Primordial Black Holes (PBHs) as dark matter

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1. Introduction to PBH dark matter

2. **PBH structure formation and evolution**
   - inflation models that produce large density perturbations
   - structure formation with PBH dark matter
   - formation and evolution of PBH binaries

3. Observational constraints on PBHs

Recap

PBHs are black holes formed from over-densities in the early Universe.

Most commonly studied mechanism: collapse of large density perturbations during radiation domination.

Dark matter candidate: non-baryonic and (for $M_{\rm PBH} \gtrsim 10^{15}\,{\rm g}$) stable.

Today

- a brief introduction to inflation
- inflation models that can produce large perturbations
- structure formation with PBH dark matter
- formation and evolution of PBH binaries

A brief introduction to inflation

Inflation: A period of accelerated expansion ($\ddot{a} > 0$) in the early Universe.

Problems with the Big Bang:

Flatness: if universe isn't exactly flat density evolves away from critical density (for which geometry is flat), to be so close to critical density today requires fine tuning of initial conditions.

Horizon: regions that have never been in causal contact have the same Cosmic Microwave Background temperature and anisotropy distribution.

Monopoles/massive relics: formed when symmetry breaks, would dominate the density of the Universe today.

Inflation solves these problems by:

driving 'initial' density extremely close to critical density

allowing currently observable universe to originate from small region (originally in causal contact)

diluting monopoles

It can also generate density perturbations:

Quantum fluctuations

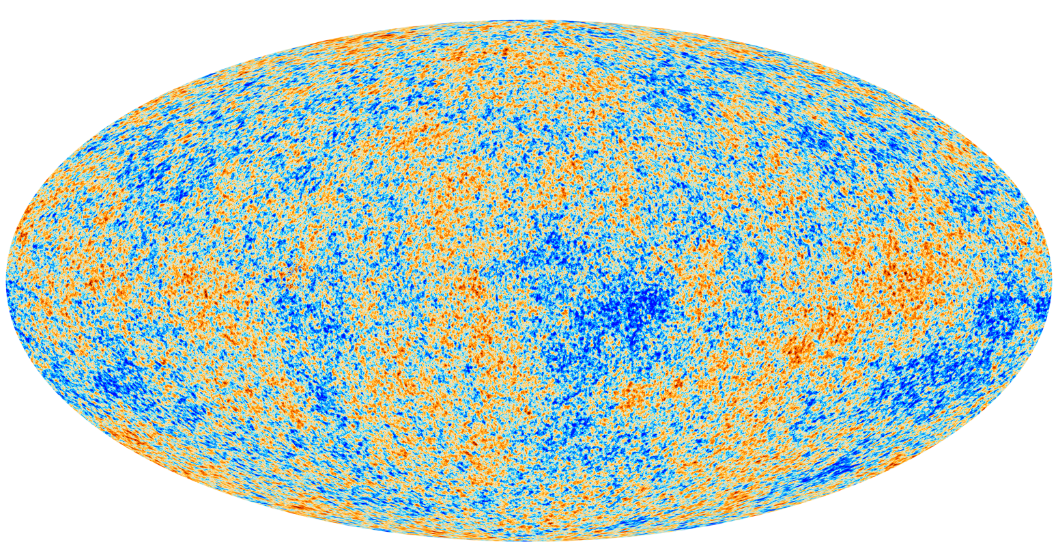
↓

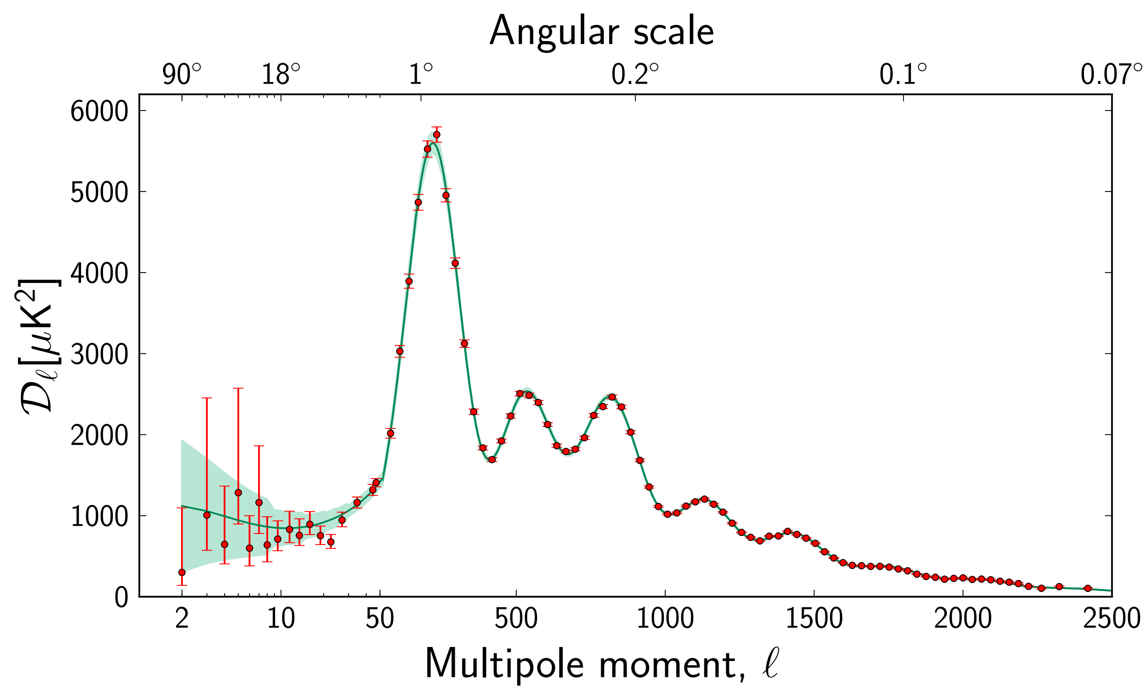
Curvature perturbations

↓

Density & temperature perturbations

which are close to scale-invariant and hence consistent with the temperature anisotropies in the cosmic microwave background radiation.





Planck

What drives inflation?

what do we need to get $\ddot{a} > 0$?

Fluid equation: $$\dot{\rho} + 3H(\rho + p) = 0$$

Acceleration equation: $$\frac{\ddot{a}}{a} = -4\pi G(\rho + 3p)$$

$$\ddot{a} > 0 \quad \Rightarrow \quad p < -\frac{1}{3}\rho$$

i.e. negative pressure!

Scalar field:

spin zero particle (unchanged under co-ordinate transformations)

common in ‘beyond standard model’ particle theories

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \qquad p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

if potential dominates: $$\rho \approx -p \approx V(\phi)$$

Scalar field dynamics-a quick overview

Friedman equation: $$H^2 = \frac{8\pi}{3m_{\rm Pl}^2}\left(V + \frac{1}{2}\dot{\phi}^2\right) \qquad H \equiv \frac{\dot{a}}{a}$$

Fluid equation: $$\ddot{\phi} + 3H\dot{\phi} = -\frac{{\rm d}V}{{\rm d}\phi}$$

[c.f. a ball rolling down a hill, with the expansion of the Universe acting as friction]

Slow roll approximation

Slow roll parameters:

$$\epsilon = \frac{m_{\rm Pl}^2}{16\pi}\left(\frac{V'}{V}\right)^2$$ slope of potential

$$\eta = \frac{m_{\rm Pl}^2}{8\pi}\frac{V''}{V}$$ curvature of potential

If $\varepsilon, |\eta| \ll 1$

$$H^2 \approx \frac{8\pi}{3m_{\rm Pl}^2}V \qquad 3H\dot{\phi} \approx -\frac{{\rm d}V}{{\rm d}\phi}$$

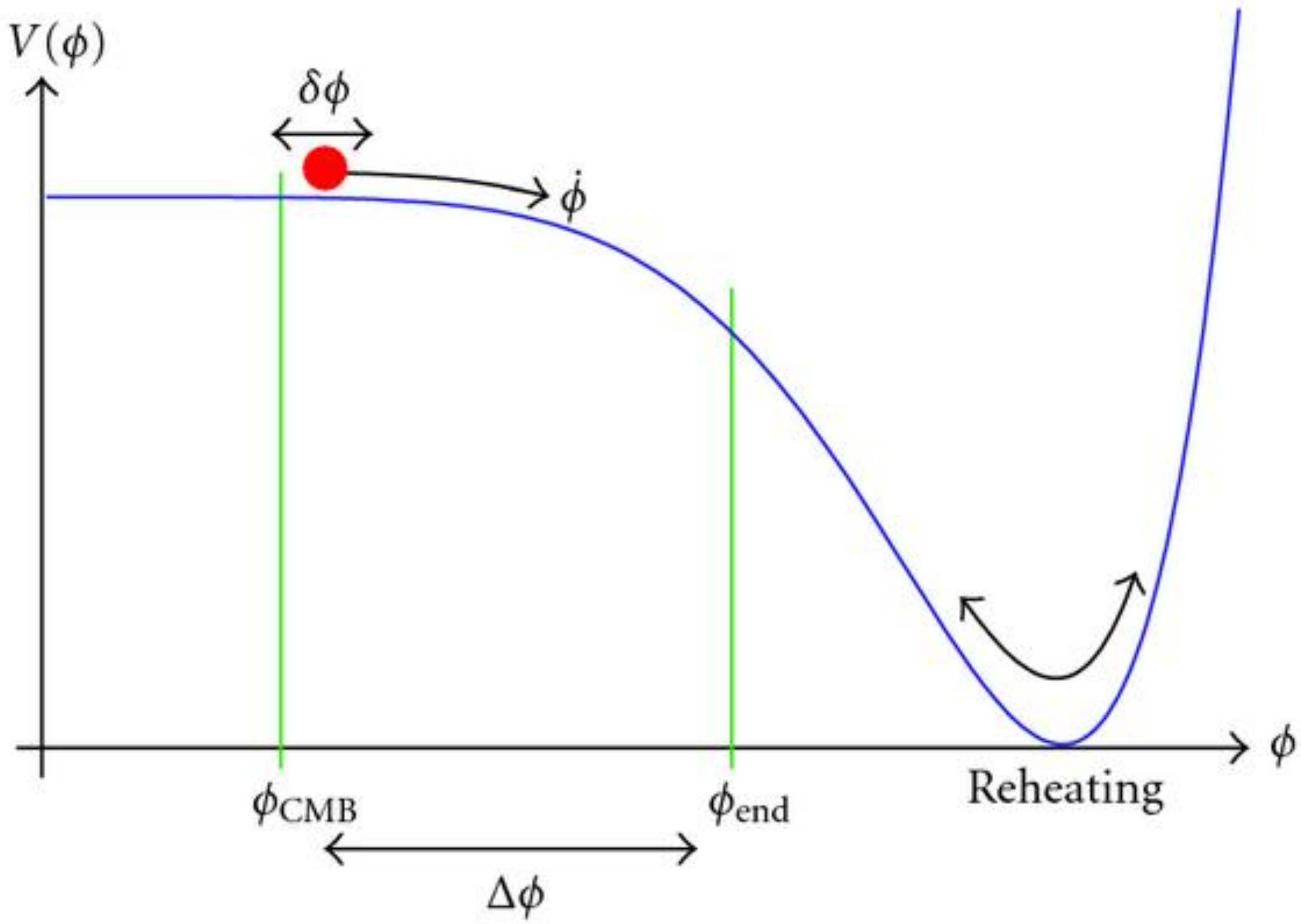
$$\varepsilon < 1 \quad \Rightarrow \quad \ddot{a} > 0$$

[nb slow roll parameters defined in terms of H, rather than V, are more useful for calculations]

Inflation ends when potential becomes too steep: $\varepsilon \approx 1$

Field oscillates around minimum of potential.

Inflaton field decays creating radiation dominated Universe (reheating).



Yadav & Wandelt

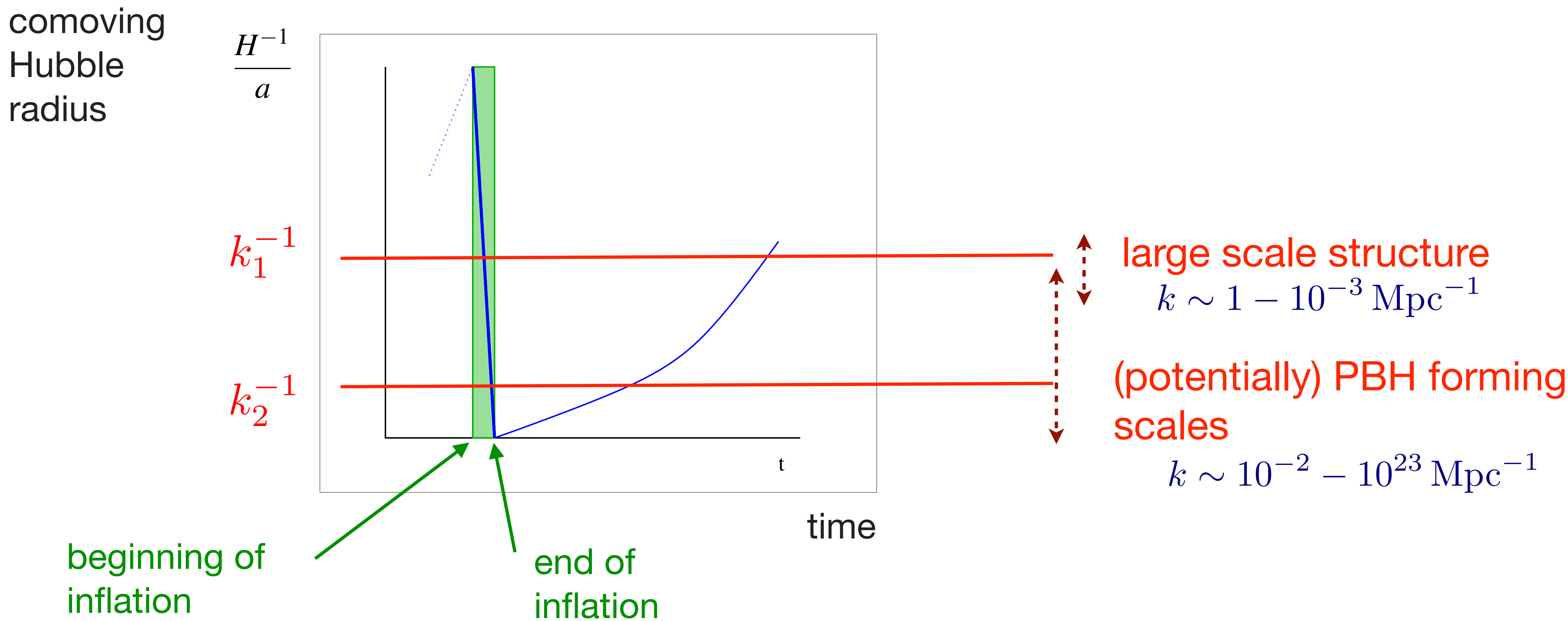
Scales probed by:

Large scale structure
& the CMB

Primordial Black Holes

$$\ddot{a} > 0 \quad \Rightarrow \quad \frac{\mathrm{d}(H^{-1}/a)}{\mathrm{d}t} < 0$$

i.e. comoving Hubble radius decreases during inflation



A scales exits the horizon during inflation when k = aH, re-enters when k = aH again (and if fluctuations are sufficiently large they collapse to form PBH soon afterwards).

Lower limit on PBH mass set by reheat temperature at the end of inflation:

$$M \sim M_{\mathrm{H}} = 10^{18}\,\mathrm{g} \left(\frac{10^7\,\mathrm{GeV}}{T} \right)^2$$

in slow roll approx primordial power spectrum: $\mathcal{P}_{\mathcal{R}}(k) \propto \frac{V^3}{(V')^2}$ $\sigma^2 \sim \mathcal{P}_{\mathcal{R}}$

power law parameterisation: $$\mathcal{P}_{\mathcal{R}}(k) = A_{\rm s} \left(\frac{k}{k_0} \right)^{n_{\rm s}(k)-1}$$

scalar spectral index: $$n_{\rm s} = 1 - 6\epsilon + 2\eta + ...$$

tensor to scalar ratio: $$r \equiv \frac{\mathcal{P}_{\rm t}(k_0)}{\mathcal{P}_{\mathcal{R}}(k_0)} = 16\epsilon$$

observations (CMB + large scale structure): Akrami et al.

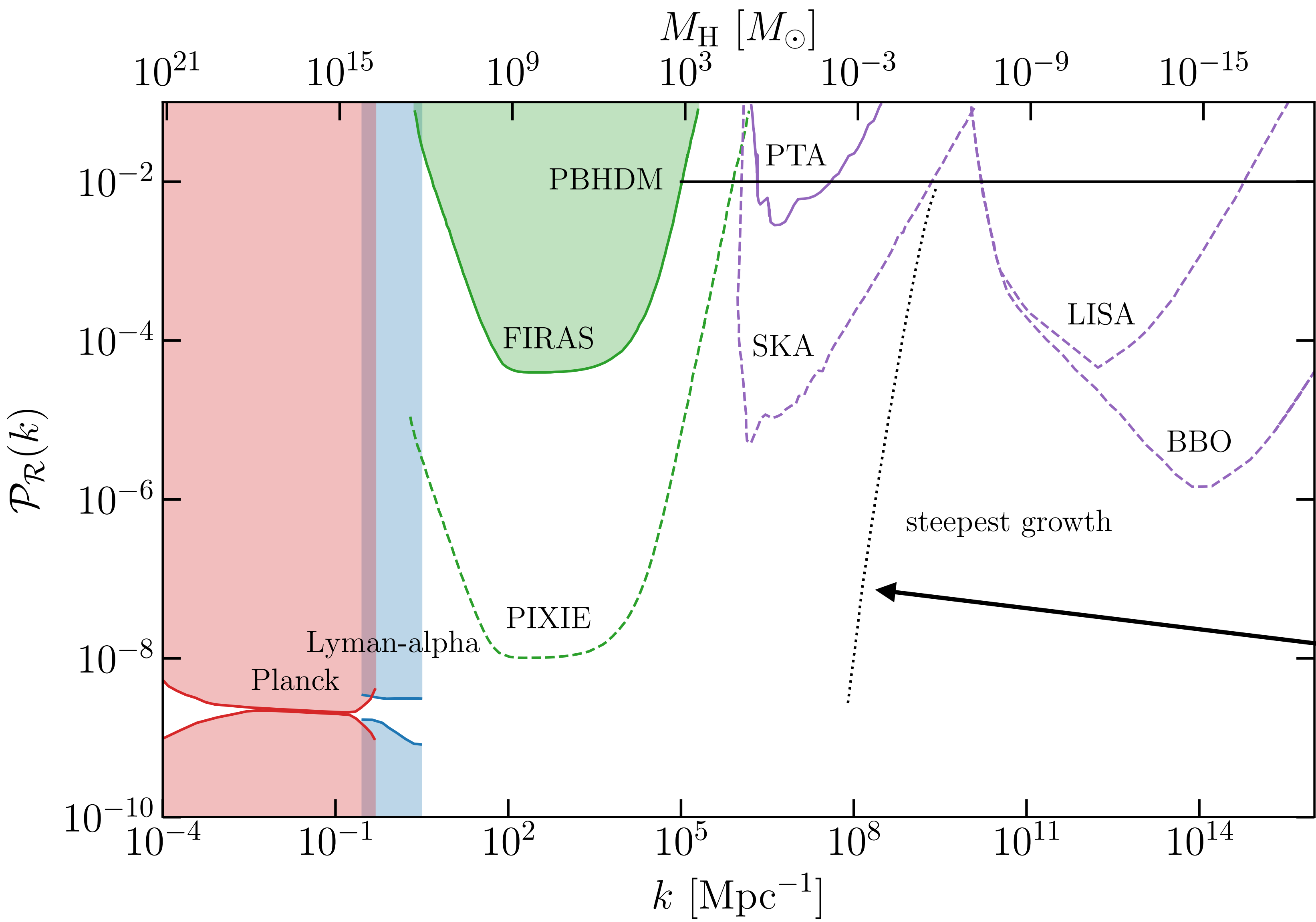
$$\ln\left(10^{10} A_{\rm s}\right) = 3.044 \pm 0.0014$$
$$n_{\rm s}|_{k_0 = 0.05\,{\rm Mpc}^{-1}} = 0.9668 \pm 0.0037$$
$$r < 0.063$$

n.b. power law expansion of power spectrum

$$n_{\rm s}(k) = n_{\rm s}|_{k_0} + \frac{1}{2} \left. \frac{{\rm d} n_{\rm s}}{{\rm d} \ln k} \right|_{k_0} \ln\left(\frac{k}{k_0} \right) + ...$$

is only valid over small range of k (fine for CMB/LSS, but not for extrapolating down to PBH forming scales Green).

$\mathcal{P}_{\mathcal{R}}(k) \sim \sigma^2$



amplitude required to form an interesting (f_{PBH} ~1) abundance of PBHs

steepest growth possible in single field inflation models

Byrnes, Cole & Patil; Carrilho, Malik, Mulryne

Green & Kavanagh
c.f. Byrnes, Cole & Patil; Chluba et al.

Constraints from CMB temperature anisotropies, Lyman-alpha forest, CMB spectral distortions and gravitational waves (______ current - - - - - future/proposed) more in lecture 3

Questions?

Inflation models that can produce large perturbations

a) single field inflation

in slow roll approx $$\mathcal{P}_{\mathcal{R}}(k) \propto \frac{V^3}{(V')^2} \propto \frac{V}{\epsilon}$$

To increase amplitude of perturbations need to decrease slope of potential. Potential then has to steepen again for inflation to end ($\epsilon \approx 1$).

For power spectrum to grow by ~7 orders of magnitude required to form PBHs, slow roll approximation has to be violated. Motohashi & Hu

As $V' \to 0$ get ultra-slow roll (USR) inflation, evolution of inflation driven by expansion rate rather than slope of potential. Standard slow roll calc. not valid, need numerical calculation of perturbations. Motohashi & Hu; Ballesteros & Taoso; Hertzberg & Yamada

Quantum diffusion (quantum kicks larger than classical evolution of field) also important. Motohashi & Hu; Ivanov; Francolini et al.; Pattison et al.; Biagetti et al.

Fastest growth that can be achieved, in principle, in single field models:

$$\mathcal{P}(k) \propto k^4$$ Byrnes, Cole & Patil

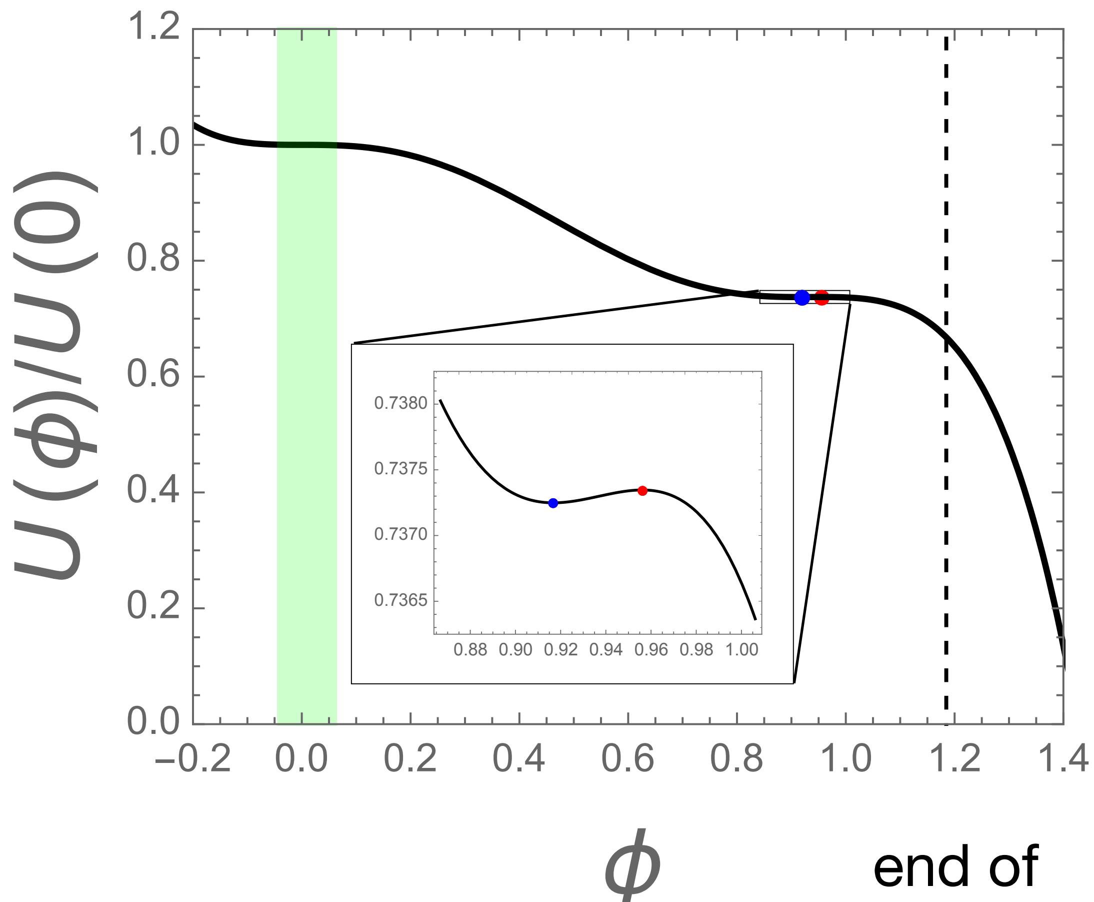
with specific form for pre-USR inflationary expansion:

$$\mathcal{P}(k) \propto k^5 (\log k)^2$$ Carrilho, Malik, Mulryne

fine-tuned potential with an inflection point, or a small local minimum

Ballesteros & Taoso; Herzberg & Yamada

potential

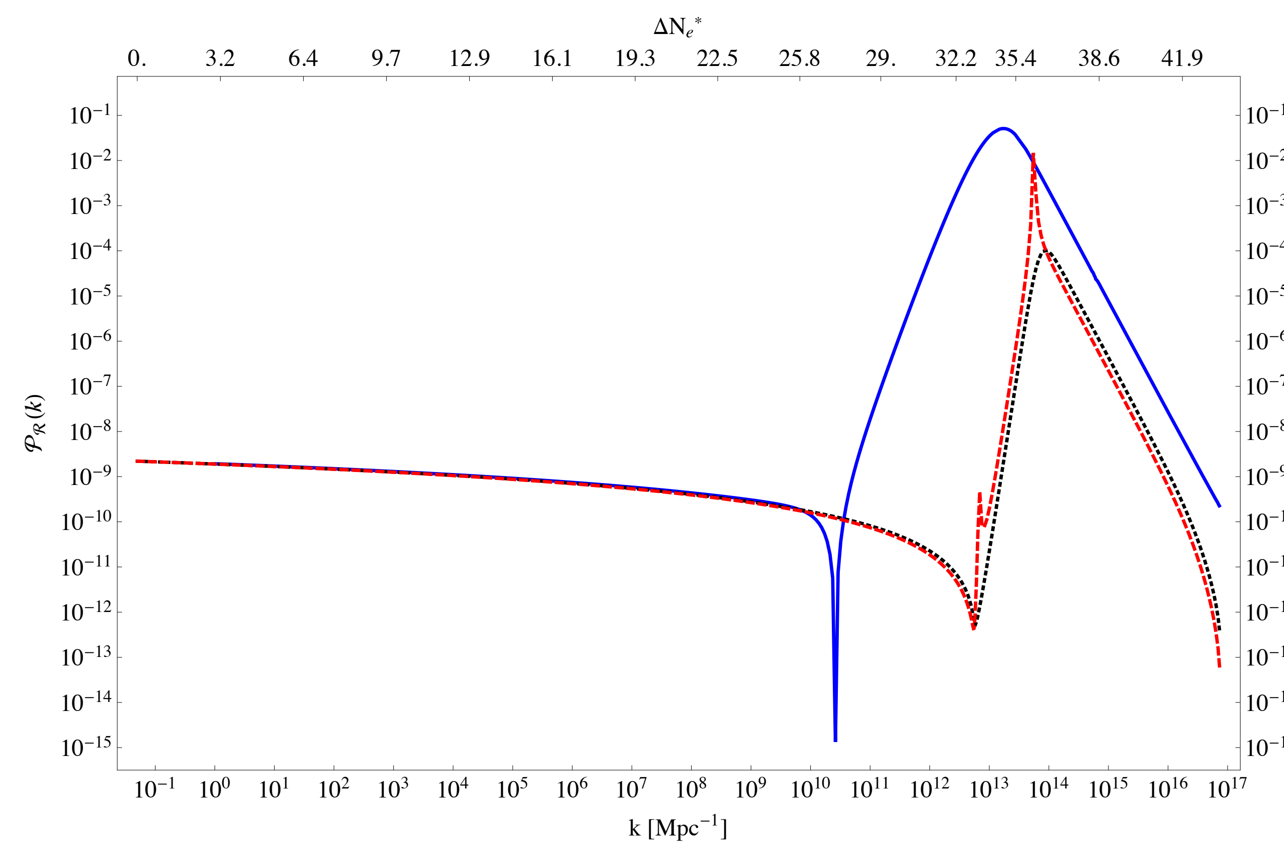


CMB/LSS

end of inflation

Herzberg & Yamada

primordial power spectrum



numerical calculation (Mukhanov-Sasaki)

slow roll approx, using Hubble parameter

slow roll approx, using potential

Ballesteros & Taoso;

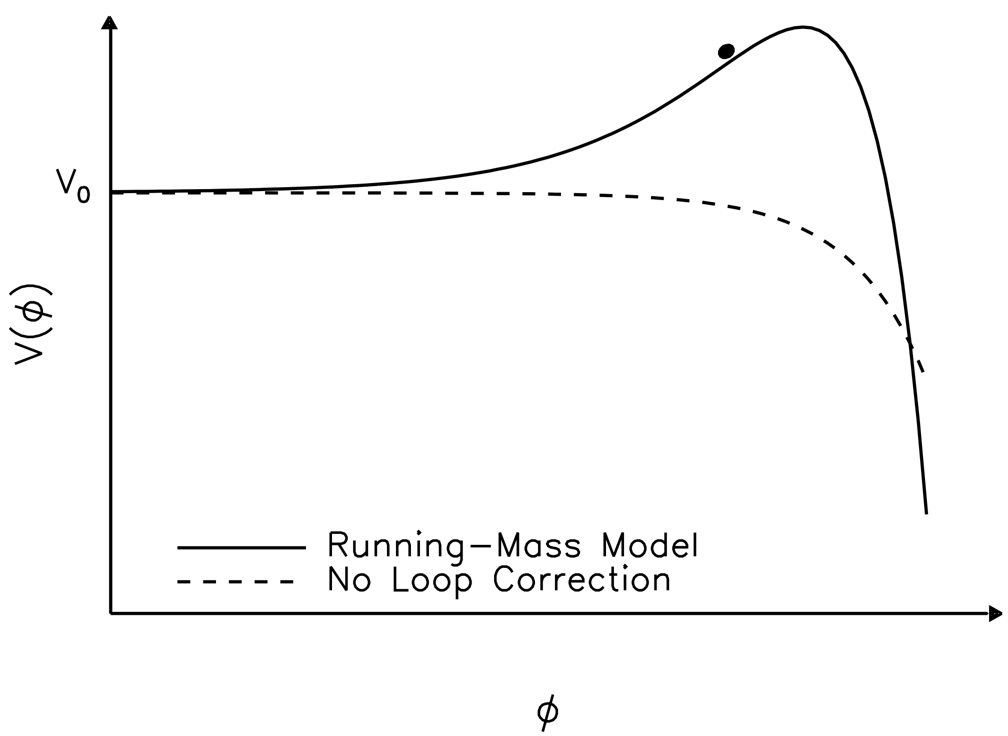
single field models with a mono-tonically growing power spectrum:

running mass inflation

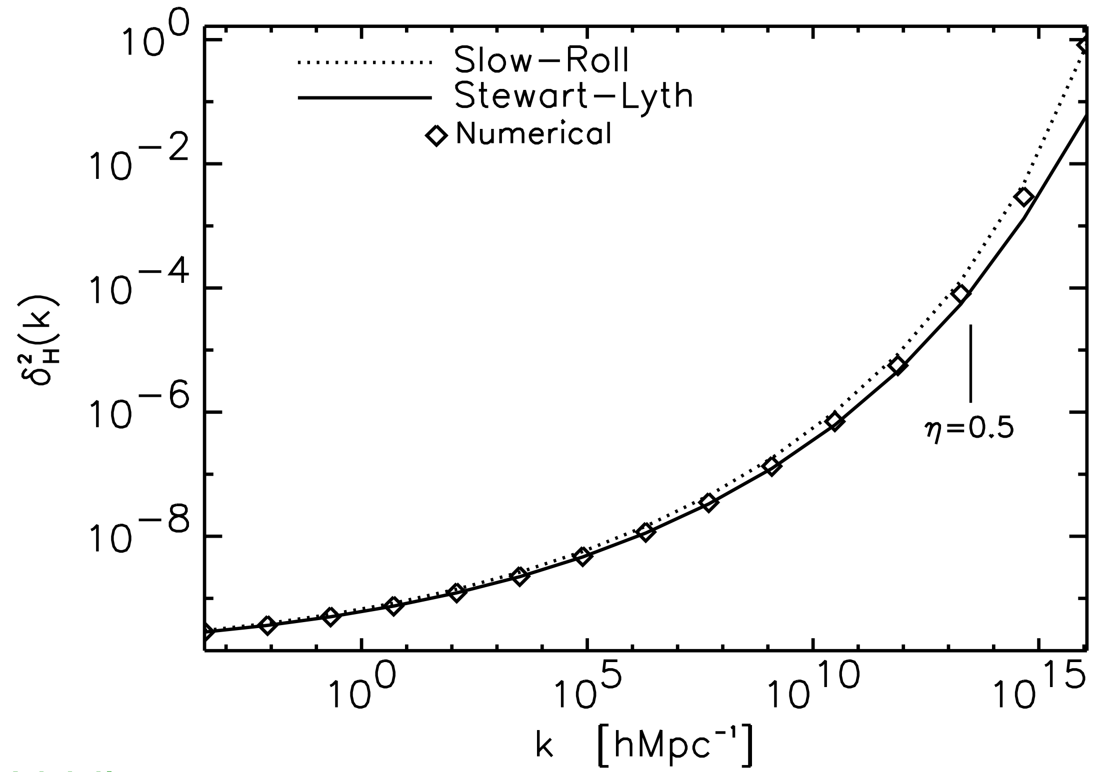
Stewart

$$V(\phi) = V_0 + \frac{1}{2} m^2(\phi) \phi^2$$

potential



primordial power spectrum



Leach, Grivell, Liddle

n.b. not a 'complete' model, need an auxiliary mechanism to end inflation.

Also: hilltop inflation Kohri, Lyth, Melchiorri; Alabidi & Kohri

Reheating process at end of inflation, can lead to amplification of perturbations

Green & Malik; Bassett & Tsujikawa; Martin, Papanikolaou & Vennin

b) multi-field models

General idea: different fields (or regions of potential with different forms) responsible for cosmological and small-scale (PBH producing) perturbations.

hybrid inflation with a mild waterfall transition

Garcia-Bellido, Linde & Wands; Garcia-Bellido & Clesse

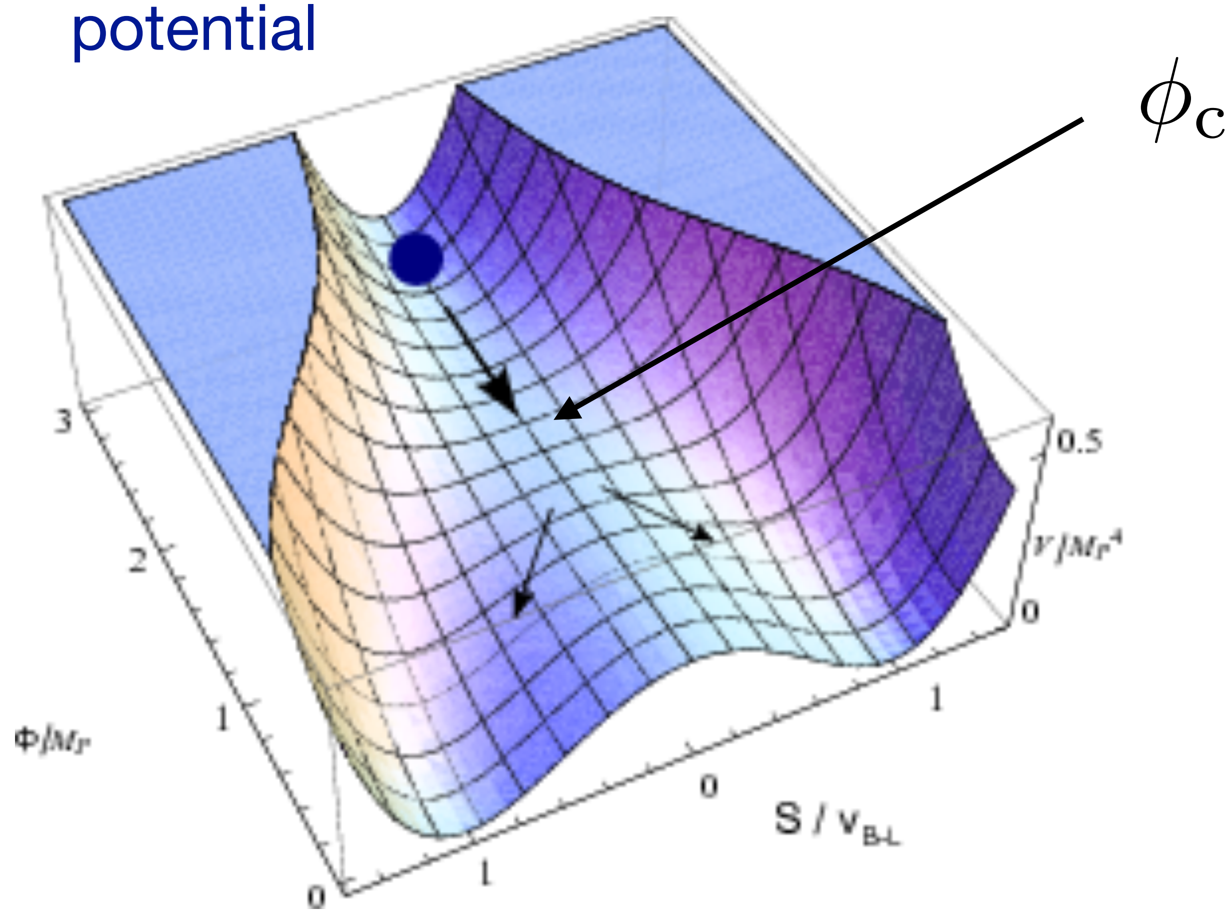
$$V(\phi,\psi) = \lambda\left[\left(1-\frac{\psi^2}{M^2}\right)^2 + \frac{(\phi-\phi_c)}{\mu_1} - \frac{(\phi-\phi_c)^2}{\mu_2^2} + \frac{2\phi^2\psi^2}{M^2\phi_c^2}\right]$$

Initially ϕ slow rolls and inflation driven by false vacuum energy of ψ.

At ϕ_c , ψ undergoes waterfall transition to global minima at $\psi = \pm$ M.

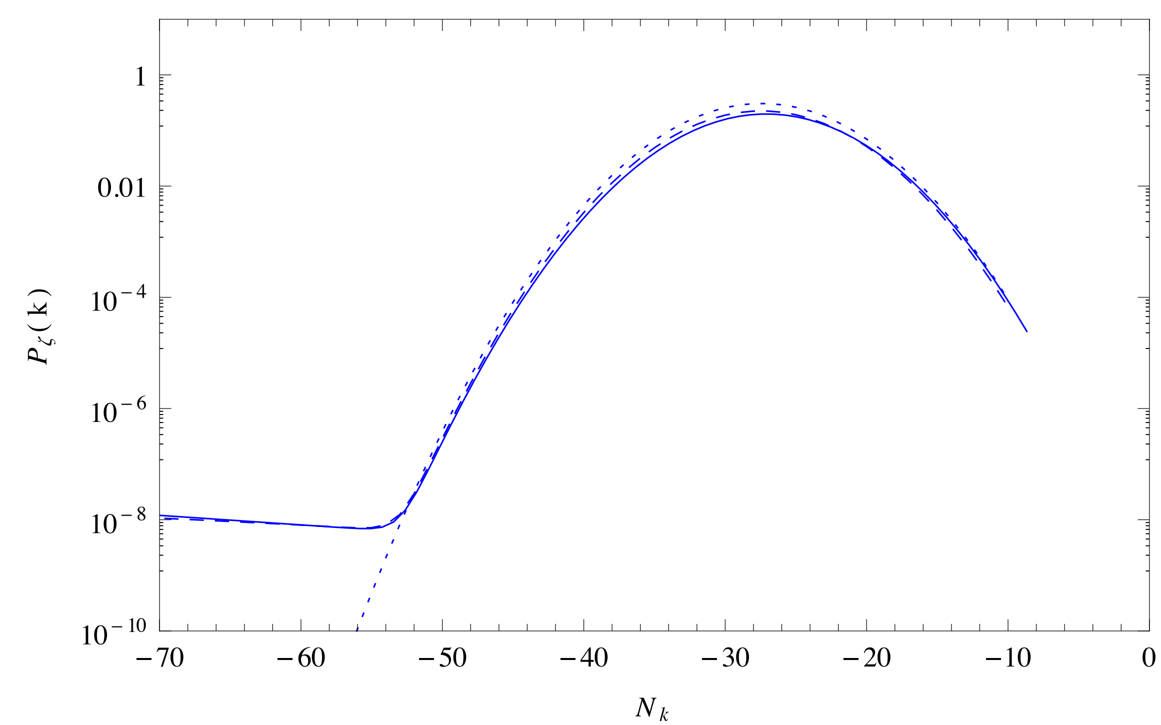
If waterfall is mild, get 2nd phase of inflation where both fields are important → isocurvature fluctuations & large, broad peak in power spectrum

potential



Buchmuller

primordial power spectrum
(as a function of number of e-foldings from end of inflation)



Clesse & Garcia-Bellido

axion-like curvaton

Kawasaki, Kitajima & Yanagida

Large scale perturbations generated by inflaton, small scale (PBH forming) perturbations by curvaton (a spectator field during inflation gets fluctuations and decays afterwards producing perturbations Lyth & Wands)

double inflation

Saito, Yokoyama & Nagata; Kannike et al.

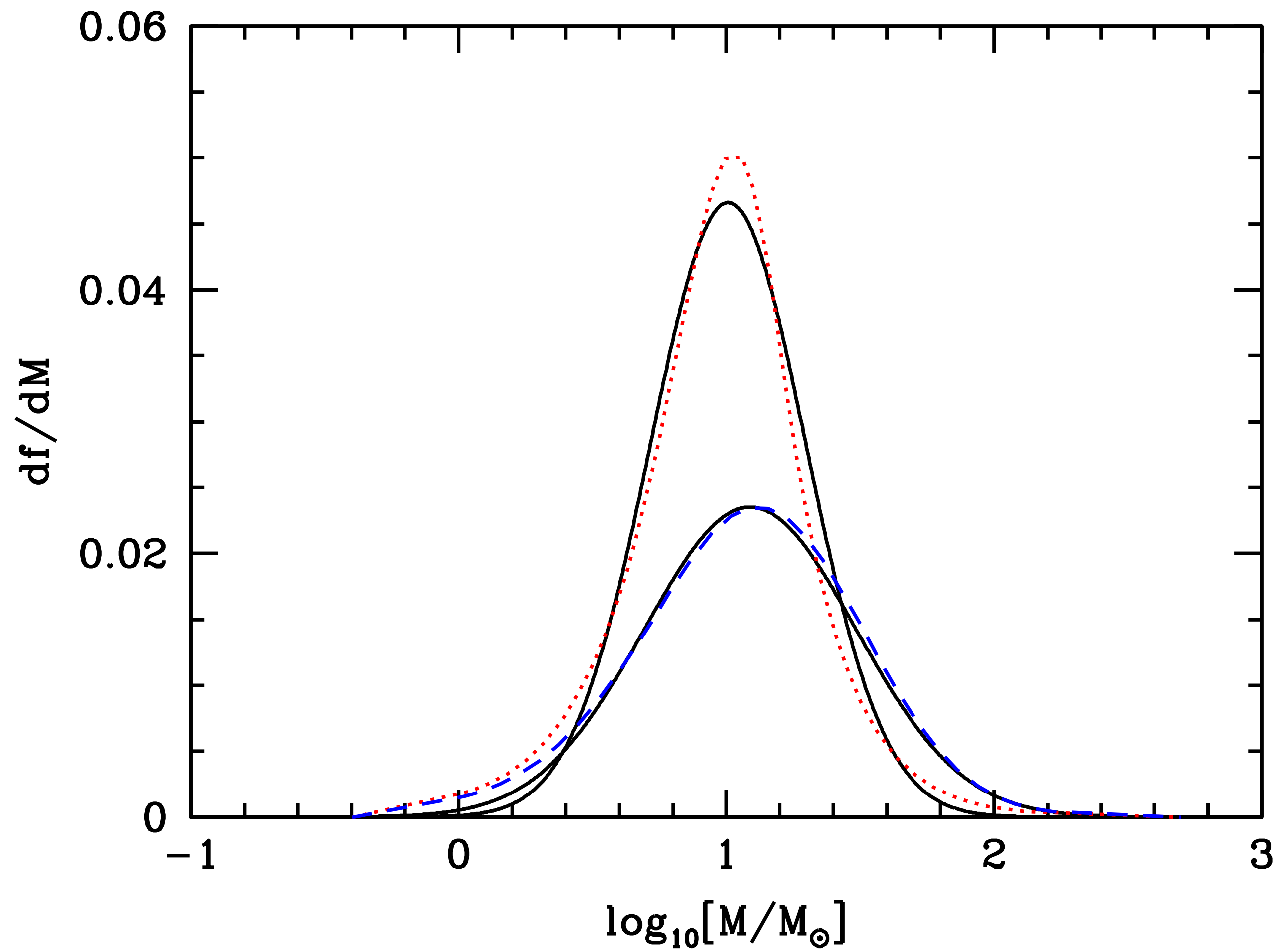
Two separate periods of inflation, perturbations on cosmological (small) scales generated by 1st (2nd) period

rapid turns in field space

Palma, Sypsas & Zenteno; Fumagalli et al.

Extended MFs produced by peak in power spectrum, well approximated by a **log-normal distribution**: Green; Kannike et al.

$$M \frac{\mathrm{d}n}{\mathrm{d}M} \propto \exp\left\{ -\frac{\left[\log\left(M/M_{\mathrm{c}}\right)\right]^2}{2\sigma^2} \right\}$$



axion-like curvaton

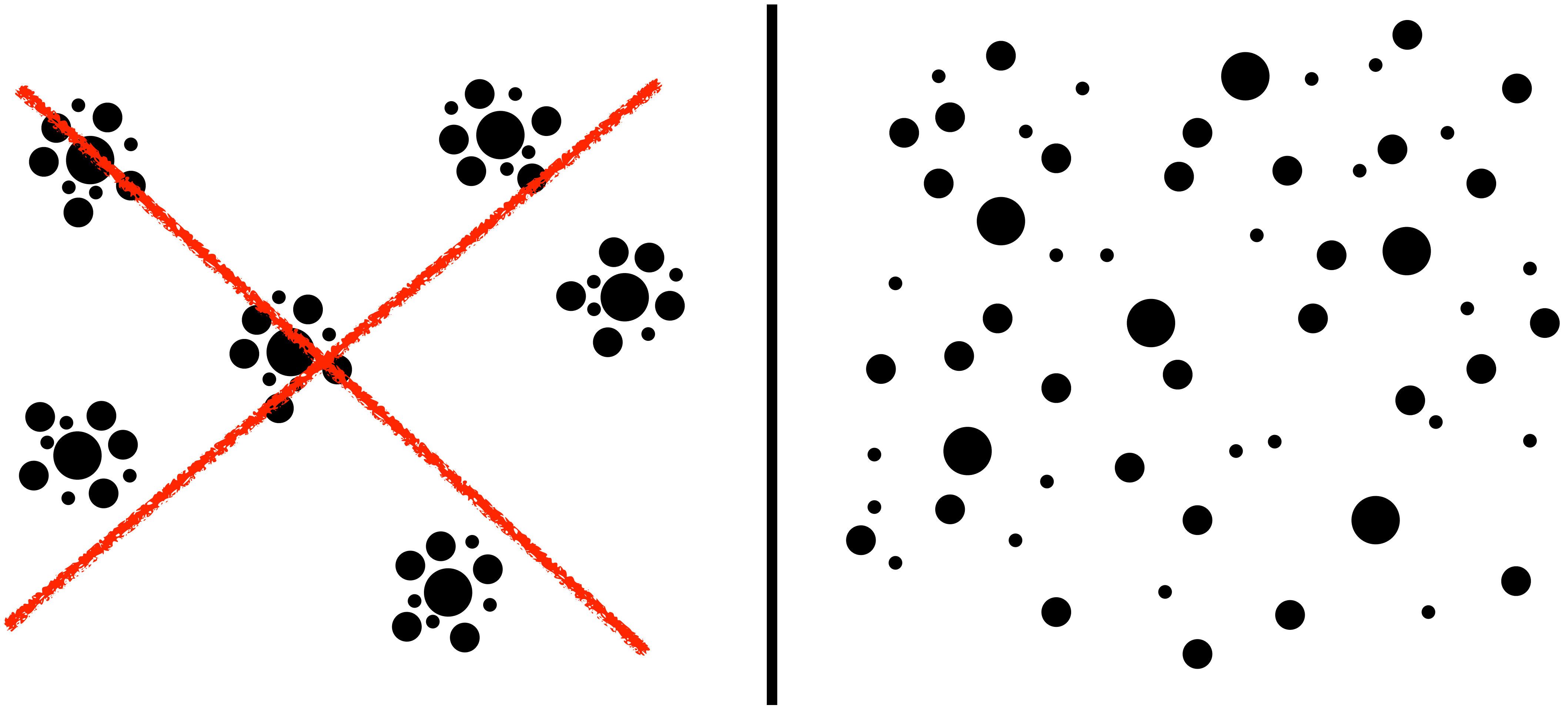
running mass inflation

Questions?

Structure formation with PBH dark matter

PBHs don't form in clusters (previous work Chisholm extrapolated an expression for the correlation function beyond its range of validity).

But if PBHs make up a large fraction of the DM, PBH clusters form shortly after matter-radiation equality. Afshordi, Macdonald & Spergel; Raidal et al.; Inman & Ali-Haïmoud; Jedamzik



Ali-Haïmoud

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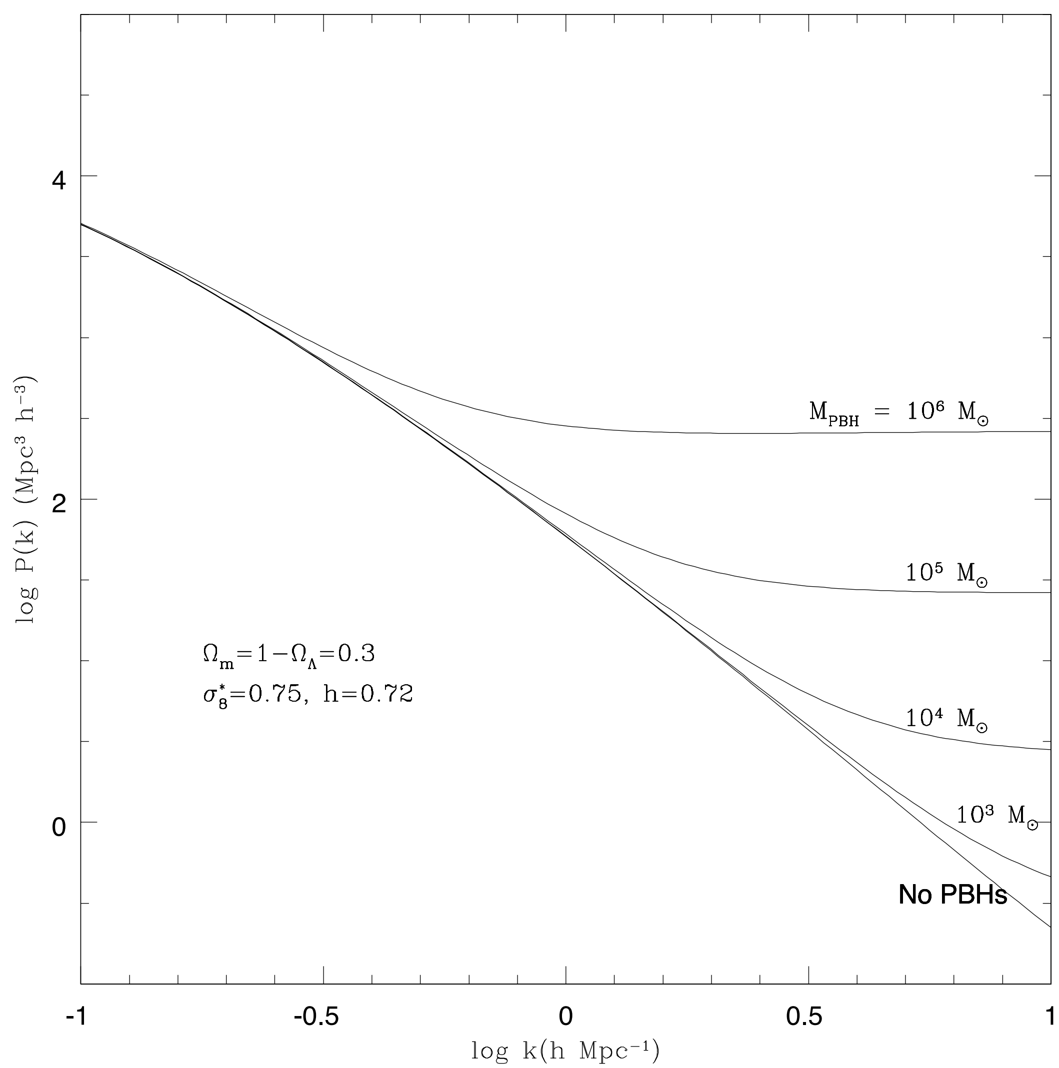
power spectrum

$P(k)$

$(\propto k^{n_s})$



increasing PBH mass

no PBHs

k = comoving wavenumber

Afshordi, Macdonald & Spergel

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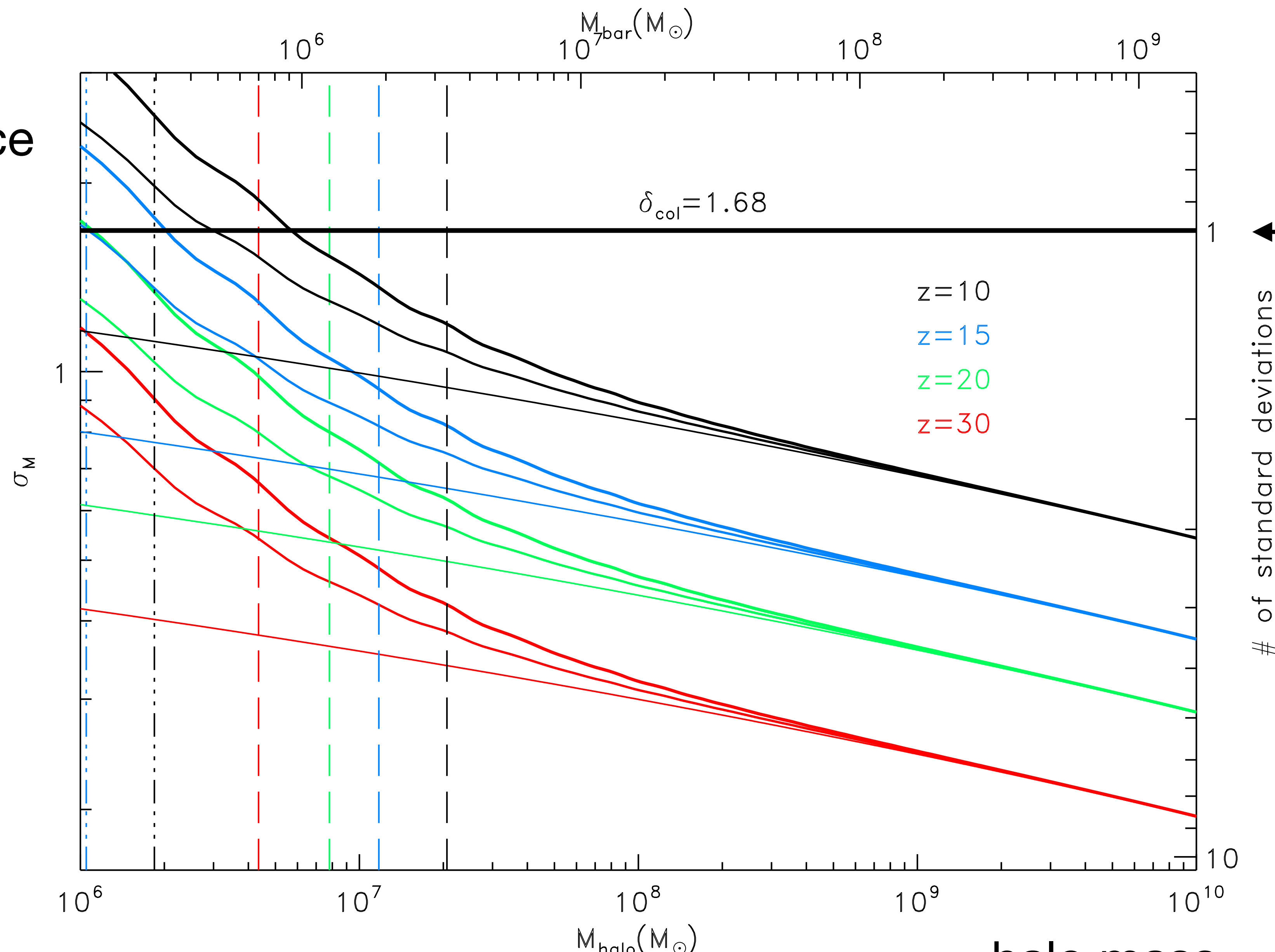
mass variance



threshold for collapse

z= 10, 15, 20, 30

$M_{PBH}=30\ M_\odot$

$M_{PBH}=15\ M_\odot$

no PBHs

halo mass

Kashlinksy

Approximate analytic calculation

c.f. Afshordi, Macdonald & Spergel; Jedamzik

PBH DM has additional isocurvature perturbations due to Poisson fluctuations in their distribution:

$$\delta(N) = \frac{\Delta N}{N} = \frac{1}{\sqrt{N}}$$

growth factor for isocurvature perturbations:

$$D(a) \approx \left(1 + \frac{3}{2}\frac{a}{a_{\rm eq}}\right)$$

spherical top hat collapse:

collapse occurs when:

$$D(a_{\rm col})\delta(N) = \delta_{\rm critical} \approx 1.69 \; ^{*}$$

final halo/cluster density:

$$\rho_{\rm cl} \approx 178 \rho_{\rm DM}(a_{\rm coll})$$

number density of PBHs in cluster:

$$n_{\rm cl} \approx 1.6 \times 10^5 \left(\frac{M_{\rm PBH}}{M_{\odot}}\right)^{-1} N^{-3/2}\,{\rm pc}^{-3}$$

radius of cluster:

$$r_{\rm cl} \approx 0.01 \left(\frac{M_{\rm PBH}}{M_{\odot}}\right)^{1/3} N^{5/6}\,{\rm pc}$$

For $M_{\rm PBH} = M_{\odot}$, N=10 (100) clusters form at $z_{\rm coll}$ ≈1200 (320) and have $r_{\rm cl}$≈ 0.06 (0.5) pc.

* for objects that collapse early in matter domination, when baryons are unclustered, $\delta_{\rm critical}$ is somewhat larger. Inman & Ali-Haïmoud

N-body simulations

Inman & Ali-Haïmoud

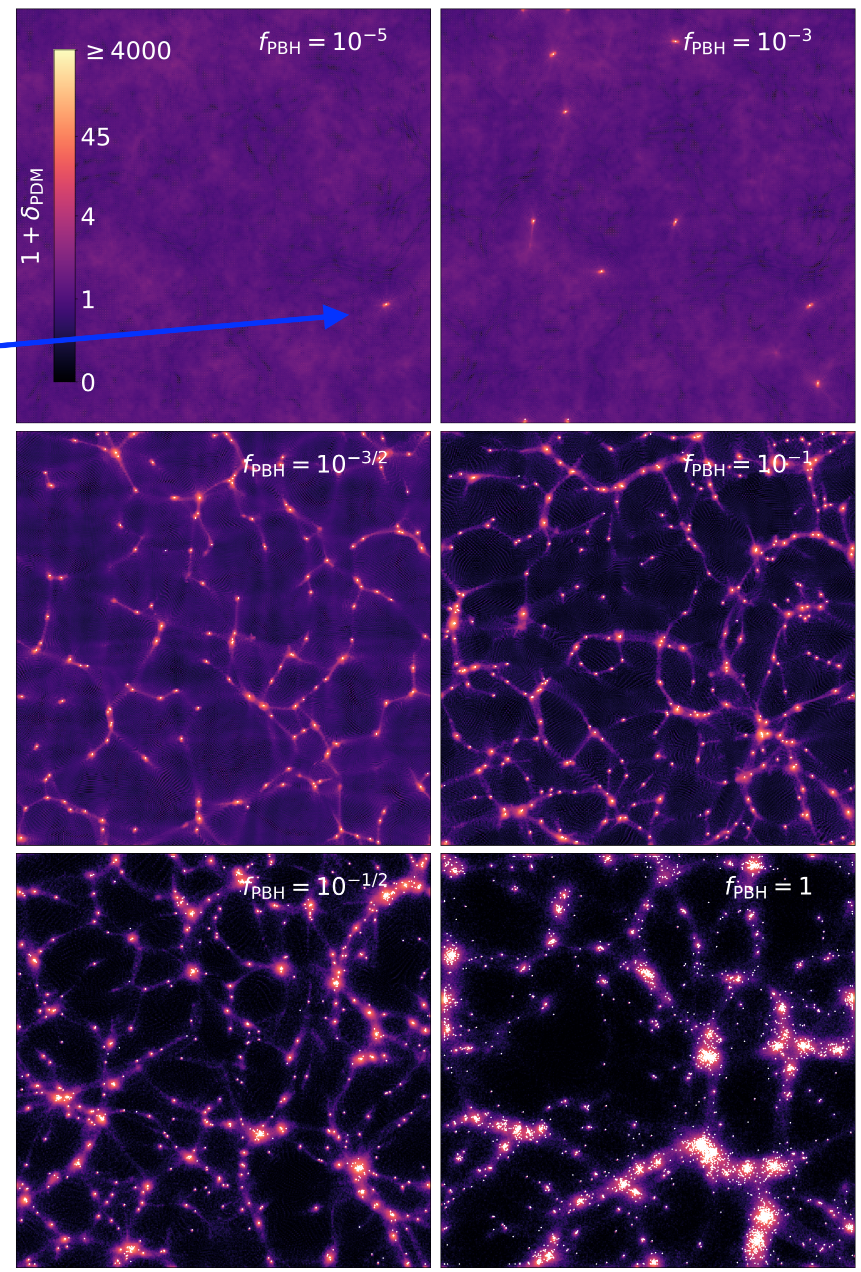
Simulate a L = 30 h^{-1} kpc box, with $M_{\rm PBH} = 20 h^{-1} M_\odot$ from radiation domination to z = 99, for $f_{\rm PBH}$ = 1 and also $f_{\rm PBH}$ < 1 + particle dark matter.

matter field at z=100

$f_{\rm PBH} = 10^{-5}$

$f_{\rm PBH} = 10^{-3}$

a PBH!



$f_{\rm PBH} = 10^{-3/2}$

$f_{\rm PBH} = 10^{-1}$

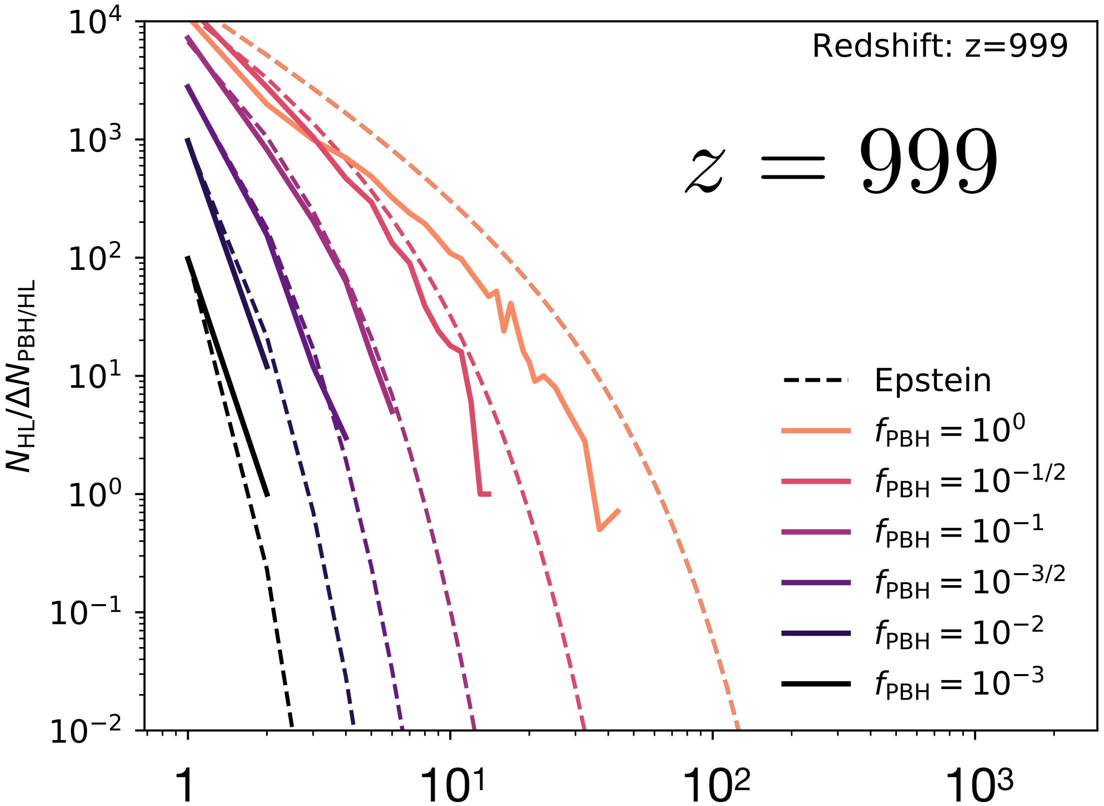
$f_{\rm PBH} = 10^{-1/2}$

$f_{\rm PBH} = 1$

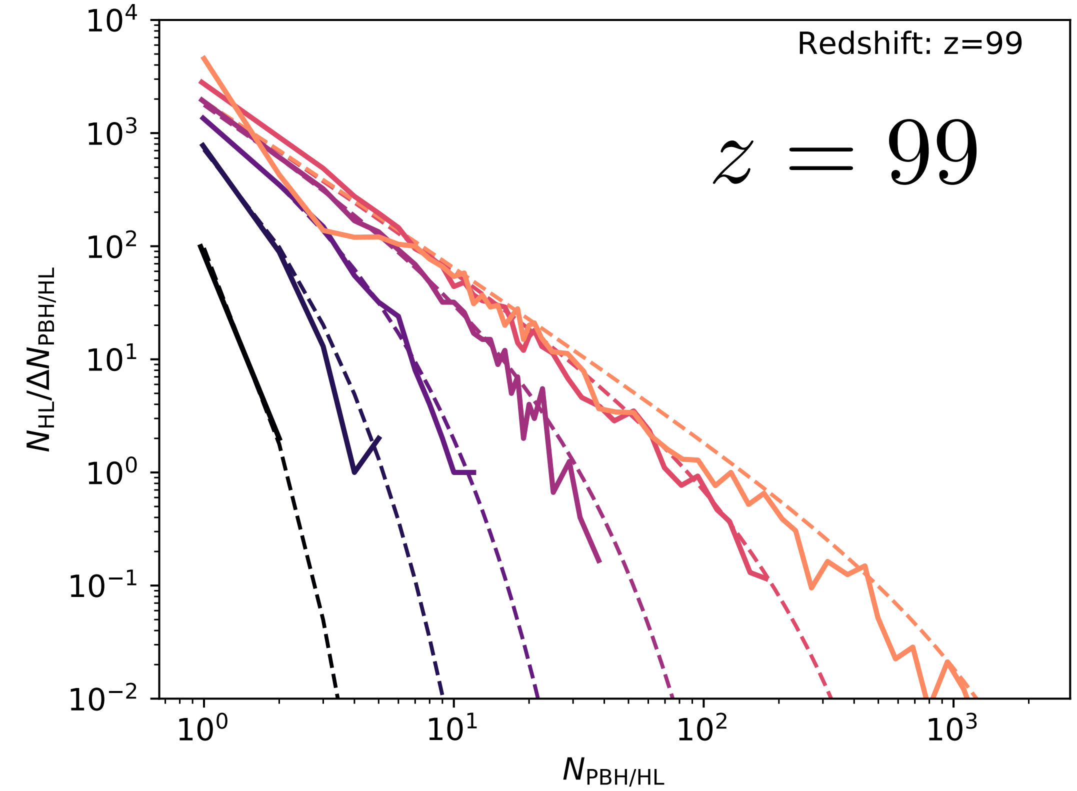
Inman & Ali-Haïmoud

halo mass function (number of halos containing a given number of PBHs)

Inman & Ali-Haïmoud



decreasing f_{PBH}



for initially Poisson distributed objects Epstein

$$N_{HL}(N) \approx \frac{\delta_\star}{\sqrt{2\pi}} \frac{N_{PBH}}{N^{3/2}} \exp\left(-N/N_\star\right)$$

N_{PBH} = total number of PBHs = $10^5 f_{PBH}$ for these simulations

$$N_\star \equiv \left[\log\left(1+\delta_\star\right) - \frac{\delta_\star}{1+\delta_\star}\right]^{-1}$$

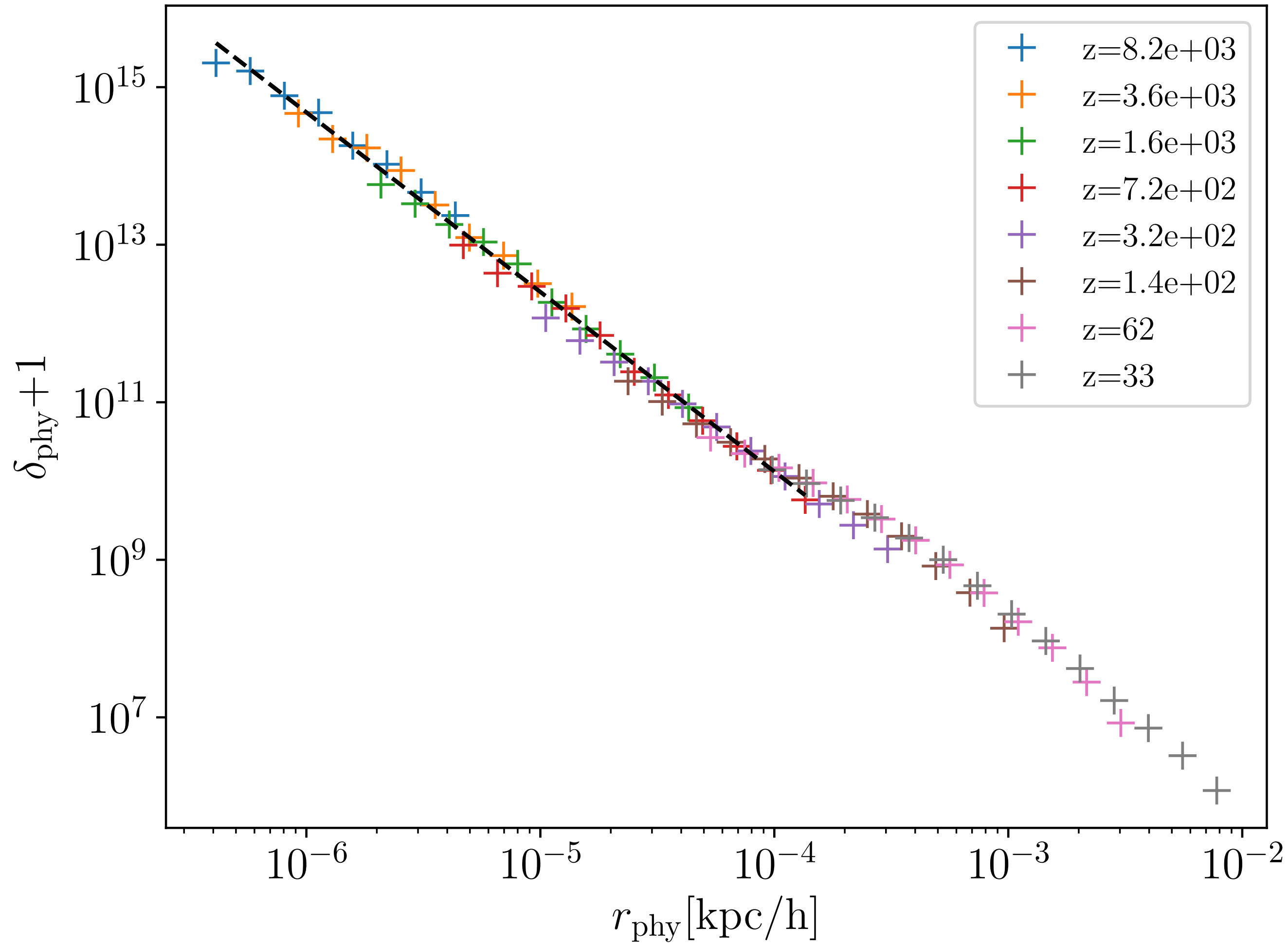
$$\delta_\star(a) = \frac{\delta_{critical}(a)}{D(a) f_{PBH}}$$ minimum initial PBH density $$\delta_\star \approx \frac{0.43\,(0.05)}{f_{PBH}} \quad \text{at} \quad z = 999\,(99)$$

mixed PBH-particle dark matter

If PBHs don't make up all of the DM ($0 < f_{\rm PBH} < 1$) then isolated PBHs accrete a halo of particle DM with a steep density profile: $\rho(r) \propto r^{-9/4}$

Mack, Ostriker & Ricotti; Adamek et al.; Inman & Ali-Haïmoud

Density profile, in physical units, formed around a $30 M_{\odot}$ PBH



Adamek et al

If the DM were a mixture of PBHs and WIMPs would get large flux of gamma-rays (and neutrinos and positrons) from WIMP annihilation in halos around PBHs: all of the DM being a mixture of WIMPs and PBHs is excluded. Lacki & Beacom

If $f_{WIMP} \sim 1$ then $f_{PBH} \lesssim 10^{-9}$.

If $f_{PBH} \sim 10^{-3}$ (if LIGO-Virgo events are PBH binary mergers) then $f_{WIMP} \lesssim 10^{-6}$.

Adamek, Byrnes, Gosenca, Hotchkiss

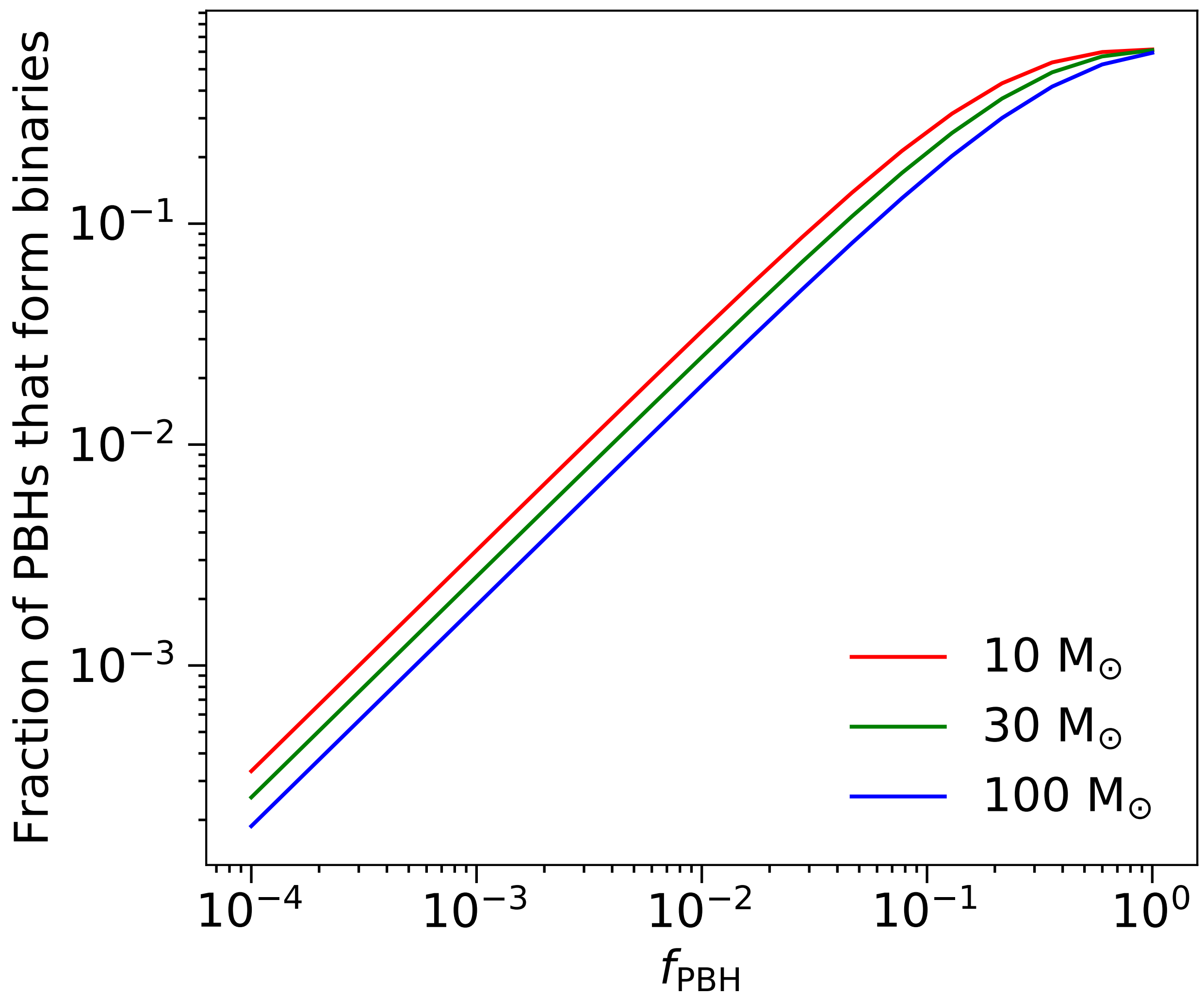
Questions?

Formation and evolution of PBH binaries

Nakamura et al.; Ioka et al; Sasaki et al.; Ali-Haïmoud, Kovetz & Kamionkowski

Two PBHs that happen to be close together can decouple from the expansion of the Universe before matter-radiation equality and form a (highly eccentric) binary (tidal forces from other PBHs prevent a head on collision).

Fraction of PBHs that form binaries, as function of f_{PBH}

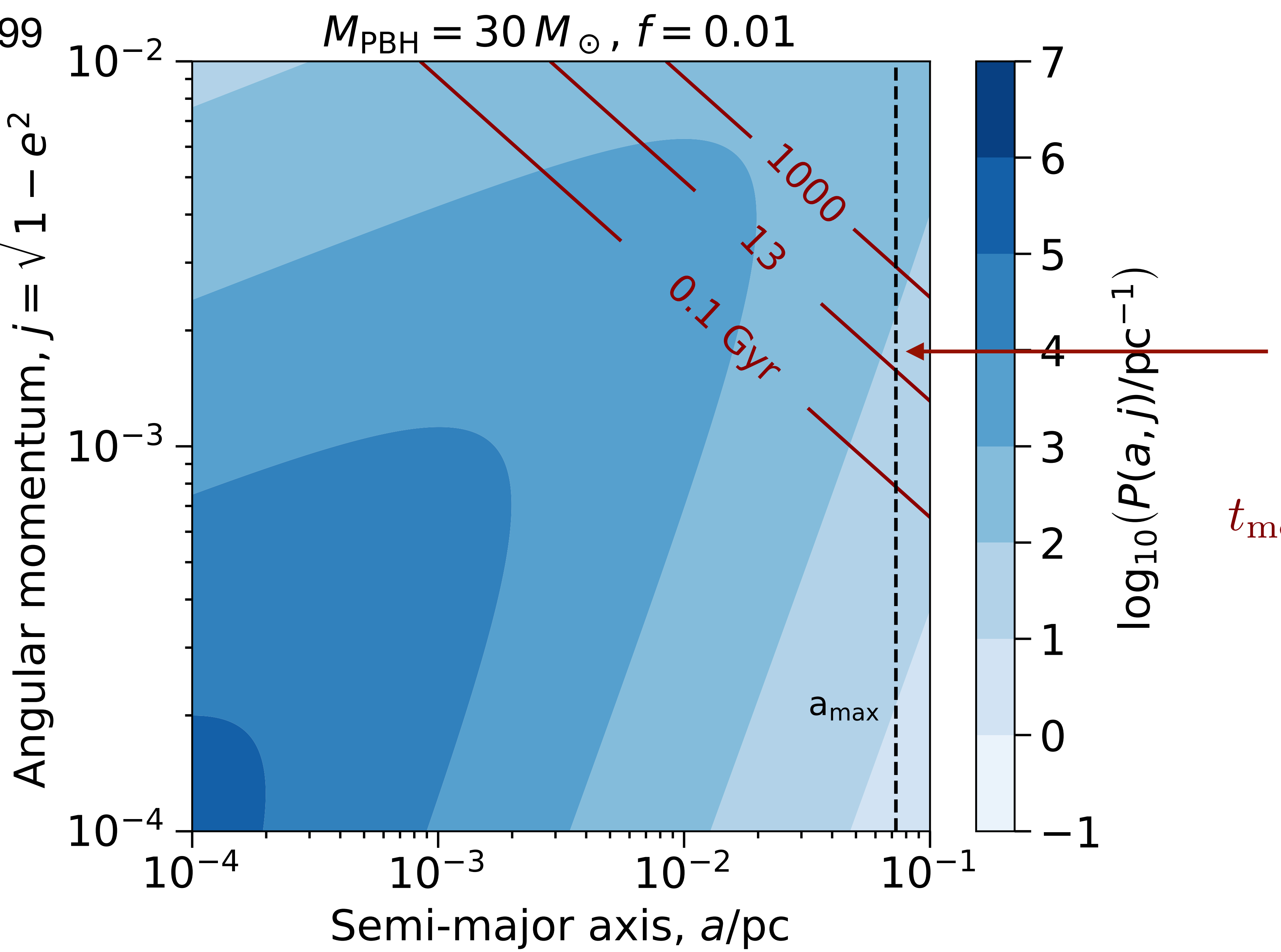


Kavanagh, Gaggero & Bertone

Probability distribution of PBH binaries semi-major axis and angular momentum

Ali-Haïmoud, Kovetz & Kamionkowski

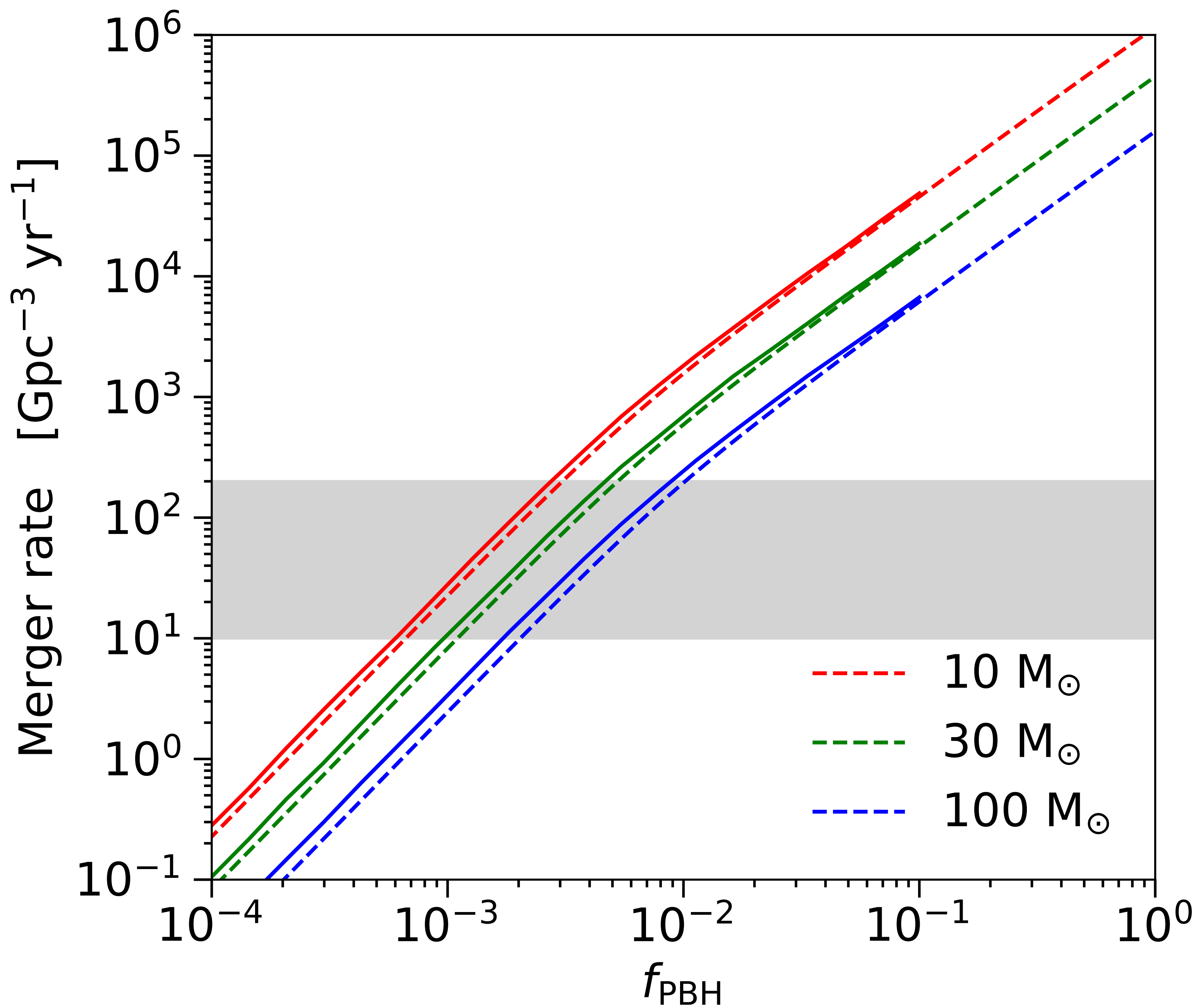
e=0.9999



$$t_{\rm merge} = \frac{3c^5}{170G^3}\frac{a^4 j^7}{M_{\rm PBH}^3}$$

Kavanagh, Gaggero & Bertone

PBH merger rate, $\mathcal{R}$, averaged z=0 to 1



LIGO-Virgo merger rate 2017

Kavanagh, Gaggero & Bertone

dashed lines: taking into account particle DM halos

Bird et al.

PBH binaries can also form in present day halos (if 2 PBHs pass close enough they can radiate enough energy in GWs to become bound).

These binaries are very tight and eccentric and have merger timescales much smaller than the Hubble time and $\mathcal{R} \sim 1 \, \mathrm{Gpc}^{-3} \mathrm{yr}^{-1}$(if $f_{\rm PBH}$=1).

Much smaller than merger rate from early forming binaries (**if** early forming binaries aren't perturbed significantly before the present day). Ali-Haïmoud, Kovetz & Kamionkowski

How do PBH-binaries evolve?

Do their semi-major axes and eccentricities (and hence merger rate) get perturbed significantly?

Effects of other PBHs and particle dark matter (if $f_{PBH} \neq 1$) are expected to be small (since binaries form very early and are tight). e.g. Ali-Haïmoud, Kovetz & Kamionkowski, Ali-Haïmoud, Kovetz & Kamionkowski

However 3-body interactions within PBH clusters could have a significant effect, on the eccentricities, and hence merger rates.
Vaskonen & Veermae; Jedamzik

Evolution of PBH clusters (and the binaries they contain) up to the present day is a (difficult) open question.
Trashorras, Garcia-Bellido & Nesseris; Tkachev, Pilipenko & Yepes

Mini-problem

Verify how the PBH cluster

- number density (n_{cl})
- radius (r_{cl})

from the spherical top-hat model scale with

- number of PBHs in the cluster (N)
- PBH mass (M_{PBH})

Approximate analytic calculation

c.f. Afshordi, Macdonald & Spergel; Jedamzik

PBH DM has additional isocurvature perturbations due to Poisson fluctuations in their distribution: $\delta(N) = \frac{\Delta N}{N} = \frac{1}{\sqrt{N}}$

growth factor for isocurvature perturbations: $D(a) \approx \left(1 + \frac{3}{2}\frac{a}{a_{\rm eq}}\right)$

spherical top hat collapse:

collapse occurs when: $D(a_{\rm col})\delta(N) = \delta_{\rm critical} \approx 1.69$ *

final halo/cluster density: $\rho_{\rm cl} \approx 178 \rho_{\rm DM}(a_{\rm coll})$

number density of PBHs in cluster: $n_{\rm cl} \approx 1.6 \times 10^5 \left(\frac{M_{\rm PBH}}{M_\odot}\right)^{-1} N^{-3/2}\,{\rm pc}^{-3}$

radius of cluster: $r_{\rm cl} \approx 0.01 \left(\frac{M_{\rm PBH}}{M_\odot}\right)^{1/3} N^{5/6}\,{\rm pc}$

Summary

- Inflation models that can produce large, PBH-forming, perturbations:

  single field, with potential fine tuned to have a local minimum
  hybrid inflation with a mild waterfall transition
  double inflation
  axion curvaton
  .
  .
  .

- PBH clusters form not long after matter-radiation equality, with more massive clusters forming at later times.

- If DM is a mixture of PBHs and particle DM, particle DM halos form around PBHs. And mixed PBH-WIMP DM is essentially already ruled out (products of WIMP annihilation in halos around PBHs would already have been observed).

- If a significant fraction of the DM is in PBH, binaries form before matter-radiation equality and (provided their orbits aren't significantly perturbed subsequently), they merge, producing gravitational waves, at an observable rate today.

Next time:

- observational constraints on PBH dark matter

Back-up slides