Primordial Black Holes (PBHs) as dark matter

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1. **Introduction to PBH dark matter**

    Recap: what do we know about dark matter

    Why PBHs?

    PBH formation: collapse of large density perturbations

    other mechanisms

2. PBH structure formation and evolution

3. Observational constraints on PBHs

(relatively) recent review papers on this topic

Sasaki, Suyama, Tanaka & Yokoyama, arXiv:1801.05235 *PBHs-perspective in GW astronomy*
Detailed review (c. 2018) of observational constraints on non-evaporated PBHs, PBH formation from large inflationary perturbations and PBH binaries as a source of GWs.

Carr, Kohri, Sendouda & Yokoyama, arXiv: 2002.12778 *Constraints on PBHs*
Very comprehensive review of constraints on PBHs of all masses, with an extensive reference list.

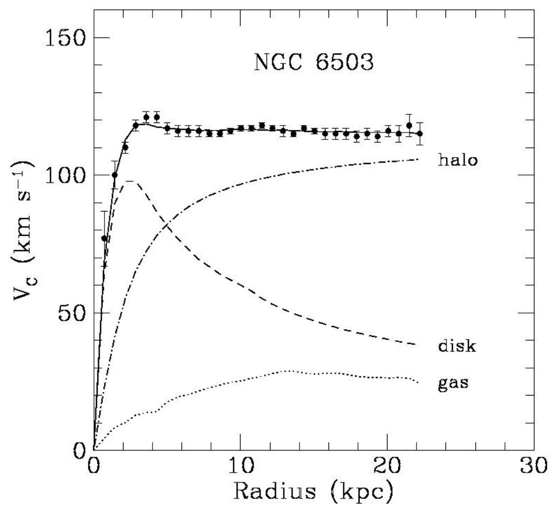
Carr & Kuhnel arXiv: 2006.02838 *PBHs as dark matter: recent developments*
Overview of various potential observational consequences of PBHs, including dark matter.

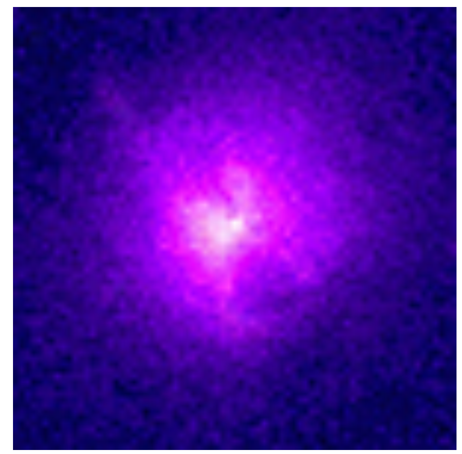
Green & Kavanagh arXiv:2007.10722 *PBHs as a dark matter candidate*
Relatively concise review, aimed at non-experts.

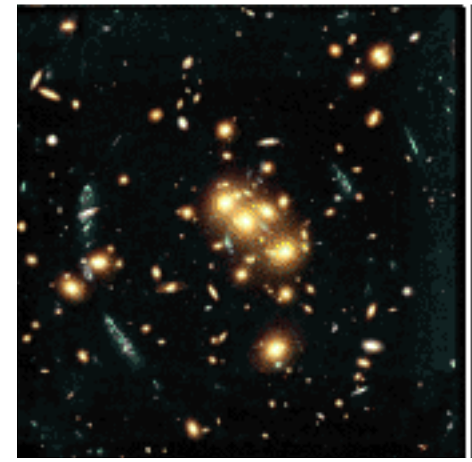
Villanueva-Domingo, Mena, Palomares-Ruiz, arXiv:2103.12087 *A brief review on primordial black holes as a dark matter candidate*
Even more concise review.

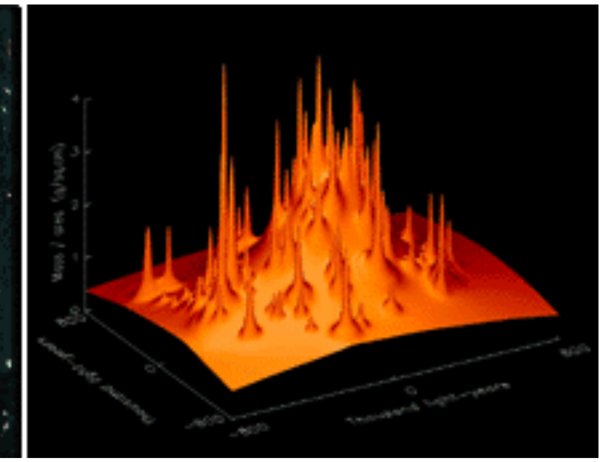
Bradley Kavanagh's PBH abundance constraint plotting code:
https://github.com/bradkav/PBHbounds

Recap: what do we know about dark matter?





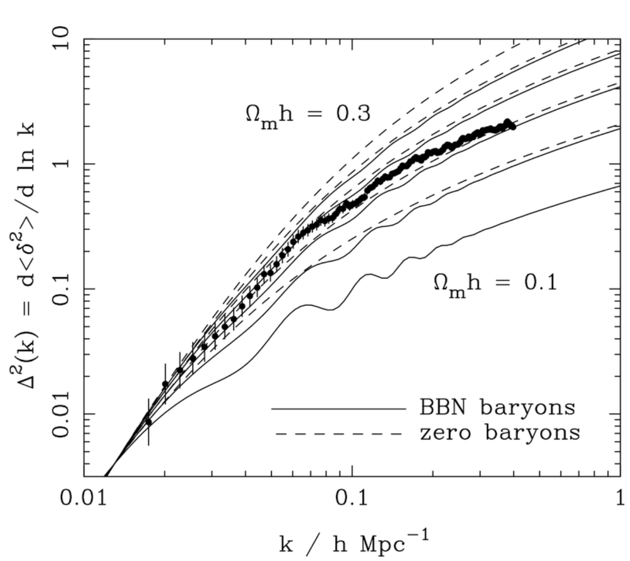


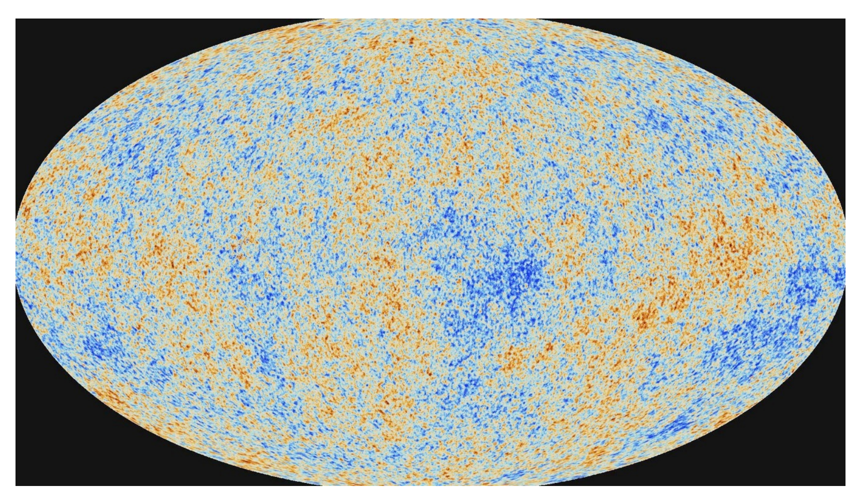


Lots of evidence for (non-baryonic cold) dark matter from diverse astronomical and cosmological observations

[galaxy rotation curves, galaxy clusters (galaxy velocities, X-ray gas, lensing), galaxy red-shift surveys, Cosmic Microwave Background (CMB)]

assuming Newtonian gravity/GR is correct.





A good dark matter candidate must be:

c.f. Taoso, Bertone & Masiero; Baudis & Profumo in Particle Data Group 2020 Review of Particle Physics

1. Cold, constraints on free-streaming length from large scale structure (LSS) e.g. Lyman-alpha forest.

2. Stable on time-scales much longer than age of universe ($\tau \gtrsim 10^{(26-27)}\,\mathrm{s}$ from Fermi-LAT observations of dwarf spheroidals).

3. Neutral

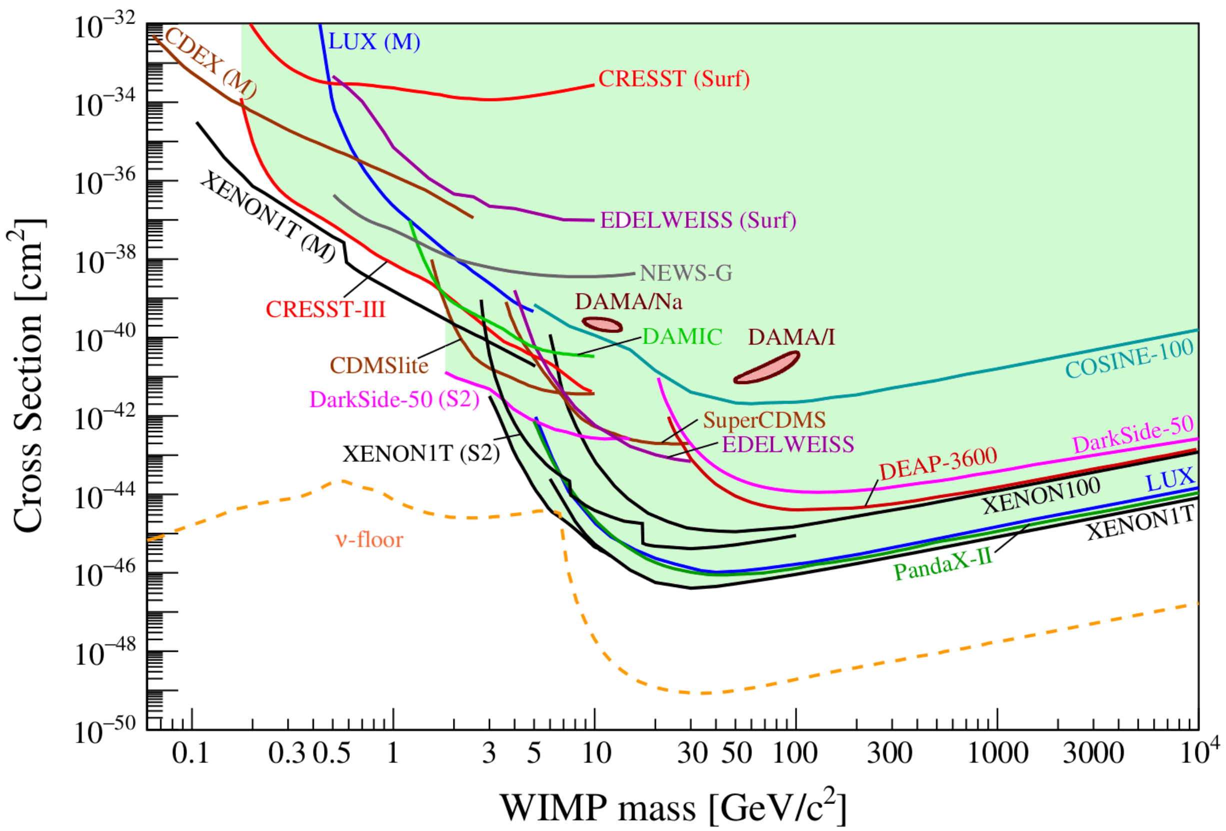
4. Non-baryonic, from nucleosynthesis, and CMB and LSS, $\Omega_{\mathrm{m}} \approx 6\Omega_{\mathrm{b}}$.

5. Weakly-interacting, limits on self-interaction cross-section from bullet cluster and shapes of DM halos.

Status of particle dark matter searches:

WIMPs

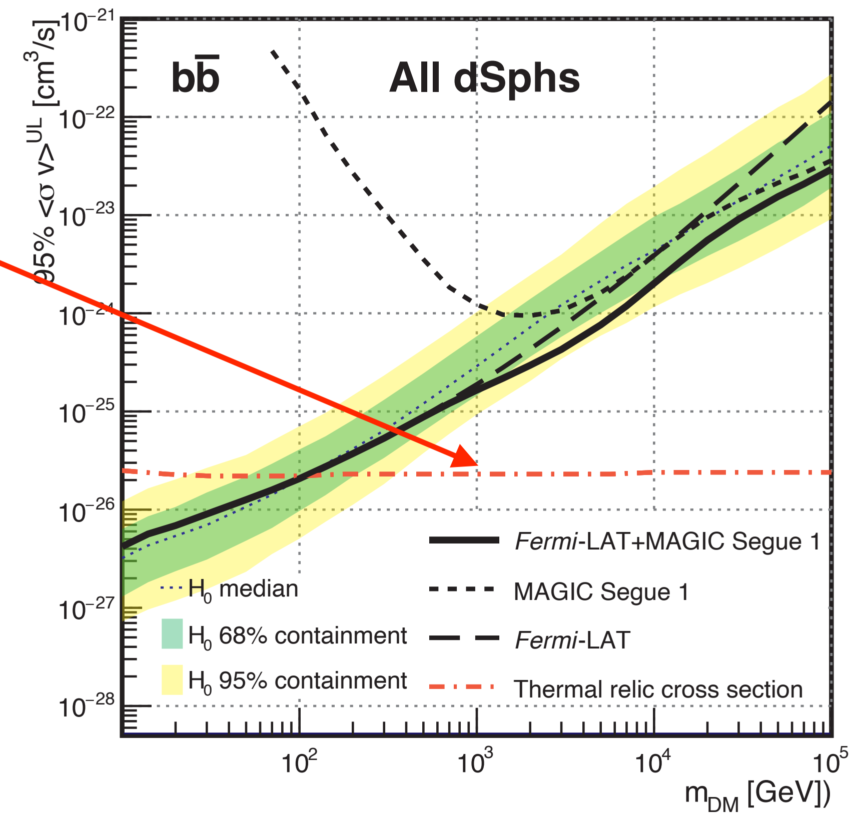
Constraints on the spin independent cross section as a function of mass from direct detection experiments.



upcoming Direct Detection of Dark Matter
ApPEC Committee report

Constraints on annihilation cross-section as a function of mass from Fermi-LAT and MAGIC.

cross-section for which a thermal relic has the observed DM density

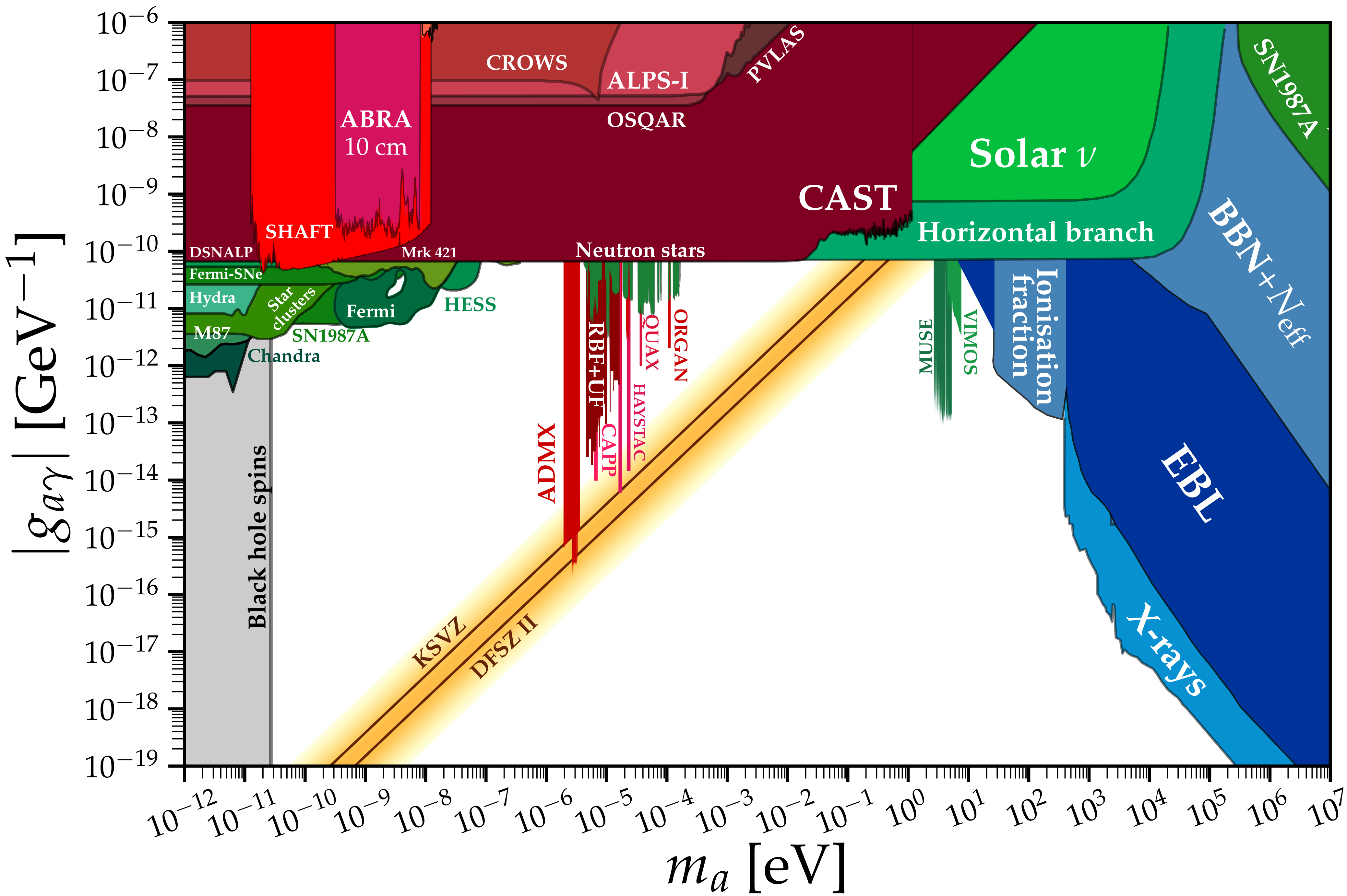


Ahnen et al.

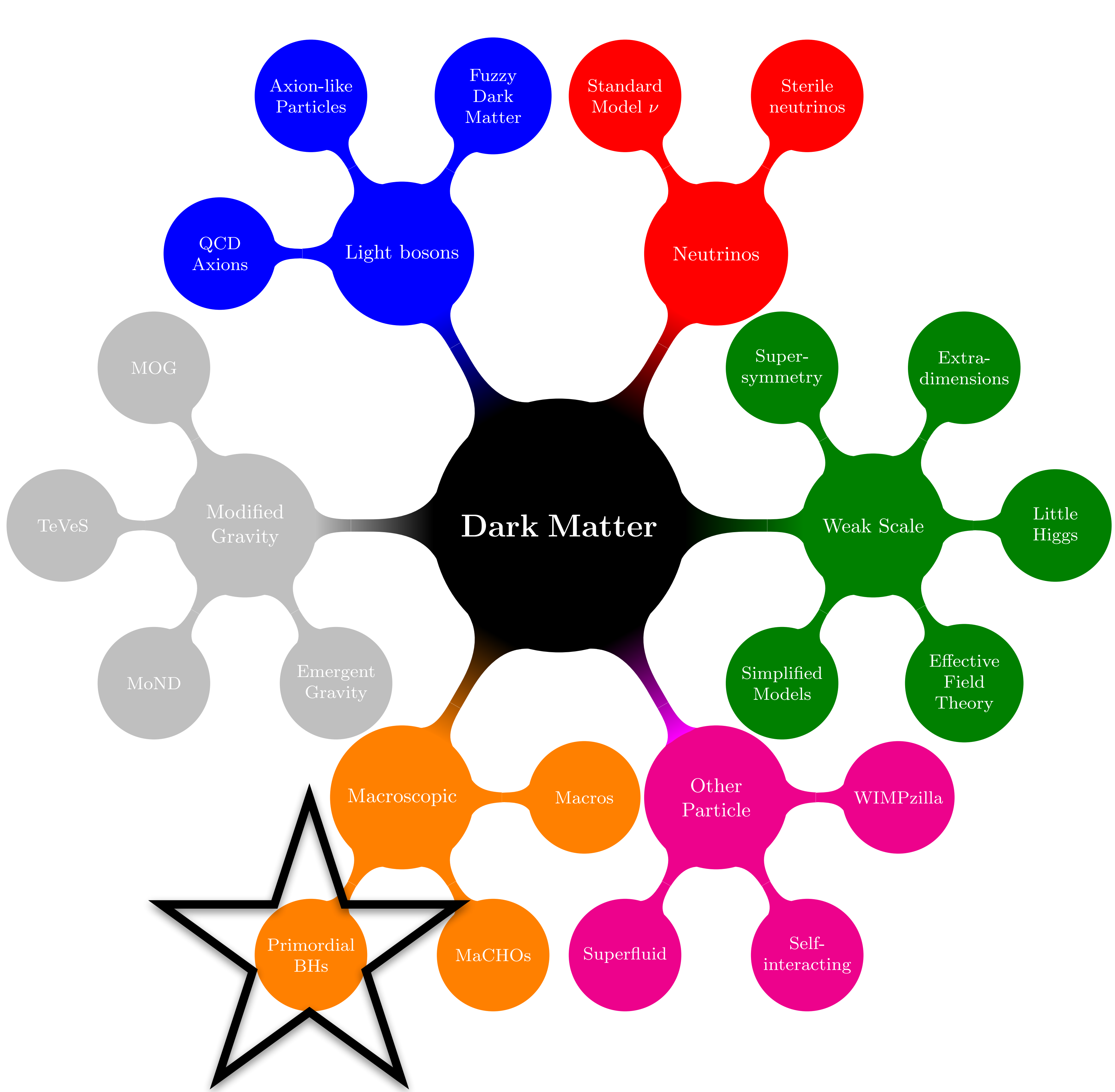
IMHO, not time to give up on WIMPs, yet, but should pursue a wider range of possibilities.

axions/ALPs

Complication of constraints on the axion-photon coupling as a function of mass



O'Hare



Bertone & Tait

Why PBHs?

Primordial Black Holes (PBHs) can form from over densities in early Universe.
Zel'dovich and Novikov; Hawking

Are non-baryonic and have lifetime longer than the age of the Universe for $M > 10^{15}$ g.
Page; MacGibbon, Carr & Page. (see lecture 3)

A DM candidate which (unlike WIMPs, axions, sterile neutrinos,…) isn't a new particle (however their formation does usually require Beyond the Standard Model physics, e.g. inflation).

n.b. Evaporation of PBHs with $M < 10^{15}$g can produce stable massive particles e.g. Fujita et al., or Planck mass relics MacGibbon, which are also DM candidates.

A brief history of PBH-dark matter

Realised already in 1970s that PBHs are a cold dark matter (DM) candidate.
Hawking; Chapline

‘PBH-MACHOs’

In mid-late 1990s MACHO collaboration observed more microlensing events towards Magellanic Clouds than expected from known stellar populations. Alcock et al.

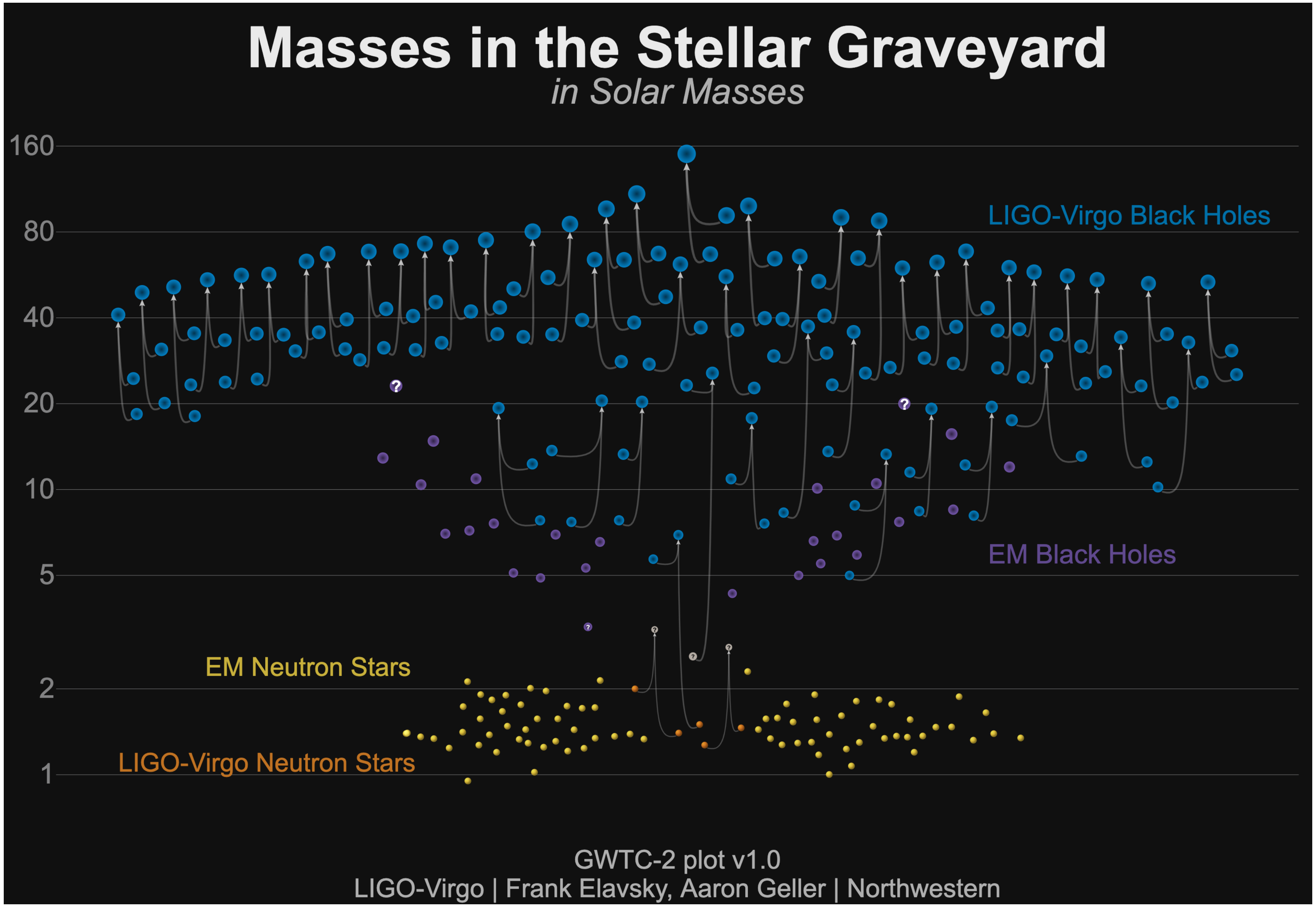
Consistent with f~0.5 of MW halo being in the form of 0.5 Solar mass objects Alcock et al. (and astrophysical compact objects, e.g. white dwarfs, ruled out by baryon budget arguments Fields, Freese & Graff).

With subsequent microlensing data from MACHO Alcock et al. and EROS Tisserand et al., planetary and stellar mass compact objects constrained to make up less than ~10% of MW halo (see lecture 3).

Nakamura et al. (1997): PBH binaries form in the early Universe and (**if** they survive to the present day) GWs from their coalescence detectable by LIGO (see lecture 2/3).

LIGO-Virgo binary BH events

Could the BHs in the LIGO-Virgo BH binaries be primordial (and also a significant component of the DM?). Bird et al.; Clesse & Garcia-Bellido; Sasaki et al.



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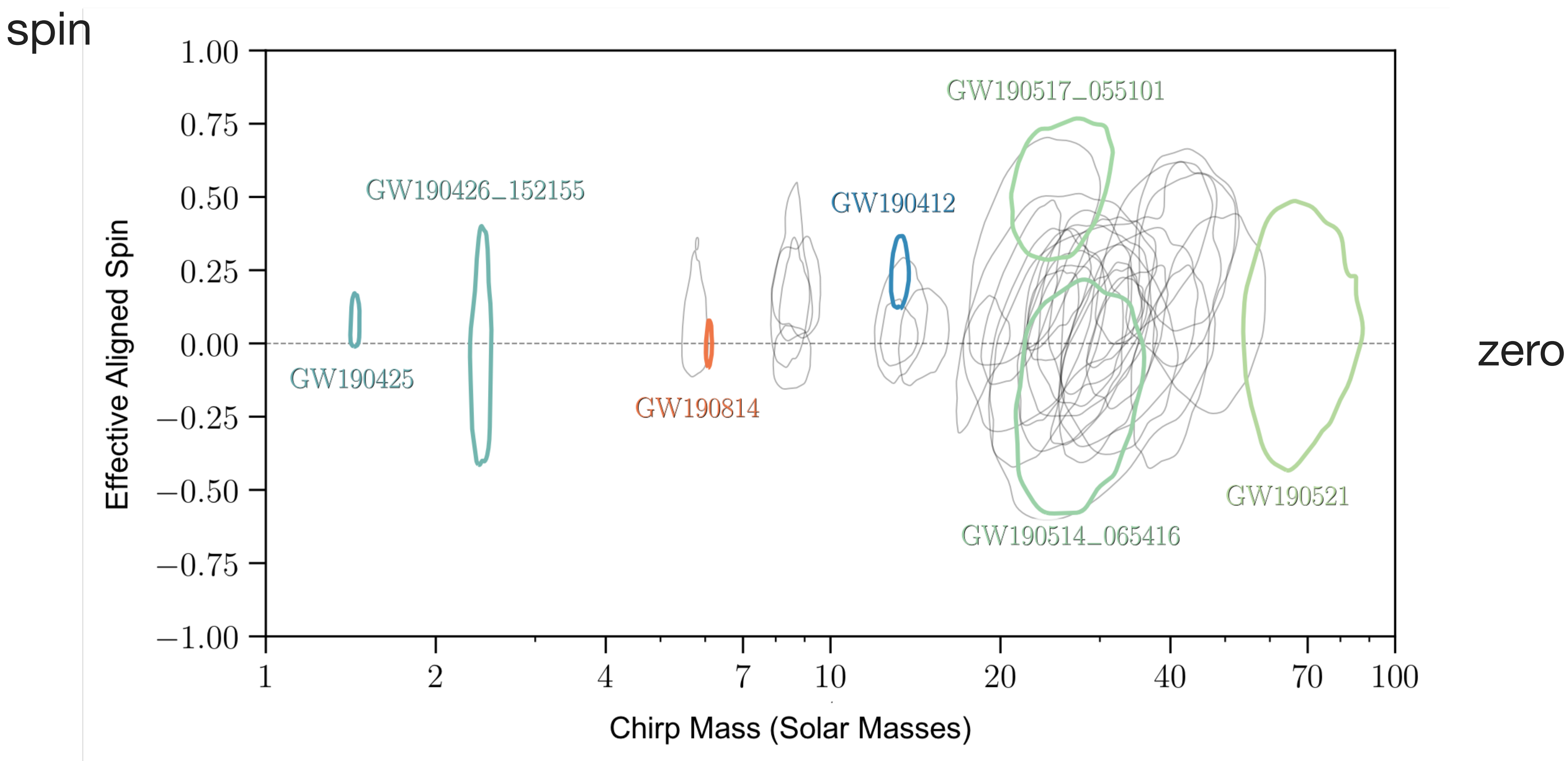


GWTC-2 Abbott et al.

PBH formation: collapse of large density perturbations

(during radiation domination)

zero-th order calculation

If a density perturbation is sufficiently large (at horizon entry) gravity can overcome pressure forces and it can collapse to form a PBH.

Zeldovich & Novikov; Hawking; Carr & Hawking; Carr

threshold for PBH formation: $$\delta \geq \delta_{\rm c} \sim w = \frac{p}{\rho} = \frac{1}{3}$$

$$\delta \equiv \frac{\rho - \bar{\rho}}{\bar{\rho}}$$ density contrast (at horizon crossing)

PBH mass roughly equal to horizon mass M_H (mass contained within horizon):

$$M_{\rm H} = \frac{4\pi}{3} \rho (cH^{-1})^3 = \frac{c^3}{2GH} = \frac{tc^3}{G}$$

$$M_{\rm PBH} \sim 10^{15}\,{\rm g} \left(\frac{t}{10^{-23}\,{\rm s}} \right) \sim M_{\odot} \left(\frac{t}{10^{-6}\,{\rm s}} \right)$$

Spin: small (PBHs form from rare high peaks in the density field, that are spherically symmetric)

Mirbabayi et al.; de Luca et al.

$$a = \frac{|\mathbf{S}|}{G M_{\rm PBH}^2} \lesssim 0.01$$

initial PBH mass fraction (fraction of universe in regions dense enough to form PBHs):

$$\beta(M) = \left(\frac{\rho_{\rm PBH}}{\rho_{\rm tot}}\right)_{\rm i} \approx \int_{\delta_{\rm c}}^{\infty} P(\delta(M_{\rm H}))\, {\rm d}\delta(M_{\rm H})$$

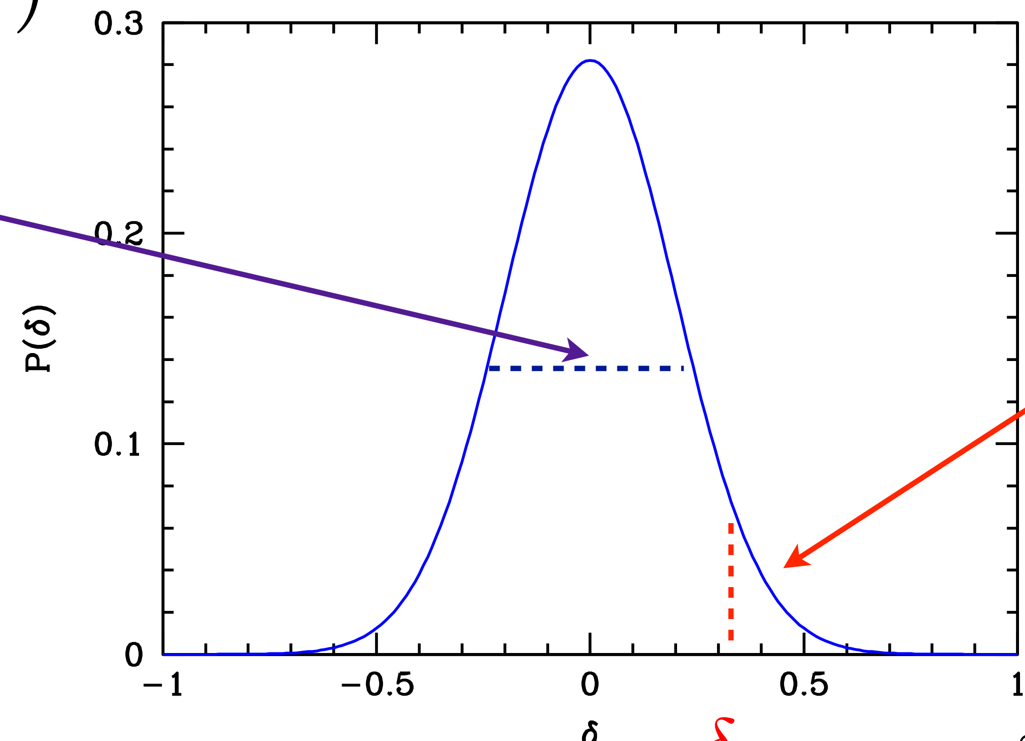
$\delta(M_{\rm H})$ density contrast, smoothed on a scale $M_{\rm H}$

assuming a gaussian probability distribution: $\beta(M) = {\rm erfc}\left(\frac{\delta_{\rm c}}{\sqrt{2}\sigma(M_{\rm H})}\right)$

$$\sim \sigma(M_{\rm H}) \exp\left(-\frac{\delta_{\rm c}^2}{2\sigma^2(M_{\rm H})}\right)$$

$P(\delta)$

$\sigma(M_{\rm H})$ (mass variance)
typical size of fluctuations



PBH forming fluctuations

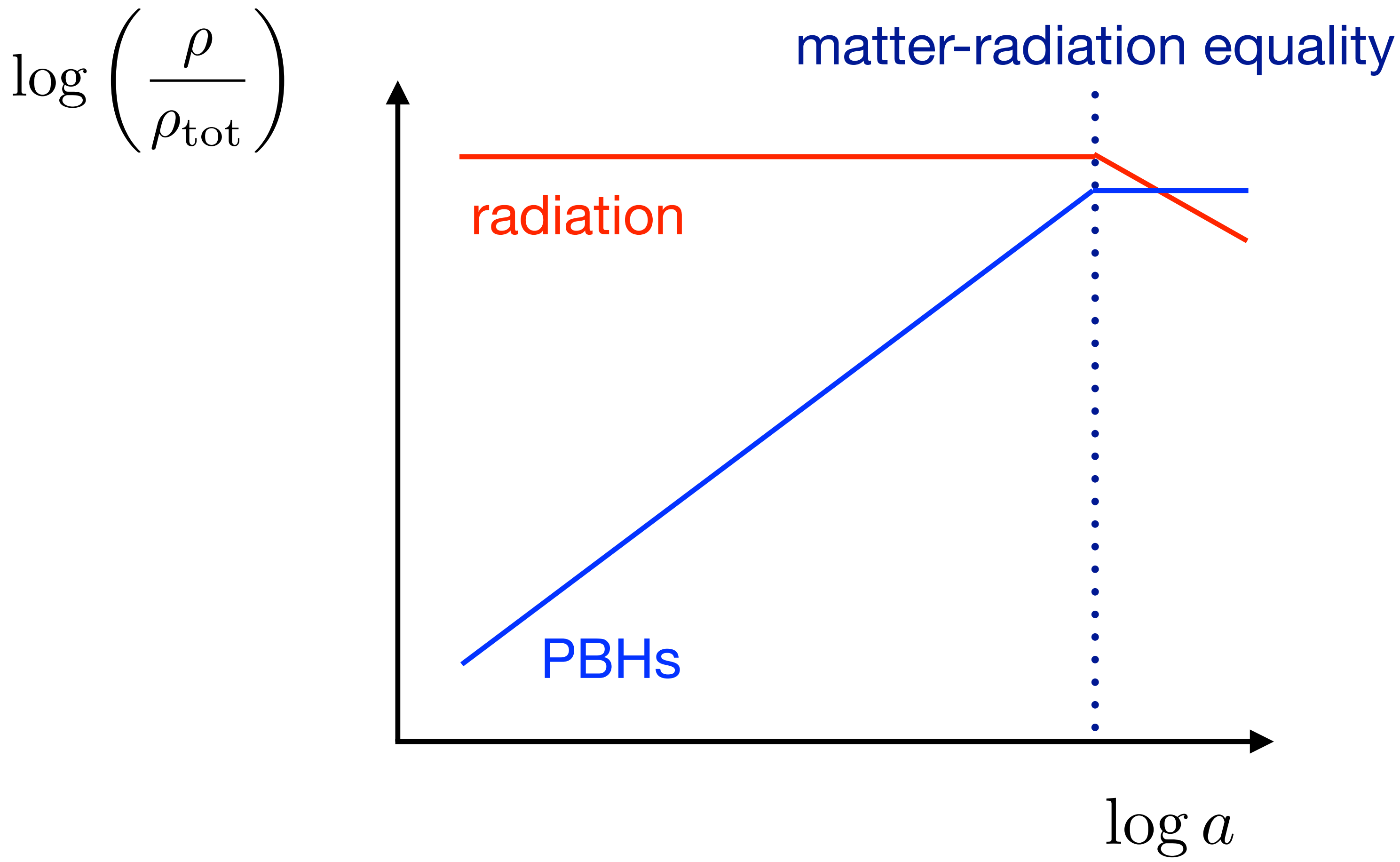
$\delta_{\rm c}$ δ

but in fact β must be small, hence $\sigma \ll \delta_{\rm c}$

PBH abundance

Since PBHs are matter, during radiation domination the fraction of energy in PBHs grows with time:

$$\frac{\rho_{\rm PBH}}{\rho_{\rm rad}} \propto \frac{a^{-3}}{a^{-4}} \propto a$$



Relationship between **PBH initial mass fraction, β**, and **fraction of DM in form of PBHs, f_{PBH}**:

$$\beta(M) \sim 10^{-9} f_{\rm PBH} \left(\frac{M}{M_\odot}\right)^{1/2}$$

i.e. initial mass fraction must be small, but non-negligible.

On Cosmic Microwave Background (CMB) scales the primordial perturbations have amplitude $\sigma(M_{\rm H}) \sim 10^{-5}$

If the primordial perturbations are very close to scale-invariant the number of PBHs formed will be completely negligible:

$$\beta(M) = {\rm erfc}\left(\frac{\delta_{\rm c}}{\sqrt{2}\sigma(M_{\rm H})}\right) \approx {\rm erfc}\left(10^{5}\right) \sim \exp\left(-10^{10}\right) \ll 1$$

To form an interesting number of PBHs amplitude of primordial perturbations must be 2-3 orders of larger on small scales than on cosmological scales **and** fine-tuned.

Calculating the mass variance, σ, from the power spectrum of the primordial curvature perturbation

Blais et al.; Josan, Green & Malik

power spectrum of primordial curvature perturbation (see lecture 2)

$$\mathcal{P}_{\mathcal{R}}(k) \equiv \frac{k^3}{2\pi^2} \langle |\mathcal{R}_k| \rangle$$

mass variance:

e.g. Liddle & Lyth

$$\sigma^2(R) = \int_0^\infty W^2(kR) \mathcal{P}_\delta(k,t) \frac{\mathrm{d}k}{k}$$

$$\sigma^2(R) = \frac{16}{81} \int_0^\infty (kR)^4 W^2(kR) \mathcal{P}_{\mathcal{R}}(k) T^2(kR/\sqrt{3}) \, \frac{\mathrm{d}k}{k}$$

transfer function (describes growth of perturbations on sub-horizon scales):

$$T(y) = 3\,\frac{(\sin y - y\cos y)}{y^3}$$

W(kR) = Fourier transform of window function used to smooth density contrast

For a locally scale-invariant power spectrum ($\mathcal{P}_{\mathcal{R}}(k) = A_{\rm PBH}$): $\sigma^2(R) = b A_{\rm PBH}$

with b =1.1, 0.09 and 0.05 for real-space top-hat, Gaussian and k-space top-hat window functions Ando, Inomata & Kawasaki

Questions?

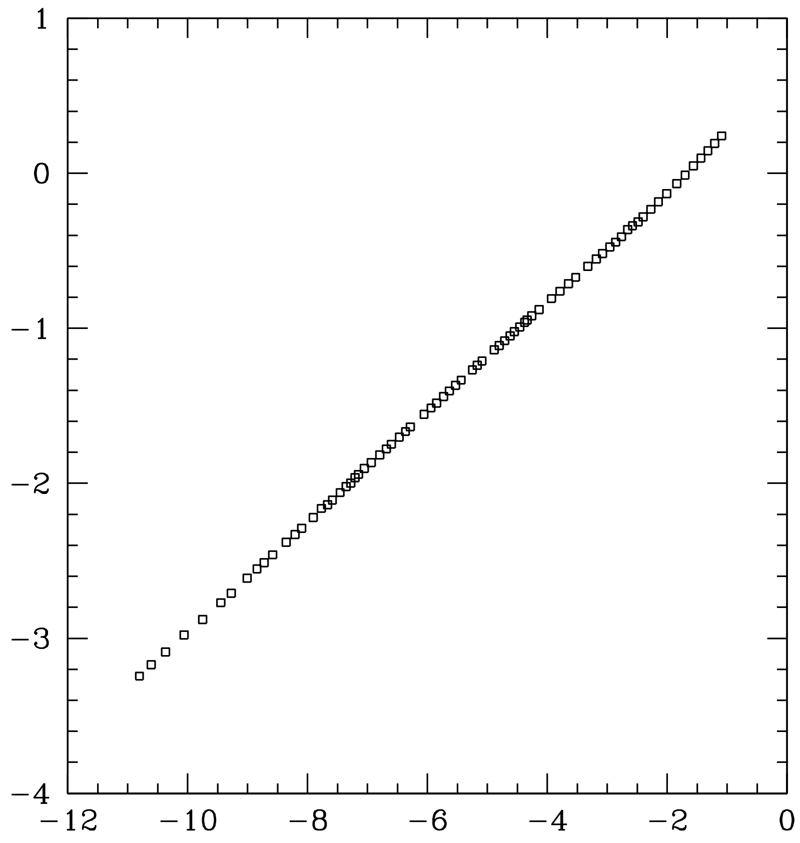
refinements to zero-th order calculation

critical collapse

Choptuik; Evans & Coleman; Niemeyer & Jedamzik

BH mass depends on size of fluctuation it forms from:

$$M = kM_{\rm H}(\delta - \delta_{\rm c})^{\gamma}$$



using numerical simulations (with appropriate initial conditions) find k=4.02, γ=0.357, $\delta_{\rm c}$ = 0.45

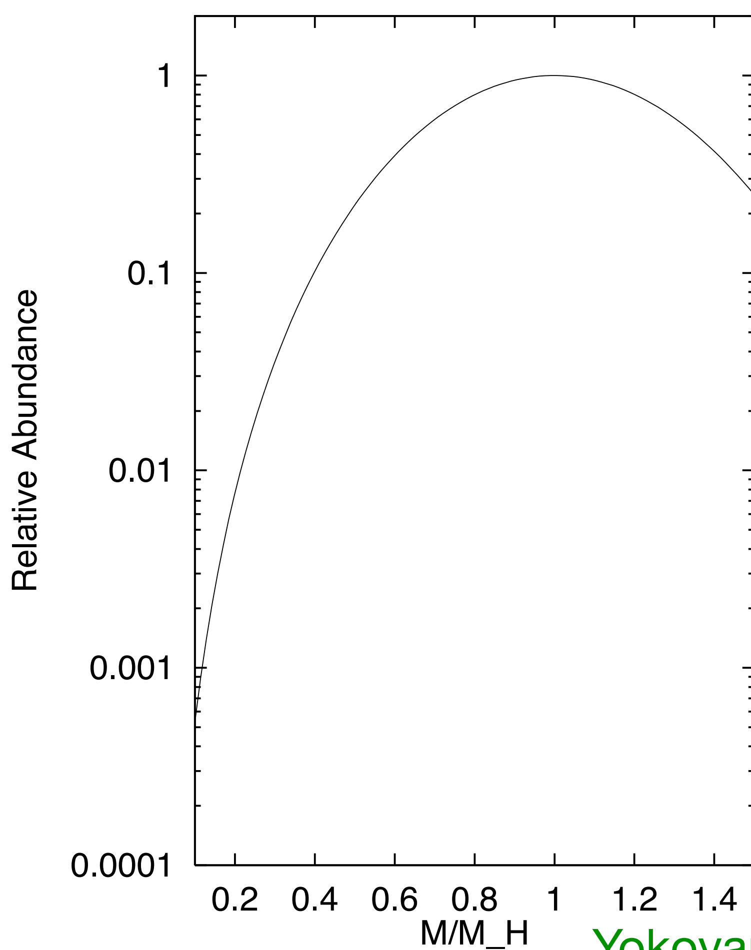
Musco, Miller & Polnarev

Get PBHs with range of masses produced even if they all form at the same time (so we don't expect the PBH MF to be a delta-function):

Niemeyer & Jedamzik:

$$\beta \approx \int_{\delta_c}^{\infty} \frac{M}{M_H} P(\delta)\, d\delta \qquad M = k M_H (\delta - \delta_c)^{\gamma}$$

$$\frac{d\beta}{d\ln M} = \frac{M}{M_H} P(\delta) \frac{d\delta}{d\ln M} \propto \left(\frac{M}{M_H}\right)^{-1+1/\gamma} \exp\left[-(1+\gamma)\left(\frac{M}{M_H}\right)^{1/\gamma}\right]$$



Yokoyama

M/M_H

threshold for collapse, δ_c

In fact depends on shape of perturbation (which depends on shape of primordial power spectrum), smaller for broad shapes (where pressure gradients negligible).

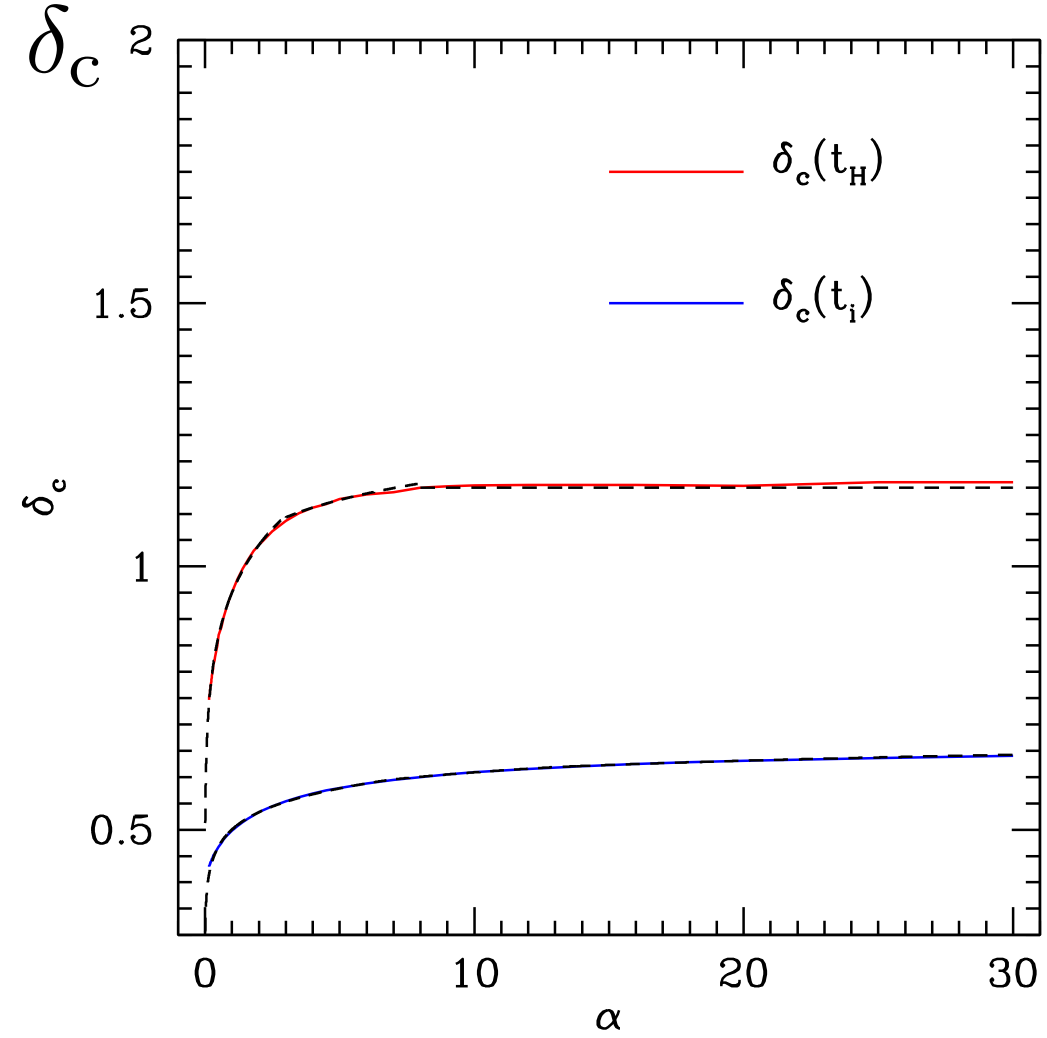
Harada,Yoo & Kohri; Germani & Musco; Musco; Escriva, Germani & Sheth;

Universal criterion for PBH formation: a cosmological perturbation can form a PBH if the peak value of the compaction function, which quantifies the gravitational potential, exceeds a threshold. Escriva, Germani & Sheth.

Recent work Musco et al. taking into account non-linearities arising from relationship between curvature perturbation and density perturbation, and from horizon crossing:

shape parameter "=" width of peak of compaction function

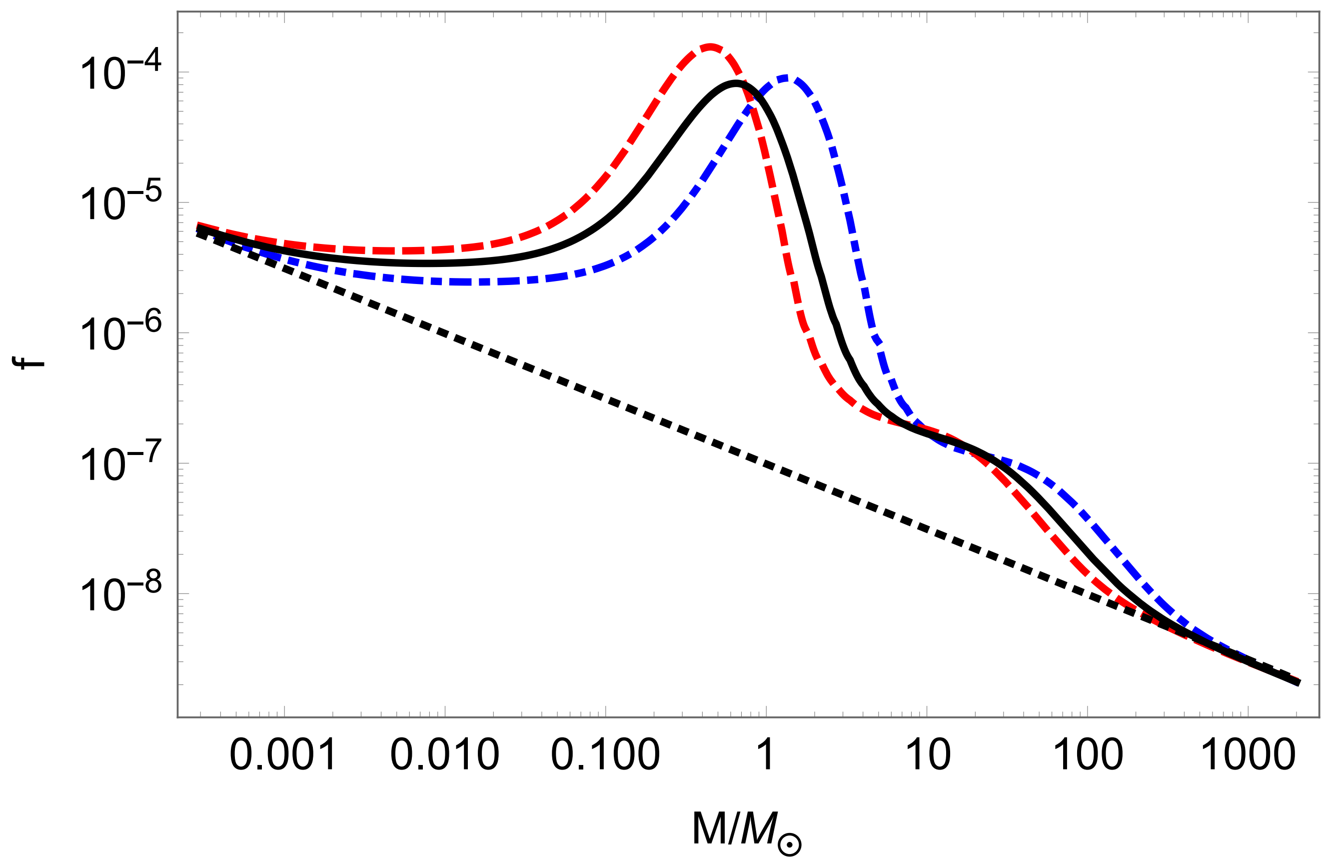
δ_c



shape parameter

Threshold for collapse is reduced (so PBH abundance increased) at phase transitions e.g. the QCD phase transition when the horizon mass is ~Solar mass. Jedamzik

Using new lattice calculation of QCD phase transition Byrnes et al. transition find a 2 order of magnitude enhancement in β (but perturbations still need to be larger than on cosmological scales):



Byrnes et al.

non-gaussianity (of probability distribution of density perturbations)

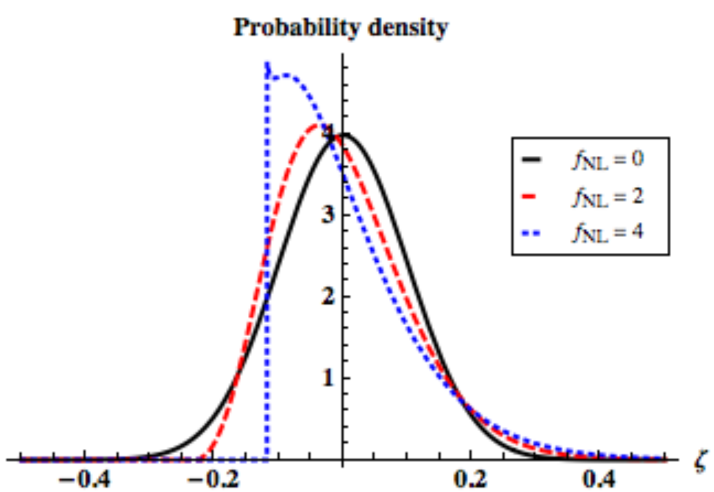
Since PBHs form from rare large density fluctuations, changes in the shape of the tail of the probability distribution (i.e. non-gaussianity) can significantly affect the PBH abundance. Bullock & Primack; Ivanov;… Francolini et al.

e.g. non zero f_{NL}

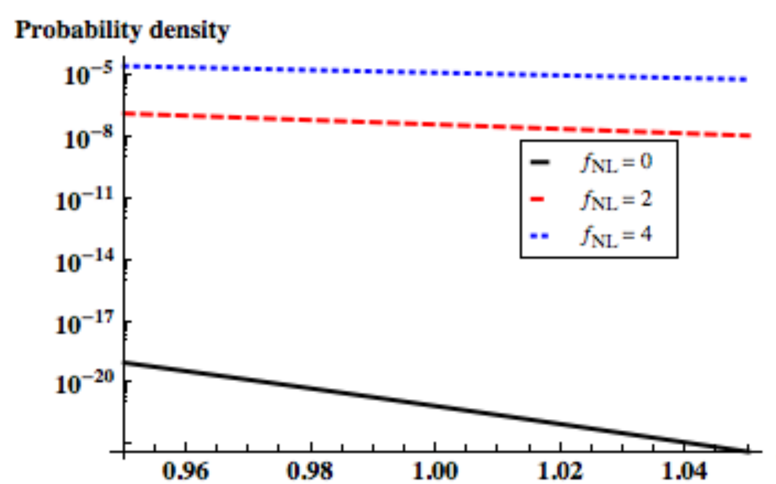
$$f_{\rm NL} \approx \frac{\mathcal{B}}{\mathcal{P}^2} \sim \frac{\mathcal{B}}{\sigma^4}$$

← bispectrum (Fourier transform of 3-point correlation function)

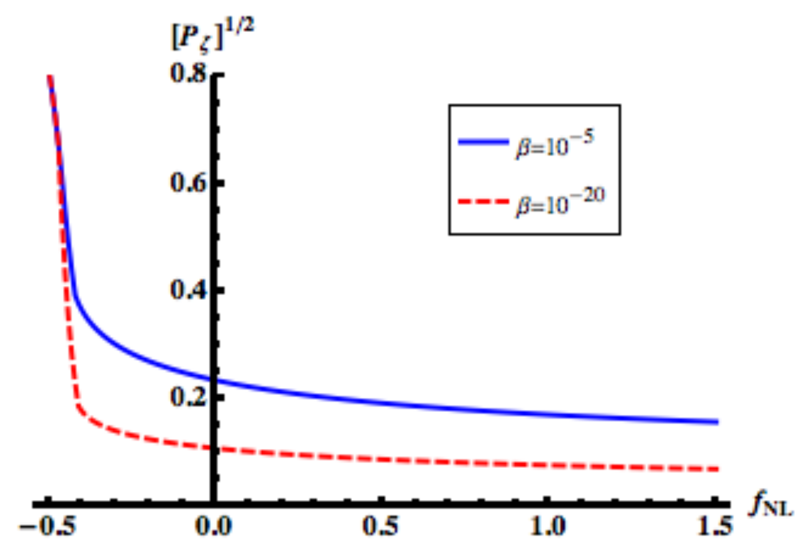
probability dist of curvature perturbations f_{NL}= 0, 2, 4



PBH forming tail of probability dist f_{NL}= 0, 2, 4



Dependence of σ on f_{NL} $\beta = 10^{-5}$ and 10^{-20}



Young & Byrnes

Relationship between density perturbations and curvature perturbations is non-linear, so even if curvature perturbations are gaussian (large) density perturbations won’t be. Kawasaki & Nakatsuka; De Luca et al.; Young, Musco & Byrnes

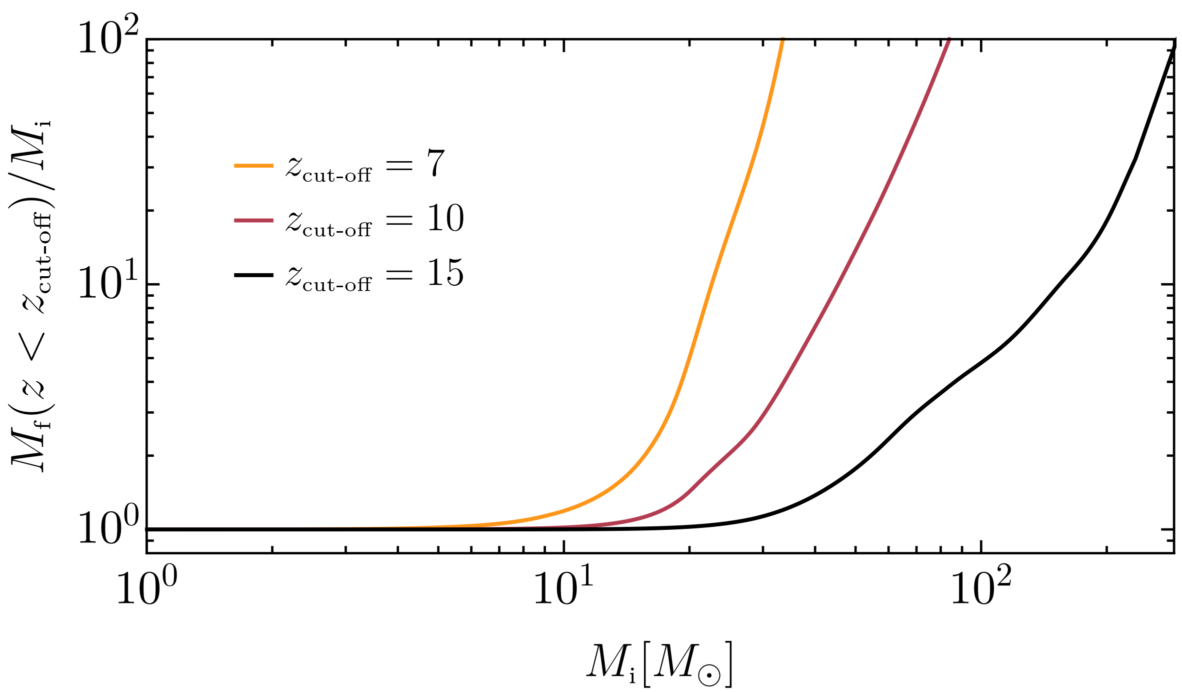
accretion

Accretion may significantly increase the mass and spin of PBHs with $M_{PBH} \gtrsim 10 \, M_\odot$

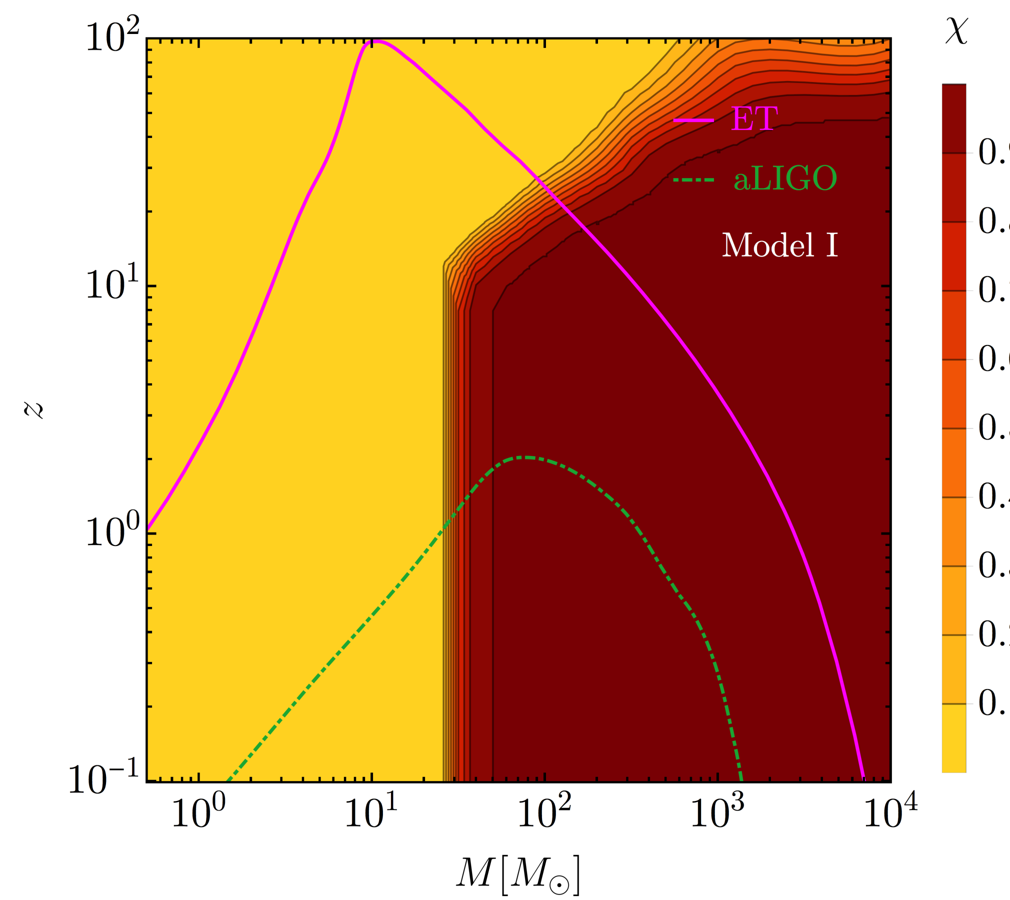
Postnov & Mitichkin; de Luca et al.

Size of effect depends on accretion physics e.g. is PBH isolated or in a binary?,
if $f_{PBH} \neq 1$ PBHs accrete a particle DM halo (see lecture 2),…

Relative increase in mass,
as a function of initial mass



Dependence of spin, χ,
on mass and redshift



de Luca et al.

is PBH mass (without accretion) constant??

PBHs usually treated as constant Schwarzschild masses, however Boehm et al. argue that:

in early Universe expansion of Universe has important effect on BH physics

for the Thakurta metric that consistently describes BHs in an expanding the BHs have a time-dependent effective mass

this affects the formation and coalescence rates of PBH binaries (see lecture 2)

But see see de Luca et al. (appendix B arXiv:2009.04731) for counter arguments.

PBH formation during an early period of matter domination

Between nucleosynthesis (t~1s) and matter-radiation equality, Universe is radiation dominated. However can have epoch of matter-radiation before nucleosynthesis due to e.g. long-live particles dominating and then decaying.

During matter domination PBHs can form from smaller fluctuations (no pressure to resist collapse). In this case fluctuations must be sufficiently spherically symmetric and homogeneous.

If $\sigma \lesssim 0.05$, initial abundance of PBHs (β) is larger than during radiation domination.

Yu, Khlopov & Polnarev; Harada et al. Kokubu et al.

$$\beta = \beta_{\rm inhom} \times \beta_{\rm aniso}$$

$\beta_{\rm inhom} \approx 3.7\sigma^{3/2}$ Kokubu et al.

$\beta_{\rm aniso} \approx 0.056\sigma^{5}$ Harada et al.

$$\beta \approx 0.21\sigma^{13/2}$$ Kokubu et al.

Relationship between initial (β) and present day ($f_{\rm PBH}$) abundances of PBHs also changed.

Angular momentum plays a significant role and initial spin is large: $a \gtrsim 0.5$.

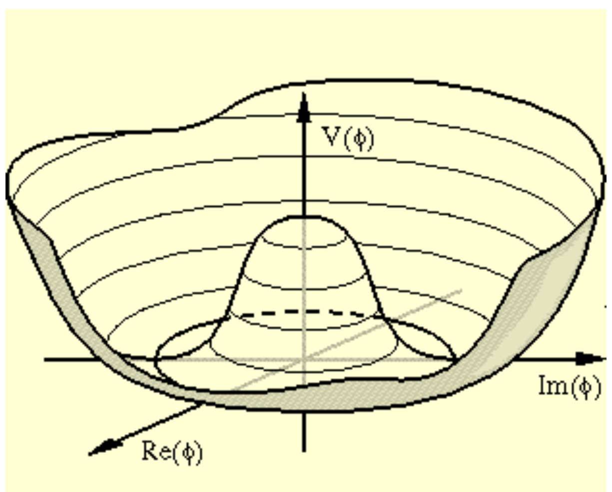
Questions?

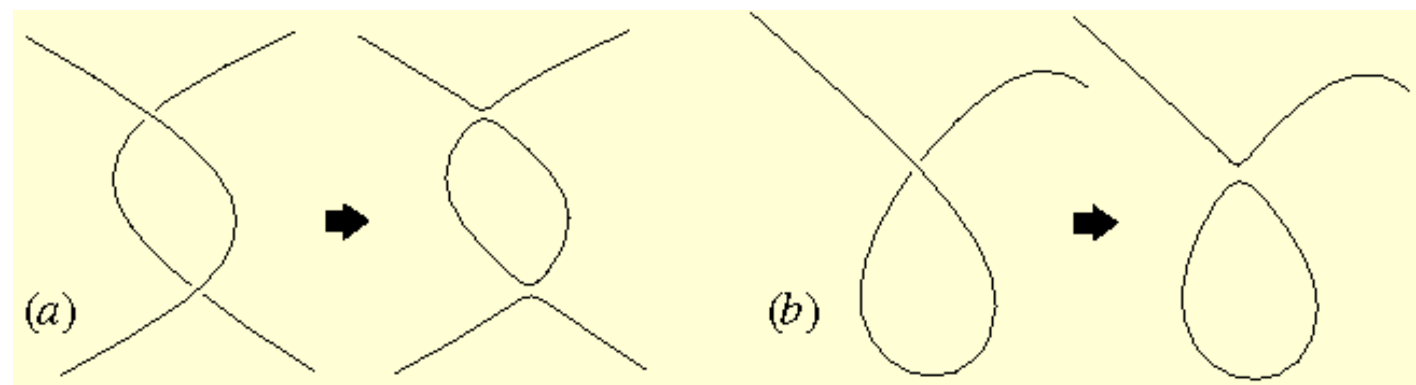
PBH formation: (some) other mechanisms

Collapse of cosmic string loops
Hawking; Polnarev & Zemboricz;

Cosmic strings are 1d topological defects formed during symmetry breaking phase transition.

String intercommute producing loops.





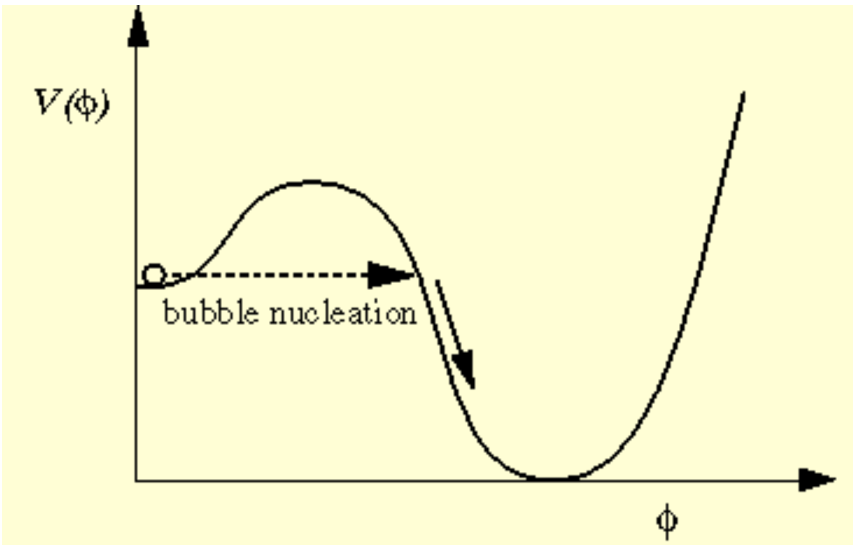
Small probability that loop will get into configuration where all dimensions lie within Schwarzschild radius (and hence collapse to from a PBH with mass of order the horizon mass at that time).

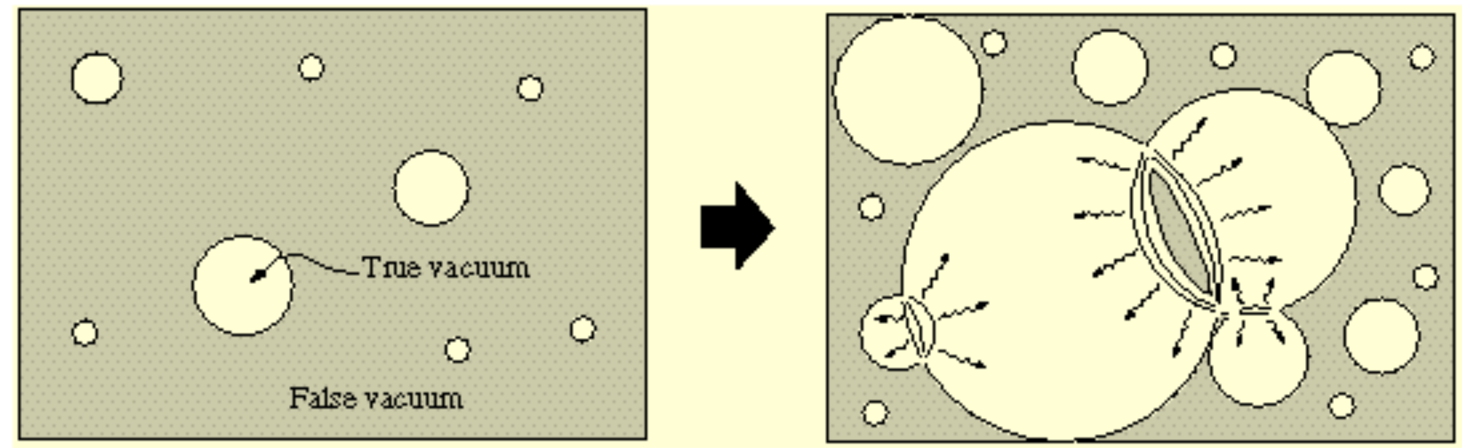
Probability is time independent, therefore PBHs have extended mass spectrum: $M^{-5/2}$

Bubble collisions

Hawking

1st order phase transitions occur via the nucleation of bubbles.





PBHs can form when bubbles collide (but bubble formation rate must be fine tuned).

PBH mass is of order horizon mass at phase transition.

Fragmentation of inflaton scalar condensate into oscillons/Q-balls

Cotner & Kusenko; Cotner, Kusenko & Takhistov

Scalar field with flat potential forms condensate at end of inflation, fragments into lumps (oscillons/Q-balls) which can come to dominate universe and have large density fluctuations that can produce PBHs.

Mass smaller than horizon mass and spin can be of order 1.

Mini-problem

Calculate:

i) the initial PBH mass fraction β
ii) the mass variance (~amplitude of primordial perturbations) σ

if PBHs with mass

a) $M_\odot$
b) 10^{15} g

make up all of the dark matter (f_{PBH}=1).

$$\beta(M) = \mathrm{erfc}\left(\frac{\delta_{\mathrm{c}}}{\sqrt{2}\sigma(M_{\mathrm{H}})}\right)$$

$$\beta(M) \sim 10^{-9} f_{\mathrm{PBH}} \left(\frac{M}{M_\odot}\right)^{1/2}$$

suggest taking δ_{c} =0.5

Bonus problem: show where the mass dependence in the relationship between β and f_{PBH} comes from.

Summary

- Primordial Black Holes (PBHs) can form in the early Universe and are a (non-baryonic cold) dark matter (DM) candidate.

- Most 'popular' mechanism: collapse of large density perturbations, during radiation formation

  - To produce a non-negligible number of PBHs, fluctuations have to be ~3 orders of magnitude larger than on cosmological scales.

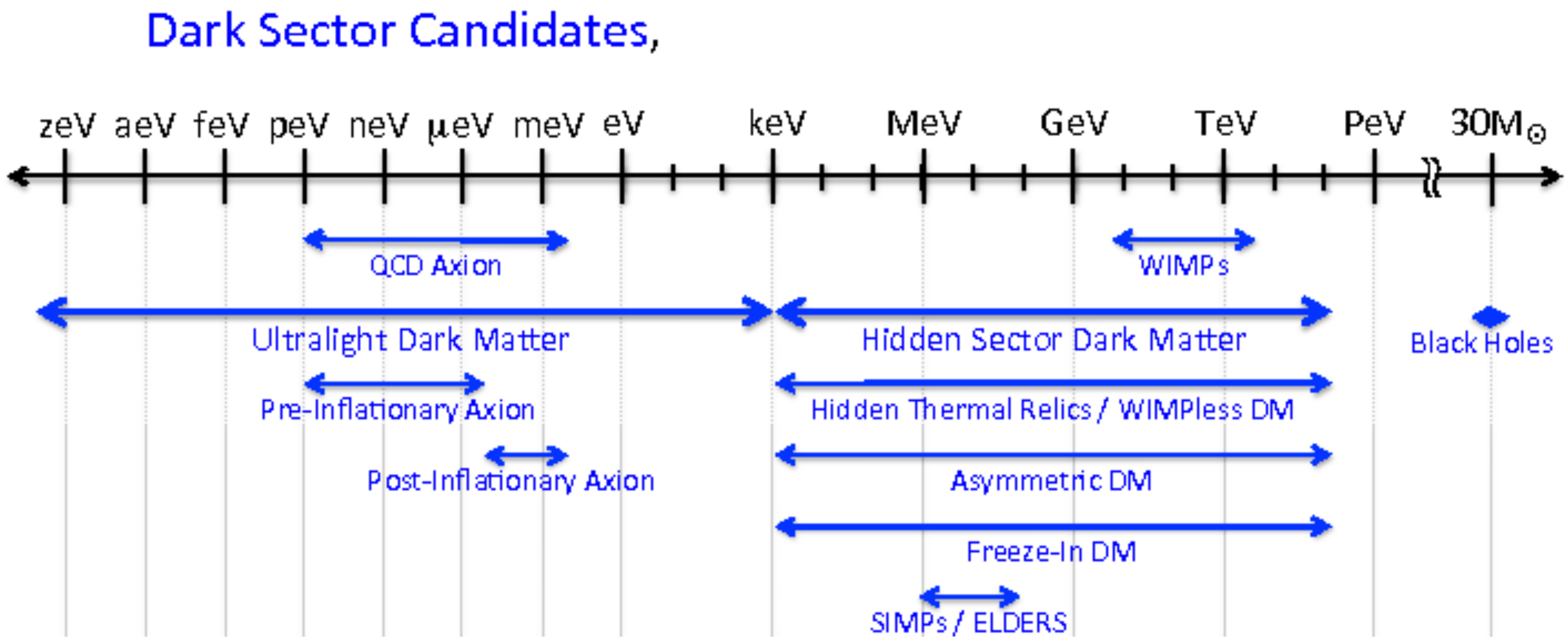
  - In this case PBHs have an extended mass function and small spin.

- Other mechanisms: bubble collisions, cosmic string loop collapse, domain wall collapse, scalar condensate fragmentation,…

Next time:

- inflation models that can produce large perturbations
- structure formation with PBH dark matter
- formation and evolution of PBH binaries

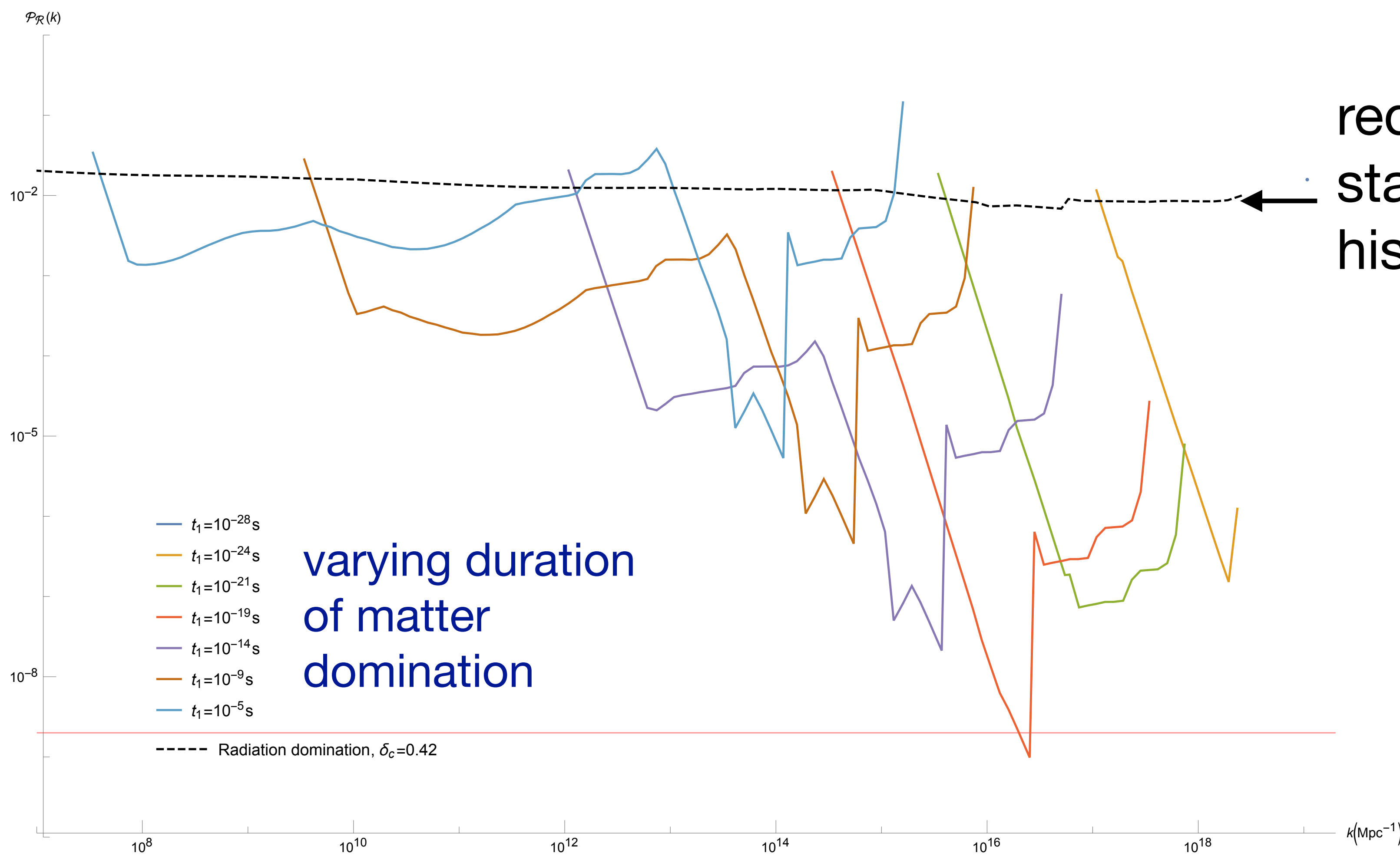
Back up slides



US Cosmic Horizons report

The increase in the amplitude of the perturbations required for PBHs to make up all of the dark matter is reduced Georg, Sengör & Watson; Georg & Watson; Carr, Tenkanen & Vaskonen; Cole & Byrnes:

Primordial curvature perturbation power spectrum



varying duration of matter domination

required amplitude for standard cosmological history

amplitude on CMB scales

Cole & Byrnes

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