primordial GW generated during Inflation:

$$\ddot{h}_{ij} + 3H\,\dot{h}_{ij} + \frac{k^2}{a^2} h_{ij} = 16\pi\, G\, \Pi_{ij}^{TT}\,, \tag{1}$$

Solution:

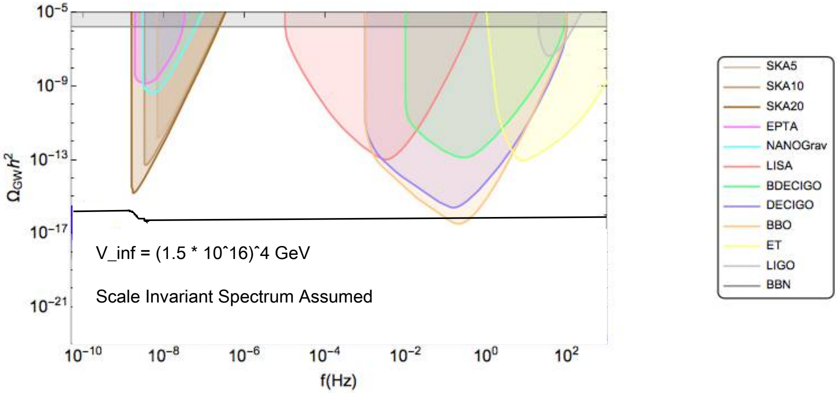
$$h_{ij}(t,\vec{x}) = \sum_P \int \frac{d^3k}{(2\pi)^3}\, h^P(t,\vec{k})\, \epsilon_{ij}^P(\vec{k})\, e^{i\,\vec{k}\cdot\vec{x}}, \tag{2}$$

$$h_{\vec{k}}^P = h_{\vec{k},\,0}^P\, U(t,k)\,, \tag{3}$$

$$\Pi_{ij} = \frac{T_{ij} - p\, g_{ij}}{a^2} \tag{4}$$

$$\Omega_{GW}(\eta,k) = \frac{1}{12\, a^2(\eta)\, H^2(\eta)} \mathcal{P}_T(k)\, \left[U'(\eta,\, k)\right]^2 \tag{5}$$

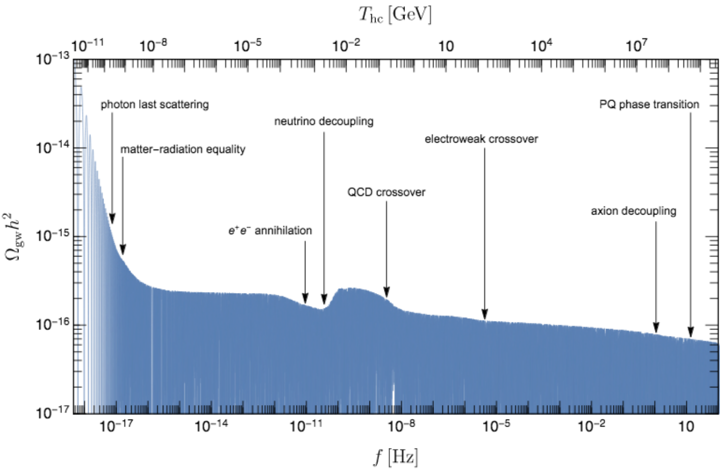
On the GW sensitivity Map



$P_T = \frac{2}{3\pi^2}\frac{V_{inf}}{M_{pl}^4}$. UV-completion: Trans-Planckian Censorship may constrain V_{inf}.

Signal-building: What non-standard cosmology enhances the signal to be detected ?

On the GW sensitivity Map

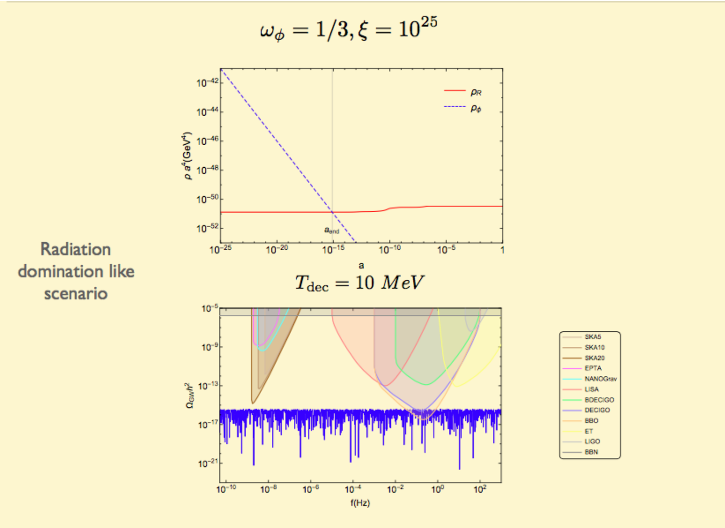


Impact on PGW spectrum from thermal history of the Universe.
Ringwald (2020)

Non-standard Cosmology

- History of the Universe before BBN is unknown.
- Non-standard cosmology predicts scalar field ϕ and its energy density dominates in the Early Universe.
- Its Equation-of-state ω_ϕ.
- Modifies the Hubble expansion: $H^2 \propto \rho_\phi \propto a^{-\frac{3}{2}(1+\omega_\phi)}$.
- PGW relic for modes coming into the horizon for modes coming inside the horizon during the ϕ-dominated era $\Omega_{GW} \propto k^{-2\frac{1+3\omega}{1-3\omega}}$.
- Independent parameter $\xi = \frac{\rho_\phi}{\rho_R}$.

On the GW sensitivity Map

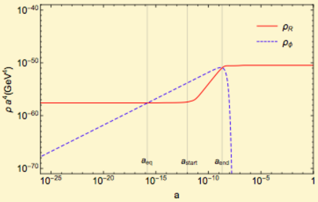


Bernal (2019) [Fazollah, GGI, Florence 2019]

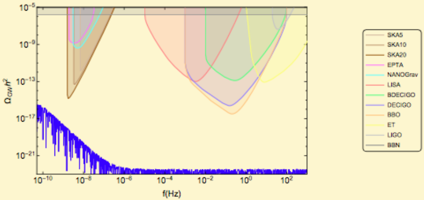
On the GW sensitivity Map

Modulus or matter domination like scenario

$$\omega_\phi = 0, \xi = 10^{-11}$$

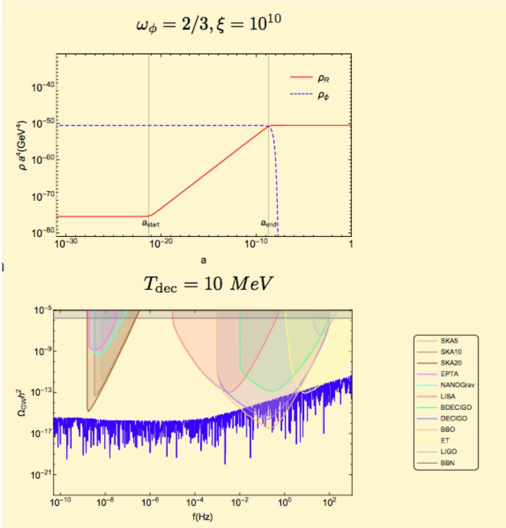


$$T_{\rm dec} = 10\ MeV$$



[Fazollah, GGI, Florence 2019]

On the GW sensitivity Map

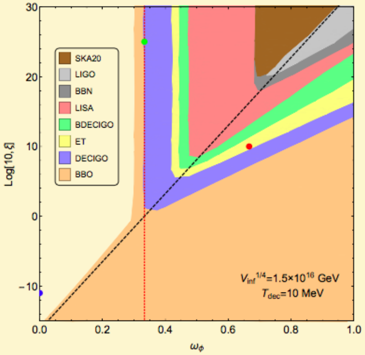


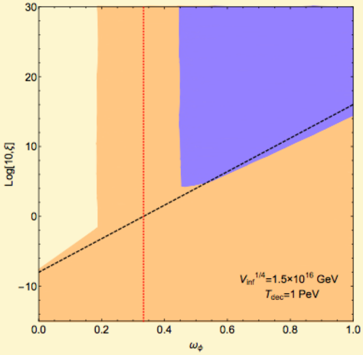
[Fazollah, GGI, Florence 2019]

On the GW sensitivity Map

Scanning over the parameter space of $[\omega_\phi, \xi]$:

$T_{\rm dec} = 10\ MeV$ $\qquad$ $T_{\rm dec} = 1\ PeV$





[Fazollah, GGI, Florence 2019]

Modified Gravity - - Theory

Why modify Einstein gravity ?

- *Theoretical:* Einstein Gravity is non-renormalizable. Quadratic Extensions makes it re-normalizable but have ghosts in the theory. Infinite-derivative extensions make it ghost-free. *Theoretical:* First-order formalisms. Allowing more symmetries.
- *Phenomenological (UV or early universe):* Inflation, dark matter.
- *Phenomenological (IR or late time):* Dark energy.

All these extensions predict modified expansion history of the universe in terms of modified Hubble.
We will try to analyse phenomenologically first.

Modified Gravity

- Modify cosmological expansion (motivated from modified gravity theories)

$$H_{MC}(T) = A(T) H_{GR}(T)\,, \tag{6}$$

- Strategy:

$$A(T) = 1 + \eta \left(\frac{T}{T_*}\right)^{\nu}\,, \tag{7}$$

  where T_* is a parameter with dimensions of the temperature, and $\{\eta, \nu\}$ are free parameters.

$$A(T) = \begin{cases} 1 + \eta \left(\frac{T}{T_*}\right)^{\nu} \tanh \frac{T - T_{re}}{T_{re}} & \text{for} \quad T > T_{BBN} \\ 1 & \text{for} \quad T \leq T_{BBN} \end{cases} \tag{8}$$

- ν labels cosmological models:
  1. $\nu = 2$ in Randall-Sundrum type II brane cosmology, $\nu = 1$ in kination models,
  2. $\nu = 0$ in cosmologies with an overall boost of the Hubble expansion rate like in the case of a large number of additional relativistic degrees of freedom in the thermal plasma
  3. $\nu = -1$ in scalar-tensor (ST) cosmology,
  4. $\nu = 2/n - 2$ in $f(x)$ cosmology, with $f(x) = x + \alpha x^n$, with $x = R, \mathcal{T}$ where R and $\mathcal{T}$ stand for the scalar curvature and the scalar torsion, respectively.

Catena (2003)

Regimes

In the range of frequencies $f \ll f_{\rm re}$, or equivalently for temperatures $T \ll T_{\rm re}$, cosmology should converge to GR, and therefore before the onset of BBN one has that

$$H(a) = H_{\rm GR}(a) = H(a_{\rm re}) \left(\frac{a_{\rm re}}{a}\right)^2 , \tag{9}$$

where $a_{\rm re}$ is the scale factor at $T = T_{\rm re}$. The PGW relic density

$$\Omega_{\rm GW}(\tau_0, k) = \frac{\mathcal{P}_T(k)}{24} \left(\frac{a_{\rm re}}{a_0}\right)^4 \left(\frac{H(a_{\rm re})}{H_0}\right)^2 \propto \mathcal{P}_T(k) , \tag{10}$$

showing the same scale dependence as the primordial tensor power spectrum $\mathcal{P}_T(k)$, as expected from the standard cosmology.

On the GW sensitivity Map

In the range of frequency $f_{\mathrm{re}} \ll f$ the amplification factor A plays a major role.
$\nu > 0$ **Case:** If ν takes positive values, the Hubble rate can be expressed as

$$H(a) \simeq H(a_{\mathrm{re}}) \left(\frac{a_{\mathrm{re}}}{a}\right)^{2+\nu}, \tag{11}$$

which allows to express the PGW relic density

$$\Omega_{\mathrm{GW}}(\tau_0,\, k) = \frac{\mathcal{P}_T(k)}{24\, a_0^4\, H_0^2} \left[H(a_{\mathrm{re}})\, k^{\nu}\, a_{\mathrm{re}}^{2+\nu}\right]^{\frac{2}{1+\nu}} \propto \mathcal{P}_T(k)\, k^{\frac{2\nu}{1+\nu}}. \tag{12}$$

The PGW spectrum gains an extra factor $k^{\frac{2\nu}{1+\nu}}$, and is therefore blue-tilted with respect to the original tensor power spectrum. This enhancement in the PGW spectrum can alternatively be understood by examining the friction term

$$2\,\frac{a'}{a} = \frac{4}{1+3\omega}\,\frac{1}{\tau} \simeq \frac{2}{1+\nu}\,\frac{1}{\tau}. \tag{13}$$

With respect to the standard case where the Universe is dominated by radiation ($\omega = 1/3$), the friction term is reduced and therefore the PGW spectrum is enhanced for $\omega > 1/3$ or equivalently $\nu > 0$.

On the GW sensitivity Map

$\nu = 0$ **Case:** The Hubble rate is enhanced by a constant factor $A = 1 + \eta$. The PGW spectrum is therefore not distorted, just showing an overall shift of A^2:

$$\Omega_{\mathrm{GW}}(\tau_0, k) \simeq \frac{(1+\eta)^2}{24} \mathcal{P}_T(k) \left[\frac{a_{\mathrm{hc}}}{a_0}\right]^4 \left[\frac{H_{\mathrm{hc}}}{H_0}\right]^2 \propto \mathcal{P}_T(k)\,. \tag{14}$$

On the GW sensitivity Map

If ν takes negatives values, both for low ($f \ll f_{\rm re}$) and high frequencies ($f \gg f_{\rm re}$) the amplification factor tends to 1. However, it is interesting to note that A reaches a maximum at $f = \bar{f} \gtrsim f_{\rm re}$ given by

$$A(\bar{f}) \simeq \eta \left(\frac{T_{\rm re}}{T_\star} \right)^{\nu} . \tag{15}$$

The PGW spectrum has the same tilt as the original tensor power spectrum, but featuring a characteristic bump at $k = \bar{k} = 2\pi \bar{f}$, with an amplitude given by

$$\Omega_{\rm GW}(\tau_0, \bar{k}) \simeq \frac{1}{24} \eta^2 \left(\frac{T_{\rm re}}{T_\star} \right)^{2\nu} \mathcal{P}_T(\bar{k}) \left[\frac{a_{\rm hc}}{a_0} \right]^4 \left[\frac{H_{\rm hc}}{H_0} \right]^2 . \tag{16}$$

Conclusions: The effect of the modified cosmologies could give a localized boost, an overall boost, or a change in the frequency dependence, for $\nu < 0$, $\nu = 0$, or $\nu > 0$, respectively. As will be evident from the plots here-after.

On the GW sensitivity Map

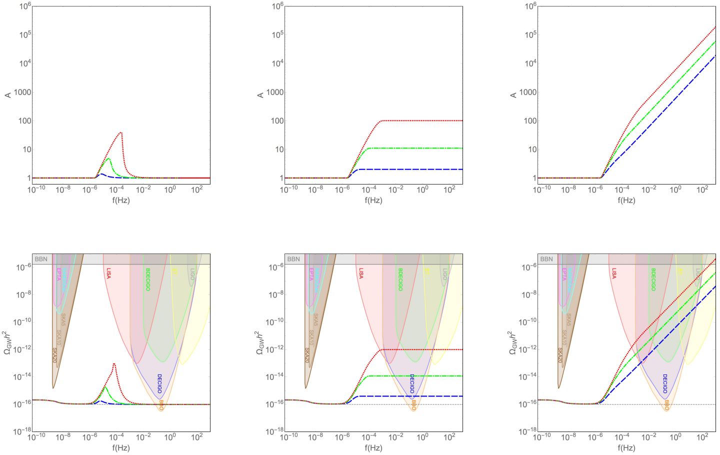


Figure: *A (upper panels) and PGW spectrum $\Omega_{GW}h^2$ (lower panels) as a function of the frequency f for $T_\star = T_{re} = 100$ GeV and $\eta = 1$ (blue dashed lines), $\eta = 10$ (green dot-dashed lines), $\eta = 100$ (red dotted lines), and $\nu = -1$ (left panels), $\nu = 0$ (central panels), $\nu = 1$ (right panels).*

On the GW sensitivity Map

ST theories are defined by the action $S \equiv S_{\mathrm{ST}} + S_{\mathrm{m}}$

$$S_{\mathrm{ST}} = \frac{1}{16\pi\, G_*} \int d^4x \sqrt{-g}\, [F(\phi)\, R(g) - Z(\phi)\, g^{\mu\nu}\, \partial_\mu\phi\, \partial_\nu\phi - 2V(\phi)]\,, \qquad (17)$$

where R is the Ricci scalar, F and Z are arbitrary dimensionless functions of the field ϕ (also dimensionless), and $S_{\mathrm{m}} = S_{\mathrm{m}}[\psi_{\mathrm{m}}, g_{\mu\nu}]$ is the matter action (here ψ_m denotes the matter fields that couple to the metric tensor $g_{\mu\nu}$).

ST theory

The conformal transformation

$$g_{\mu\nu} = A_C^2(\phi_*)\, g_{*\mu\nu} \,, \tag{18}$$

together with the change of variables

$$\begin{aligned} \left(\frac{d\phi_*}{d\phi}\right)^2 &= \frac{3}{4}\left[\frac{d\ln F(\phi)}{d\phi}\right]^2 + \frac{Z(\phi)}{2\,F(\phi)}\,, && (19)\\ A_C(\phi_*) &= F^{-\frac{1}{2}}(\phi)\,, && (20)\\ V_*(\phi_*) &= \frac{V(\phi)}{2\,F^2(\phi)}\,, && (21) \end{aligned}$$

yield the action in the Einstein frame

$$S_{\rm ST} = \frac{1}{16\pi G_*}\int d^4x_* \sqrt{-g_*}\left[R_*(g_*) - 2g_*^{\mu\nu}\partial_\mu\phi_*\partial_\nu\phi_* - 4V_*(\phi_*)\right], \tag{22}$$

while $S_{\rm m} = S_{\rm m}[\psi_{\rm m},\, A_C^2\, g_{*\mu\nu}]$.

Plot - - STT

The FLRW cosmological field equations in the Einstein frame

$$3H_*^2 \equiv 3\left(\frac{\dot{a}_*}{a_*}\right)^2 = 8\pi G_* \rho_* + \dot{\phi}_*^2 + 2V_*(\phi_*)\,, \tag{23}$$

$$3\frac{\ddot{a}_*}{a_*} = -4\pi G_*(\rho_* + 3p_*) - 2\dot{\phi}_*^2 + 2V_*(\phi_*)\,, \tag{24}$$

$$\ddot{\phi}_* + 3H_*\,\dot{\phi}_* + \frac{dV_*}{d\phi_*} = -4\pi G_*\,\alpha(\phi_*)\,(\rho_* - 3p_*)\,, \tag{25}$$

where the dots denote derivatives with respect to the time variable t_*. Deviations of ST theories from GR are parameterized by

$$\alpha(\phi_*) \equiv \frac{d\ln A_C(\phi_*)}{d\phi_*}\,, \tag{26}$$

where in the limit $\alpha \to 0$, A_C becomes a constant, the two frames coincide, and therefore the ST theory reduces to GR.
$N = \ln(a_*/a_{*0})$ (subindex '0' labels quantities evaluated at present). Relation between Jordan and Einstein frames are

$$a = A_C(\phi_*)\,a_*\,, \quad dt = A_C(\phi_*)\,dt_*\,, \quad \rho = \frac{\rho_*}{A_C^4(\phi_*)}\,, \quad p = \frac{p_*}{A_C^4(\phi_*)}\,. \tag{27}$$

Final Equations (setting $V_* = 0$):

$$H = \frac{A_C(\phi_*)}{A_C(\phi_{*0})} \frac{1 + \alpha(\phi_*) \frac{d\phi_*}{dN}}{\sqrt{1+\alpha^2(\phi_{*0})} \sqrt{1 - \frac{1}{3}\left(\frac{d\phi_*}{dN}\right)^2}} H_{\rm GR} \,, \tag{28}$$

$$\frac{2}{3 - \left(\frac{d\phi_*}{dN}\right)^2} \frac{d^2\phi_*}{dN^2} + [1-\omega] \frac{d\phi_*}{dN} + \alpha(\phi_*) [1 - 3\omega] = 0 \,, \tag{29}$$

where $\omega = \omega(T)$ varies from $1/3$ to -1 after reheating. In particular, during radiation-domination era, its evolution is given by the variation of the effective number of degrees of freedom

$$\omega(T) = \frac{4}{3} \frac{h(T)}{g(T)} - 1 \,. \tag{30}$$

Let us note that ω has to be understood as the equation-of-state parameter of the SM bath. That means that after neutrino decoupling, only photons and electrons/positrons contribute to ω.

Plot - - STT

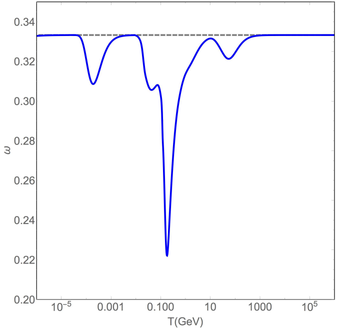


Figure: *Evolution of the equation-of-state parameter ω with respect to the photon temperature T. For reference, $\omega = 1/3$ corresponding to radiation domination is also shown.*

Funnels at $T \simeq 0.5$ MeV, $T \simeq 150$ MeV, and $T \simeq 100$ GeV correspond to the neutrino decoupling, QCD crossover, and electroweak crossover.

Fixing the Transformation

We fix the transformation:

$$A_C(\phi_*) = e^{\frac{1}{2}\beta\,\phi_*^2}, \tag{31}$$

which implies that $\alpha(\phi_*) = \beta\,\phi_*$. In our numerical study, the ST model is fully set by fixing β, the initial value of the field $\phi_{*\text{in}}$ and its derivative $(d\phi/dN)_{*\text{in}}$, at a high temperature $T_{\text{in}} = 10^{14}$ GeV. For the sake of simplicity, here we focus on the case $(d\phi/dN)_{*\text{in}} = 0$. Additionally, the specific choice of T_{in} is not important, as long as it is much higher than the electroweak scale. In fact, as it will be seen, for $T \gg T_{\text{EW}}$ the field ϕ_* does not evolve.

Evolution of scalar field

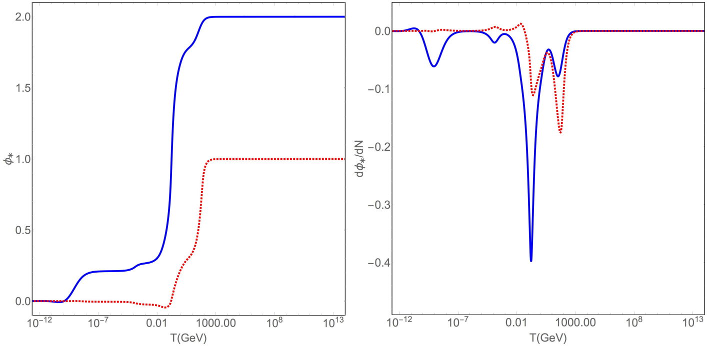


Figure: *Evolution of ϕ_* (left panel) and $d\phi_*/dN$ (right panel) with respect to the frequency f, for the benchmark points $[\beta,\ \phi_{*in}] = [1,\ 2]$ (blue solid line) and $[5,\ 1]$ (red dotted line). We also took $T_{in} = 10^{14}$ GeV and $(d\phi_*/dN)_{in} = 0$.*

Reduces to GR at late times.

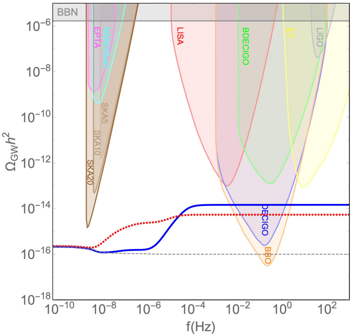


Figure: *Scalar-Tensor Gravity: PGW spectrum, for the benchmark points* $[\beta,\ \phi_{*in}] = [1,\ 2]$ *(blue solid line) and* $[5,\ 1]$ *(red dotted line), and assuming a scale invariant primordial tensor spectrum* $(n_T = 0)$ *with a tensor-to-scalar ratio* $r = 0.07$. *We also took* $T_{in} = 10^{14}$ *GeV and* $(d\phi_*/dN)_{in} = 0$.

High-frequency GW detectors will be able to detect such PGW in presence of Scalar-Tensor dominated cosmological era.

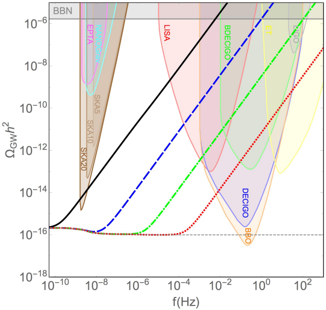
Braneworld Cosmology

In the braneworld cosmology, the Friedmann equation for a spatially flat Universe

$$H^2 = \frac{8\pi G}{3}\rho\left(1+\frac{\rho}{\sigma}\right), \tag{32}$$

where ρ is the SM energy density and the parameter σ is the brane tension which is related to 5-dimensional Planck mass M_5 as

$$\sigma \equiv 96\pi\, G\, M_5^6\,. \tag{33}$$



Conclusion: Gravitational Waves

- Huge gap in our understanding between the end of inflation and the beginning of radiation-dominated era.
- GW detectors will be probing the pre-BBN era. Cosmological history can be verifiable.
- Primordial GW propagation is modified in non-standard cosmological history.
- Primordial GW propagation is modified in alternative gravity theories.
- Reduced Hubble friction causes enhancement of PGW amplitude and large signal in the detectors.
- Scalar-tensor cosmological model predicts PGW at high-frequency detectors.
- Braneworld predicts signals even at lower frequency pulsar timing array GW detectors as well.
- Joint analysis from various detectors will be able to distinguish between various non-standard & modified cosmological histories.
- GW detectors are probing the theory of gravity at larger frequencies, which corresponds to tests of gravity at smaller scales.

NanoGrav GW Detection

NanoGrav recently detected GW events. Many cosmic sources have been proposed. The GW spectrum nicely fits cosmic strings origin hypothesis.

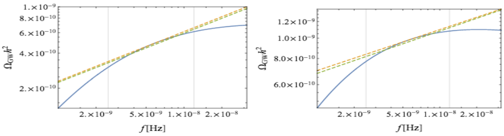
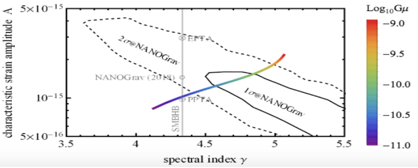


Figure 1. *Cosmic string spectra (solid blue curves) together with our fitted power laws for* $G\mu = 4 \times 10^{-11}$, *and* $G\mu = 10^{-10}$. *The green dashed lines show the results of numerically fitting the curves, while the orange lines result from the simple logarithmic derivative in Eq. (3.3). The thin grey lines indicate the frequency range of interest that was used in the NANOGrav linear fit.*



Ellis (2020)

Thank You