Do we really need to fix the gauge?

VLAD-MIHAI MANDRIC

X NExT PHD WORKSHOP

MARCH 2021

SUPERVISOR: PROF. TIM MORRIS

UNIVERSITY OF Southampton

Gauge-fixing procedure

- The main object of interest in a gauge theory is the partition function

$$Z = \int_{\Phi/\mathcal{G}} \mathcal{D}\{A_\mu\}\, e^{-S[\{A_\mu\}]}$$

- In order to make sense of the path integral one has to implement a gauge-fixing procedure $G^a[A_\mu(x)] = f^a(x)$:

$$Z \propto \int_{\Phi} \mathcal{D}A_\mu\, \mathcal{D}\bar{c}\, \mathcal{D}c\, e^{-S[\phi]-S_{gh}[\phi]-S_{gf}[\phi]}$$

$$S_{gh} = \int_x \int_y \bar{c}^a(x)\, \frac{\delta G^a(x)}{\delta \omega^b(y)}\, c^b(y) \qquad S_{gf} = \int_x \frac{1}{2\xi} \left(G^a(x)\right)^2$$

- However, there are some drawbacks:
  - Non-physical auxiliary fields (ghosts) → extra vertices and propagators
  - Gauge-fixing → Gribov ambiguity

    [Gribov 78']
  - Gauge invariance is broken

Is there a way to avoid all this?

- Short answer: YES, there is.

- The Exact Renormalization Group (ERG) provides the right framework to achieve this.

- ERG is a non-perturbative tool which can be used, in principle, to solve quantum or statistical field theories

The Flow Equation

- The new flow equation can be written as

$$\Lambda D_\Lambda S = \frac{\hbar}{2} Tr[G^{-1}\Lambda D_\Lambda G \; - H^{-1}\Lambda D_\Lambda H]$$

[Falls 21']

where $\Lambda D_\Lambda = \Lambda\partial_\Lambda + \mathcal{L}_\Psi$ is the total RG derivative

$$\Lambda D_\Lambda S = \Lambda\partial_\Lambda S + \Psi^a S_{,a}$$

$$\Lambda D_\Lambda G_{ab} = \Lambda\partial_\Lambda G_{ab} + \Psi^c G_{ab,c} + \Psi^c{}_{,a} G_{bc} + \Psi^c{}_{,b} G_{ac}$$

$$\Lambda D_\Lambda H_{\alpha\beta} = \Lambda\partial_\Lambda H_{\alpha\beta} + \Psi^c H_{\alpha\beta,c}$$

- In view of the novelty of the previously presented construction, it is desirable to test the formalism first.

- The simplest way to do this is to compute the universal (i.e. scheme independent) coefficients of the beta function.

- The 1-loop coefficient of the beta function β_1 has been explicitly computed for both SU(N) Yang-Mills and Gravity, but the analysis presented in the paper cannot be extended to compute the 2-loop coefficient of the beta function. [Falls 21']

- Our aim is to compute both coefficients of the beta function for SU(N) Yang-Mills using a Vertex Expansion of the action S.

Vertex Expansion

- Any gauge invariant quantity can be expanded in traces of fields and products of traces:

$$S = \sum_{n=2}^{\infty} \frac{1}{n} \int_{x_1} \dots \int_{x_n} S_{\mu_1 \dots \mu_n}(x_1, \dots, x_n)\ tr\left(A_{\mu_1} \dots A_{\mu_n}\right) +$$
$$\sum_{n,m=2}^{\infty} \frac{1}{nm} \int_{x_1} \dots \int_{x_n} \int_{y_1} \dots \int_{y_n} S_{\mu_1 \dots \mu_n, \nu_1 \dots \nu_m}(x_1, \dots, x_n; y_1, \dots, y_m)\ tr\left(A_{\mu_1} \dots A_{\mu_n}\right) tr\left(A_{\nu_1} \dots A_{\nu_m}\right) + \dots$$

- Since gauge invariance is preserved at all stages during the flow, we can derive a set of 'Naive' Ward Identities for the Vertex Functions:

$$p_{1_{\mu_1}} S_{\mu_1 \dots \mu_n}(p_1, \dots, p_n) = S_{\mu_2 \dots \mu_n}(p_1 + p_2, \dots, p_n) - S_{\mu_2 \dots \mu_n}(p_2, \dots, p_1 + p_n)$$

- We can expand the ERG Kernels in a similar fashion and derive a separate set of Ward identities for them as well.

- Our strategy is:

  - Expand the action S and the ERG Kernels in $\hbar$

  - At each level perform an expansion in vertices

  - Then solve the flow equations order by order and use Ward identities to express *n-point functions* in terms of *m(<n)-point functions*.

Concluding Remarks

- We don’t really need to fix the gauge.
- The recently constructed ERG preserves gauge invariance at all stages during the flow, avoids using a gauge-fixing procedure and does not require the addition of any auxiliary fields.
- The same ERG formalism can be used to describe Gravity only in terms of the dynamical metric $g_{\mu\nu}$ without the need to introduce any background metric. (i.e. background independence)
- However, all these advantages are introduced at the expense of a more involved flow equation.
- Further Work:
  - finish computing $\beta_1 \rightarrow$ compute $\beta_2 \rightarrow$ what can we say about Gravity using this background-independent approach