Interquark Potentials in Lattice QCD

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Lattice Requirements

- None really, from Lattice QCD specifically
- Visualise a regular cubic lattice in 3D
- Know that the temperature of a simulation can be changed

HAL QCD Method I

Starting with the Schrödinger equation for radially symmetric S-wave states:

$$\left(-\frac{1}{2\mu}\nabla_r^2 + V_\Gamma(r) \right)\psi_j(r) = E_j\psi_j(r), \qquad (1)$$

we will try and calculate the potential $V(r)$ for the system of a single $b\bar{b}$ or $c\bar{c}$ meson.

HAL QCD Method II

On the lattice we can calculate the two point correlation function for a meson with one constituent (quark or antiquark) placed at, for example, x and the other at $x + r$, where x and r are just arbitrary points on the lattice.
This is the correlation function $C_\Gamma(\mathbf{r}, \tau)$, where $\mathbf{r}$ is the vector distance between the quark and antiquark, τ is the Euclidean time, and Γ dictates the channel: vector, pseudoscalar, etc.
To tie this back to the Schrödinger equation we need a wavefunction, so we can rehash our correlation function (the data we get from lattice simulations) to the Nambu Bethe Salpeter (NBS) wavefunction

$$C_\Gamma(\mathbf{r}, \tau) = \Sigma_j \frac{\psi_j^*(\mathbf{0})\psi_j(\mathbf{r})}{2E_j} e^{-E_j \tau}. \tag{2}$$

HAL QCD Method III

Without more than a few lines of derivatives and algebra we arrive at the final expression

$$V_\Gamma(r) = \frac{\frac{1}{2\mu}\nabla_r^2 C_\Gamma(\mathbf{r},\tau) - \frac{\partial C_\Gamma(\mathbf{r},\tau)}{\partial \tau}}{C_\Gamma(\mathbf{r},\tau)}. \tag{3}$$

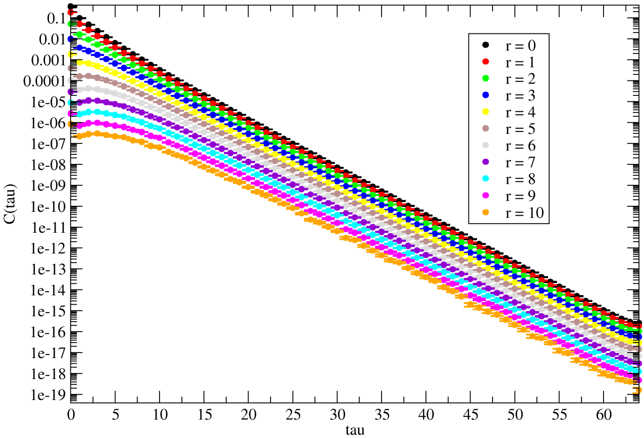
This then, relates the potential between the quark and antiquark in a meson to a quantity that can be measured on the lattice. The only non-triviality remaining is the derivatives. The output data from the lattice simulations, $C_\Gamma(\mathbf{r},\tau)$, is just a complex value for every point in the lattice, so there is no analytic form for the derivatives (I will talk later about an improvement to this in current/future work). For now though, we will make do with finite differences.

HAL QCD Method IV

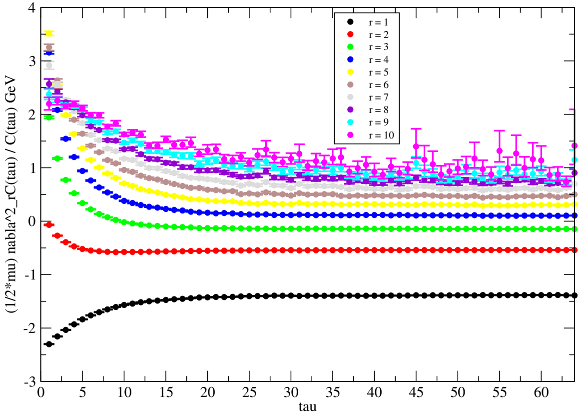
$$
\begin{aligned}
\nabla_r^2 f(r) &\equiv \left[\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}\right] f(r) \\
&\to \left[\frac{\delta_{r',r+a_s} - 2\delta_{r',r} + \delta_{r',r-a_s}}{a_s^2} + \frac{\delta_{r',r+a_s} - \delta_{r',r-a_s}}{r a_s}\right] f(r'), \\
\frac{\delta}{\delta\tau} f(\tau) &\to \left[\frac{f(\tau+a_\tau) - f(\tau-a_\tau)}{2a_\tau}\right].
\end{aligned} \tag{4}
$$

Where a_s and a_τ are the unit lengths in the space and time directions.

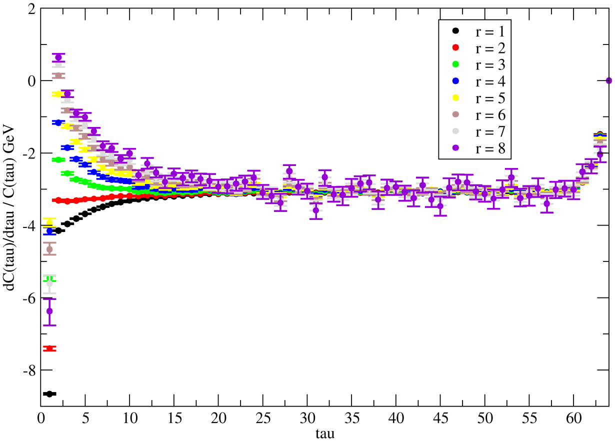
Correlation Functions



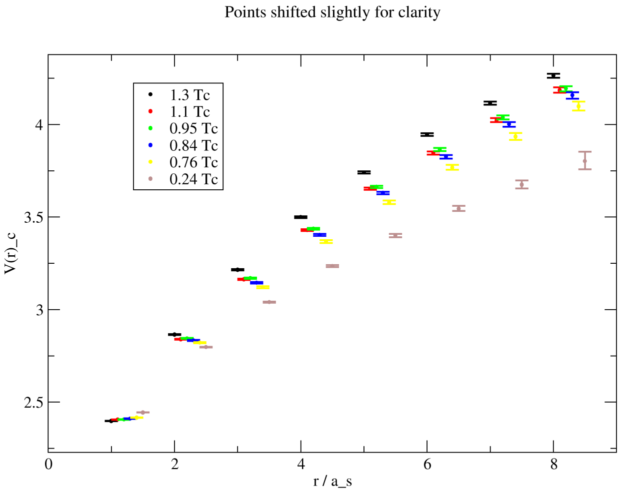
Spatial derivative



Time derivative



V(r) - Interquark potential for the pseudoscalar $c\hat{c}$ meson



Future Work

- Only a subset of the possible x,y,z combinations were used, we are now using all
- If the momentum space correlator is used, as opposed to the position space as used here, then there is an analytic form of the spatial derivative, and thus an improvement on the noisy nearest neighbours derivatives
- Likewise, there is an analytic form of the time derivative that could have been used in this work already, but will be used in the future work

Thanks, any questions?