Precision calculations for the LHC

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UK Research and Innovation

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Ask me anything!

Me: Jonas Lindert

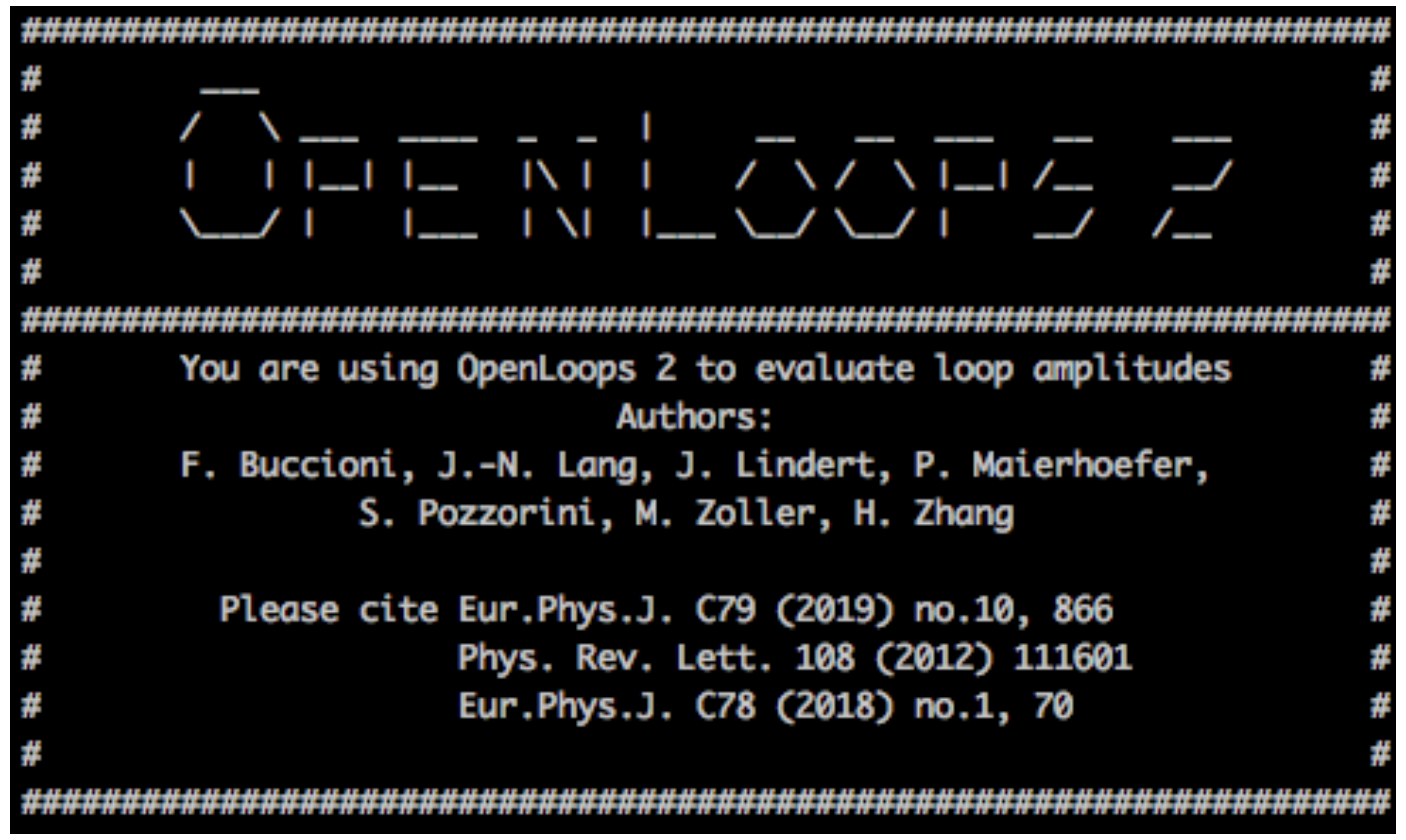
Steps into HEP-PH:

- Undergrad at RWTH Aachen, Germany (MSc in 2010).
  MSc thesis on: *SUSY Parameter Determination at the LHC*
- PhD at Max-Planck Institute for Physics, Munich in 2013
  PhD thesis: *Aspects of SUSY phenomenology at the LHC*
- PostDoc at University of Zurich, Switzerland 2013-2016
- PostDoc at Durham University 2016-2019
- Since 2019 STFC Ernest Rutherford Fellow at Sussex

BSM

↓

SM precision



EW physics   Higgs physics

SM backgrounds   Top physics

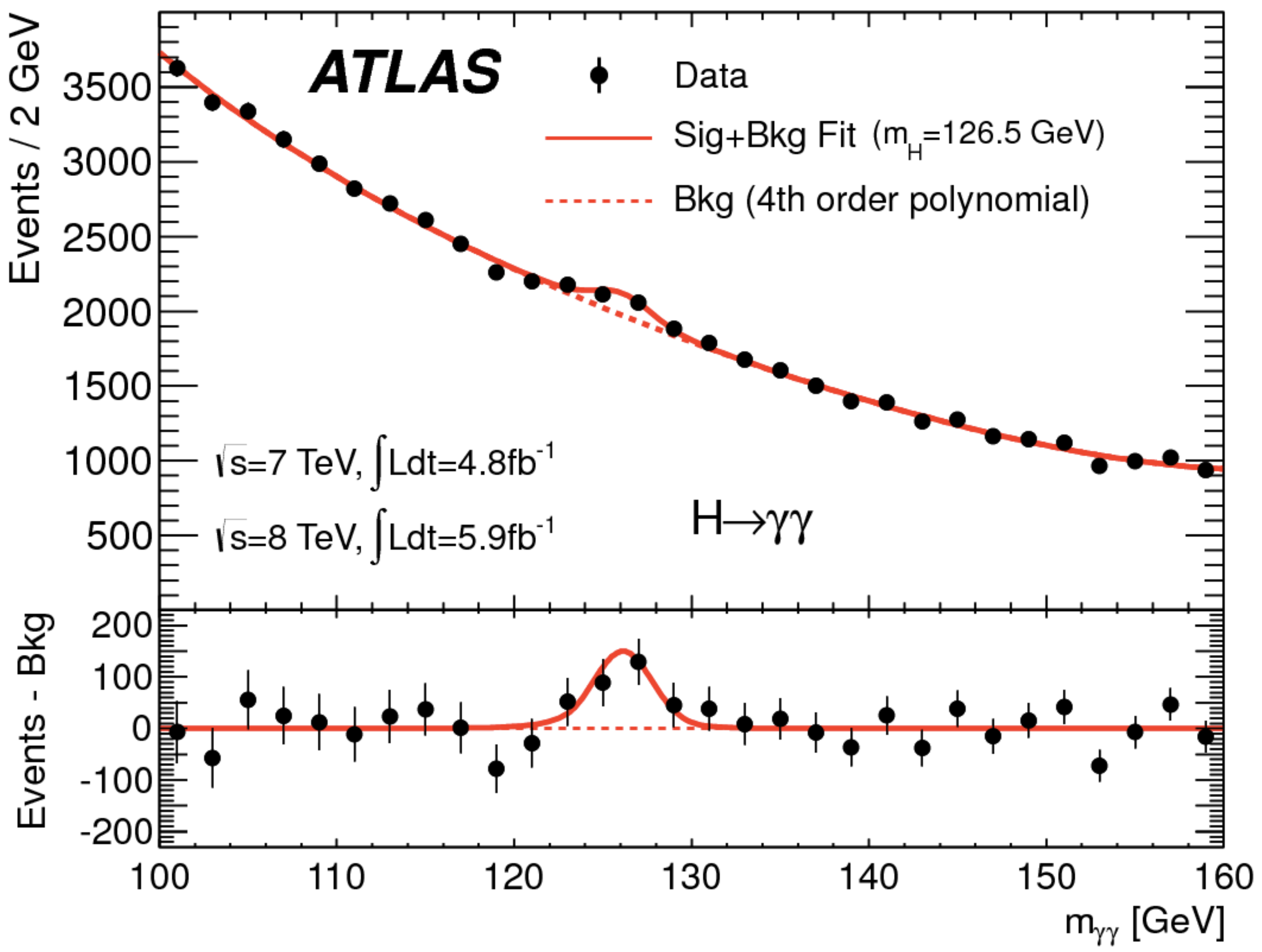
OpenLoops

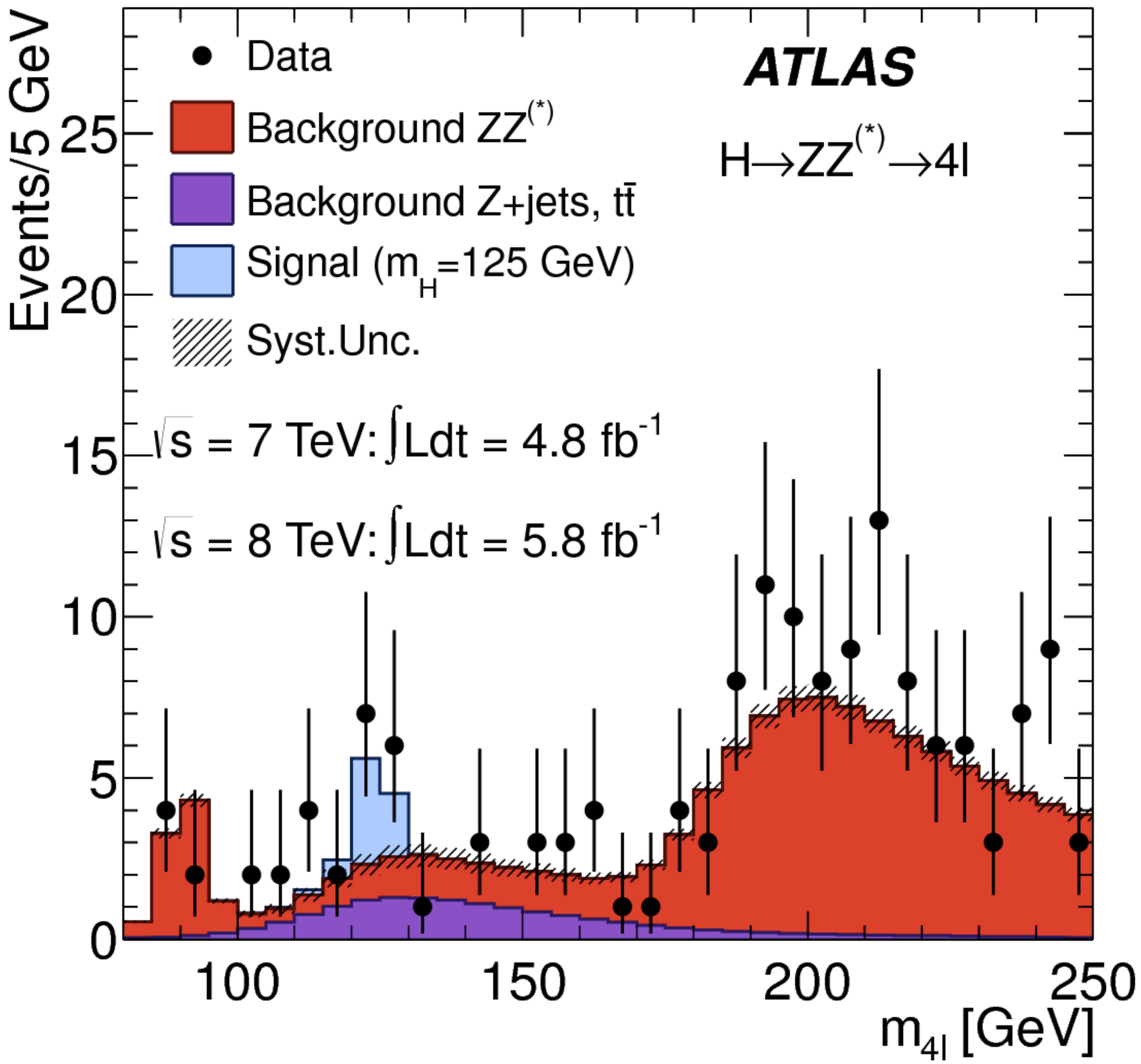
LHC

ILC   NLO QCD+EW   NNLO

EFT   NLO-PS

From a pheno perspective finding the Higgs was "easy"…





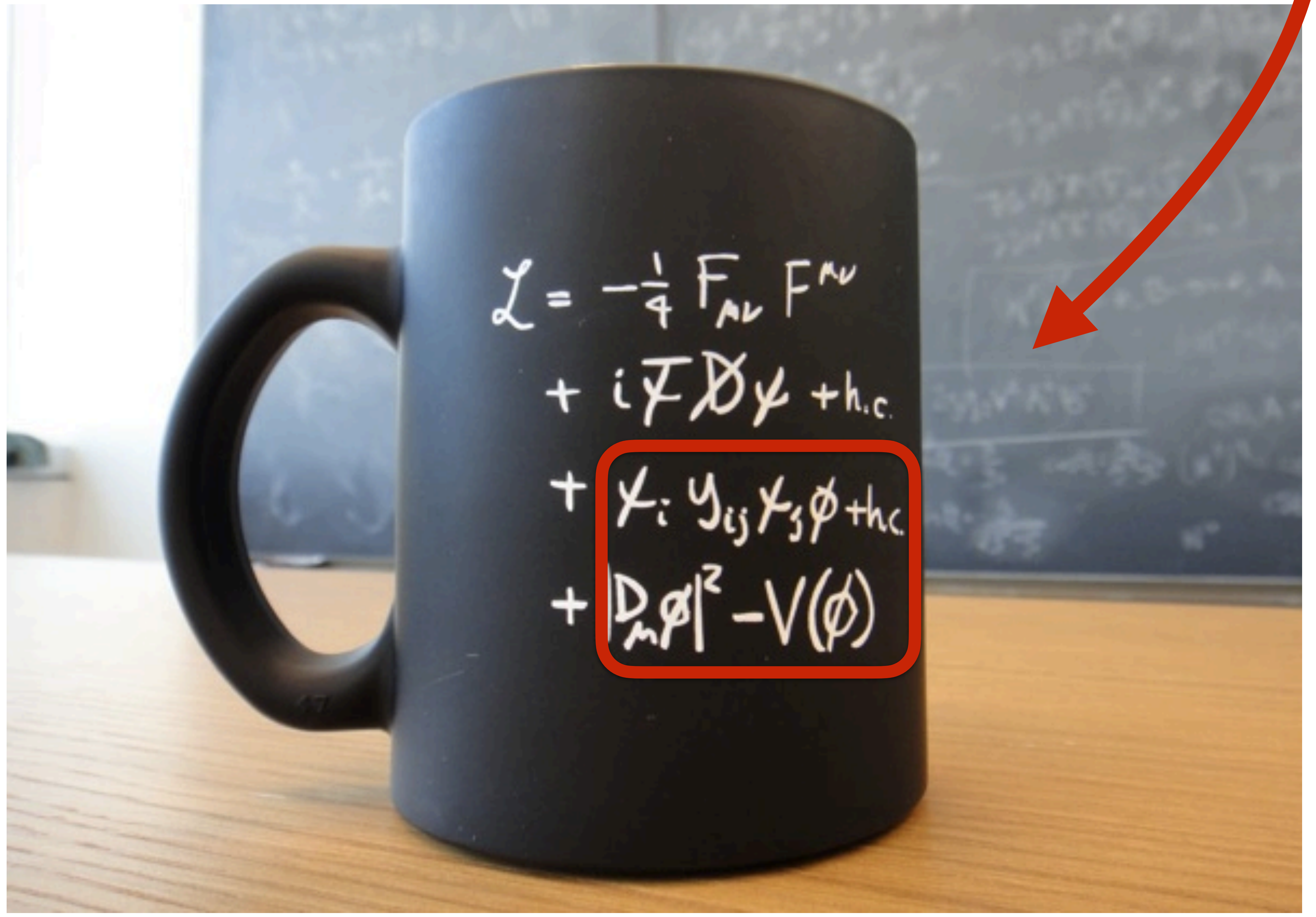
- Higgs at 125 GeV allowed for very clean discovery in **γγ** & 4l channels
- Bump hunting: little to no theoretical input needed.

…understanding the Higgs and its properties is tough!

Is the S(125 GeV) really the SM Higgs?

- CP properties? Is there a small CP-odd admixture?
- Precise couplings with vector-bosons/fermions as in SM?
- what is the Higgs width? Is there a significant invisible decay?
- only one Higgs doublet?
- what is the Higgs potential? self-coupling?

➡ precision is key!



The motivation for BSM searches are as compelling as ever

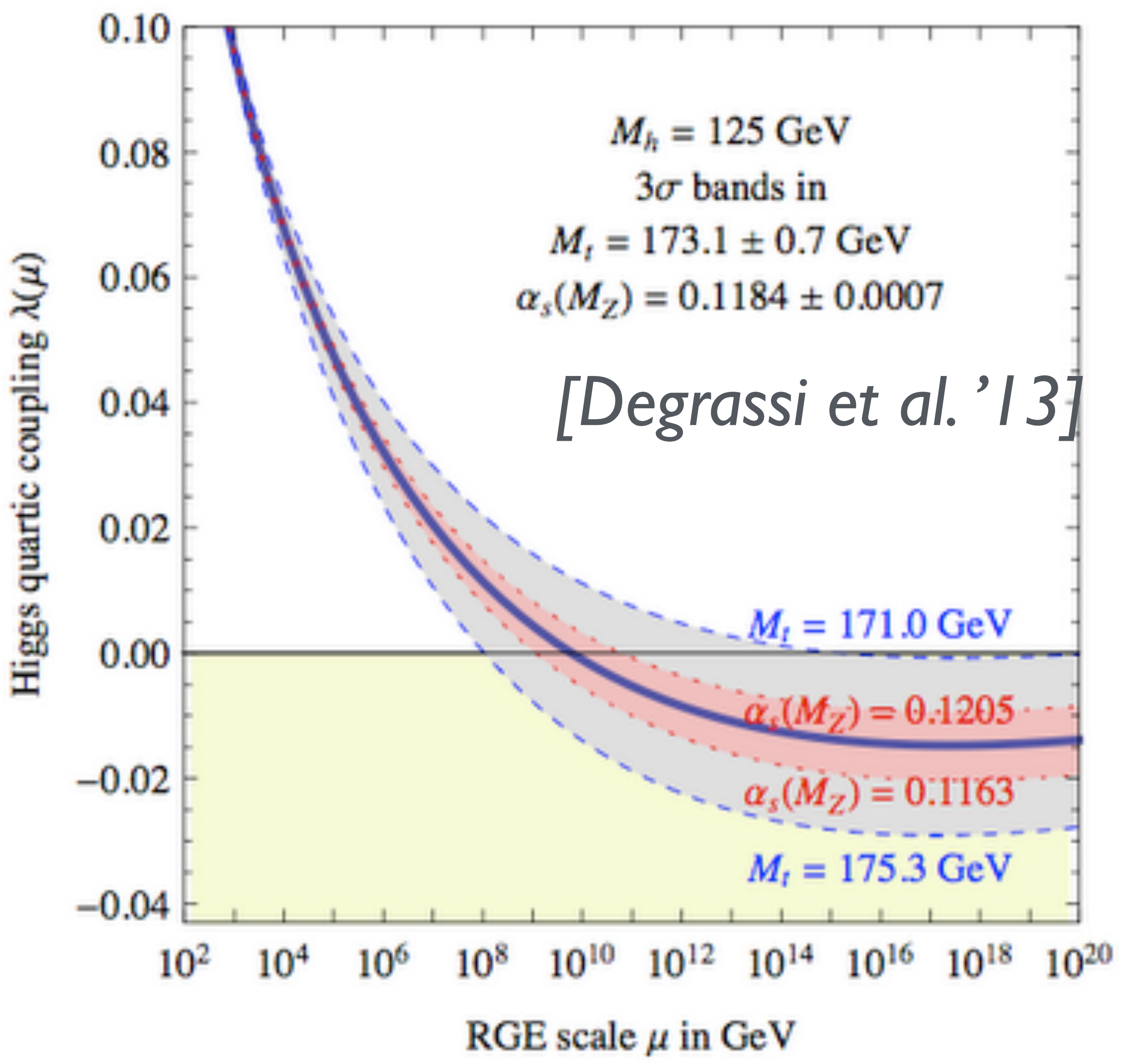
EW vacuum stability

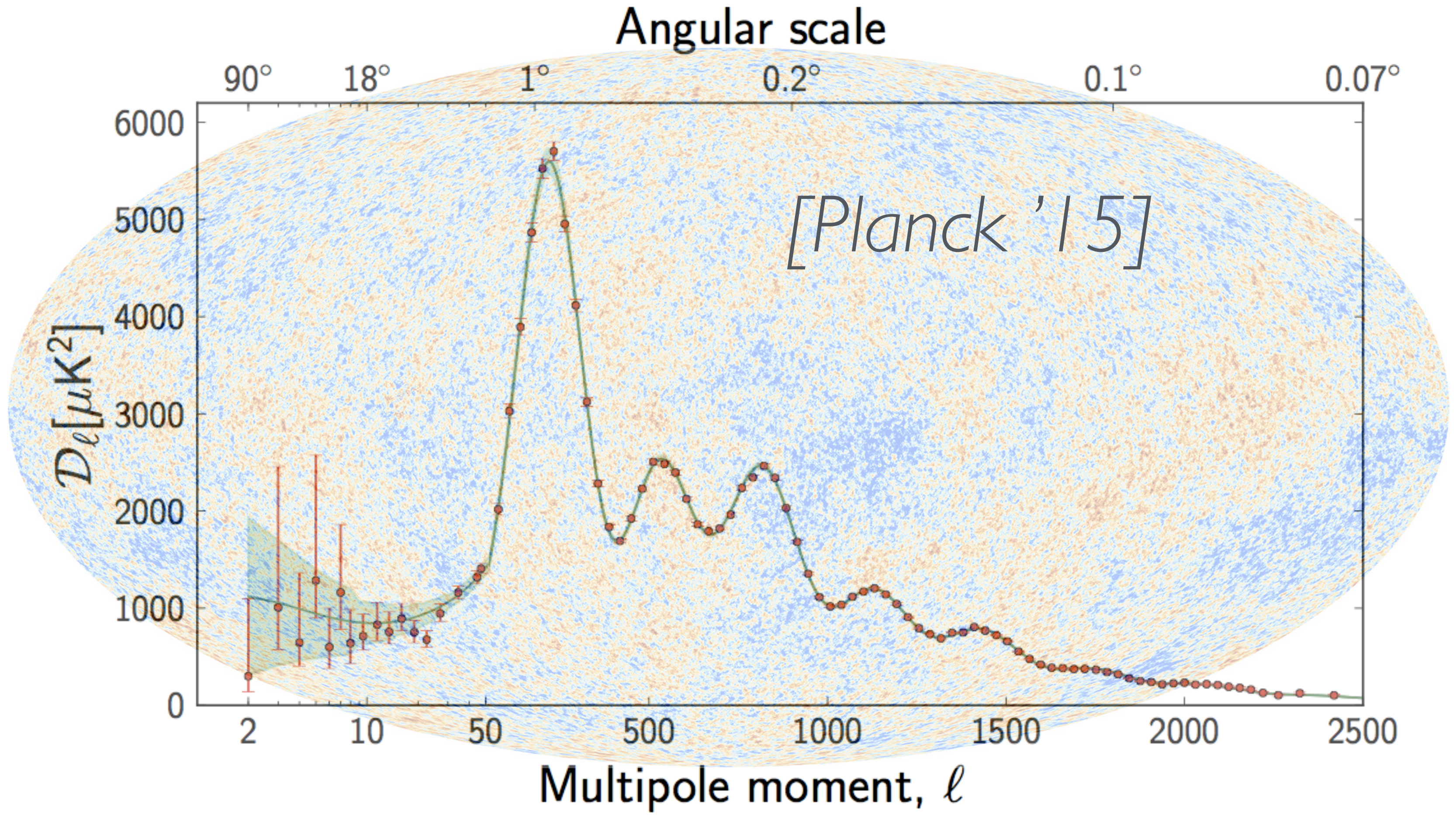
Dark Matter

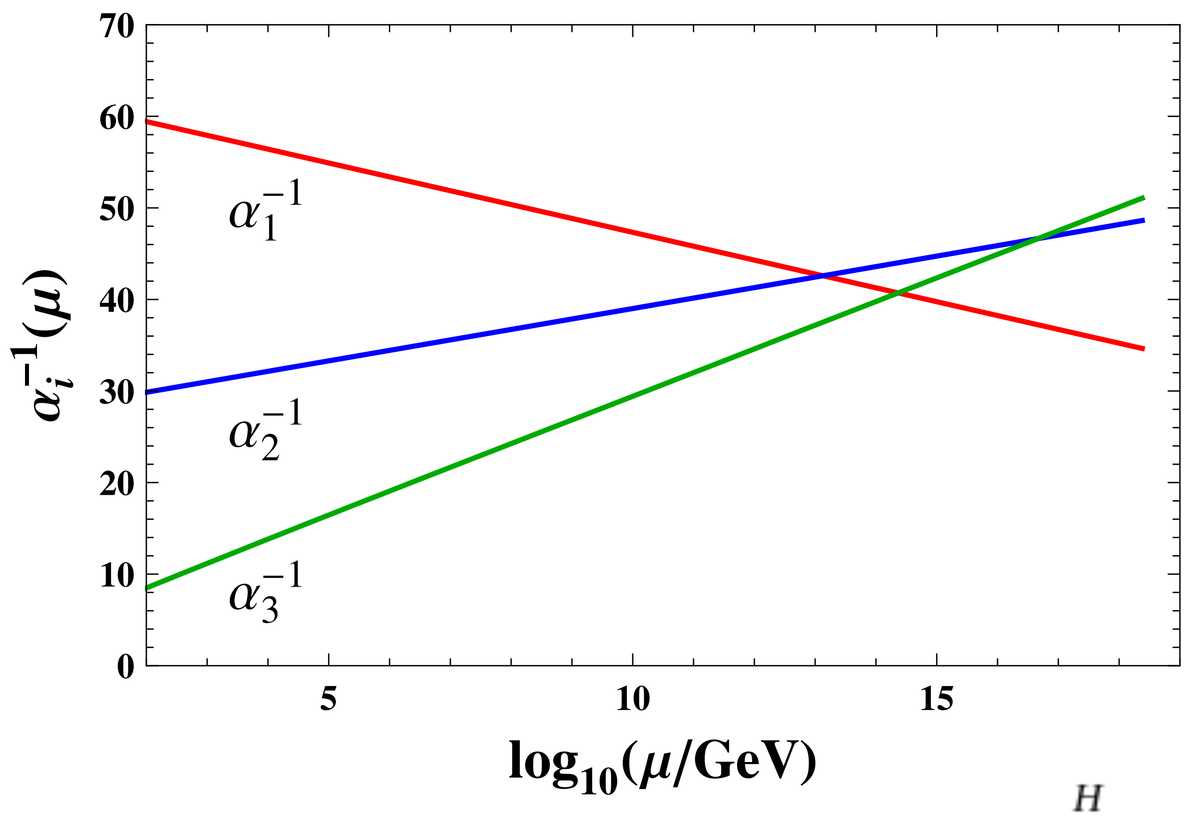
GUT unification

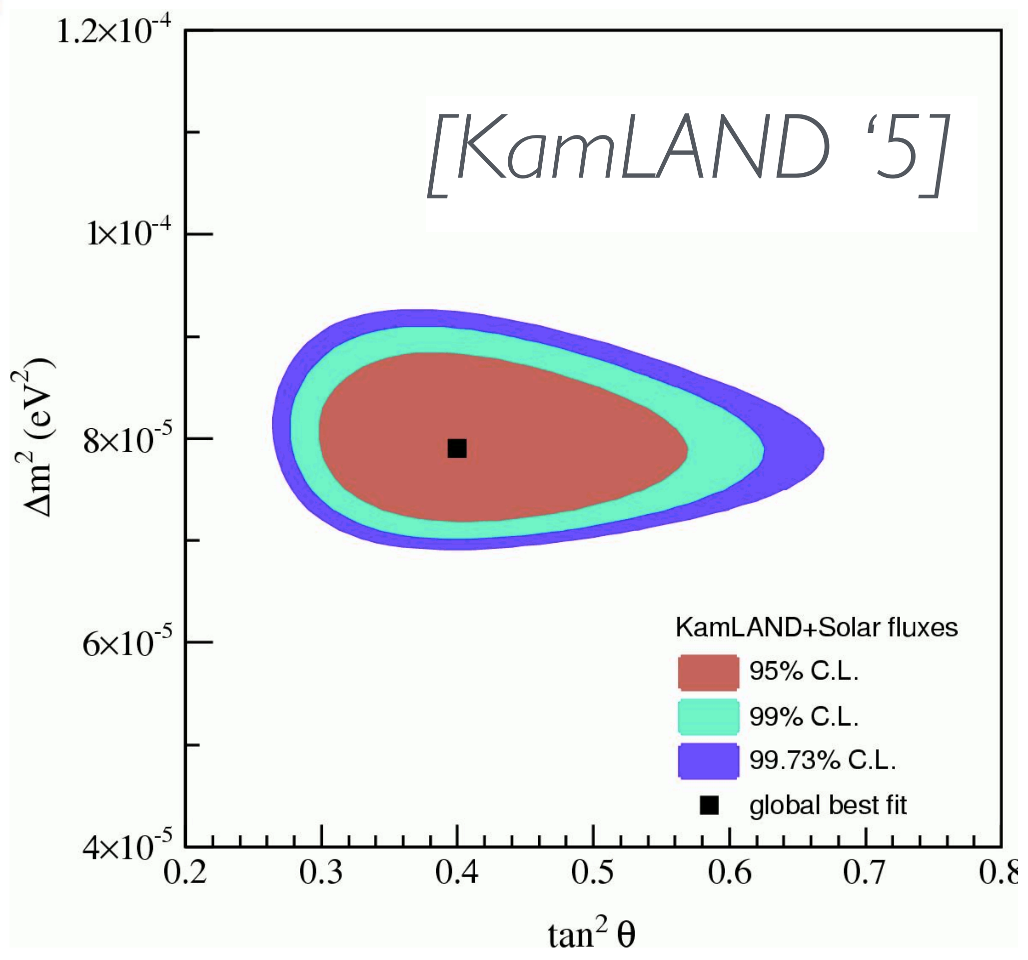
Neutrino masses

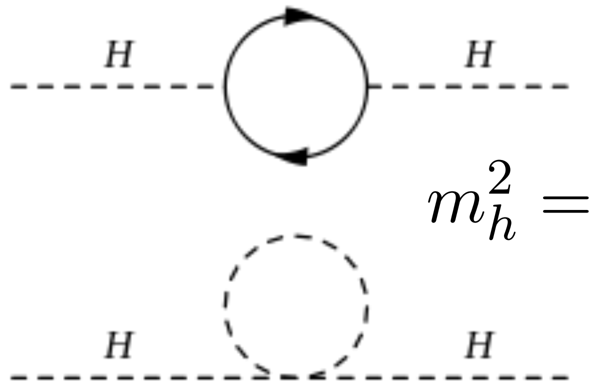
Hierarchy problem







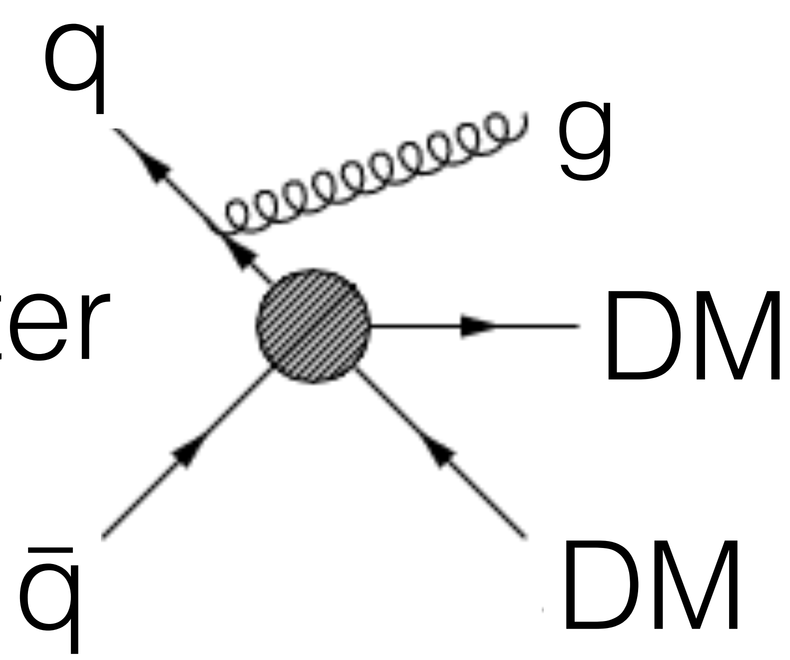




$$m_h^2 = (m_h^0)^2 + \frac{3\Lambda_{\rm UV}^2}{8\pi v^2}(m_h^2 + 2m_W^2 + m_Z^2 - 4m_t^2)$$

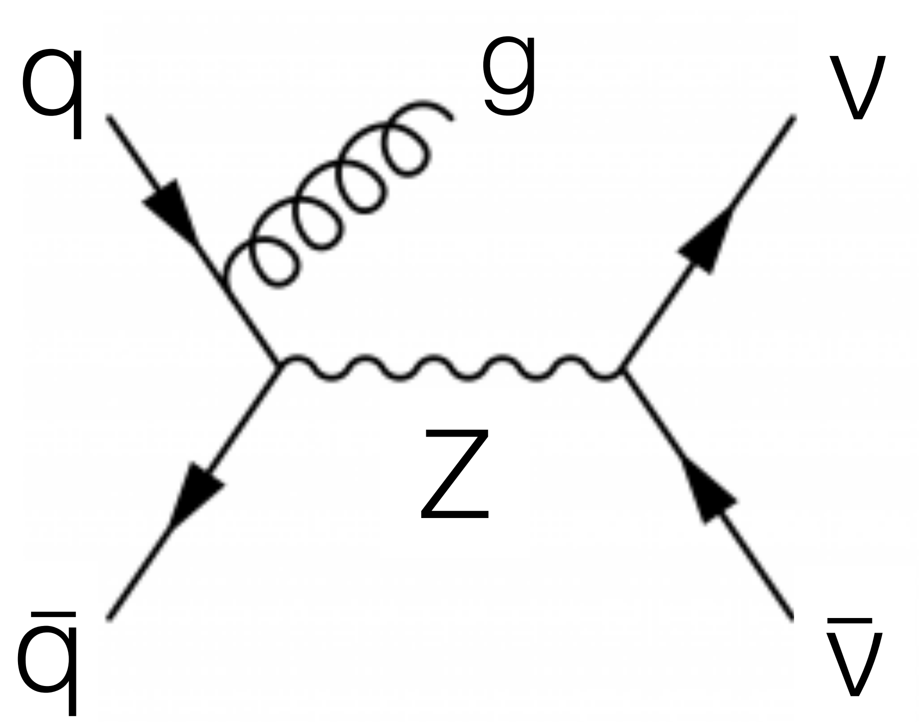
Direct searches for new physics: overwhelming SM backgrounds

e.g.
Dark Matter

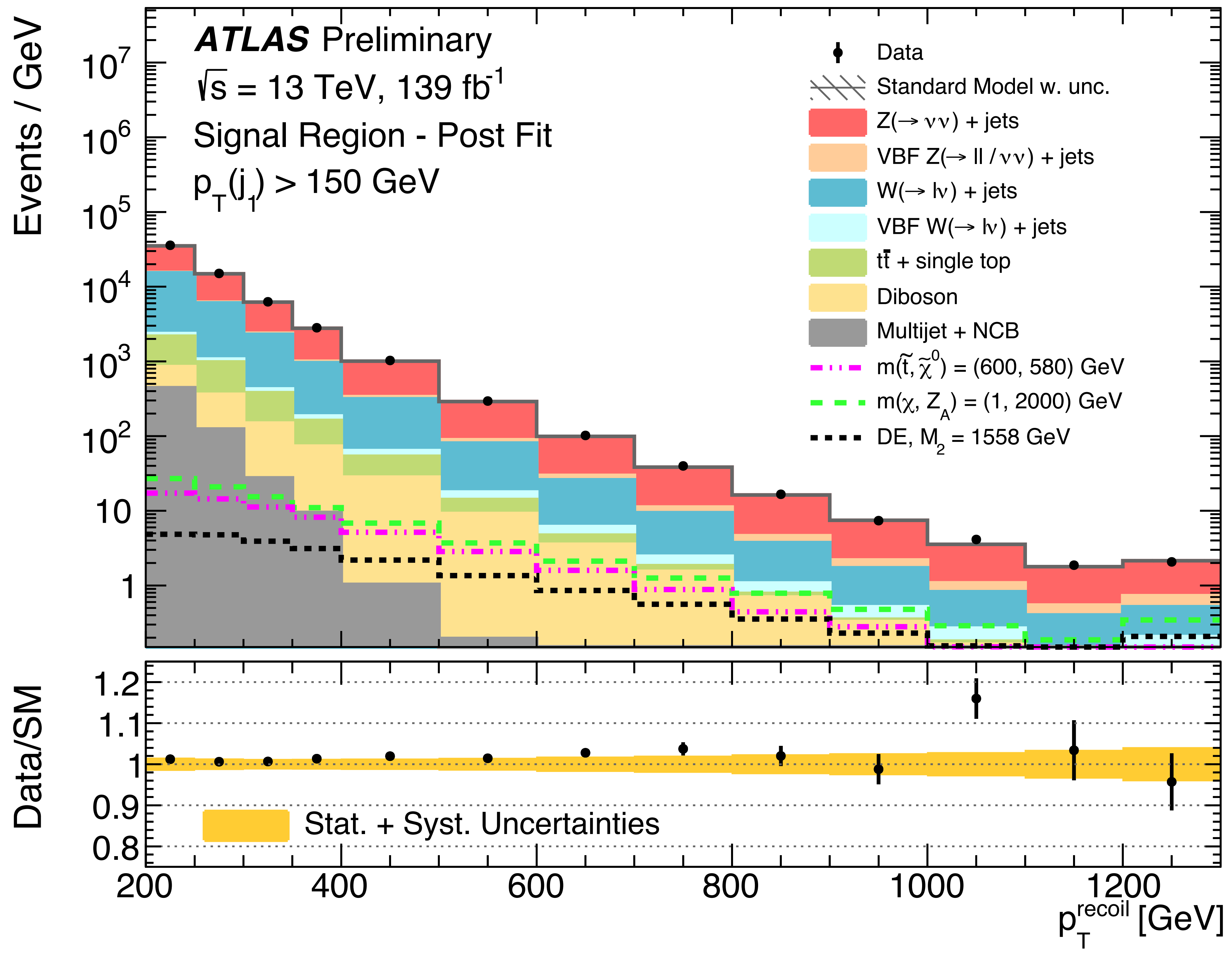


invisible in
detectors

vs.

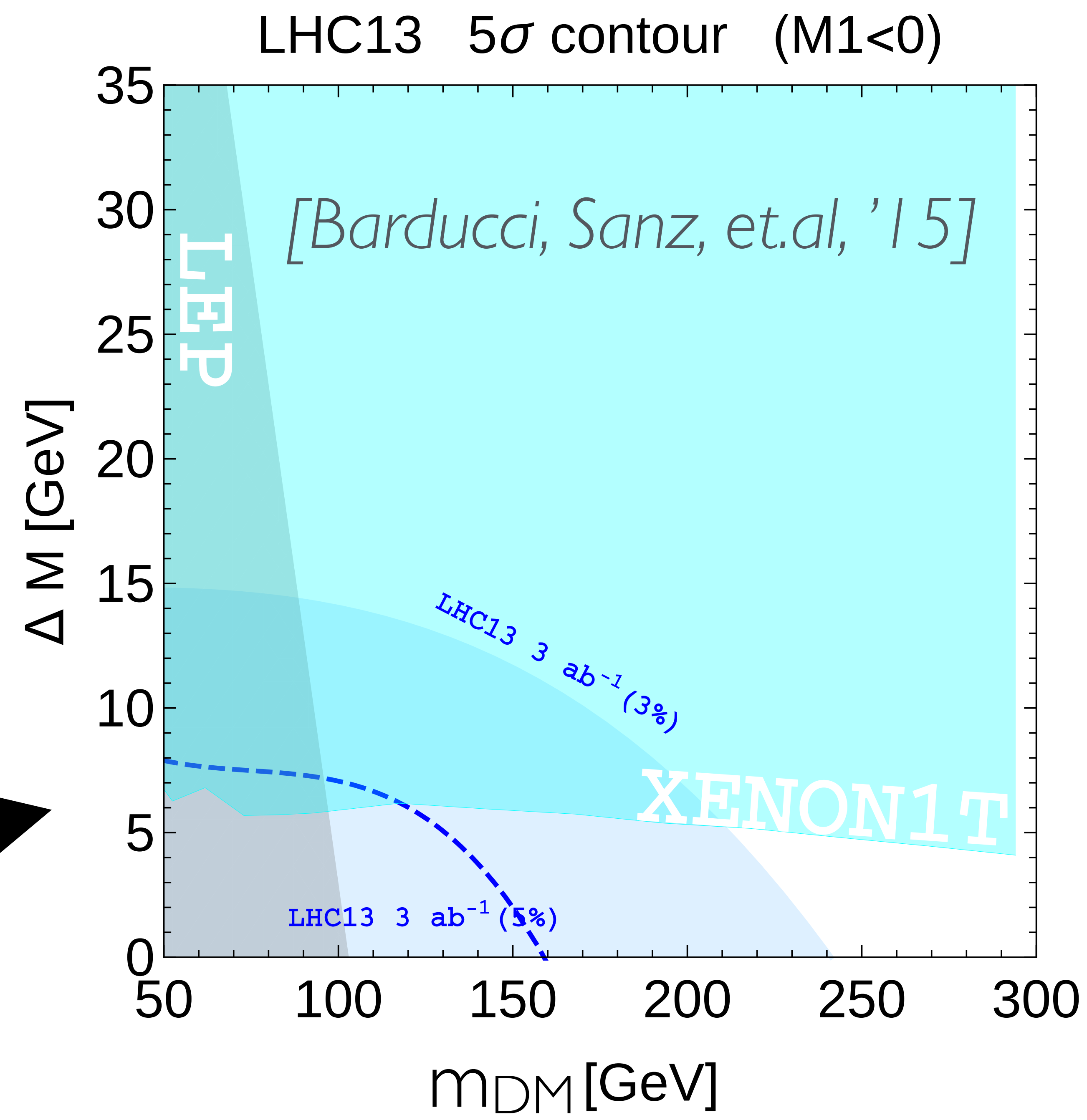


invisible in
detectors



Thanks to state-of-the-art
theory predictions+uncertainties
for SM backgrounds
[JML, et.al., '17]

few percent!

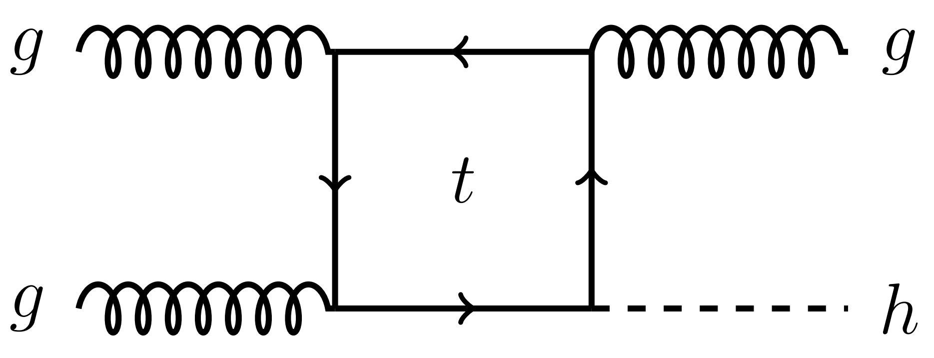


→Theory precision is key to harness full potential of LHC data!

Indirect searches: disentangling very small effects

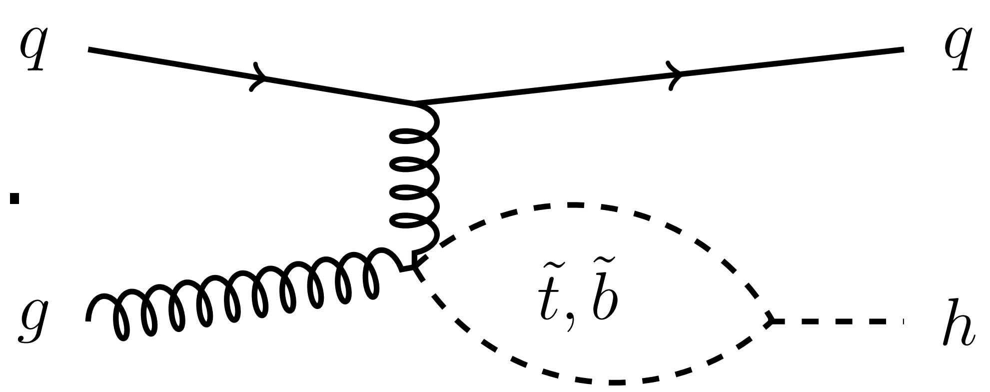
e.g.

SM



vs.

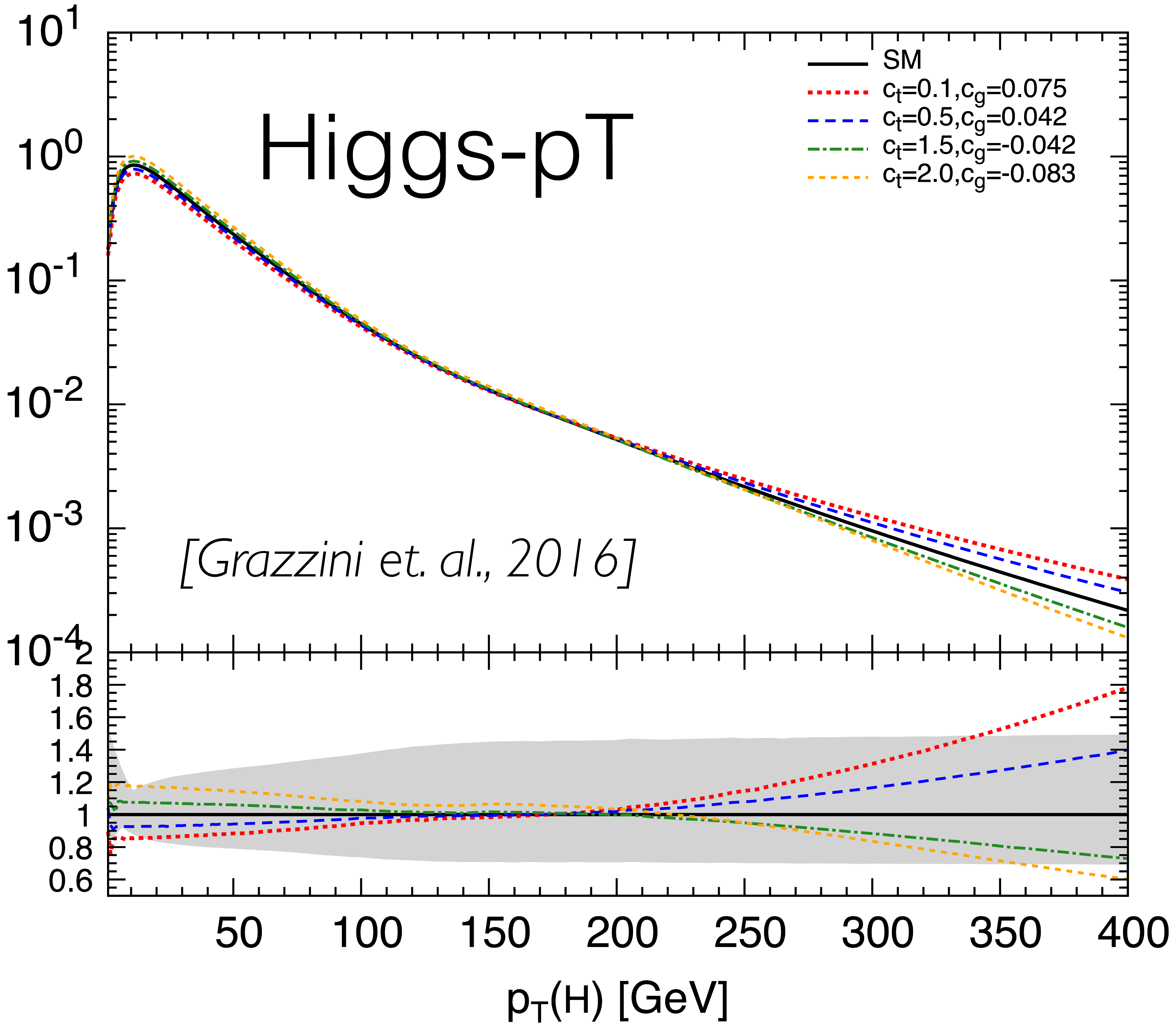
New physics



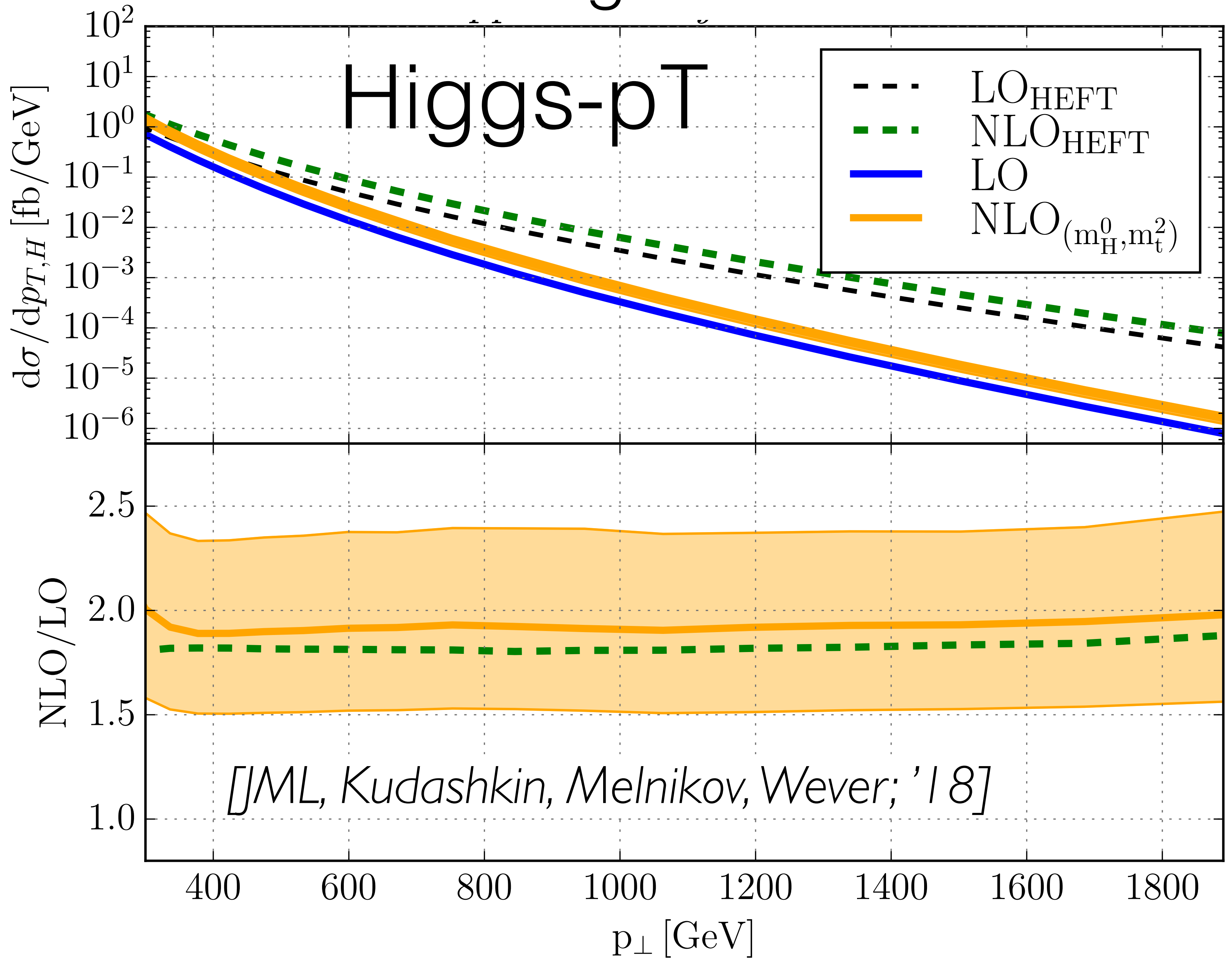
Look for BSM effects in small deviations from SM predictions:

→ Higgs processes natural place to look at

→ **very good control on theory necessary!**



next-to-leading order corrections



→Theory precision opens the door to new analysis strategies!

Perturbative expansion for diboson production

$$d\sigma = d\sigma_{\rm LO} + \alpha_S\, d\sigma_{\rm NLO}$$

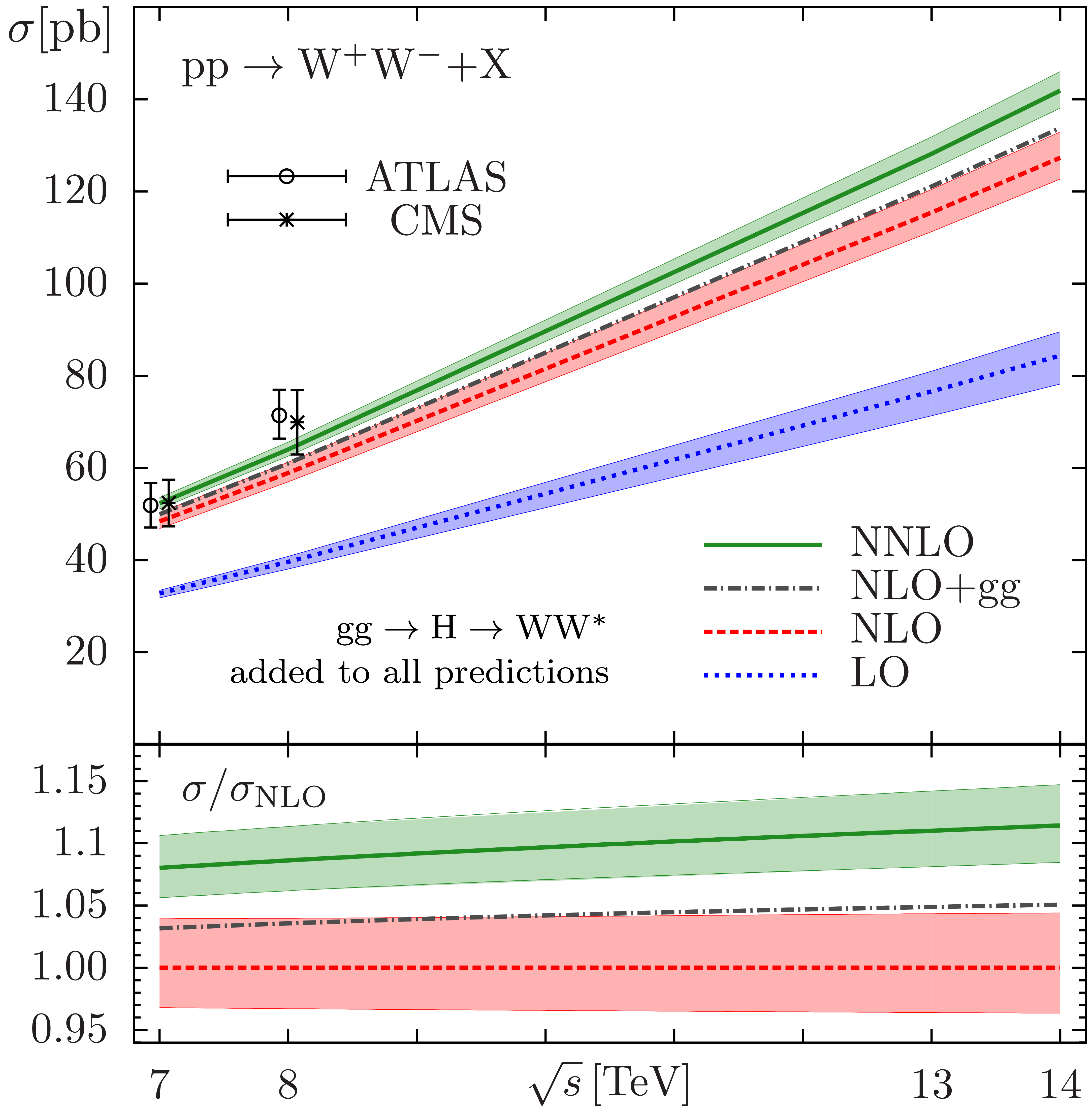
NLO QCD O(100%)

$$+\alpha_S^2\, d\sigma_{\rm NNLO}$$

NNLO QCD O(10%)

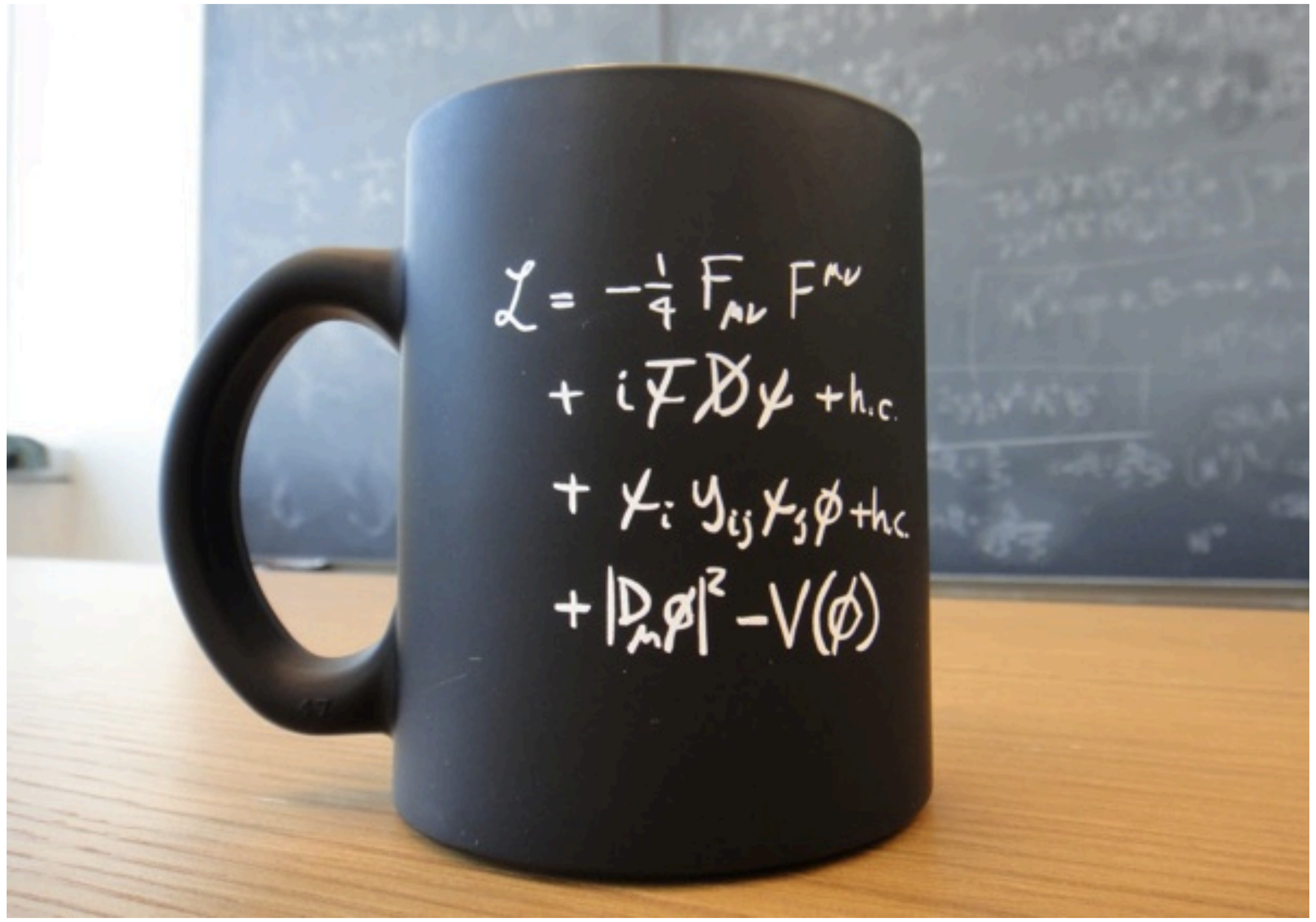
$$+\alpha_S^3\, d\sigma_{\rm NNLO} + \ldots$$

N3LO QCD O(1%)



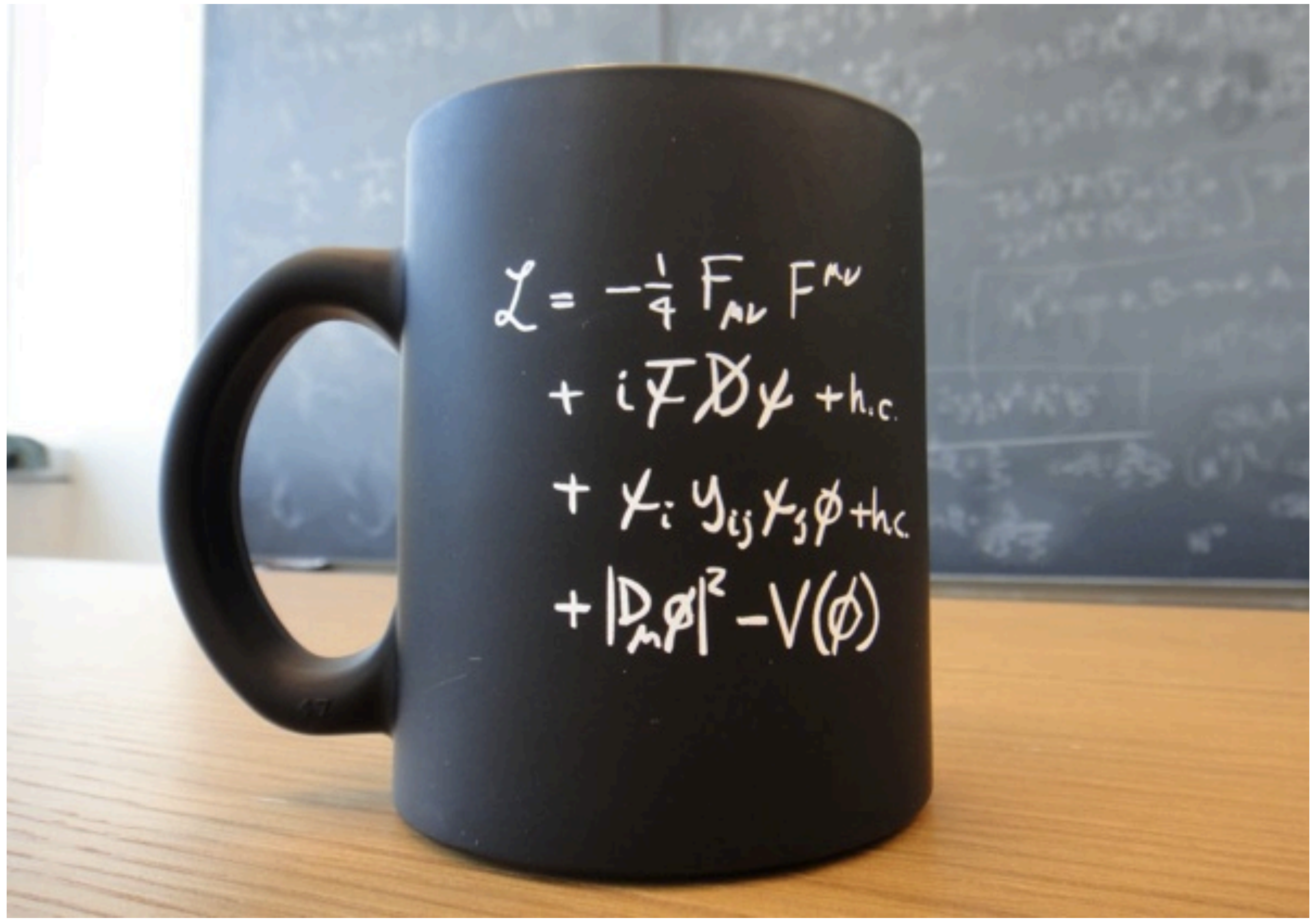
➡Higher-order predictions mandatory for reliable predictions

Theoretical Predictions for the LHC



$$\hookrightarrow |\mathcal{M}|^2 \hookrightarrow \sigma$$

Theoretical Predictions for the LHC



- Expand SU(2) Higgs field around vacuum in either unitary or Feynman gauge
- Transform SU(3)xSU(2)xU(1) gauge fields and goldstones into mass eigenstates (g,W,Z,γ)
- Expand covariant derivates including non-abelian structures
- Add ghost fields

$$\hookrightarrow |\mathcal{M}|^2 \rightarrow \sigma$$

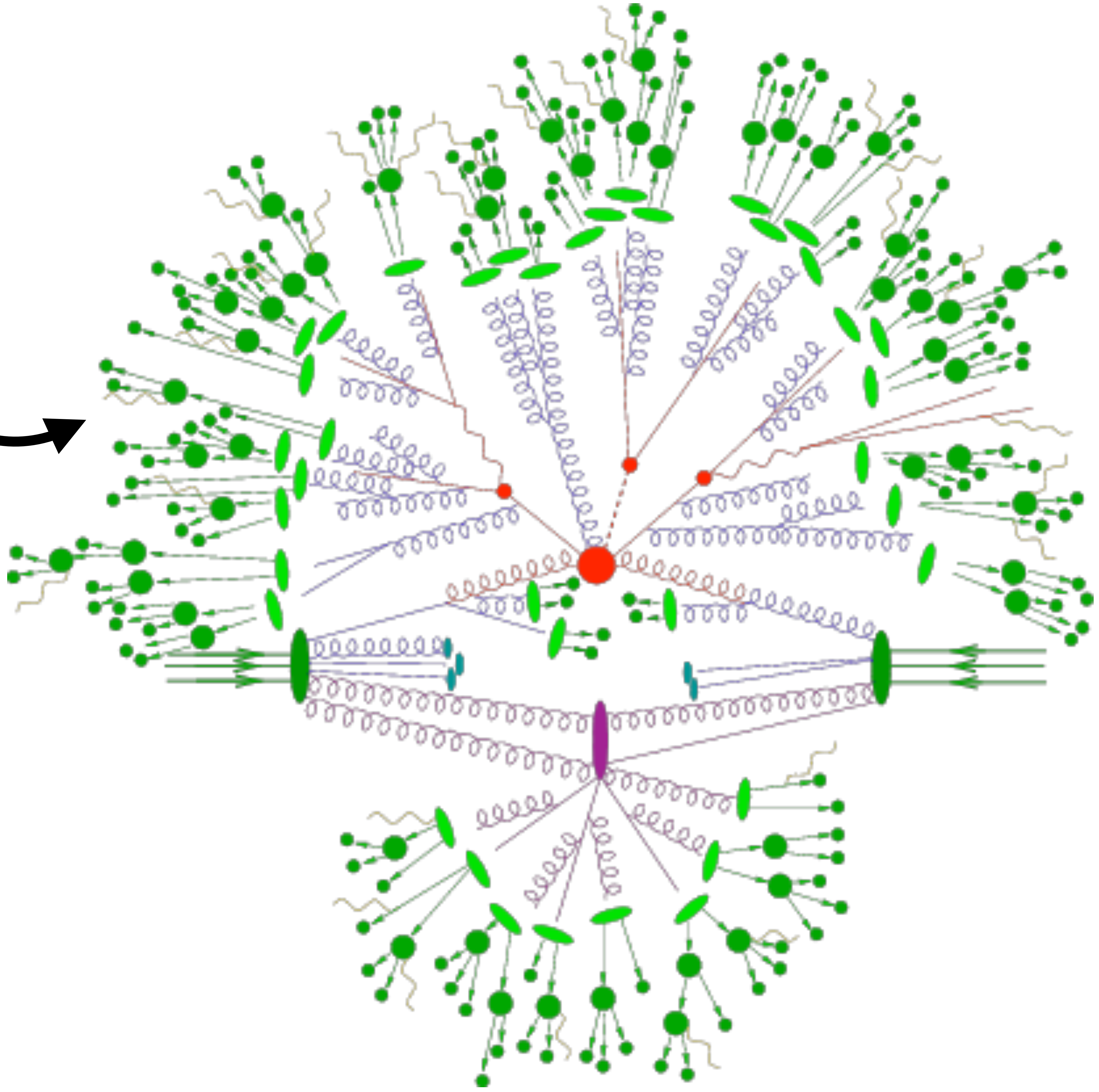
Theoretical Predictions for the LHC

$$
\begin{gathered}
\mathcal{L}_{SM} = -\tfrac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \tfrac{1}{4}g_s^2 f^{abc}f^{ade}g^b_\mu g^c_\nu g^d_\mu g^e_\nu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - \\
M^2 W^+_\mu W^-_\mu - \tfrac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \tfrac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \tfrac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - igc_w(\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)) - \\
igs_w(\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + A_\mu(W^+_\nu \partial_\nu W^-_\mu - \\
W^-_\nu \partial_\nu W^+_\mu)) - \tfrac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \tfrac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - \\
Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w(A_\mu Z^0_\nu(W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu) - \tfrac{1}{2}\partial_\mu H \partial_\mu H - 2M^2\alpha_h H^2 - \partial_\mu\phi^+\partial_\mu\phi^- - \tfrac{1}{2}\partial_\mu\phi^0\partial_\mu\phi^0 - \\
\beta_h\left(\tfrac{2M^2}{g^2} + \tfrac{2M}{g}H + \tfrac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)\right) + \tfrac{2M^4}{g^2}\alpha_h - \\
g\alpha_h M\,(H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-) - \\
\tfrac{1}{8}g^2\alpha_h\,(H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2H^2) - \\
gMW^+_\mu W^-_\mu H - \tfrac{1}{2}g\tfrac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \\
\tfrac{1}{2}ig\left(W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)\right) + \\
\tfrac{1}{2}g\left(W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) + W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)\right) + \tfrac{1}{2}g\tfrac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) + \\
M\,(\tfrac{1}{c_w}Z^0_\mu\partial_\mu\phi^0 + W^+_\mu\partial_\mu\phi^- + W^-_\mu\partial_\mu\phi^+) - ig\tfrac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + igs_w MA_\mu(W^+_\mu\phi^- - \\
W^-_\mu\phi^+) - ig\tfrac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + igs_w A_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \\
\tfrac{1}{4}g^2W^+_\mu W^-_\mu\,(H^2 + (\phi^0)^2 + 2\phi^+\phi^-) - \tfrac{1}{8}g^2\tfrac{1}{c_w^2}Z^0_\mu Z^0_\mu\,(H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2\phi^+\phi^-) - \\
\tfrac{1}{2}g^2\tfrac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) - \tfrac{1}{2}ig^2\tfrac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \tfrac{1}{2}g^2s_wA_\mu\phi^0(W^+_\mu\phi^- + \\
W^-_\mu\phi^+) + \tfrac{1}{2}ig^2s_wA_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\tfrac{s_w}{c_w}(2c_w^2 - 1)Z^0_\mu A_\mu\phi^+\phi^- - \\
g^2s_w^2A_\mu A_\mu\phi^+\phi^- + \tfrac{1}{2}ig_s\,\lambda^a_{ij}(\bar{q}^\sigma_i\gamma^\mu q^\sigma_j)g^a_\mu - \bar{e}^\lambda(\gamma\partial + m^\lambda_e)e^\lambda - \bar{\nu}^\lambda(\gamma\partial + m^\lambda_\nu)\nu^\lambda - \bar{u}^\lambda_j(\gamma\partial + \\
m^\lambda_u)u^\lambda_j - \bar{d}^\lambda_j(\gamma\partial + m^\lambda_d)d^\lambda_j + igs_wA_\mu\left(-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \tfrac{2}{3}(\bar{u}^\lambda_j\gamma^\mu u^\lambda_j) - \tfrac{1}{3}(\bar{d}^\lambda_j\gamma^\mu d^\lambda_j)\right) + \\
\tfrac{ig}{4c_w}Z^0_\mu\{(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar{d}^\lambda_j\gamma^\mu(\tfrac{4}{3}s_w^2 - 1 - \gamma^5)d^\lambda_j) + \\
(\bar{u}^\lambda_j\gamma^\mu(1 - \tfrac{8}{3}s_w^2 + \gamma^5)u^\lambda_j)\} + \tfrac{ig}{2\sqrt{2}}W^+_\mu\left((\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)U^{lep}{}_{\lambda\kappa}e^\kappa) + (\bar{u}^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)\right) + \\
\tfrac{ig}{2\sqrt{2}}W^-_\mu\left((\bar{e}^\kappa U^{lep\dagger}{}_{\kappa\lambda}\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\kappa\lambda}\gamma^\mu(1+\gamma^5)u^\lambda_j)\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_e(\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa}(1-\gamma^5)e^\kappa) + m^\lambda_\nu(\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa}(1+\gamma^5)e^\kappa\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_e(\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa}(1+\gamma^5)\nu^\kappa) - m^\kappa_\nu(\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa}(1-\gamma^5)\nu^\kappa\right) - \tfrac{g}{2}\tfrac{m^\lambda_\nu}{M}H(\bar{\nu}^\lambda\nu^\lambda) - \\
\tfrac{g}{2}\tfrac{m^\lambda_e}{M}H(\bar{e}^\lambda e^\lambda) + \tfrac{ig}{2}\tfrac{m^\lambda_\nu}{M}\phi^0(\bar{\nu}^\lambda\gamma^5\nu^\lambda) - \tfrac{ig}{2}\tfrac{m^\lambda_e}{M}\phi^0(\bar{e}^\lambda\gamma^5e^\lambda) - \tfrac{1}{4}\,\bar{\nu}_\lambda\,M^R_{\lambda\kappa}\,(1-\gamma_5)\hat{\nu}_\kappa - \\
\tfrac{1}{4}\,\bar{\nu}_\lambda\,M^R_{\lambda\kappa}\,(1-\gamma_5)\hat{\nu}_\kappa + \tfrac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_d(\bar{u}^\lambda_jC_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + m^\lambda_u(\bar{u}^\lambda_jC_{\lambda\kappa}(1+\gamma^5)d^\kappa_j\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_d(\bar{d}^\lambda_jC^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m^\kappa_u(\bar{d}^\lambda_jC^\dagger_{\lambda\kappa}(1-\gamma^5)u^\kappa_j\right) - \tfrac{g}{2}\tfrac{m^\lambda_u}{M}H(\bar{u}^\lambda_ju^\lambda_j) - \\
\tfrac{g}{2}\tfrac{m^\lambda_d}{M}H(\bar{d}^\lambda_jd^\lambda_j) + \tfrac{ig}{2}\tfrac{m^\lambda_u}{M}\phi^0(\bar{u}^\lambda_j\gamma^5u^\lambda_j) - \tfrac{ig}{2}\tfrac{m^\lambda_d}{M}\phi^0(\bar{d}^\lambda_j\gamma^5d^\lambda_j) + \bar{G}^a\partial^2G^a + g_sf^{abc}\partial_\mu\bar{G}^aG^bg^c_\mu + \\
\bar{X}^+(\partial^2 - M^2)X^+ + \bar{X}^-(\partial^2 - M^2)X^- + \bar{X}^0(\partial^2 - \tfrac{M^2}{c_w^2})X^0 + \bar{Y}\partial^2Y + igc_wW^+_\mu(\partial_\mu\bar{X}^0X^- - \\
\partial_\mu\bar{X}^+X^0) + igs_wW^+_\mu(\partial_\mu\bar{Y}X^- - \partial_\mu\bar{X}^+Y) + igc_wW^-_\mu(\partial_\mu\bar{X}^-X^0 - \\
\partial_\mu\bar{X}^0X^+) + igs_wW^-_\mu(\partial_\mu\bar{X}^-Y - \partial_\mu\bar{Y}X^+) + igc_wZ^0_\mu(\partial_\mu\bar{X}^+X^+ - \\
\partial_\mu\bar{X}^-X^-) + igs_wA_\mu(\partial_\mu\bar{X}^+X^+ - \\
\partial_\mu\bar{X}^-X^-) - \tfrac{1}{2}gM\left(\bar{X}^+X^+H + \bar{X}^-X^-H + \tfrac{1}{c_w^2}\bar{X}^0X^0H\right) + \tfrac{1-2c_w^2}{2c_w}igM\left(\bar{X}^+X^0\phi^+ - \bar{X}^-X^0\phi^-\right) + \\
\tfrac{1}{2c_w}igM\left(\bar{X}^0X^-\phi^+ - \bar{X}^0X^+\phi^-\right) + igMs_w\left(\bar{X}^0X^-\phi^+ - \bar{X}^0X^+\phi^-\right) + \\
\tfrac{1}{2}igM\left(\bar{X}^+X^+\phi^0 - \bar{X}^-X^-\phi^0\right).
\end{gathered}
$$

$$\hookrightarrow |\mathcal{M}|^2 \rightarrow \sigma$$

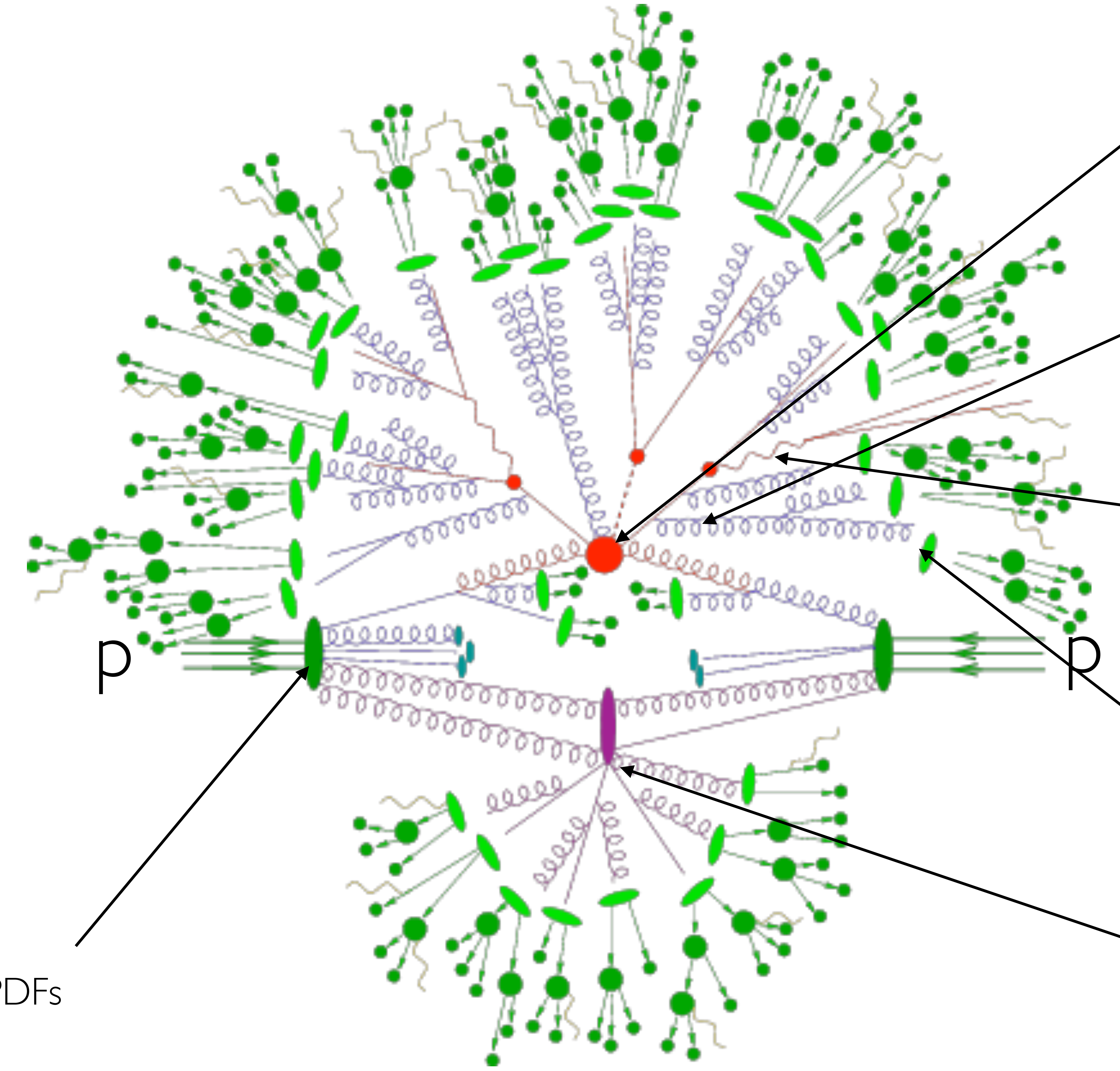
Theoretical Predictions for the LHC

$$
\begin{gathered}
\mathcal{L}_{SM} = -\tfrac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \tfrac{1}{4}g_s^2 f^{abc}f^{ade}g^b_\mu g^c_\nu g^d_\mu g^e_\nu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - \\
M^2 W^+_\mu W^-_\mu - \tfrac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \tfrac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \tfrac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - igc_w(\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)) - \\
igs_w(\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + A_\mu(W^+_\nu \partial_\nu W^-_\mu - \\
W^-_\nu \partial_\nu W^+_\mu)) - \tfrac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \tfrac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - \\
Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w(A_\mu Z^0_\nu(W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu) - \tfrac{1}{2}\partial_\mu H \partial_\mu H - 2M^2\alpha_h H^2 - \partial_\mu\phi^+\partial_\mu\phi^- - \tfrac{1}{2}\partial_\mu\phi^0\partial_\mu\phi^0 - \\
\beta_h\left(\tfrac{2M^2}{g^2} + \tfrac{2M}{g}H + \tfrac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)\right) + \tfrac{2M^4}{g^2}\alpha_h - \\
g\alpha_h M\left(H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-\right) - \\
\tfrac{1}{8}g^2\alpha_h\left(H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2H^2\right) - \\
gMW^+_\mu W^-_\mu H - \tfrac{1}{2}g\tfrac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \\
\tfrac{1}{2}ig\left(W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)\right) + \\
\tfrac{1}{2}g\left(W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) + W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)\right) + \tfrac{1}{2}g\tfrac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) + \\
M\left(\tfrac{1}{c_w}Z^0_\mu\partial_\mu\phi^0 + W^+_\mu\partial_\mu\phi^- + W^-_\mu\partial_\mu\phi^+\right) - ig\tfrac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + igs_w MA_\mu(W^+_\mu\phi^- - \\
W^-_\mu\phi^+) - ig\tfrac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + igs_w A_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \\
\tfrac{1}{4}g^2 W^+_\mu W^-_\mu\left(H^2 + (\phi^0)^2 + 2\phi^+\phi^-\right) - \tfrac{1}{8}g^2\tfrac{1}{c_w^2}Z^0_\mu Z^0_\mu\left(H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2\phi^+\phi^-\right) - \\
\tfrac{1}{2}g^2\tfrac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) - \tfrac{1}{2}ig^2\tfrac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \tfrac{1}{2}g^2 s_w A_\mu\phi^0(W^+_\mu\phi^- + \\
W^-_\mu\phi^+) + \tfrac{1}{2}ig^2 s_w A_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\tfrac{s_w}{c_w}(2c_w^2 - 1)Z^0_\mu A_\mu\phi^+\phi^- - \\
g^2 s_w^2 A_\mu A_\mu\phi^+\phi^- + \tfrac{1}{2}ig_s\lambda^a_{ij}(\bar{q}^\sigma_i\gamma^\mu q^\sigma_j)g^a_\mu - \bar{e}^\lambda(\gamma\partial + m_e^\lambda)e^\lambda - \bar{\nu}^\lambda(\gamma\partial + m_\nu^\lambda)\nu^\lambda - \bar{u}^\lambda_j(\gamma\partial + \\
m_u^\lambda)u^\lambda_j - \bar{d}^\lambda_j(\gamma\partial + m_d^\lambda)d^\lambda_j + igs_w A_\mu\left(-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \tfrac{2}{3}(\bar{u}^\lambda_j\gamma^\mu u^\lambda_j) - \tfrac{1}{3}(\bar{d}^\lambda_j\gamma^\mu d^\lambda_j)\right) + \\
\tfrac{ig}{4c_w}Z^0_\mu\{(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar{d}^\lambda_j\gamma^\mu(\tfrac{4}{3}s_w^2 - 1 - \gamma^5)d^\lambda_j) + \\
(\bar{u}^\lambda_j\gamma^\mu(1 - \tfrac{8}{3}s_w^2 + \gamma^5)u^\lambda_j)\} + \tfrac{ig}{2\sqrt{2}}W^+_\mu\left((\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)U^{lep}{}_{\lambda\kappa}e^\kappa) + (\bar{u}^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)\right) + \\
\tfrac{ig}{2\sqrt{2}}W^-_\mu\left((\bar{e}^\kappa U^{lep\dagger}{}_{\kappa\lambda}\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\kappa\lambda}\gamma^\mu(1+\gamma^5)u^\lambda_j)\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^+\left(-m_e^\kappa(\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa}(1-\gamma^5)e^\kappa) + m_\nu^\lambda(\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa}(1+\gamma^5)e^\kappa\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^-\left(m_e^\lambda(\bar{e}^\lambda U^{lep\dagger}_{\lambda\kappa}(1+\gamma^5)\nu^\kappa) - m_\nu^\kappa(\bar{e}^\lambda U^{lep\dagger}_{\lambda\kappa}(1-\gamma^5)\nu^\kappa\right) - \tfrac{g}{2}\tfrac{m_\nu^\lambda}{M}H(\bar{\nu}^\lambda\nu^\lambda) - \\
\tfrac{g}{2}\tfrac{m_e^\lambda}{M}H(\bar{e}^\lambda e^\lambda) + \tfrac{ig}{2}\tfrac{m_\nu^\lambda}{M}\phi^0(\bar{\nu}^\lambda\gamma^5\nu^\lambda) - \tfrac{ig}{2}\tfrac{m_e^\lambda}{M}\phi^0(\bar{e}^\lambda\gamma^5 e^\lambda) - \tfrac{1}{4}\bar{\nu}_\lambda M^R_{\lambda\kappa}(1-\gamma_5)\hat{\nu}_\kappa - \\
\tfrac{1}{4}\bar{\nu}_\lambda M^R_{\lambda\kappa}(1-\gamma_5)\hat{\nu}_\kappa + \tfrac{ig}{2M\sqrt{2}}\phi^+\left(-m_d^\kappa(\bar{u}^\lambda_j C_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + m_u^\lambda(\bar{u}^\lambda_j C_{\lambda\kappa}(1+\gamma^5)d^\kappa_j\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^-\left(m_d^\lambda(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m_u^\kappa(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1-\gamma^5)u^\kappa_j\right) - \tfrac{g}{2}\tfrac{m_u^\lambda}{M}H(\bar{u}^\lambda_j u^\lambda_j) - \\
\tfrac{g}{2}\tfrac{m_d^\lambda}{M}H(\bar{d}^\lambda_j d^\lambda_j) + \tfrac{ig}{2}\tfrac{m_u^\lambda}{M}\phi^0(\bar{u}^\lambda_j\gamma^5 u^\lambda_j) - \tfrac{ig}{2}\tfrac{m_d^\lambda}{M}\phi^0(\bar{d}^\lambda_j\gamma^5 d^\lambda_j) + \bar{G}^a\partial^2 G^a + g_s f^{abc}\partial_\mu\bar{G}^a G^b g^c_\mu + \\
\bar{X}^+(\partial^2 - M^2)X^+ + \bar{X}^-(\partial^2 - M^2)X^- + \bar{X}^0(\partial^2 - \tfrac{M^2}{c_w^2})X^0 + \bar{Y}\partial^2 Y + igc_w W^+_\mu(\partial_\mu\bar{X}^0 X^- - \\
\partial_\mu\bar{X}^+ X^0) + igs_w W^+_\mu(\partial_\mu\bar{Y}X^- - \partial_\mu\bar{X}^+ Y) + igc_w W^-_\mu(\partial_\mu\bar{X}^- X^0 - \\
\partial_\mu\bar{X}^0 X^+) + igs_w W^-_\mu(\partial_\mu\bar{X}^- Y - \partial_\mu\bar{Y}X^+) + igc_w Z^0_\mu(\partial_\mu\bar{X}^+ X^+ - \\
\partial_\mu\bar{X}^- X^-) + igs_w A_\mu(\partial_\mu\bar{X}^+ X^+ - \\
\partial_\mu\bar{X}^- X^-) - \tfrac{1}{2}gM\left(\bar{X}^+ X^+ H + \bar{X}^- X^- H + \tfrac{1}{c_w^2}\bar{X}^0 X^0 H\right) + \tfrac{1-2c_w^2}{2c_w}igM\left(\bar{X}^+ X^0\phi^+ - \bar{X}^- X^0\phi^-\right) + \\
\tfrac{1}{2c_w}igM\left(\bar{X}^0 X^-\phi^+ - \bar{X}^0 X^+\phi^-\right) + igMs_w\left(\bar{X}^0 X^-\phi^+ - \bar{X}^0 X^+\phi^-\right) + \\
\tfrac{1}{2}igM\left(\bar{X}^+ X^+\phi^0 - \bar{X}^- X^-\phi^0\right).
\end{gathered}
$$



$$\hookrightarrow |\mathcal{M}|^2 \hookrightarrow \sigma$$

Theoretical Predictions for Hadron Colliders



Hard (perturbative) scattering process
- N(N)LO QCD + EW

QCD Parton Shower

Covered in these lectures

QED Parton shower

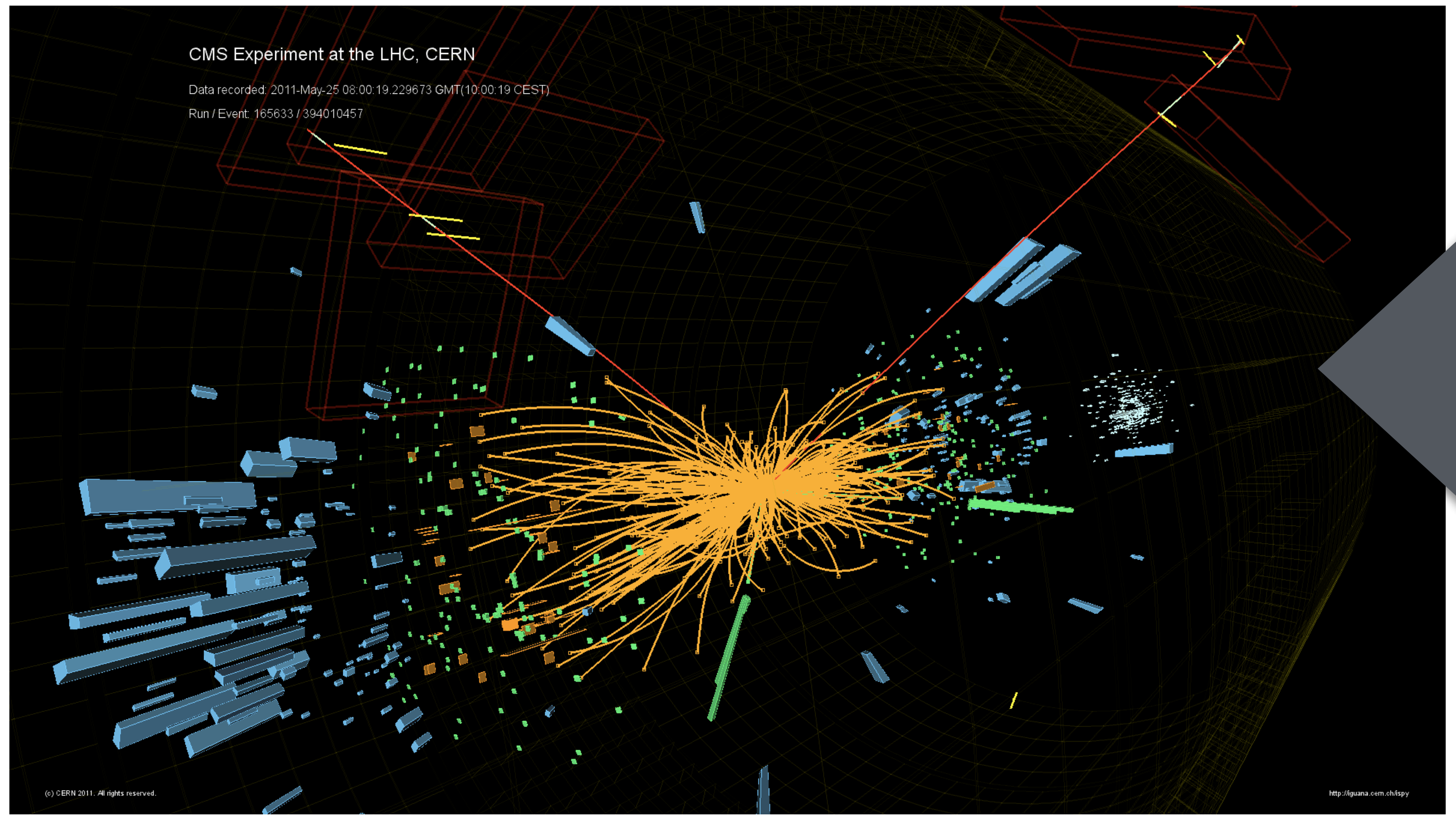
Hadronization/fragmentation/
hadron decays

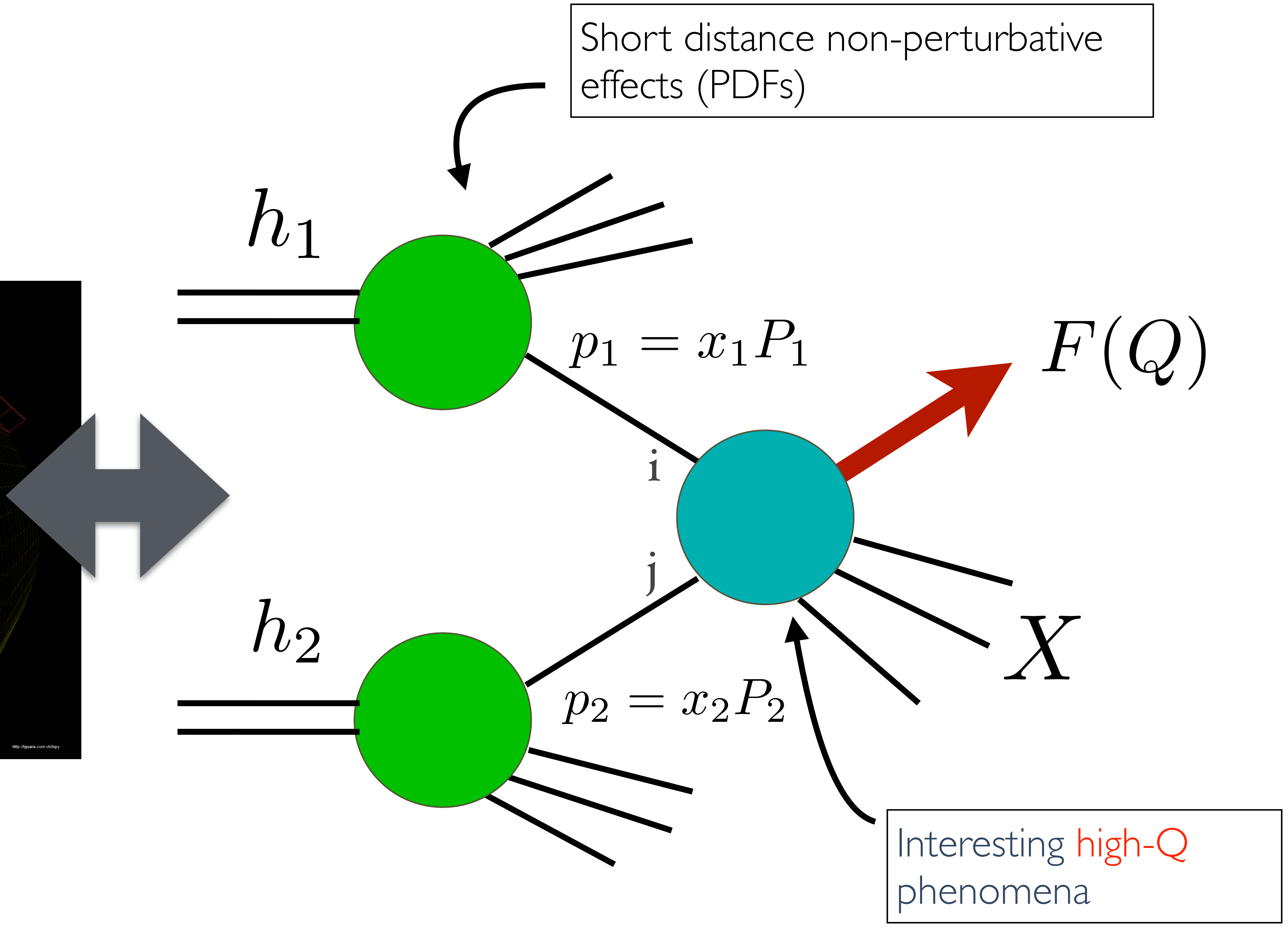
Soft underleying event

PDFs

Path to precision at the LHC

- Key: QCD factorization





$$d\sigma = \sum_{ij} \int dx_1 dx_2 f_1^{(P_1)}(x_1) f_2^{(P_2)}(x_2) d\hat{\sigma}_{ij}(x_1 x_2 s)$$

Sum over all partons

Path to precision

$$d\sigma = \sum_{ij} \int dx_1 dx_2 f_1^{(P_1)}(x_1) f_2^{(P_2)}(x_2) d\hat{\sigma}_{ij}(x_1 x_2 s)$$

Sum over all partons

Parton distributions:

- (At LO:) Probability for finding a quark or gluon with a certain momentum fraction in a hadron
- universal but not perturbatively computable

➡ determine via fit against data

Hard partonic cross section:

- process dependent but computable in perturbation theory

Perturbative expansion

- Expansion in a small coupling **α**:

$$d\sigma = d\sigma(\alpha^n) + d\sigma(\alpha^{n+1}) + d\sigma(\alpha^{n+2}) + d\sigma(\alpha^{n+3}) + \ldots$$

LO NLO NNLO N3LO

- at the LHC consider in particular **α** = **α**$_S$ (QCD coupling), but also **α** = **α**$_{EW}$ (EW coupling) relevant → later!

- In QCD running strong coupling: $\alpha_S = \alpha_S(\mu) = \frac{1}{b_0 \ln \frac{\mu^2}{\Lambda^2}} + \ldots$

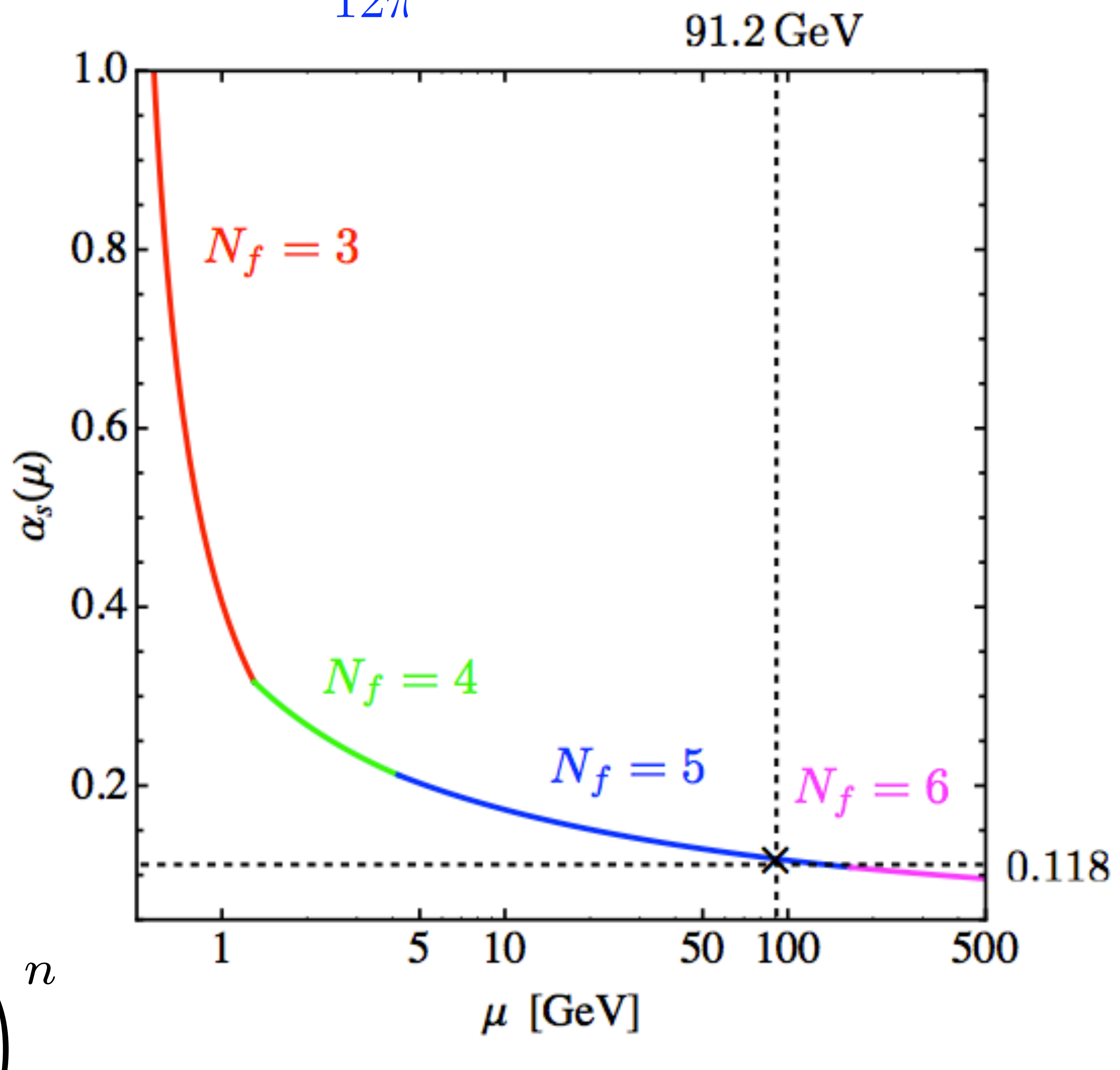
$$d\sigma^{\rm LO}(\mu) = \alpha_S(\mu)^n A^{\rm LO}$$

$$\to d\sigma^{\rm LO}(\mu') = \alpha_S(\mu')^n A^{\rm LO} = \alpha_S(\mu)^n \left(1 + n b_0 \alpha_S(\mu) \ln \frac{\mu^2}{\mu'^2} + \ldots\right) A^{\rm LO}$$

- So the change of scale is an NLO effect (∝**α**$_S$).

- At LO the normalisation is not under control: $\frac{d\sigma^{\rm LO}(\mu)}{d\sigma^{\rm LO}(\mu')} = \left(\frac{\alpha_S(\mu)}{\alpha_S(\mu')}\right)^n$

$$b_0 = \frac{11 N_c - 4 n_f T_R}{12\pi}$$



Perturbative expansion

- At NLO we have:

$$d\sigma^{\mathrm{NLO}}(\mu) = \alpha_S(\mu)^n A^{\mathrm{LO}} + \alpha_S(\mu)^{n+1}\left(A^{\mathrm{NLO}} - n b_0 \ln\frac{\mu^2}{Q_0^2}\right) + \dots$$

- So the NLO result compensates the LO scale dependence and the residual dependence is NNLO!

➡ **Include higher-order corrections in order to reduce scale dependence!**

- That means scale dependence can be regarded as higher-order effect, but can be very relevant! (in particular for large n)
- Normalisation starts being under control at NLO: **compensation mechanism**
- Note: a good scale choice automatically resums large logarithms to all orders, while a bad one spuriously introduces large logs and ruins the perturbative expansion
- Scale variation is conventionally used to estimate the **theory uncertainty**

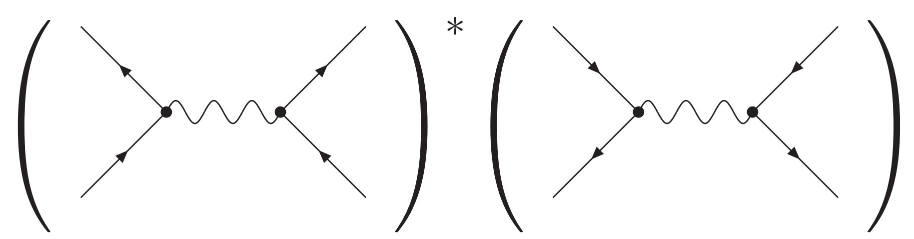
LO Ingredients

- LO partonic cross section for a 2→n process can be written as

$$d\hat{\sigma}_{\rm LO} = \frac{1}{2s}\int d\Phi_n |\mathcal{M}_{\rm LO}|^2$$

$$\int d\Phi_n = (2\pi)^4 \delta^{(4)}\left(P - \sum_{i=1}^{n} q_i\right) \prod_{i=1}^{n} \frac{d^3 q_i}{(2\pi)^3 2E_i}$$ n-particle phase-space

$\mathcal{M}_{\rm LO}$ LO matrix element: tree-level → $|\mathcal{M}_{\rm LO}|^2$



$s = P^2 = (\hat{p}_1 + \hat{p}_2)^2$ squared centre-of-mass energy of hard process

- Integration over phase space by Monte Carlo methods
  - ➡ any distribution/histogram can be determined simultaneously
  - ➡ Monte Carlo events can be unweighted
- Integration over phase space analytically
  - ➡ very fast evaluation
  - ➡ analytical structure of the result can be investigated

NLO Ingredients

- NLO partonic cross section for a 2→n process can be written as

$$d\hat{\sigma}_{\mathrm{NLO}} = \frac{1}{2s}\int d\Phi_n \left[|\mathcal{M}_{\mathrm{LO}}|^2 + 2\mathrm{Re}\{\mathcal{M}_{\mathrm{LO}}\mathcal{M}^*_{\mathrm{NLO,V}}\}\right] + \frac{1}{2s}\int d\Phi_{n+1}|\mathcal{M}_{\mathrm{NLO,R}}|^2$$

NLO = B + V +R

$\int d\Phi_{n(+1)}$ n or n+1 particle phase space

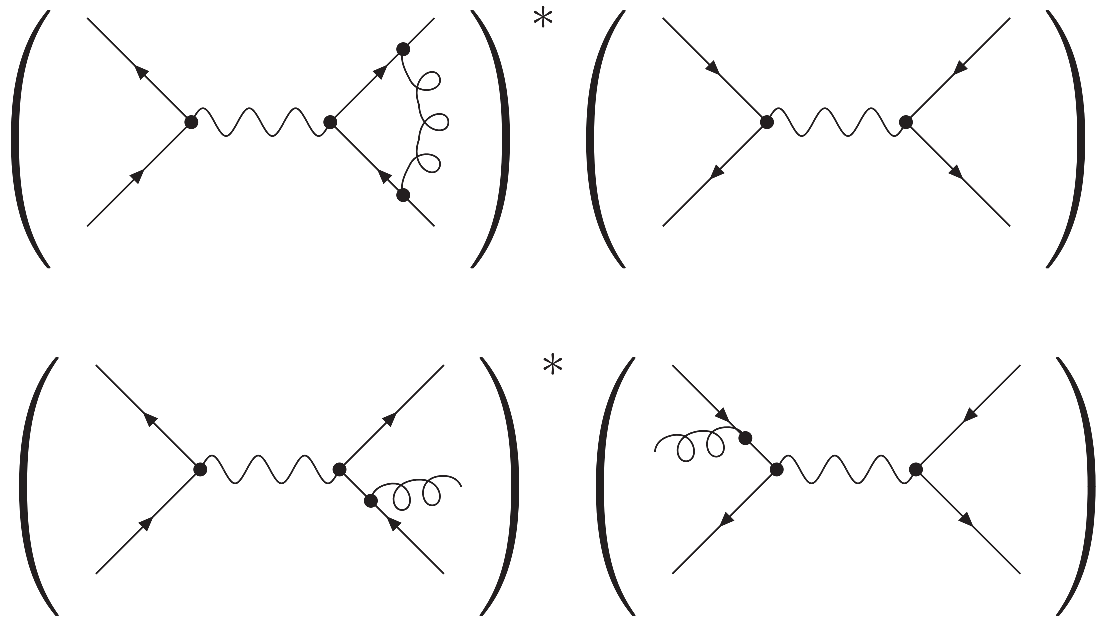
$\propto \alpha \begin{cases} \mathcal{M}_{\mathrm{NLO,V}} \\ \mathcal{M}_{\mathrm{NLO,R}} \end{cases}$

$\mathcal{M}_{\mathrm{NLO,V}}$ **virtual** one-loop matrix element

$\mathcal{M}_{\mathrm{NLO,R}}$ **real** tree-level matrix element

$\mathrm{Re}\{\mathcal{M}_{\mathrm{LO}}\mathcal{M}^*_{\mathrm{NLO,V}}\}$

$|\mathcal{M}_{\mathrm{NLO,R}}|^2$



Note: real radiation might open up new partonic channels!

NNLO Ingredients

- NNLO partonic cross section for a 2→n process can be written as

$$d\hat{\sigma}_{\rm NNLO} = \frac{1}{2s}\int d\Phi_n \left[|\mathcal{M}_{\rm LO}|^2 + 2{\rm Re}\{\mathcal{M}_{\rm LO}\mathcal{M}^*_{\rm NLO,V}\} + 2{\rm Re}\{\mathcal{M}_{\rm LO}\mathcal{M}^*_{\rm NNLO,V}\}\right]$$

NNLO = B + V + V2 + …

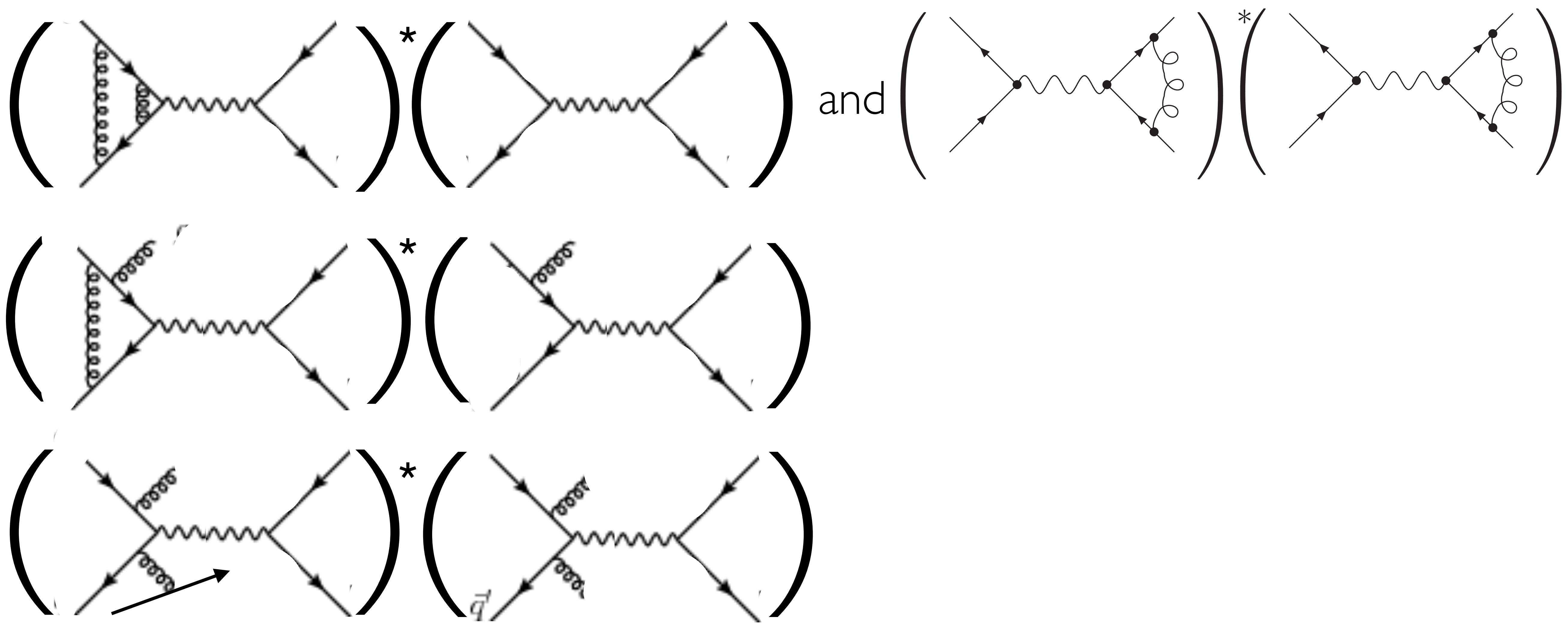
$$+\frac{1}{2s}\int d\Phi_{n+1}\left[|\mathcal{M}_{\rm NLO,R}|^2 + 2{\rm Re}|\mathcal{M}_{\rm NLO,R}\mathcal{M}^*_{\rm NNLO,RV}|\right] + \frac{1}{2s}\int d\Phi_{n+2}|\mathcal{M}_{\rm NNLO,RR}|^2$$

+ R + RV + RR

$\int d\Phi_{n(+1)}$ n, n+1, n+2 particle phase space

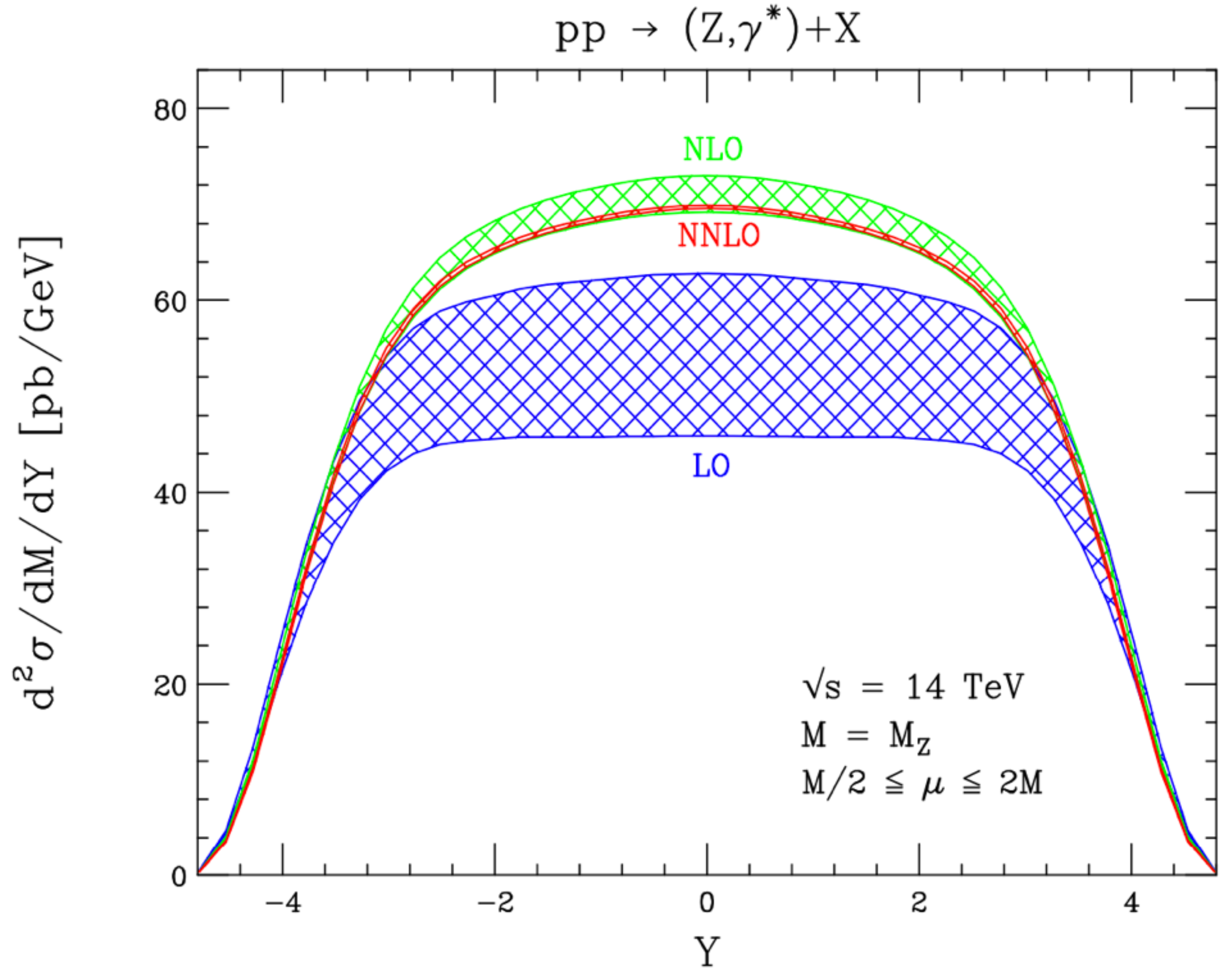
ΔNLO $\propto \alpha$
- $\mathcal{M}_{\rm NLO,V}$ **virtual** one-loop matrix element
- $\mathcal{M}_{\rm NLO,R}$ **real** tree-level matrix element

ΔNNLO $\propto \alpha^2$
- $M_{\rm NNLO,V}$ **double-virtual** two-loop matrix element
- $M_{\rm NNLO,RV}$ **real-virtual** one-loop matrix element
- $\mathcal{M}_{\rm NNLO,RR}$ **double-real** tree-level matrix element



Convergence of the perturbative expansion: Drell-Yan

[Anastasiou et al.,2003]

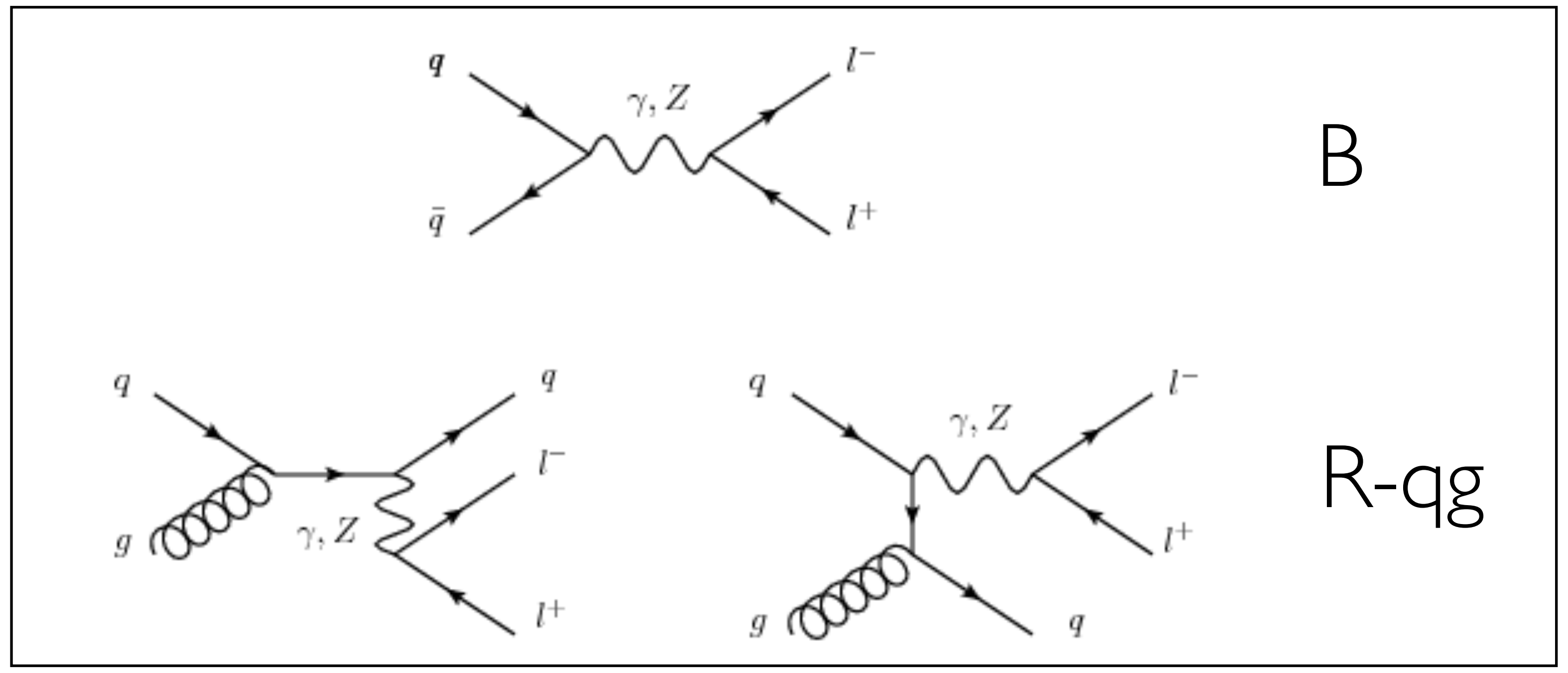


- NNLO calculation first performed for the inclusive cross section *[Van Neerven et al., 1990]*
  →NNLO/NLO at the few percent level
- Rapidity distribution: 13 years later!
- Bands obtained by studying scale variations varied in $\mu=[m_Z/2,2m_Z]$
- LO and NLO bands do not overlap!
  ➡Error estimate at LO largely underestimated!
- large contribution coming from qg channel that opens up at NLO
- NLO and NNLO bands do overlap
  ➡Reliable error estimate only when all partonic channels contribute

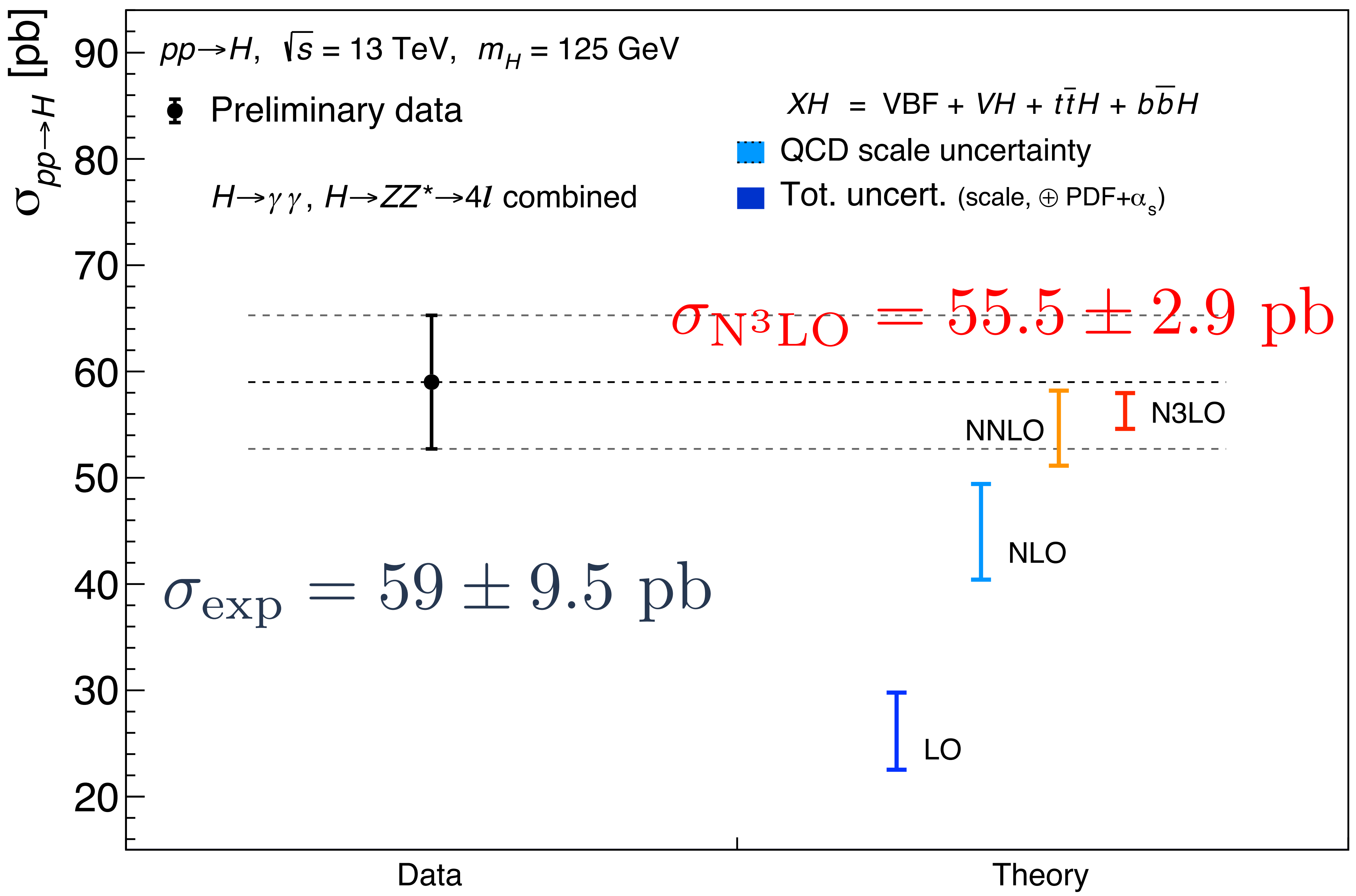
➡Higher-orders are crucial for reliable predictions

➡Use these precision predictions to

- stress-test the SM: QCD and EW
- determine parameters and PDFs!



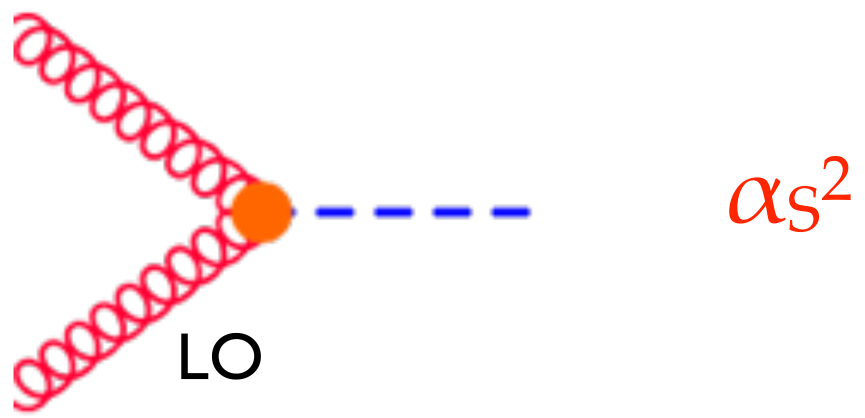
Convergence of the perturbative expansion: inclusive Higgs

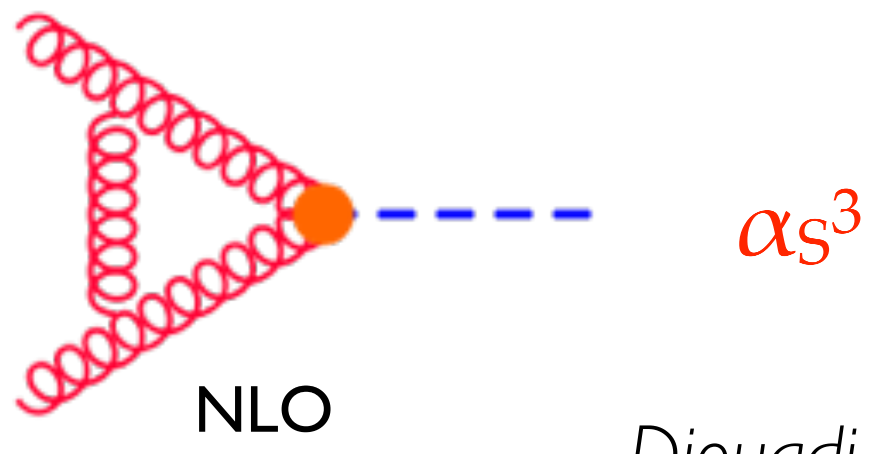


➡ Error estimate at LO largely underestimated!

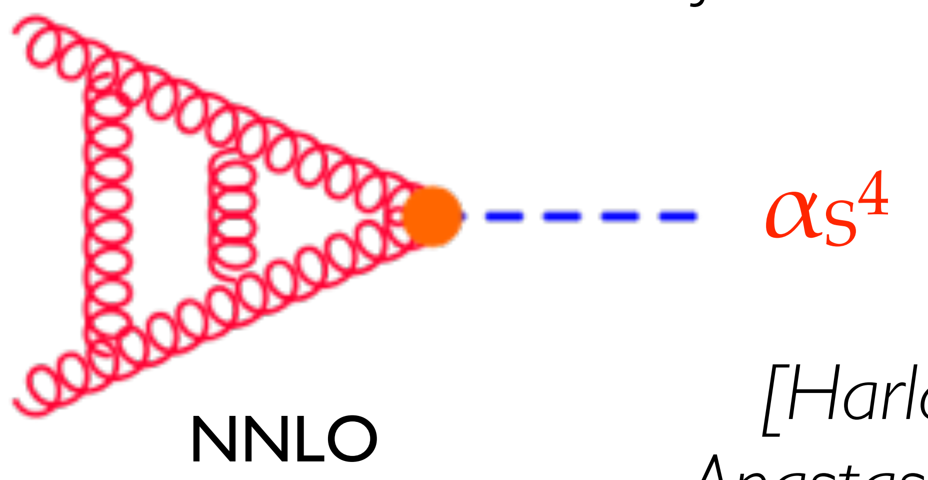
➡ N3LO ~ 2 LO

➡ Higher-orders are crucial for reliable predictions and precision tests of Higgs properties

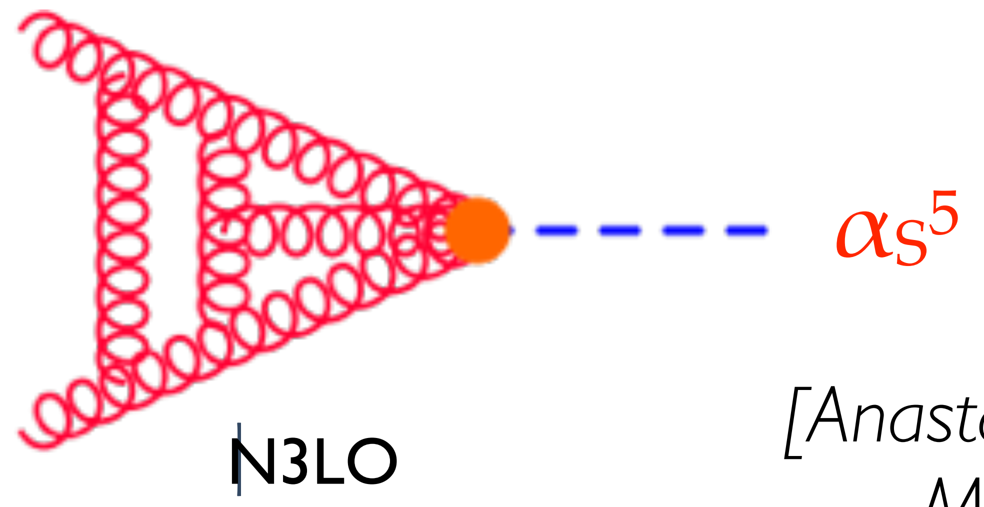




[Dawson; 1991
Djouadi, Spira, Zerwas;1991]

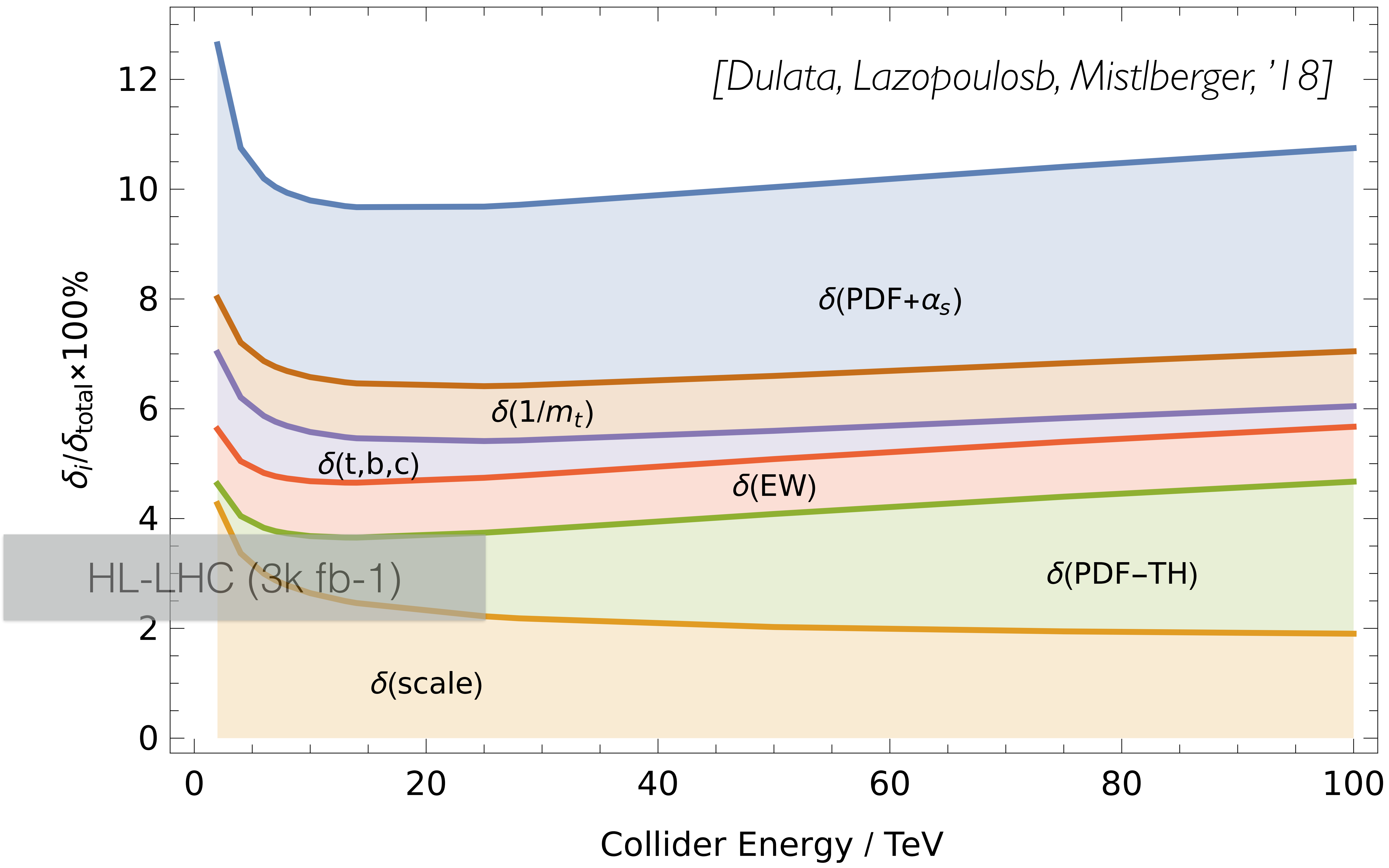


[Harlander, Kilgore; 2002
Anastasiou, Melnikov;2002]

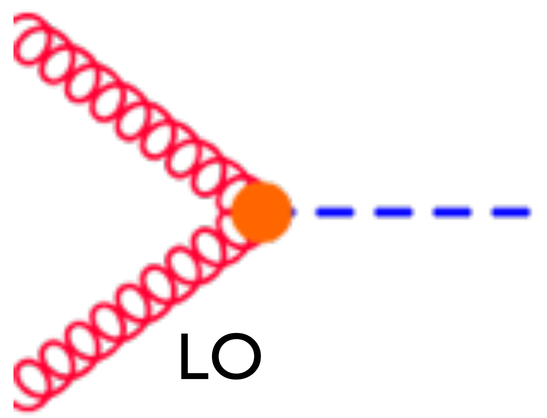


[Anastasiou et al.;2015
Mistlberger; 2018]

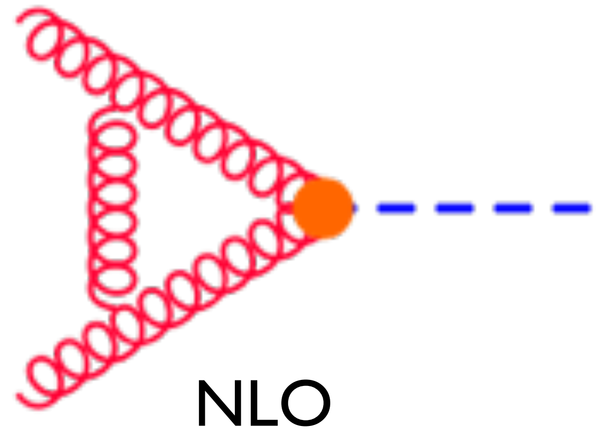
Convergence of the perturbative expansion: inclusive Higgs up to N3LO



➡ At this level: crucial to investigate any possible uncertainty beyond naive scale variations

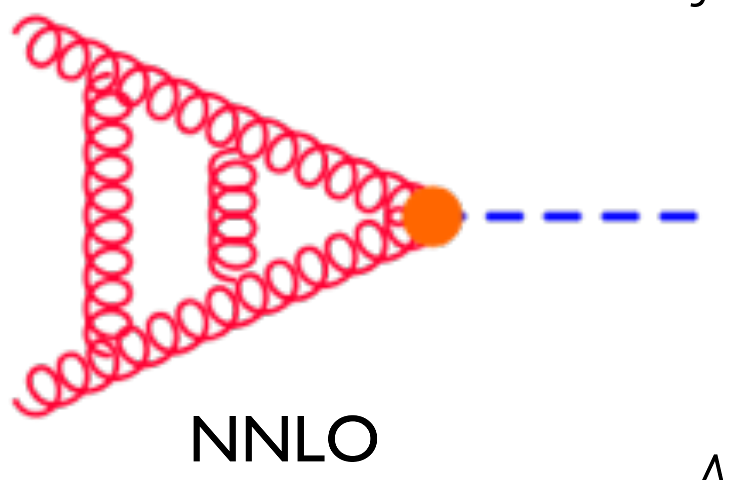


$\alpha_S{}^2$



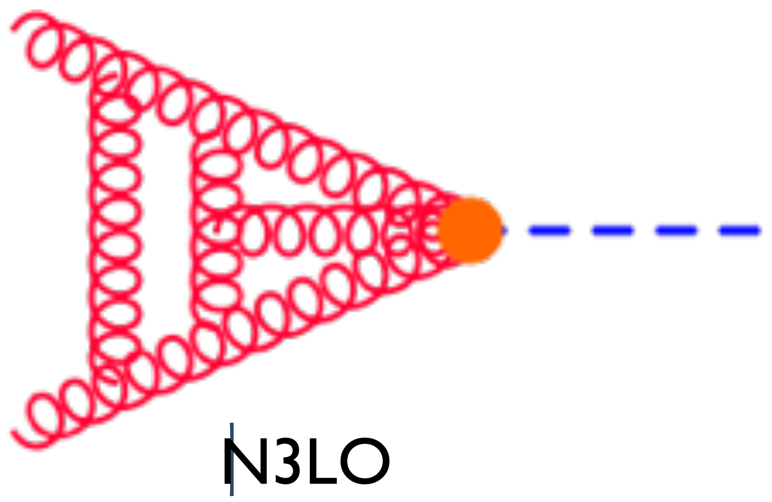
$\alpha_S{}^3$

[Dawson; 1991
Djouadi, Spira, Zerwas;1991]



$\alpha_S{}^4$

[Harlander, Kilgore; 2002
Anastasiou, Melnikov;2002]



$\alpha_S{}^5$

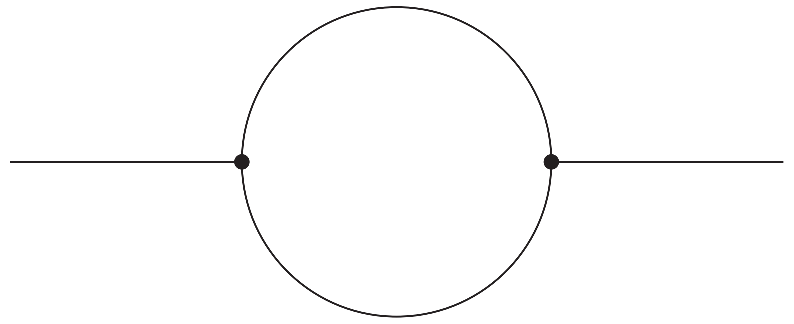
[Anastasiou et al.;2015
Mistlberger; 2018]

NLO computations

UV & IR divergences

- naively starting from NLO the predictions are divergent.

- Virtual loop diagrams are UV & IR divergent, e.g.:



$$\int \frac{d^4k}{(2\pi)^4}\frac{1}{(k^2)^2} = \frac{1}{(4\pi)^2}\int_0^\infty dk^2 \frac{1}{k^2} = \frac{1}{(4\pi)^2}\int_0^\infty \frac{dx}{x}$$

This (and many other) integral diverges at

- $k^2 \rightarrow \infty$ (**UV-divergence**)
- $k^2 \rightarrow 0$ (**IR-divergence**)

➡ Use **dimensional regularisation** to regulate these UV- and IR-divergences: D=4 → D=4-2**ε**

$$\int \frac{d^4l}{(2\pi)^4} \rightarrow \mu^{2\epsilon}\int \frac{d^dl}{(2\pi)^d}\,, \quad d = 4 - 2\epsilon < 4$$

Note: in order to preserve the correct dimensions a mass scale **μ** is needed (regularisation scale)

➡ Divergences are transformed into poles in ε, e.g.:

$$\int_0^1 \frac{dx}{x} \rightarrow \int_0^1 \frac{dx}{x^{1-\epsilon}} = \frac{1}{\epsilon}$$

→ **UV poles**

→ **IR poles**

➡ This "dim-reg" procedure is gauge invariant & Lorentz invariant, in contrast to many other regularisation procedures (photon/gluon mass, cut-offs, Pauli-Villard,…)

UV Renormalisation

- Dim-reg: UV & IR divergences are transformed into poles in ε

- **Renormalisation:** Parameters appearing in the Lagrangian are not observed quantities, but "bare" quantities.

  ➡ **absorb UV-divergences** via redefinition ("renormalization") of all couplings and masses, e.g. $\alpha_s^{\mathrm{ren}} = Z_{\alpha_s} \alpha_s^{\mathrm{bare}}$

  ➡ "Renormalisation constants" are related to self-energies and can be defined according to different schemes. Note: all scheme have to absorb the same divergences.

◉On-shell scheme:

$$\Sigma(p^2 = m^2) = 0, \quad \frac{\partial \Sigma(p^2)}{\partial p^2}\Big|_{p^2=m^2} = 0 \qquad \Sigma = \text{self-energy}$$

- ‣ standard scheme for the renormlization of masses & wave-functions

◉$\overline{\text{MS}}$-scheme ("modified minimal subtraction scheme"):

- ‣ standard scheme for the renormlization of α_S
- ‣ in dim-reg, poles always appear in the combination $\frac{1}{\epsilon} + \ln(4\pi) - \gamma_E$
- ‣ $\overline{\text{MS}}$: subtract this combination and replace bare coupling with renormalized one

The Kinoshita-Lee-Nauenberg theorem

- After the renormalization procedure IR poles remain.
- However there is the very fundamental (based on the structure of the S-matrix) Kinoshita-Lee-Nauenberg (KLM) theorem *[Kinoshita; '62, Lee, Nauenber; '64]*:

Measurable quantities, summed over indistinguishable states are free from IR divergences.

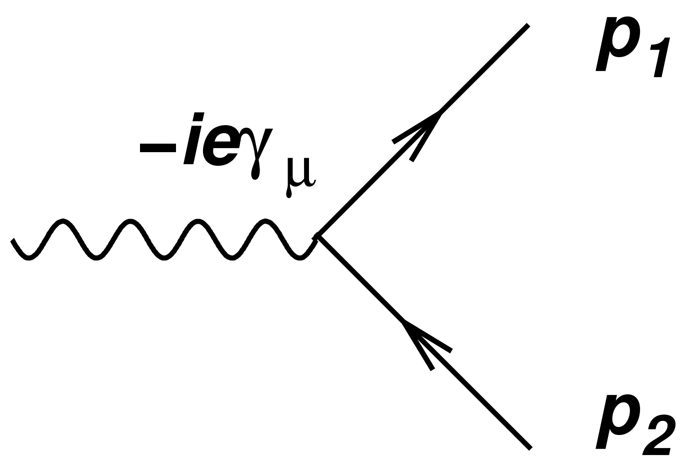
$\sim$

For IR-safe observables IR divergences in the virtual corrections cancel with corresponding divergences in the (indistinguishable) real radiation

IR divergences

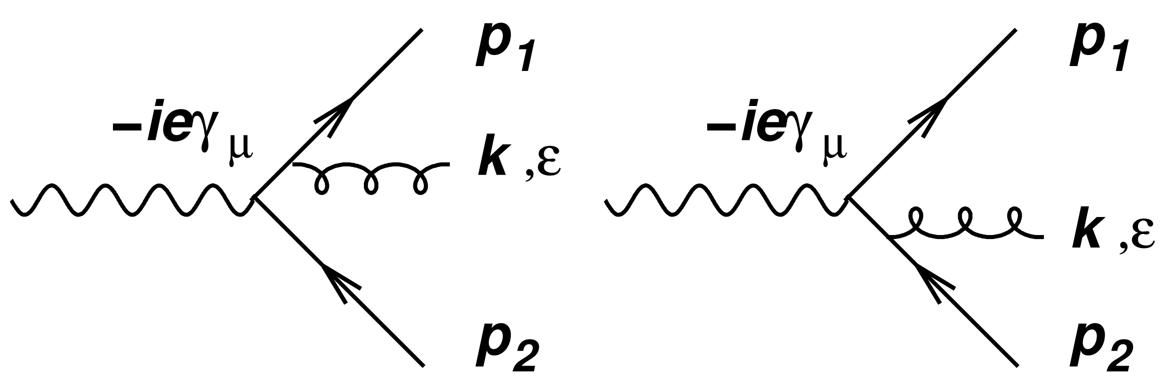
Consider as example $\gamma^* \to q\bar{q}$ (dijet production in e+e-):

LO



$$\mathcal{M}_{q\bar{q}} = -\bar{u}(p_1) i e_q \gamma_\mu v(p_2)$$

NLO-real



$$\mathcal{M}_{q\bar{q}g} = \bar{u}(p_1) i g_s \not{\epsilon} t^A \frac{i}{\not{p}_1 + \not{k}} i e_q \gamma_\mu v(p_2)$$

$$- \bar{u}(p_1) i e_q \gamma_\mu \frac{i}{\not{p}_2 + \not{k}} i g_s \not{\epsilon} t^A v(p_2)$$

$$\simeq \bar{u}(p_1) i e_q \gamma_\mu t^A v(p_2)\, g_s \left(\frac{p_1.\epsilon}{p_1.k} - \frac{p_2.\epsilon}{p_2.k} \right)$$

gluon *soft* $\equiv k \ll p_{1,2}$

(ignore terms suppressed by powers of k)

use: $\not{p} v(p) = 0,$
$\not{p}\not{k} + \not{k}\not{p} = 2p.k$

IR divergences

Squared amplitude:

$$|M^2_{q\bar{q}g}| \simeq \sum_{A,\mathrm{pol}} \left| \bar{u}(p_1) i e_q \gamma_\mu t^A v(p_2)\, g_s \left(\frac{p_1.\epsilon}{p_1.k} - \frac{p_2.\epsilon}{p_2.k} \right) \right|^2$$

$$= -|M^2_{q\bar{q}}| C_F g_s^2 \left(\frac{p_1}{p_1.k} - \frac{p_2}{p_2.k} \right)^2 = |M^2_{q\bar{q}}| C_F g_s^2 \frac{2p_1.p_2}{(p_1.k)(p_2.k)}$$

LO

➡ real radiation ME factorizes for soft gluon

Add the phase-space:

$$d\Phi_{q\bar{q}g}|M^2_{q\bar{q}g}| \simeq (d\Phi_{q\bar{q}}|M^2_{q\bar{q}}|) \underbrace{\frac{d^3\vec{k}}{2E(2\pi)^3} C_F g_s^2 \frac{2p_1.p_2}{(p_1.k)(p_2.k)}}_{d\mathcal{S}}$$

➡ real radiation phase-space factorizes for soft gluon

hard underlying contribution

soft-gluon emission:

$$d\mathcal{S} = EdE\, d\cos\theta \frac{d\phi}{2\pi} \cdot \frac{2\alpha_s C_F}{\pi} \frac{2p_1.p_2}{(2p_1.k)(2p_2.k)}$$

Factorization!

$\theta \equiv \theta_{p_1 k}$
$\phi =$ azimuth

IR divergences

Consider soft-gluon emission: $d\mathcal{S} = EdE\, d\cos\theta\, \frac{d\phi}{2\pi} \cdot \frac{2\alpha_s C_F}{\pi} \frac{2p_1.p_2}{(2p_1.k)(2p_2.k)}$

Rewrite "eikonal" in terms of E and **θ**: $\frac{2p_1.p_2}{(2p_1.k)(2p_2.k)} = \frac{1}{E^2(1-\cos^2\theta)}$

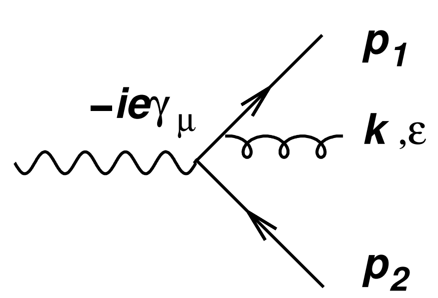
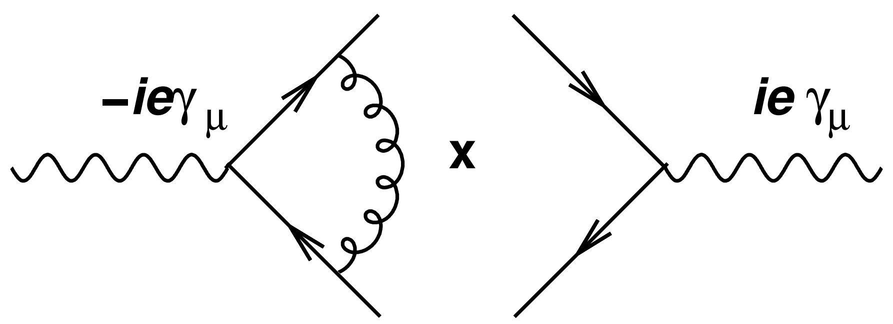
Thus we get:

$$d\mathcal{S} = \frac{2\alpha_s C_F}{\pi} \frac{dE}{E} \frac{d\theta}{\sin\theta} \frac{d\phi}{2\pi}$$

- diverges for E → 0: **soft divergence**
- diverges for **θ** → 0 or **θ** → **π**: **collinear divergence**

Note: the structure of these IR divergences is universal! → (N)NLO subtractions; → Parton-shower splitting kernels

For IR-safe observables we have (KLN theorem):

$\int d\Phi_3 \,\Big|$  $\Big|^2$ + $\int d\Phi_2$  ~ finte

R V

NLO Subtraction

- In an **analytical calculation** (of the phase-space) the IR divergences in the reals and the virtuals can be regularised in dim-reg → Corresponding **ε**-poles cancel explicitly in the sum R+V.
- This is not possible when the phase-space integration is performed with **MC methods**. V and R live in different phase-spaces, thus the integration has to be performed before they can be added:

$$d\hat{\sigma}_{\mathrm{NLO}} = \frac{1}{2s}\int d\Phi_n \left[|\mathcal{M}_{\mathrm{LO}}|^2 + 2\mathrm{Re}\{\mathcal{M}_{\mathrm{LO}}\mathcal{M}^*_{\mathrm{NLO,V}}\}\right] + \frac{1}{2s}\int d\Phi_{n+1}|\mathcal{M}_{\mathrm{NLO,R}}|^2 \quad \sim \text{finte}$$

∞ ∞

- Possible solutions:

  phase space slicing: global subtraction

  subtraction methods: local subtraction

Phase space slicing

- Idea: cut-off the real radiation phase-space in the IR, e.g. $E > \delta E_{cut}$ and $1 - \cos\Delta\theta > \delta\theta_{cut}$

$$d\hat{\sigma}_{\mathrm{NLO}} = \frac{1}{2s}\int d\Phi_n \left[|\mathcal{M}_{\mathrm{LO}}|^2 + 2\mathrm{Re}\{\mathcal{M}_{\mathrm{LO}}\mathcal{M}^*_{\mathrm{NLO,V}}\}\right] + \frac{1}{2s}\int d\Phi_{n+1}|\mathcal{M}_{\mathrm{NLO,R}}|^2$$

~-1/ε

$$\frac{1}{2s}\int_{>\delta E_{\mathrm{cut}},>\delta\theta_{\mathrm{cut}}} d\Phi_{n+1}|\mathcal{M}_{\mathrm{NLO,R}}|^2 + \frac{1}{2s}\int_{<\delta E_{\mathrm{cut}},<\delta\theta_{\mathrm{cut}}} d\Phi_{n+1}|\mathcal{M}_{\mathrm{NLO,R}}|^2$$

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Integrate numerically

$\sim \ln^2(\delta E_{cut}) + \ln^2(\delta\theta_{cut})$ = finite

soft/eikonal approximation

$$\frac{1}{2s}\int d\Phi_n dS|\mathcal{M}_{\mathrm{LO}}|^2$$

Integrate *dS* analytically

$\sim 1/\varepsilon - \ln^2(\delta E_{cut}) - \ln^2(\delta\theta_{cut})$

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$\sim -1/\varepsilon$

$$\frac{1}{2s}\int_{>\delta E_{\rm cut},>\delta\theta_{\rm cut}} d\Phi_{n+1}|\mathcal{M}_{\rm NLO,R}|^2 + \frac{1}{2s}\int_{<\delta E_{\rm cut},<\delta\theta_{\rm cut}} d\Phi_{n+1}|\mathcal{M}_{\rm NLO,R}|^2$$

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Integrate dS analytically

$\sim 1/\varepsilon - \ln^2(\delta E_{cut}) - \ln^2(\delta\theta_{cut})$

ε-poles cancel pointwise in LO PS

= finite and independent of δE_{cut} & $\delta\theta_{cut}$

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$\sim -1/\varepsilon$

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= finite and independent of δE_{cut} & $\delta\theta_{\text{cut}}$

- However: cancellation (possibly several orders of magnitude) of δE_{cut} & $\delta\theta_{\text{cut}}$ dependence happens only numerically!

➡ Very bad numerical convergence!

➡ At least at NLO not used anymore.

Phase space slicing

- Idea: cut-off the real radiation phase-space in the IR,

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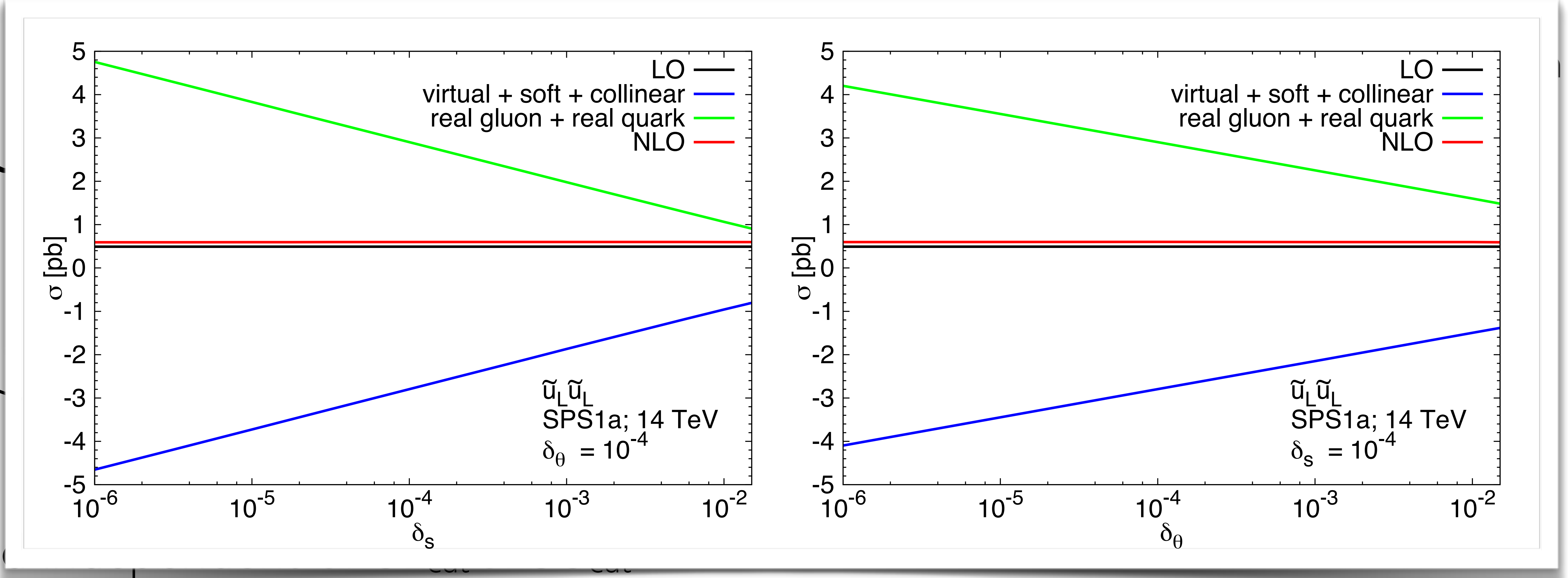
~-1/ε

$$\frac{1}{2s}\int_{>\delta E_{\rm cut},>\delta\theta_{\rm cut}} d\Phi_{n+1}|\mathcal{M}_{\rm NLO,R}|^2 + \frac{1}{2s}\int_{<\delta E_{\rm cut},<\delta\theta_{\rm cut}} d\Phi_{n+1}|\mathcal{M}_{\rm NLO,R}|^2$$

ε-poles cancel pointwise in LO PS



= finite an

- However: cancellation (possibly several orders of magnitude) of δE_{cut} & $\delta\theta_{cut}$ dependence happens only numerically!

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NLO subtraction

- Add and subtract a term S that cancels locally the divergences in the real radiation (e.g. based on the soft/eikonal approximation):

$$d\hat{\sigma}_{\text{NLO}} = \frac{1}{2s}\int d\Phi_n \left[|\mathcal{M}_{\text{LO}}|^2 + 2\text{Re}\{\mathcal{M}_{\text{LO}}\mathcal{M}^*_{\text{NLO,V}}\}\right] + \frac{1}{2s}\int d\Phi_{n+1}|\mathcal{M}_{\text{NLO,R}}|^2$$

$$= \frac{1}{2s}\int d\Phi_n \left[|\mathcal{M}_{\text{LO}}|^2 + 2\text{Re}\{\mathcal{M}_{\text{LO}}\mathcal{M}^*_{\text{NLO,V}}\}\right] + \frac{1}{2s}\int d\Phi_{n+1}|\mathcal{M}_{\text{NLO,R}}|^2 \underbrace{-S + S}_{0}$$

- Integrate this subtraction term analytically over the emission phase space: $I = \int d\Phi_1 S$

$$= \frac{1}{2s}\int d\Phi_n \left[|\mathcal{M}_{\text{LO}}|^2 + \underbrace{2\text{Re}\{\mathcal{M}_{\text{LO}}\mathcal{M}^*_{\text{NLO,V}}\} + I}_{\text{finite}}\right] + \frac{1}{2s}\int d\Phi_{n+1}\underbrace{|\mathcal{M}_{\text{NLO,R}}|^2 - S}_{\text{finite}}$$

The subtraction term S

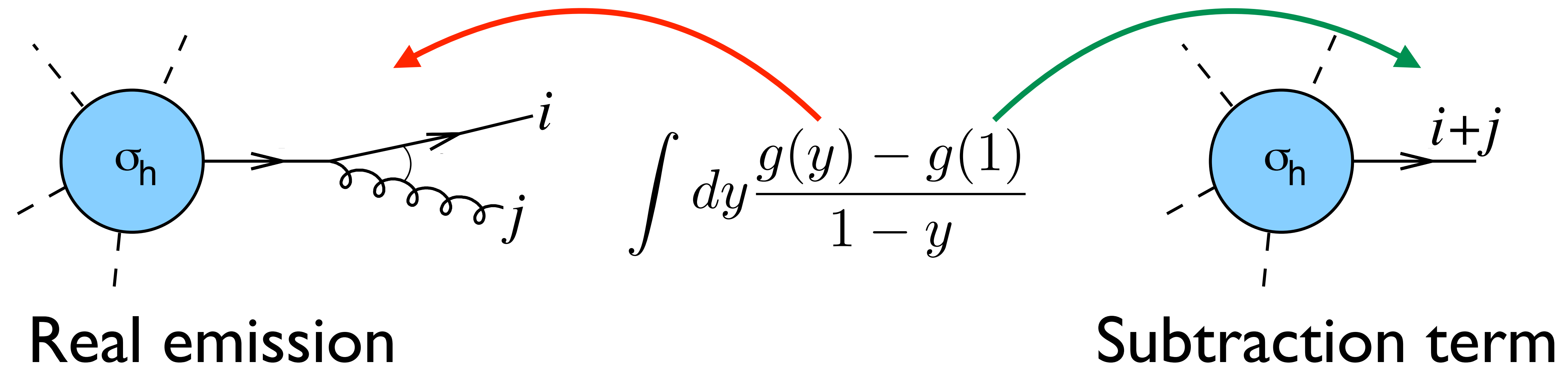
The subtraction term S should be chosen such that:

- it matches the singular behaviour of R
- it can be integrated numerically in a convenient way
- it can be integrated analytically over one-particle subspace.
  In d dimensions this yields to explicit poles in regulator.
- It is universal, i.e. process independent (overall factor times Born)

Procedure systematized in the seminal papers of

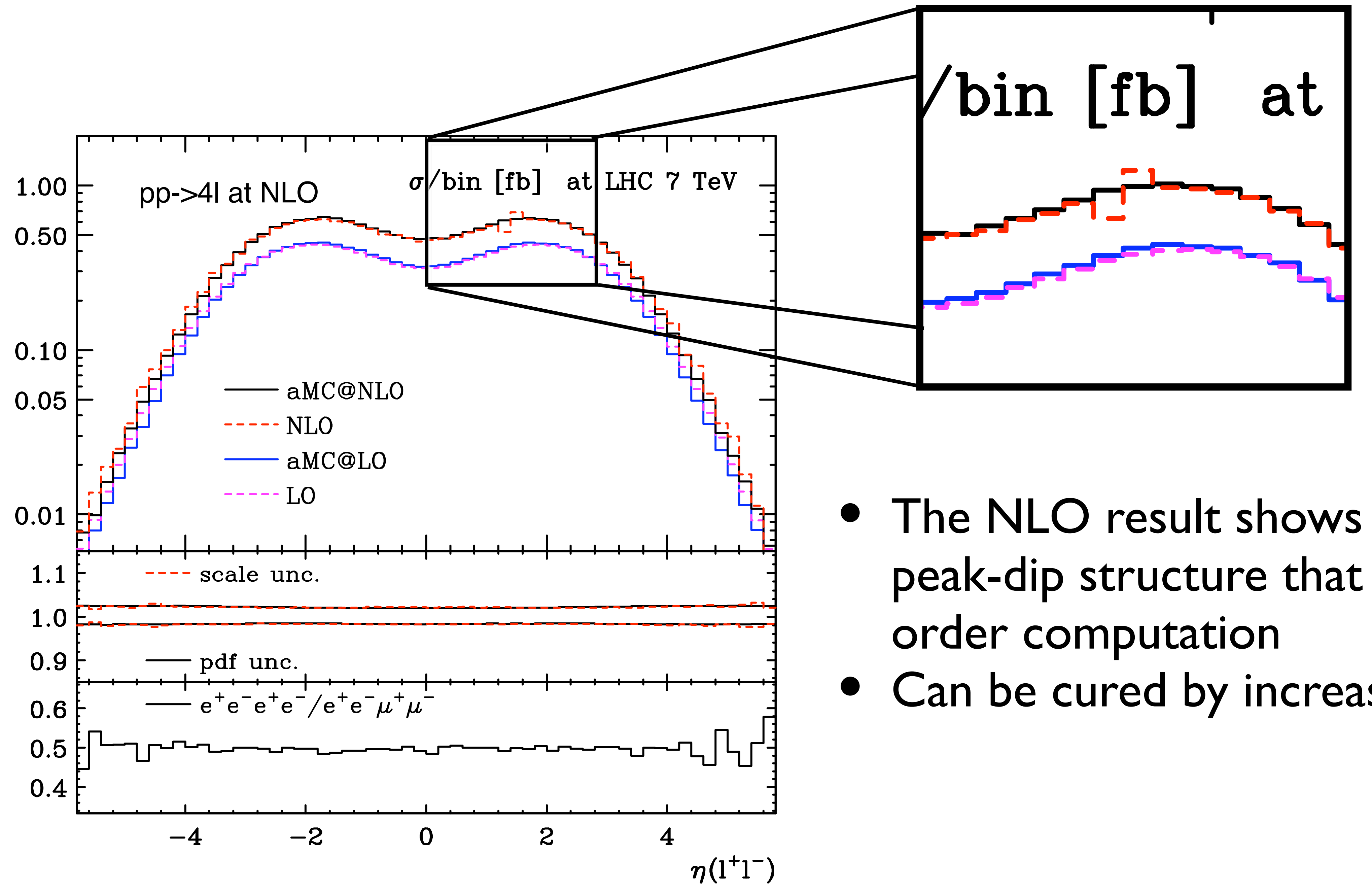
- Catani-Seymour (**dipole/CS subtraction**, '96): Sherpa, Herwig7, HELAC-NLO, Munich
- Frixione-Kunszt-Signer (**FKS subtraction**, '96): POWHEG-BOX, MadGraph_aMC@NLO

Recoil and misbinning



- Assuming *i* and *j* are on-shell in the real emission event, in the subtraction event the combined particle *i+j* has to be on-shell
- *i+j* can only be on-shell when other particles are reshuffled

→It can happen that event and counter event end up in different histogram bins



- The NLO result shows the typical peak-dip structure that hampers fixed-order computation
- Can be cured by increasing the statistics

→Difficult to achieve arbitrary resolution with numerical integration.

Subtraction schemes

Dipole CS subtraction

- widely used
- automated in Sherpa, MadDipole, Helac-NLO
- Scaling of subtraction terms: N^3
- Recoil (momentum shift) of an *emitter* taken by one specific *spectator*
- Proven to work efficiently for simple and complicated processes

FKS subtraction

- Somewhat less popular
- automated in MadGraph5_aMC@NLO, POWHEG-BOX
- Scaling of subtraction terms: N^2
- Recoil (momentum shift) of an *emitter* taken by all particles
- Proven to work efficiently for simple and complicated processes

The Kinoshita-Lee-Nauenberg theorem

- After the renormalization procedure IR poles remain.
- However there is the very fundamental (based on the structure of the S-matrix) Kinoshita-Lee-Nauenberg (KLM) theorem *[Kinoshita; '62, Lee, Nauenber; '64]*:

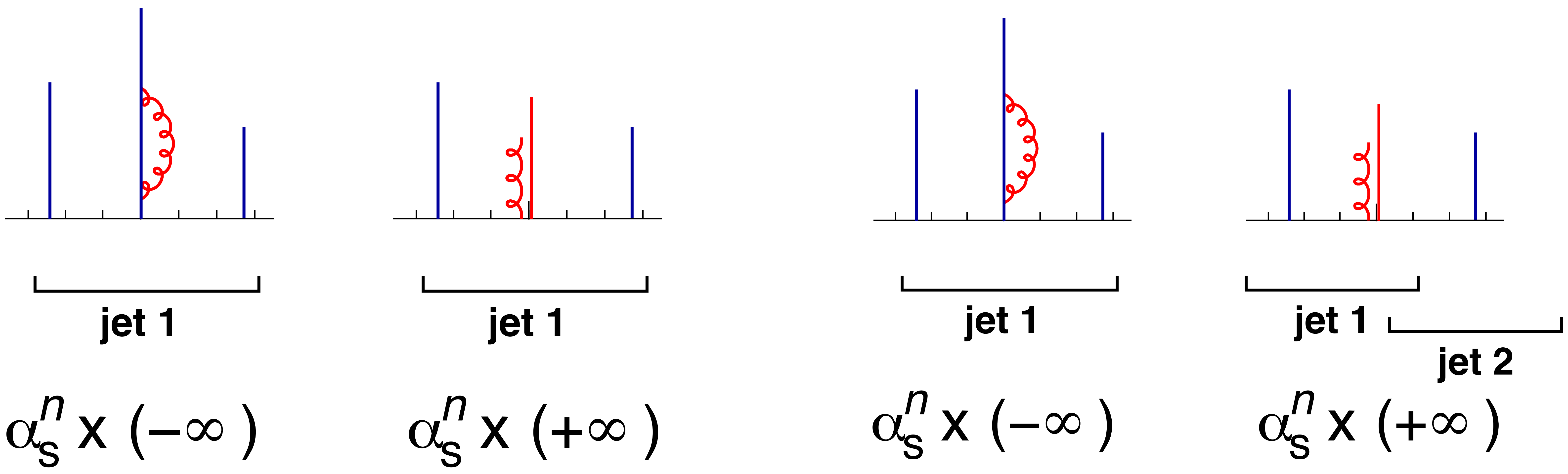
Measurable quantities, summed over indistinguishable states are free from IR divergences.

$\sim$

For IR-safe observables IR divergences in the virtual corrections cancel with corresponding divergences in the (indistinguishable) real radiation

IR safety

IR safe | IR unsafe



Infinities cancel | **Infinities do not cancel**

In the IRC unsafe algorithm, a collinear splitting leads to a different set of final state jets and thus to the lack of cancellation of soft and collinear divergences (KLN theorem)

An observable O is infrared and collinear safe if

$$\mathcal{O}_{n+1}(k_1, k_2, \ldots, k_i, k_j, \ldots k_n) \to \mathcal{O}_n(k_1, k_2, \ldots k_i + k_j, \ldots k_n)$$

whenever one of the ki/kj becomes soft or ki and kj are collinear

i.e. the observable is insensitive to emission of soft particles or to collinear splittings.

IR safety: examples

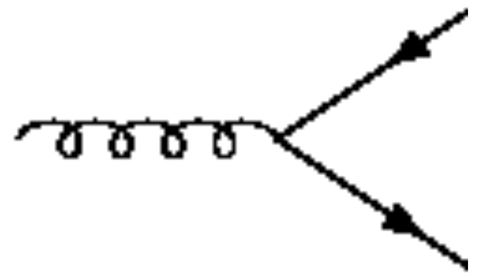
Are these observables IR safe?

- energy of the hardest parton in an event
- multiplicity of gluons
- momentum flow into a cone in rapidity and angle
- cross-section for producing one gluon with $E > E_{min}$ and $\theta > \theta_{min}$
- jet cross-sections

IR safety: examples

Are these observables IR safe?

- energy of the hardest parton in an event **NO**



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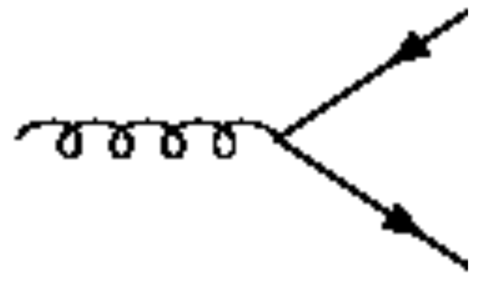

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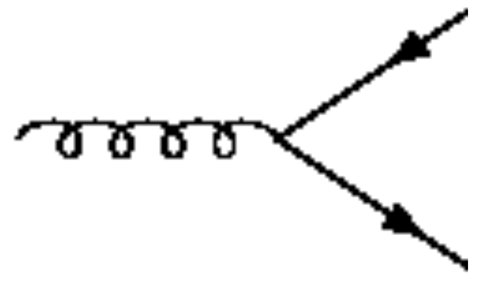

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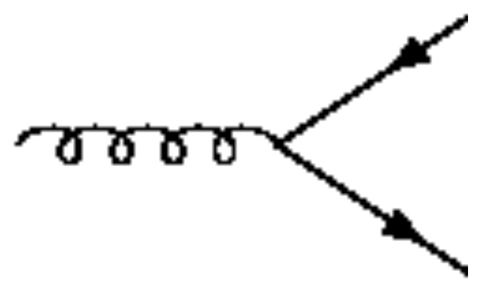

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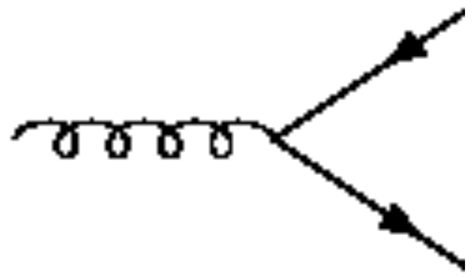
- jet cross-sections **DEPENDS**



IR safety: examples

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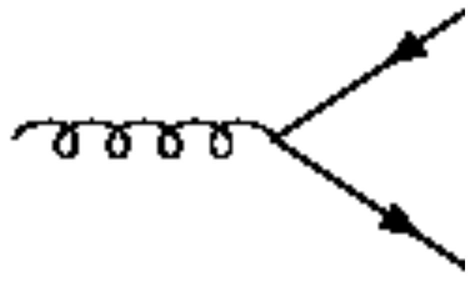

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  (i.e. all jet definitions used nowadays)

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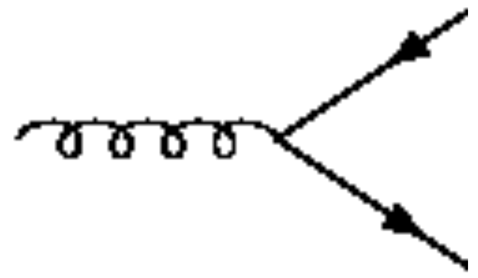


Recap

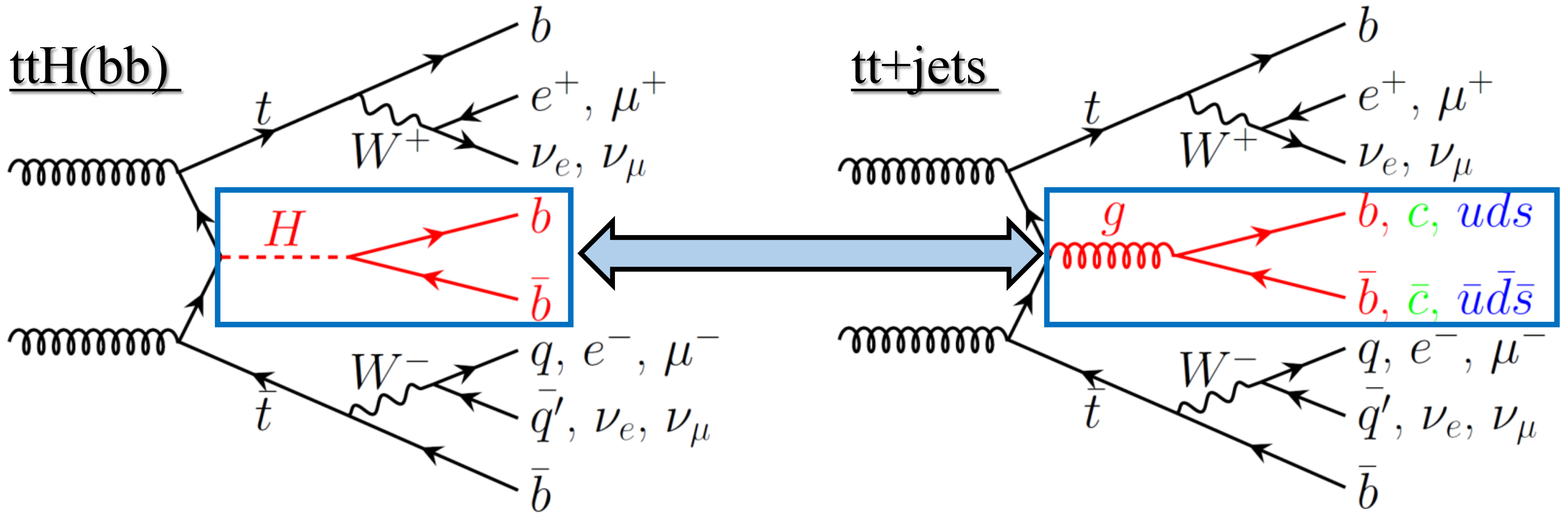
- Higher-order corrections mandatory for reliable predictions at the LHC
- NLO corrections often O(100%)
- ΔNLO=V+R
- ΔNNLO=VV+RV+RR
- KLN theorem ensures IR finiteness of corrections at each perturbative order…
- …for IR safe observables

Questions?

Motivation: NLO multileg

- For a multitude of multileg processes NLO predictions are needed for current state-of-the-art applications at the LHC.
- Consider for example ttH production, which is very important for the direct determination of the top-Yukawa coupling.
- Due to the small cross section we have to consider the tricky H ➝bb decay mode:

signal   background



- signal: $\mathcal{O}(1 \times 10^3)$ graphs
- background: $\mathcal{O}(3 \times 10^3)$ graphs
- 1-loop signal: $\mathcal{O}(2 \times 10^5)$ graphs
- 1-loop background: $\mathcal{O}(3 \times 10^5)$ graphs
- real(extra gluon) signal: $\mathcal{O}(1 \times 10^4)$ graphs
- real(extra gluon) background: $\mathcal{O}(5 \times 10^4)$ graphs

- need milions of evaluations in practical Monte Carlo calculations
- need many partonic processes
- similar for many other processes besides ttH production

➜ efficient automation needed!

Tree Matrix elements

$$\mathcal{L} \to |\mathcal{M}_{\text{tree}}|^2$$

(I) Textbook Feynman diagram construction:

1. draw all Feynman diagrams
2. put in the explicit Feynman rules and get the amplitude
3. do some algebra, simplifications
4. square the amplitude
5. sum/average over outgoing/incoming states

Automated tools for
(1-2): FeynArts/QGRAF
(3-5): FormCalc/Form/CompHEP/CalcHEP

Bottlenecks

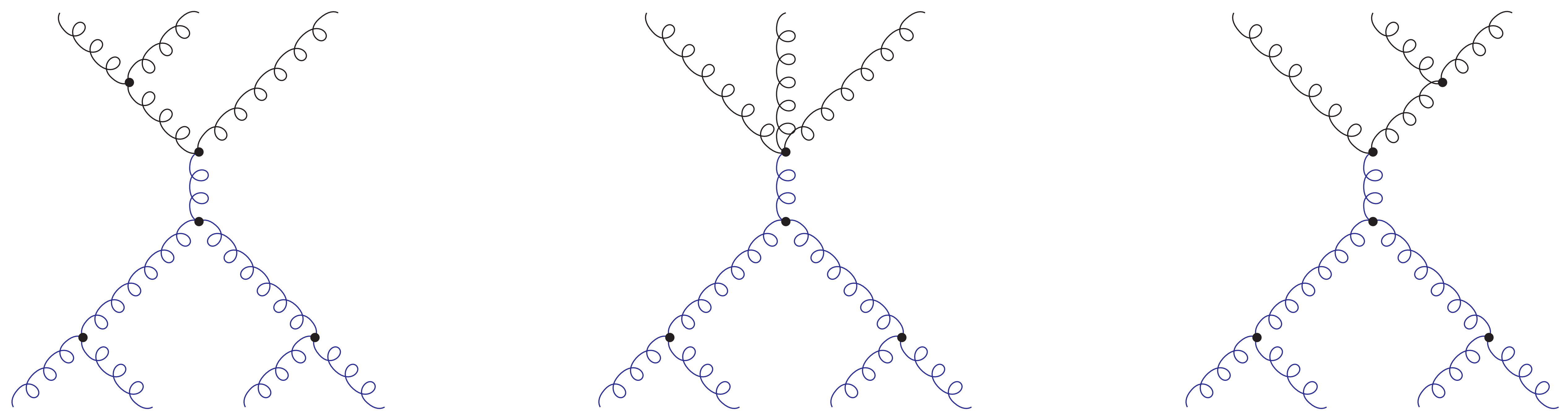
a) number of Feynman diagrams grows factorially!
b) algebra becomes more cumbersome with more particles

Number of Feynman diagrams contributing to $gg \to ng$ at tree level:

2	4
3	25
4	220
5	2485
6	34300
7	559405
8	10525900

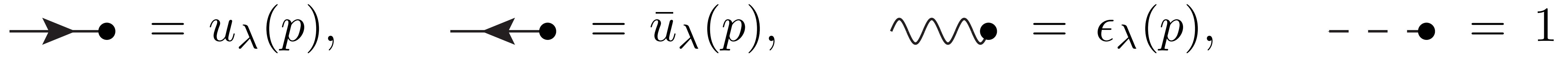
Helicity amplitudes

- The Monte Carlo integration has to be performed numerically
  ➜ only numerical representation of the amplitudes required
- Idea: construct the amplitude purely numerical (as complex numbers) from numerical representation of external wave-functions and spinors combined with vertex routines.
  ➜ squaring the amplitude becomes very cheap (squaring complex numbers is numerically cheap)
- Build algorithms that constructs amplitudes automatically
- Try to recycle sub-expressions as much as possible, e.g.



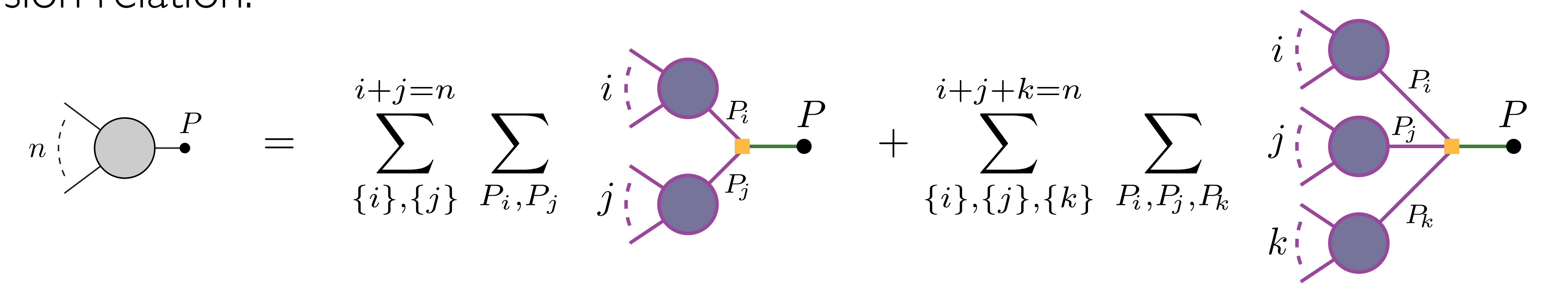
Recurrence relations

- Pioneered by *[Berends, Giele; '88]* algorithms based on recurrence relations can be designed that calculate SM tree-level helicity amplitudes with complexity $\mathcal{O}(n^4)$
- off-shell currents for external legs are wave functions



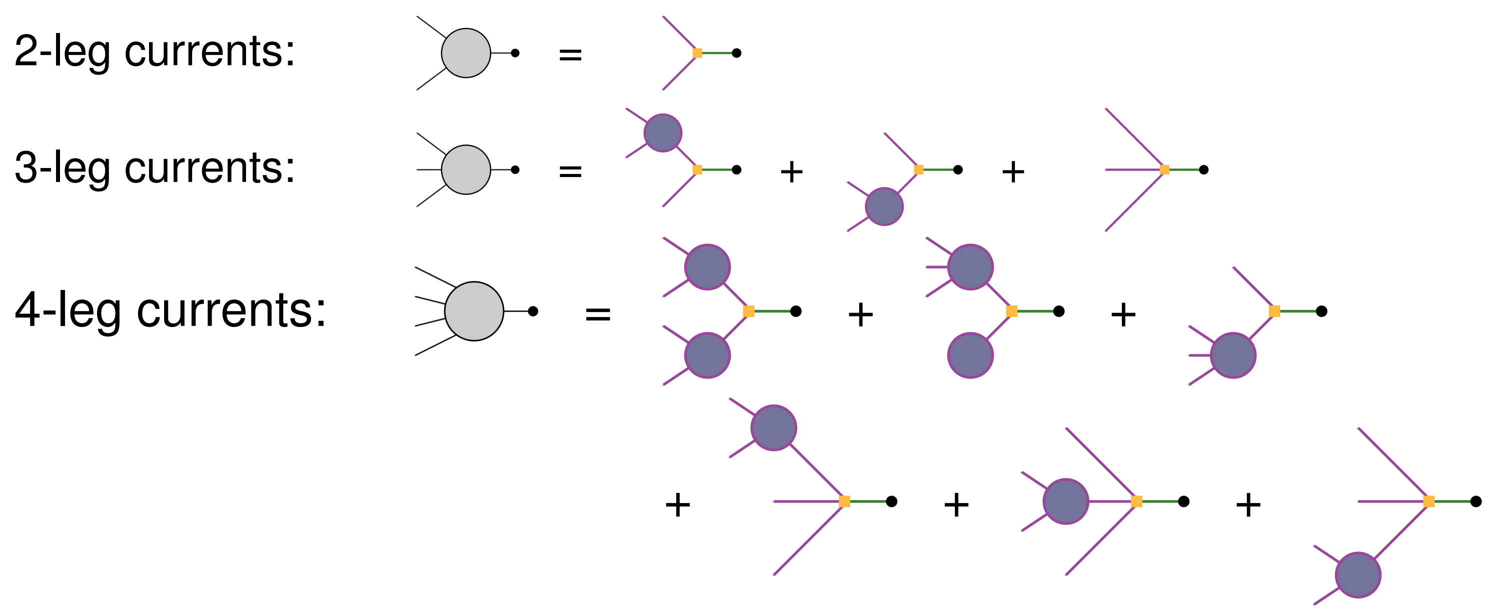
- Amplitude for process with N external particles:

- Recursion relation:



incoming currents $\times$ **vertex** $\times$ **propagator**

- In the SM we have:



No Feynman diagrams are calculated in this approach!

Recurrence relations: history

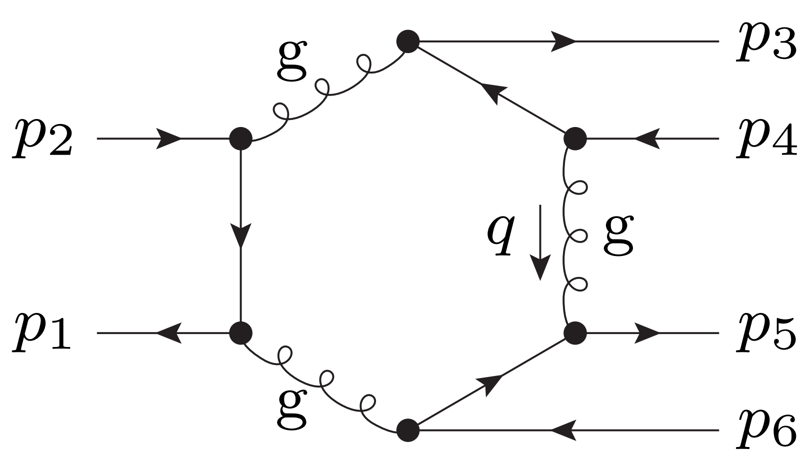
- Berends, Giele 1987: planar multi-gluon amplitudes
- Caravaglios, Moretti 1995: formulation for arbitrary lagrangians
- Draggiotis, Kleiss, Papadopoulos 1998: multi-gluon amplitudes
- Caravaglios, Mangano, Moretti, Pittau 1998: multi-jet processes
- Kanaki, Papadopoulos 1999: HELAC (standard model) →MadGraph
- Moretti, Ohl, Reuter 2001: O'Mega →Whizard
- Mangano, Moretti, Piccinini, Pittau, Polosa 2003: ALPGEN
- Gleisberg, Hoeche 2008: Comix →Sherpa

➡Used in all modern amplitude generators

Traditional NLO Matrix elements

- For one-loop matrix elements the complexity based on Feynman diagrams is worse than factorial.
- Conventional approach, e.g. $q\bar{q} \to q\bar{q}q\bar{q}$

Typical 6-point amplitude:



p_k momenta
σ_k helicities
i_k colour indices

↓ Feynman rules

$$\mathcal{M}^{\sigma_1\ldots\sigma_6}_{i_1\ldots i_6}(p_1,\ldots,p_6) = g^6 (T^aT^b)_{i_1i_2}(T^bT^c)_{i_3i_4}(T^cT^a)_{i_5i_6}$$
$$\times \int \mathrm{d}^D q \, \frac{\bar{u}(p_5,\sigma_5)\gamma^\mu(\not{q}+\not{p}_5+m_5)\gamma^\nu v(p_6,\sigma_6)}{(q^2)[(q+p_5)^2-m_5^2]}$$
$$\times \frac{\bar{v}(p_1,\sigma_1)\gamma^\nu(\not{q}+\not{p}_5+\not{p}_6-\not{p}_1+m_1)\gamma^\lambda u(p_2,\sigma_2)}{[(q+p_5+p_6)^2][(q+p_5+p_6-p_1)^2-m_1^2]}$$
$$\times \frac{\bar{u}(p_3,\sigma_3)\gamma^\lambda(\not{q}-\not{p}_4+m_3)\gamma^\mu v(p_4,\sigma_4)}{[(q-p_3-p_4)^2][(q-p_4)^2-m_3^2]}$$

Traditional NLO Matrix elements

- Define tensor integrals:

$$F^{\mu_1\cdots\mu_P}(k_1,\ldots,k_5,m_0,\ldots,m_5) = \frac{(2\pi\mu)^{4-D}}{\mathrm{i}\pi^2}\int \mathrm{d}^D q\, \frac{q^{\mu_1}\cdots q^{\mu_P}}{N_0 N_1 \ldots N_5}$$

denominator factors:
$N_i = (q+k_i)^2 - m_i^2 + \mathrm{i}\epsilon, \quad k_0 = 0$
$i = 0,\ldots,5$

- Construct decomposition of the diagram into "tensor-integrals" and "standard matrix elements"

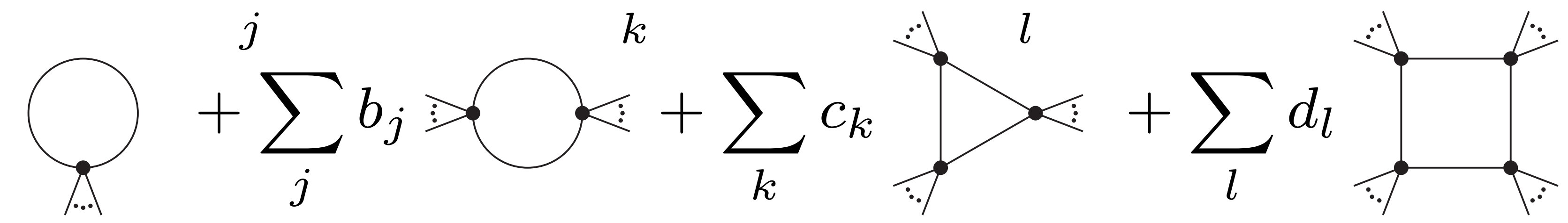
$$\mathcal{M}^{\sigma_k}_{i_k}(p_k) = \underbrace{\mathcal{C}_{i_k}}_{\text{factorised colour structure}} \sum_m \mathcal{F}_m(\{p_a\cdot p_b\})\, \underbrace{\hat{\mathcal{M}}^{\{\lambda_k\}}_m(\{p_k\})}_{\text{standard matrix elements}}$$

- "standard matrix elements": purely kinematical objects
- This decomposition can be automated, but large algebraic expressions! (e.g. FeynArts)
  ➜ **used to be a major bottleneck**
- Reduce tensor-integrals to scalar integrals:

$$\mathcal{F}_m(\{p_a\cdot p_b\}) = \sum_{j_1\ldots j_R} \mathcal{K}_{m,j_1\ldots j_R}(\{p_a\cdot p_b\})\, \overbrace{T_{j_1\ldots j_R}(\{p_a\cdot p_b\})}^{\text{tensor loop coefficients}}$$

$$T_{j_1\ldots j_R} = \sum_i a_i A_0(i) + \sum_j b_j B_0(j) + \sum_k c_k C_0(k) + \sum_l d_l D_0(l)$$

$$= \sum_i a_i \quad + \sum_j b_j \quad + \sum_k c_k \quad + \sum_l d_l$$



A0 B0 C0 D0

⇒ Every one-loop amplitude can be reduced to A0, B0, C0 and D0.

NLO Matrix elements: Solutions

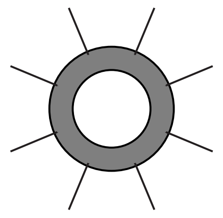
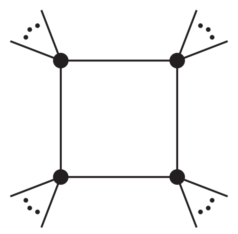
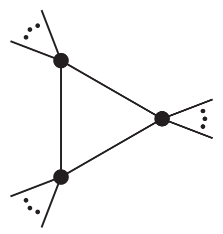
- unitarity-cut techniques (e.g. BlackHat, CutTools)
- extension of recursion-relation technique to NLO (e.g. Recola)
- combination of Feynman diagrams and recursion relations (e.g. OpenLoops)

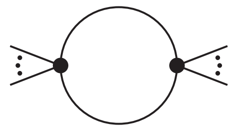
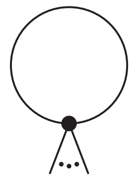
…

All methods: **numerical** ⇒ problem of numerical stability

Unitarity methods: Ideas

- Starting point: In 4 + **ε** dimensions, any one-loop amplitude can be represented by a linear combination of scalar one-loop integrals

$$\mathcal{M}^{\text{1-loop}} =$$  $$= \sum_l d_l$$  $$+ \sum_k c_k$$ 

$$+ \sum_j b_j$$  $$+ \sum_i a_i$$  $$+ R$$

$$= \sum_l d_l D_0(l) + \sum_k c_k C_0(k) + \sum_j b_j B_0(j) + \sum_i a_i A_0(i) + R$$

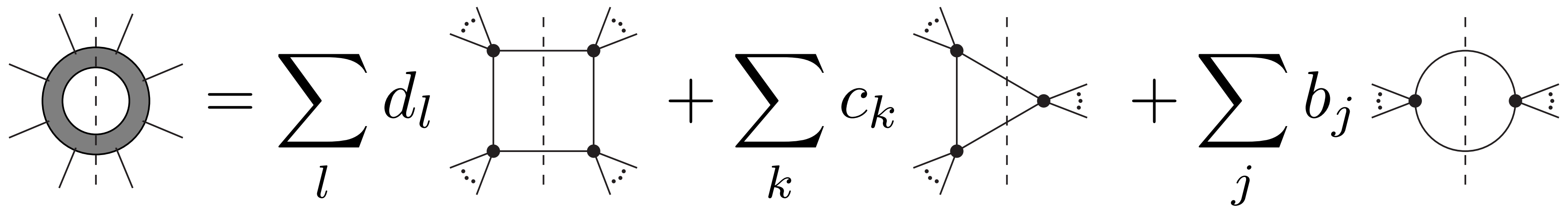
- calculation of amplitude ⇔ determination of coefficients a_i,b_j,c_k,d_l and R

- these coefficients can be determined from cuts (=on-shell propagators) of the one-loop diagram (“unitarity methods”)

Unitarity methods

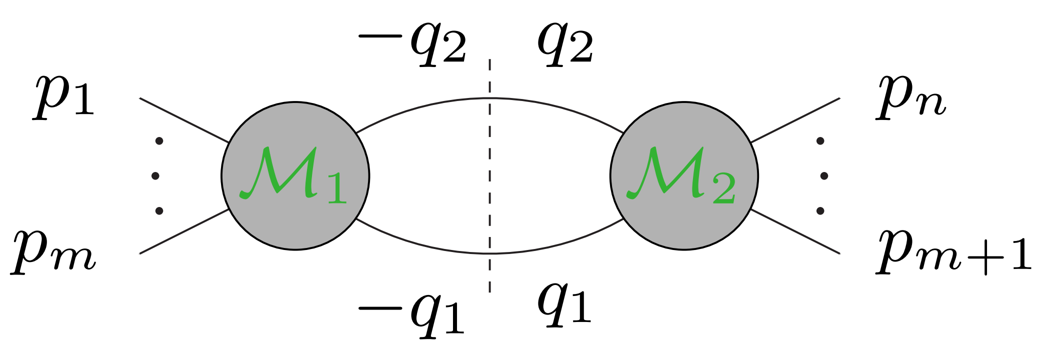
- Conventional Unitarity: only double cuts:

$-\mathrm{i}\,\mathrm{Disc}\,\mathcal{M}^{\text{1-loop}} =$



- Cut: one-loop diagram ➜ two tree diagrams

$$\frac{\mathrm{i}}{q_1^2 - m_1^2} \to 2\pi\delta^+(q_1^2 - m_1^2), \qquad \frac{\mathrm{i}}{q_2^2 - m_2^2} \to 2\pi\delta^+(q_2^2 - m_2^2)$$



$$-\mathrm{i}\,\mathrm{Disc}\,\mathcal{M}^{\text{1-loop}}(p_1,\ldots,p_n) = \int [\mathrm{d}q]\, 2\pi\delta^+(q_1^2 - m_1^2)\, 2\pi\delta^+(q_2^2 - m_2^2)$$

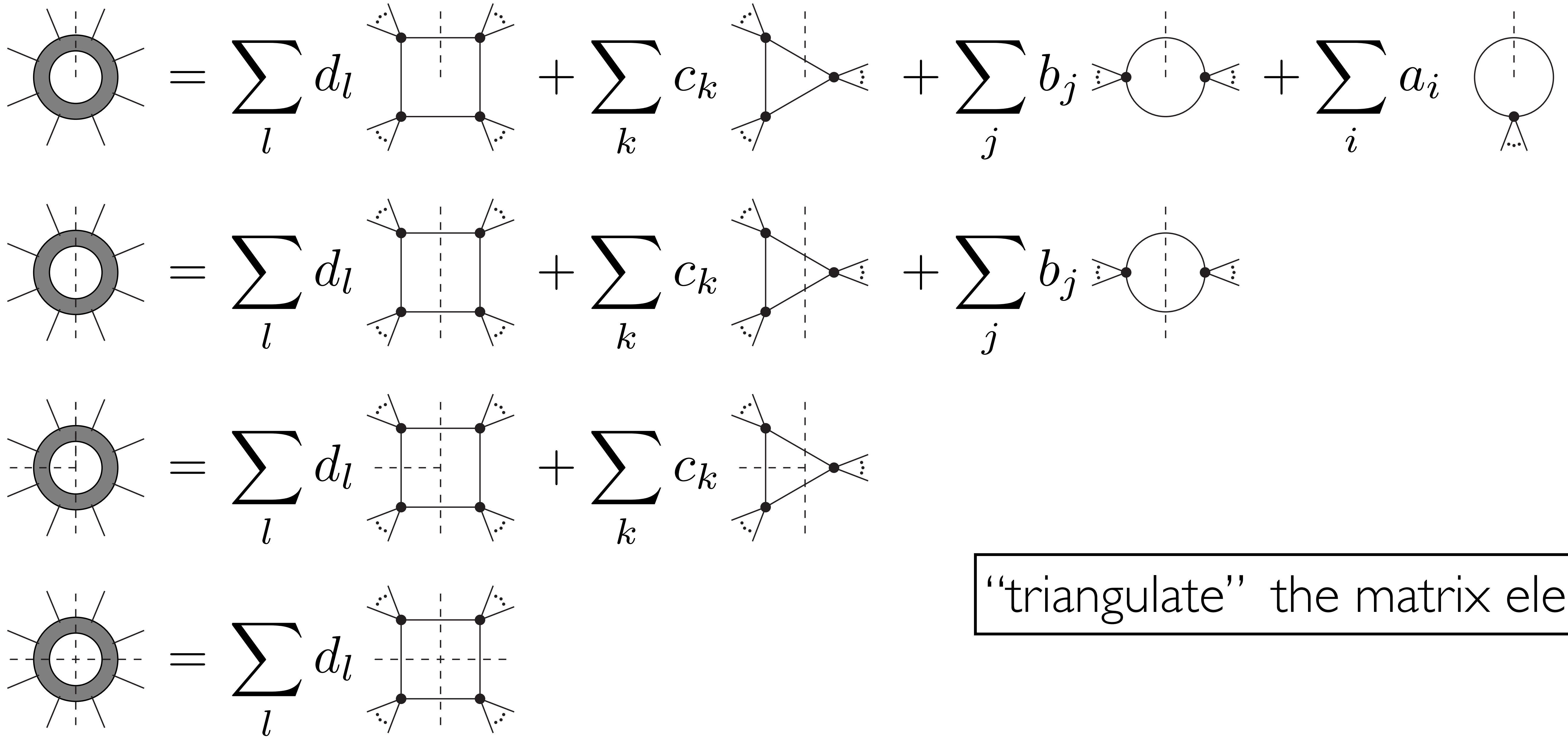
$$\times\, \mathcal{M}_1^{\text{tree}}(p_1,\ldots,p_m,q_1,-q_2) \times \mathcal{M}_2^{\text{tree}}(p_{m+1},\ldots,p_n,-q_1,q_2)$$

- relates one-loop amplitudes to products of tree amplitudes

- reconstruct the coefficients from the cuts on both sides of the equation

Generalized unitary

- Consider all kinds of cuts:

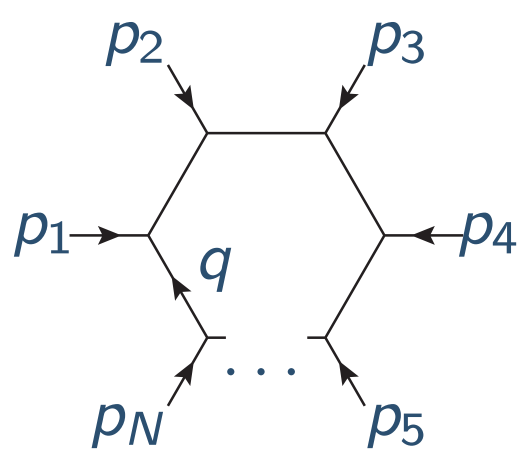


“triangulate” the matrix element

- OPP = Generalized Unitarity at the integrand-level: requires multiple evaluation of the numerator function N(q)

OpenLoops recursion

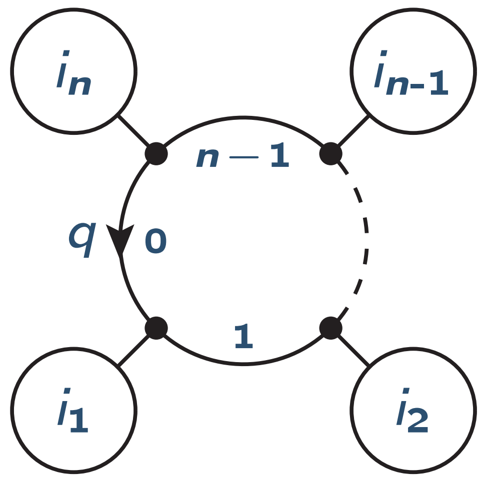
- Factorize one-loop amplitude into colour factors, tensor coefficients and tensor integrals



$$= \mathcal{C} \cdot \sum_{r=0}^{R} \mathcal{N}_r^{\mu_1 \ldots \mu_r} \cdot \int d^d q \, \frac{q_{\mu_1} \ldots q_{\mu_r}}{D_0 \, D_1 \, \ldots \, D_{N-1}}$$

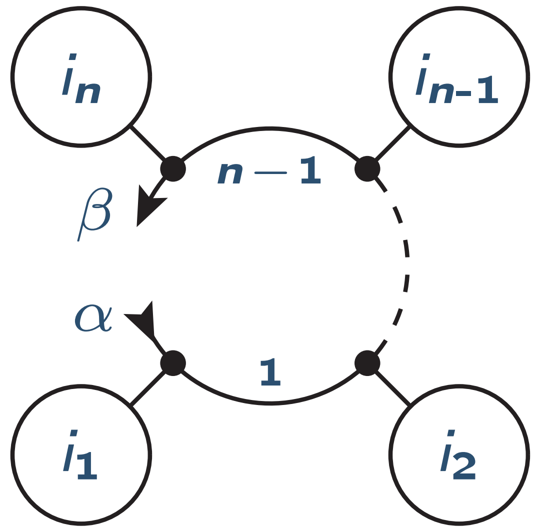
$$D_i = (q + \textstyle\sum_{\ell=0}^{i} p_\ell)^2 - m_i^2$$

- Treat one-loop diagram as ordered set of sub-trees $\mathcal{I}_n = \{i_1, \ldots, i_n\}$ connected by propagators



cut one loop propagator

$\mathcal{N}_\alpha^\beta(\mathcal{I}_n; q) =$



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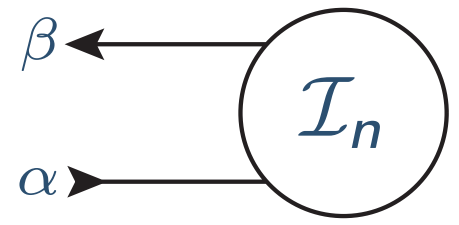
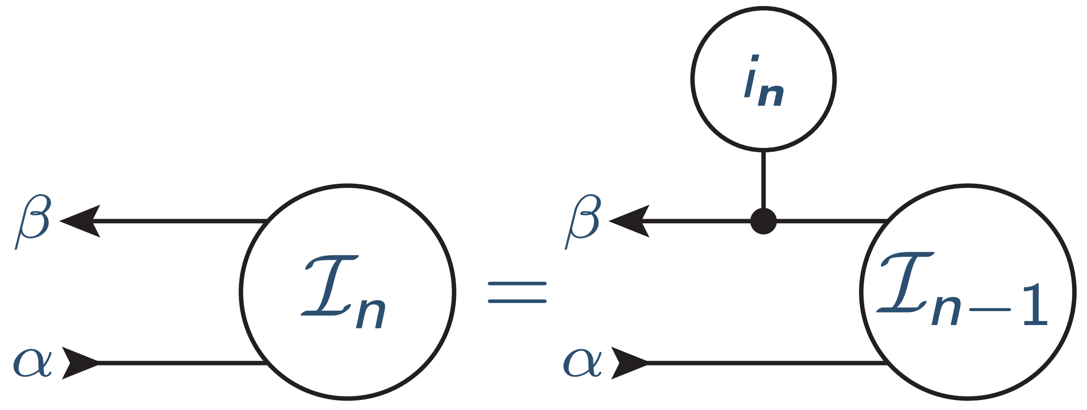


- Build numerator connecting subtrees along the loop



$$\mathcal{N}_\alpha^\beta(\mathcal{I}_n; q) = X_{\gamma\delta}^{\beta}(q) \, \mathcal{N}_\alpha^\gamma(\mathcal{I}_{n-1}; q) \, w^\delta(i_n)$$

OpenLoops recursion

- Recursively build "open loops" polynomials $\mathcal{N}^{\beta}_{\mu_1\dots\mu_r;\alpha}$
  - disentangle loop momentum q from the coefficients

$$\mathcal{N}^{\beta}_{\alpha}(\mathcal{I}_n; q) = \sum_{r=0}^{n} \mathcal{N}^{\beta}_{\mu_1\dots\mu_r;\alpha}(\mathcal{I}_n)\, q^{\mu_1} \dots q^{\mu_r} \qquad X^{\beta}_{\gamma\delta} = Y^{\beta}_{\gamma\delta} + q^{\nu} Z^{\beta}_{\nu;\gamma\delta}$$

  - recursion in d=4:

$$\mathcal{N}^{\beta}_{\mu_1\dots\mu_r;\alpha}(\mathcal{I}_n) = \left[Y^{\beta}_{\gamma\delta}\, \mathcal{N}^{\gamma}_{\mu_1\dots\mu_r;\alpha}(\mathcal{I}_{n-1}) + Z^{\beta}_{\mu_1;\gamma\delta}\, \mathcal{N}^{\gamma}_{\mu_2\dots\mu_r;\alpha}(\mathcal{I}_{n-1}) \right] w^{\delta}(i_n)$$

  - model and process independent algorithm
  - numerical implementation requires only universal building blocks, derived from the Feynma rules of the theory
- **ε**-dimensional part of the numerator x poles of the tensor integrals yield R_2 rational terms

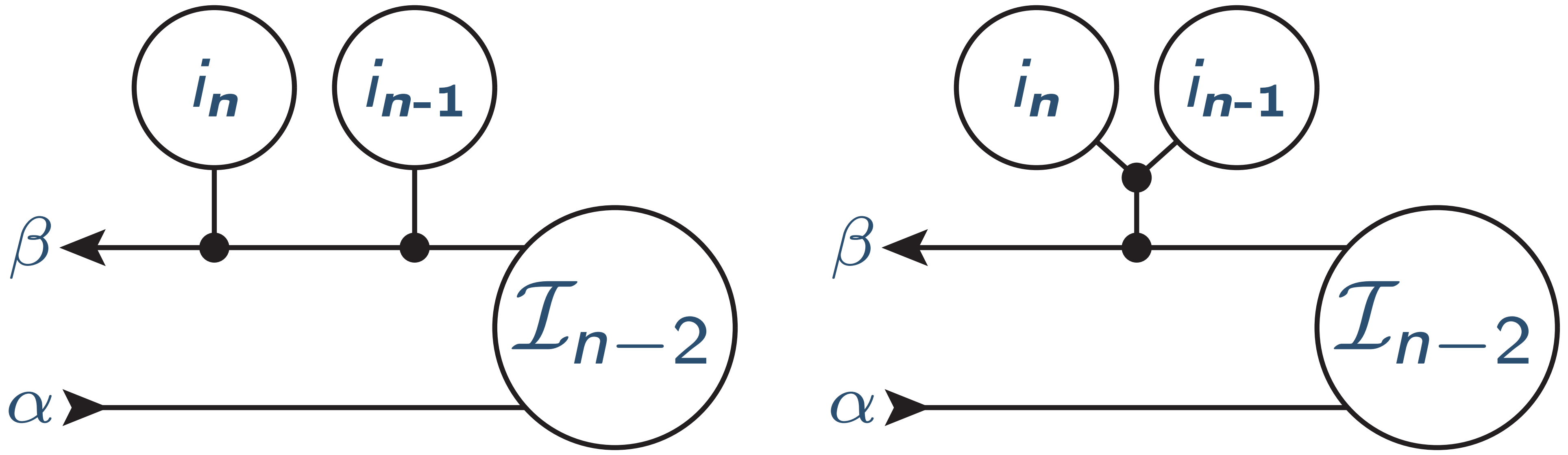
$$R_2 = ([\mathcal{N}]_{d=4-2\epsilon} - [\mathcal{N}]_{d=4}) \cdot [TI]_{UV}$$

  - numerical recursion in D=4 ➞ restore R_2 via process independent counter terms

[Draggiotis, Garzelli, Malamos, Papadopoulos, Pittau '09, '10; Shao, Zhang, Chao '11]

OpenLoops recursion: recycle loop structures

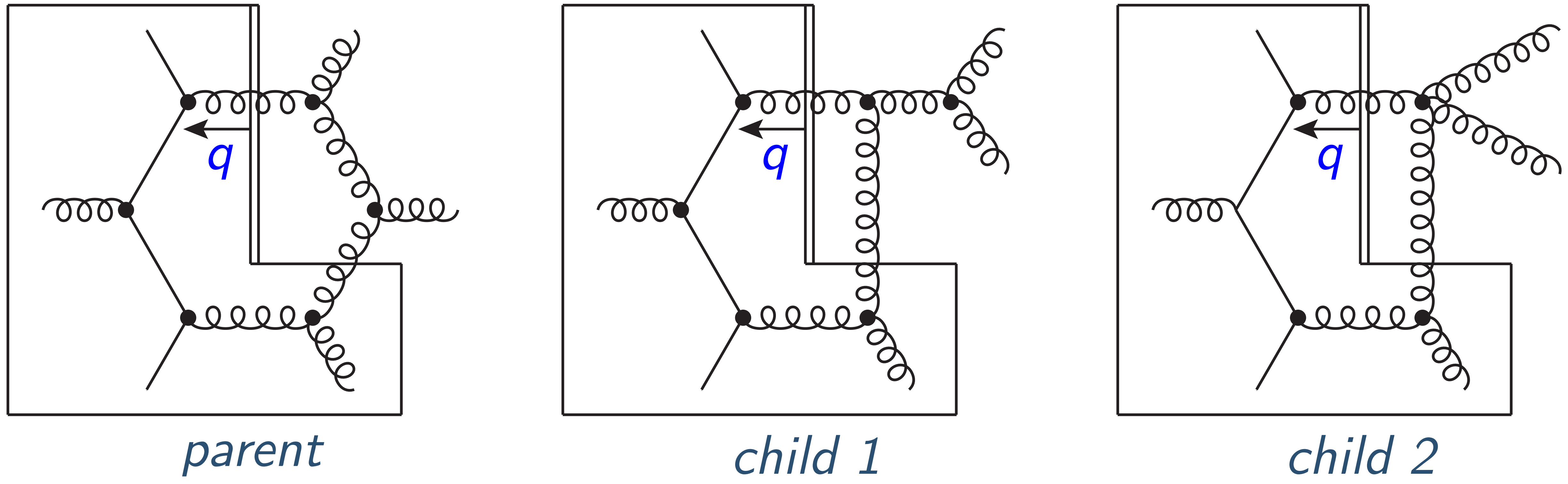
OpenLoops recycling:



Lower-point open-loops can be shared between diagrams if

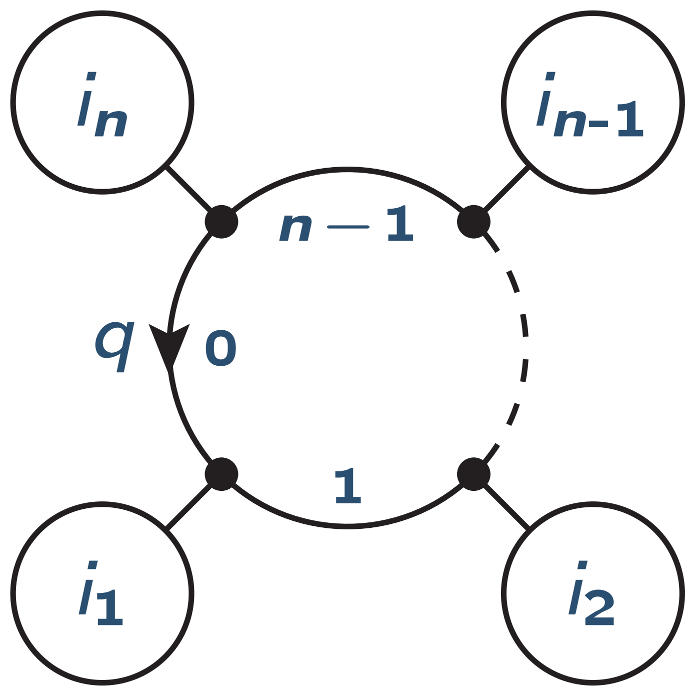
- cut is put appropriately
- direction chosen to maximise recyclability

Illustration:



Complicated diagrams require only "last missing piece"

OpenLoops recursion



$$= \int \frac{d^D \mathcal{N}(q)}{D_0 D_1 \dots D_{n-1}} = \sum_{r=0}^{R} \mathcal{N}_{\mu_1 \dots \mu_r} \underbrace{\int \frac{q^{\mu_1} \dots q^{\mu_r}}{D_0 D_1 \dots D_{n-1}}}_{\text{tensor integral}}$$

- Tensorial coefficients $\mathcal{N}^{\alpha}_{\mu_1 \dots \mu_r;\alpha}$ can directly be used with tensor integral libraries (COLLIER).

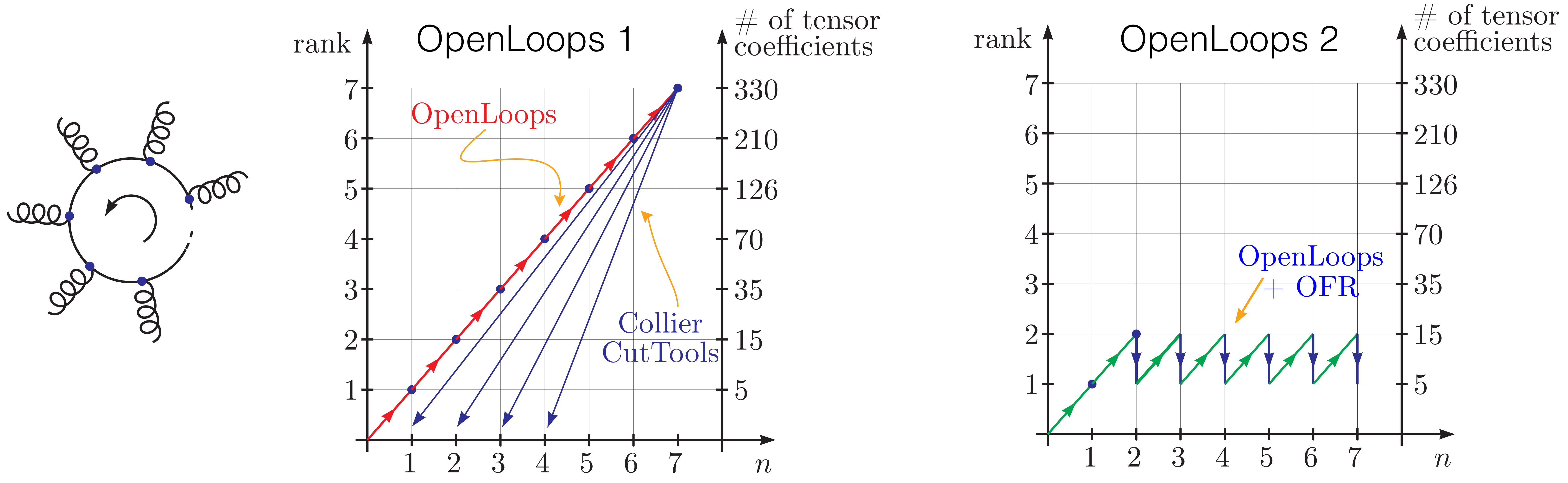
  [Denner, Dittmaier, Hofer; '16]

- Fast evaluation of $\mathcal{N}(q) = \sum \mathcal{N}_{\mu_1 \dots \mu_r} q^{\mu_1} \dots q^{\mu_r}$ at multiple q-values, required in OPP reduction methods (CutTools).

  [Ossola, Papadopolous, Pittau; '07]

On-the-fly OpenLoops reduction

- Interleaved amplitude construction and integrand reduction ⇒ "on-the-fly" (OFR) reduction
- At each Open Loops step perform "on-the-fly" rank=2 ⇒ rank=1 reduction:



- complexity associated with tensor rank remains small
- allows for very targeted stability improvements

NLO Tools

- NLO Monte-Carlo integrators (+subtraction):
  - `MadGraph_aMC@NLO (FKS)`
  - `Sherpa (CS)`
  - `POWHEG-BOX (FKS)`
  - `Herwig7 (CS)`
  - `Whizard (FKS)`
  - `HelacNLO (CS)`
  - `MUNICH (CS)`
  - …

- one-loop (& tree) amplitude provider:
  - `BlackHat (Unitarity)`
  - `MadLoop (OpenLoops)`
  - `GoSam (Unitarity & OPP)`
  - `OpenLoops (OpenLoops)`
  - `Recola (NLO Recursion)`
  - `HelacNLO (OPP)`
  - …

- integral reduction libraries:
  - `CutTools`
  - `Golem95`
  - `PJFry`
  - `COLLIER`
  - `Ninja`
  - …

- tree amplitude provider:
  - `MadGraph`
  - `Comix`
  - `Amegic`
  - `Omega`
  - …

- scaler one-loop libraries
  - `QCDLoop`
  - `OneLoop`
  - `COLLIER`
  - …

Perturbative expansion: revised

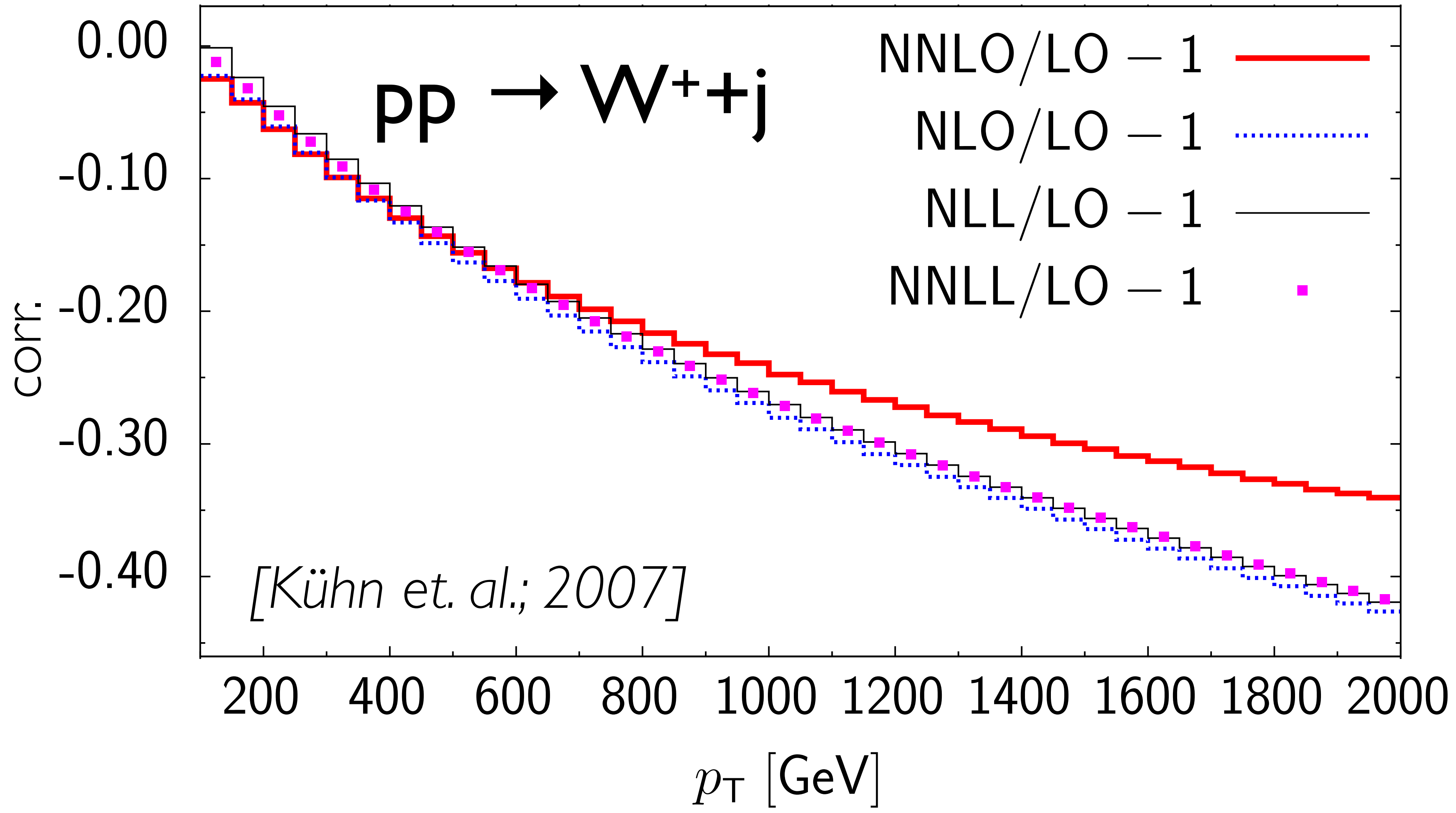
$$d\sigma = d\sigma_{\mathrm{LO}} + \alpha_S\, d\sigma_{\mathrm{NLO}} + \alpha_{\mathrm{EW}}\, d\sigma_{\mathrm{NLO\,EW}}$$

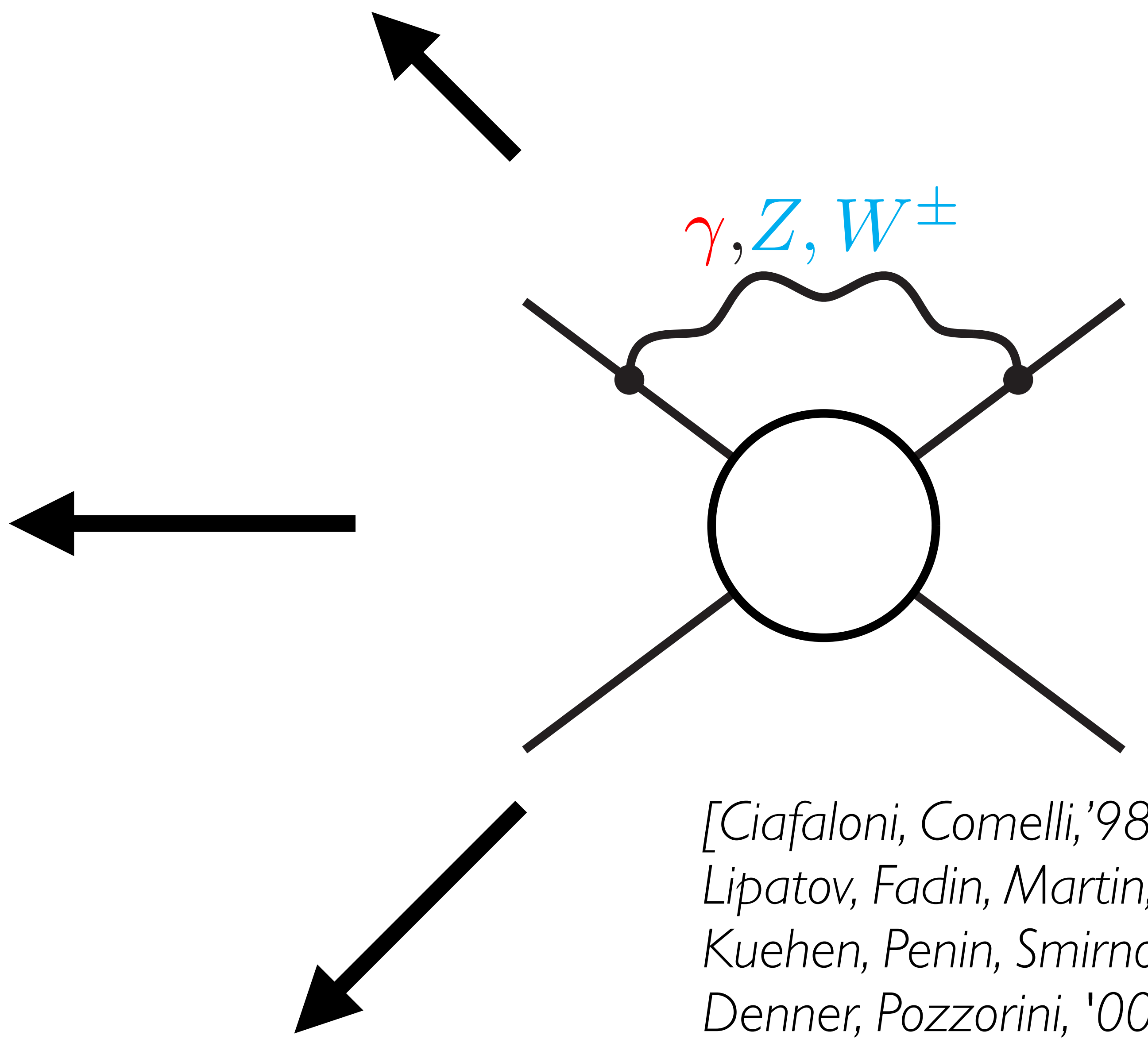
NLO QCD NLO EW

Relevance of EW higher-order corrections I

Numerically $\mathcal{O}(\alpha) \sim \mathcal{O}(\alpha_s^2) \Rightarrow$ **NLO EW ~ NNLO QCD**

Possible large (negative) enhancement due to soft/collinear **logs** from virtual EW gauge bosons:





[Ciafaloni, Comelli,'98; Lipatov, Fadin, Martin, Melles, '99; Kuehen, Penin, Smirnov, '99; Denner, Pozzorini, '00]

Universality and factorisation: *[Denner, Pozzorini; '01]*

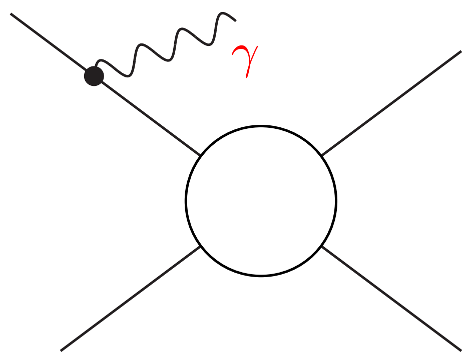
$$\delta\mathcal{M}^{1-\mathrm{loop}}_{\mathrm{LL+NLL}} = \frac{\alpha}{4\pi}\sum_{k=1}^{n}\left\{\frac{1}{2}\sum_{l\neq k}\sum_{a=\gamma,Z,W^\pm} I^a(k)I^{\bar{a}}(l)\ln^2\frac{\hat{s}_{kl}}{M^2} + \gamma^{\mathrm{ew}}(k)\ln\frac{\hat{s}}{M^2}\right\}\mathcal{M}_0$$

➜ overall large effect in the tails of distributions: p_T, m_{inv}, H_T,…

Relevance of EW higher-order corrections II

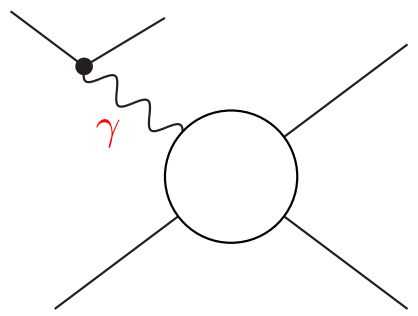
Real photon radiation

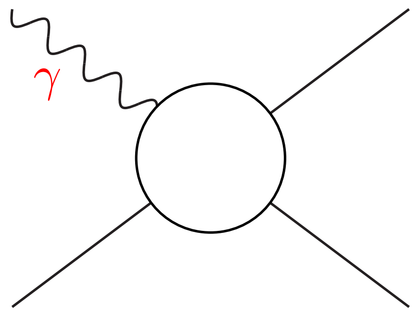
- soft/coll. photon unresolved
- needed to cancel QED singularities



Photon initial states

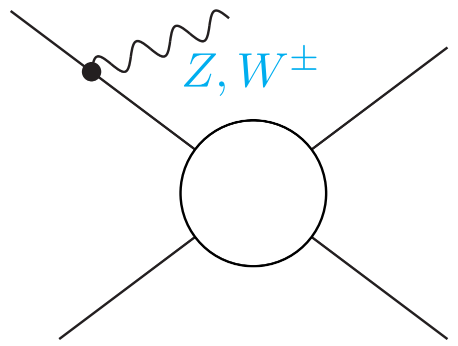
- QED factorisation needed to absorb IS photon singularities
- possible strong enhancement, e.g. for VV





Real W,Z,h radiation (HBR)

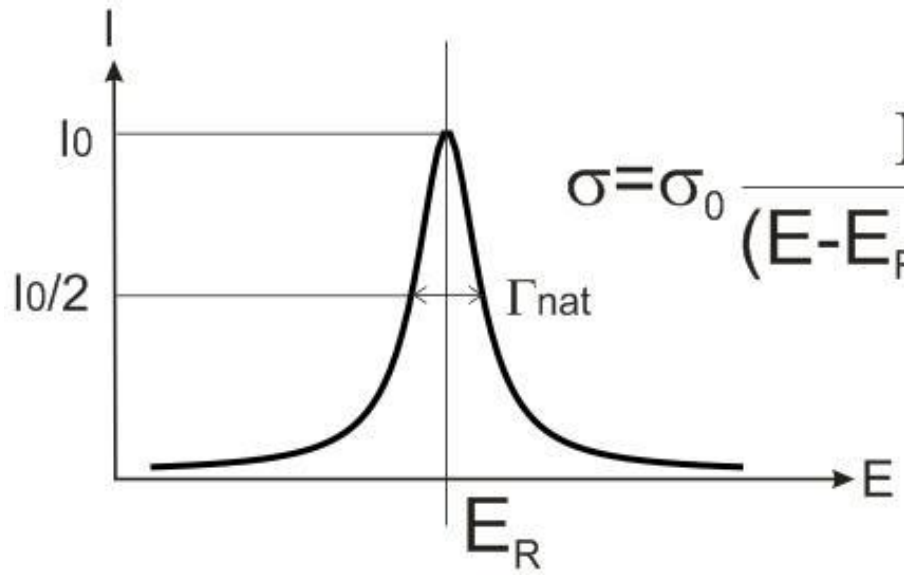
- partial cancellation with virtual Sudakov logs (KLN theorem not applicable) (strongly dependent on experimental selection)
- free from singularities $\Longrightarrow$ separate processes
- themselves receive large virtual EW corrections & inclusion requires care (double-counting issues)

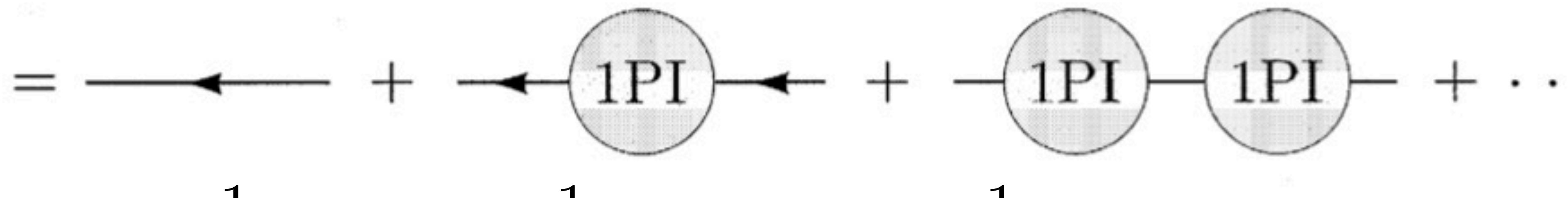


Decays of heavy particles

- Naively processes with a massive s-channel propagator diverge when $p^2 = M^2$
- Experimentally we now resonances follow Breit-Wigner (BW) shape
- Origin: all-order summation of 1PI corrections to propagator of massive particles

propagator ~ $\dfrac{1}{p^2 - k^2}$



propagator ~ 

$$= \frac{1}{p^2 - M_0} + \frac{1}{p^2 - M_0}(-i\Sigma)\frac{1}{p^2 - M_0} + \ldots$$

$$= \frac{1}{k^2 - M_0^2}\sum_{n=0}^{\infty}\left(\frac{-\Sigma(k^2)}{k^2 - M_0^2}\right)^n = \frac{1}{k^2 - M_0^2 + \Sigma(k^2)} = \frac{1}{k^2 - M_0^2 - iM_0\Gamma}$$

Optical theorem→

$$\Gamma \sim \tfrac{1}{M}\,\mathrm{Im}\,\Sigma(M^2)$$

$$\int dk^2 |M|^2 \sim \int_{-\infty}^{\infty} \frac{dk^2}{(k^2 - m^2)^2 + m^2\Gamma} \sim \text{ BW}$$

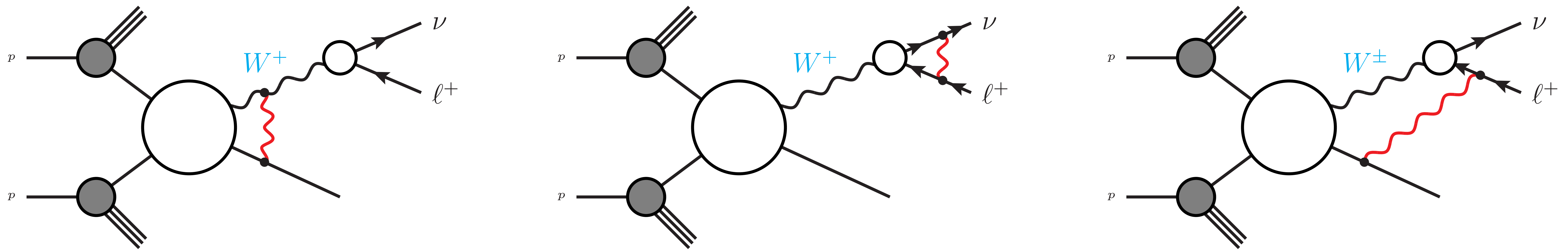
- However: this summation mixes different order of perturbation theory.
  Thus, in general it might (and will) break gauge invariance when applied naively.
- (Usually) not a problem at LO, i.e. also for not for vector bosons decays into leptons at NLO QCD

- Alternative: narrow-width approximation (NWA)
  Advantage: reduces complexity in NLO computation
  However: unable to capture off-shell effects

$$\Gamma/M \to 0: \quad \int_{-\infty}^{\infty} \frac{dk^2}{(k^2 - m^2)^2 + m^2\Gamma} = \frac{\pi}{m\Gamma}\delta(k^2 - m^2)$$

Decays of heavy particles

- Leptonic decays of gauge bosons are trivial at NLO QCD. At NLO EW corrections in production, decay and non-factorizable contributions have to be considered.



- Scheme of choice: complex-mass-scheme *[Denner, Dittmaier, et. al.]*
  - gauge invariant and exact NLO
  - **computationally very expensive**: one extra leg per two-body decay

- Analytical continuation at the level of the Lagrangian: M➞M-iΓM

➡effects propagators, incl. numerators

➡all derived couplings, incl. weak mixing angle: $\sin\theta_W^2 = 1 - \frac{M_W^2}{M_Z^2}$

➡position of the pole in the renormalisation

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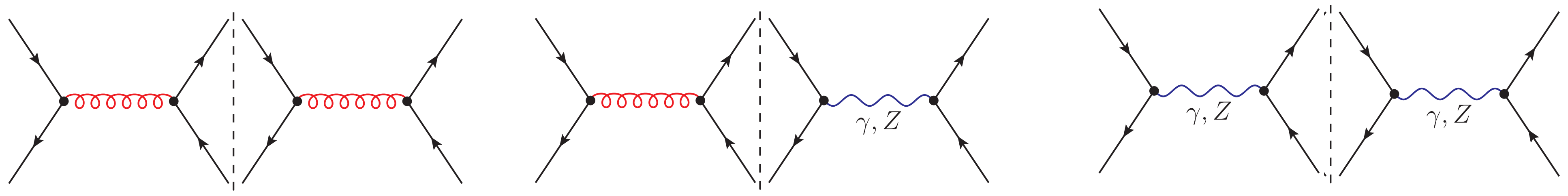
  - OneLoop
  - COLLIER
  - …

Perturbative expansion: revised II

- In general combined expansion in α_S and α necessary:

$$d\sigma = d\sigma(\alpha_s^n \alpha^m) + d\sigma(\alpha_s^{n-1}\alpha^{m+1}) + \sigma(\alpha_s^{n-2}\alpha^{m+2}) + \dots$$

LO "subleading Born contributions": LO2, LO3



Example: $q\bar{q} \rightarrow q\bar{q}$

Perturbative expansion: revised II

- In general combined expansion in α_S and α necessary:

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LO "subleading Born contributions": LO2, LO3

- also at NLO: $\mathcal{O}(\alpha_s)$ $\mathcal{O}(\alpha)$

$$\dots + \sigma(\alpha_s^{n+1}\alpha^m) + d\sigma(\alpha_s^n \alpha^{m+1}) + \sigma(\alpha_s^{n-1}\alpha^{m+2}) + \sigma(\alpha_s^{n-2}\alpha^{m+3}) + \dots$$

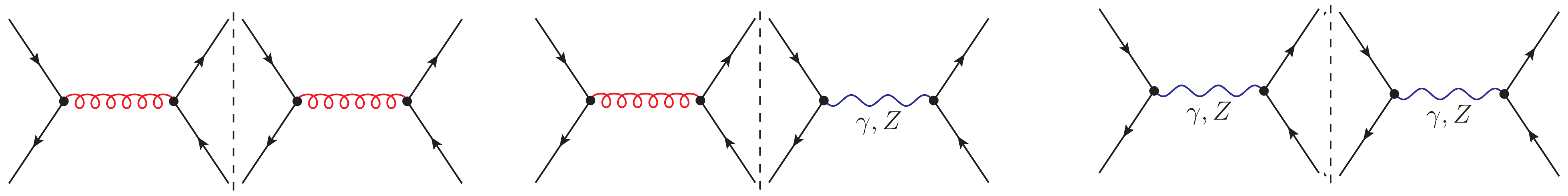
"NLO QCD" "NLO EW" "subleading one-loop contributions": NLO3, NLO4

Perturbative expansion: revised II

- In general combined expansion in α_S and α necessary:

$$d\sigma = d\sigma(\alpha_s^n \alpha^m) + d\sigma(\alpha_s^{n-1}\alpha^{m+1}) + \sigma(\alpha_s^{n-2}\alpha^{m+2}) + \ldots$$

LO "subleading Born contributions": LO2, LO3

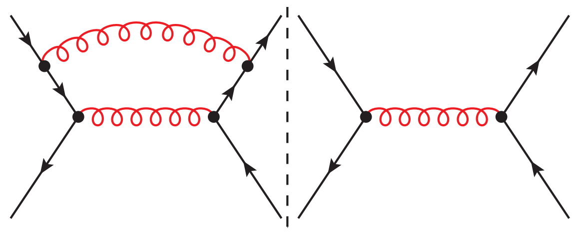


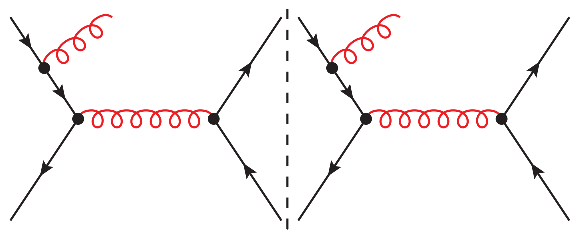
Example: $q\bar{q} \to q\bar{q}$

- also at NLO:

$$\cdots + \sigma(\alpha_s^{n+1}\alpha^m) + d\sigma(\alpha_s^n \alpha^{m+1}) + \sigma(\alpha_s^{n-1}\alpha^{m+2}) + \sigma(\alpha_s^{n-2}\alpha^{m+3}) + \ldots$$

"NLO QCD" "NLO EW" "subleading one-loop contributions": NLO3, NLO4



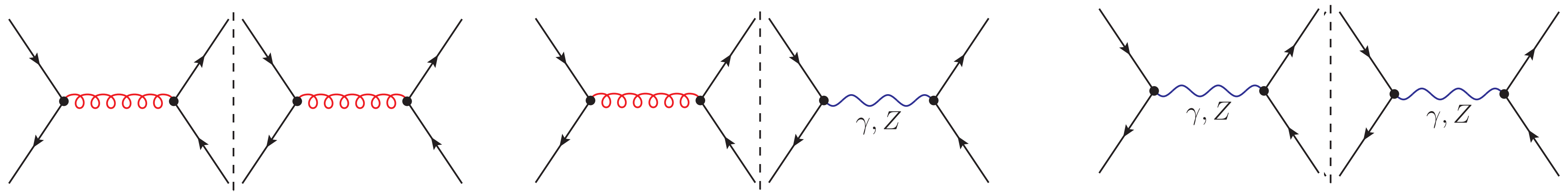


Perturbative expansion: revised II

- In general combined expansion in α_S and α necessary:

$$d\sigma = d\sigma(\alpha_s^n \alpha^m) + d\sigma(\alpha_s^{n-1}\alpha^{m+1}) + \sigma(\alpha_s^{n-2}\alpha^{m+2}) + \dots$$

LO "subleading Born contributions": LO2, LO3



Example: $q\bar{q} \rightarrow q\bar{q}$

- also at NLO:

$$\dots + \sigma(\alpha_s^{n+1}\alpha^m) + d\sigma(\alpha_s^n \alpha^{m+1}) + \sigma(\alpha_s^{n-1}\alpha^{m+2}) + \sigma(\alpha_s^{n-2}\alpha^{m+3}) + \dots$$

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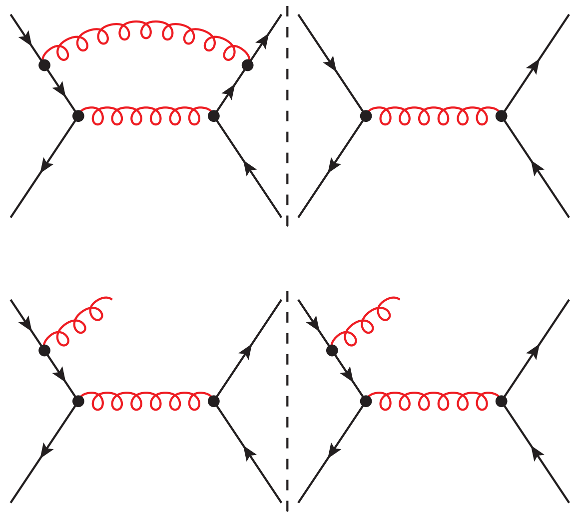

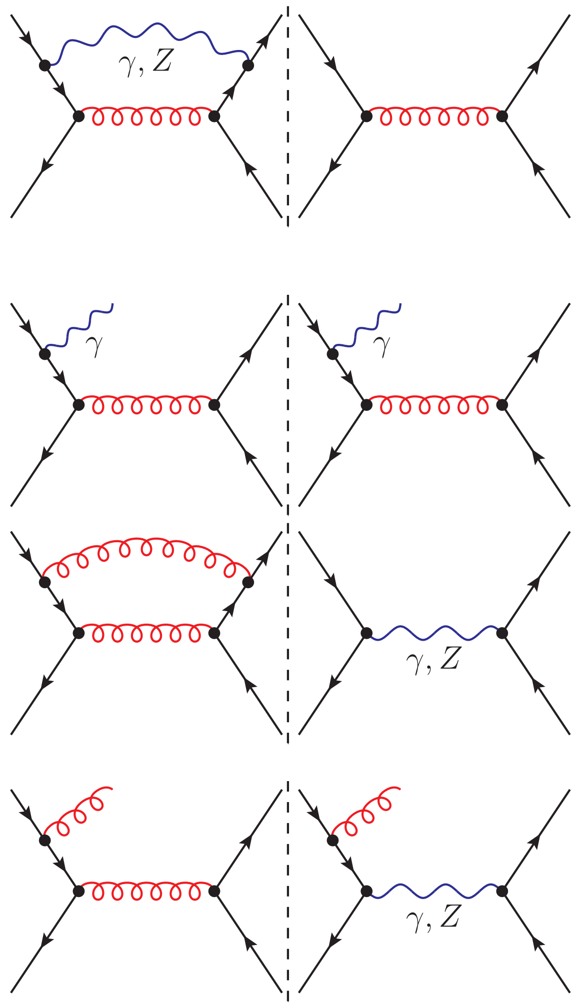

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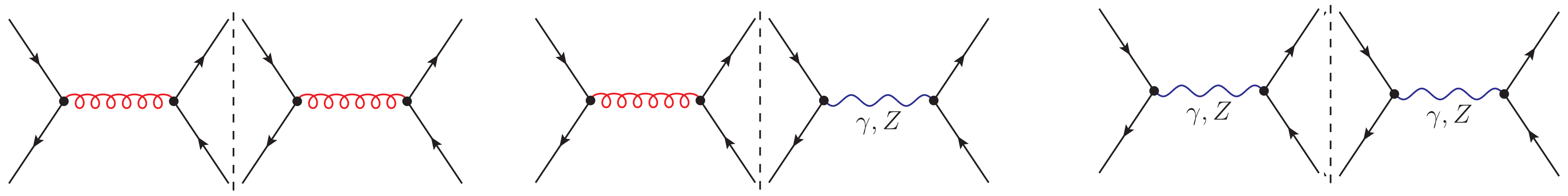

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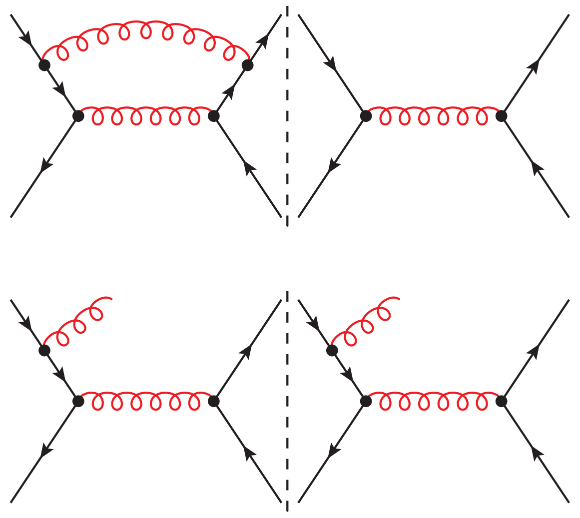
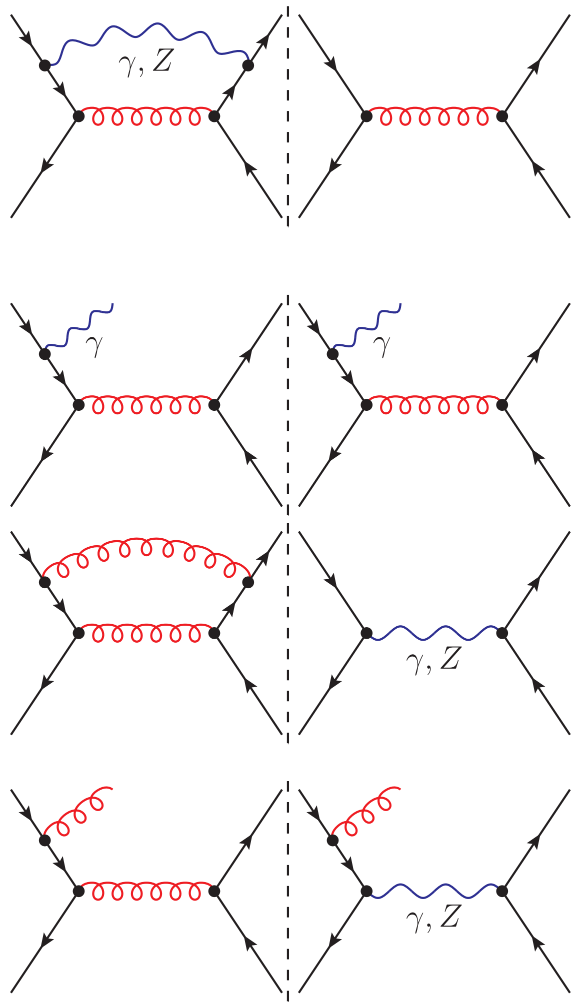
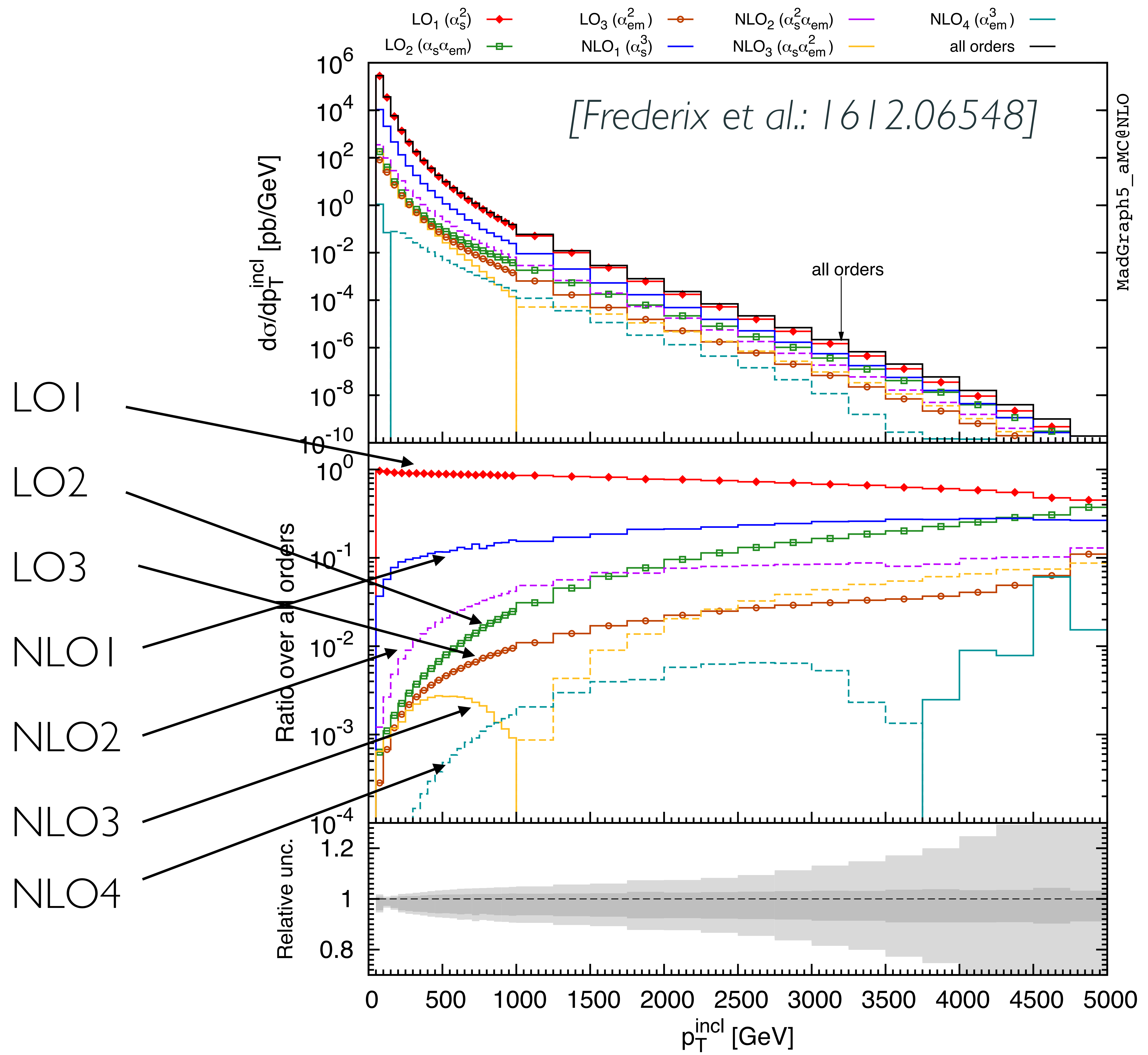




Note:

- No diagrammatic separation in NLO QCD and EW
- An IR finite & gauge invariant result is only obtained including all virtual and real contributions of a given perturbative order.

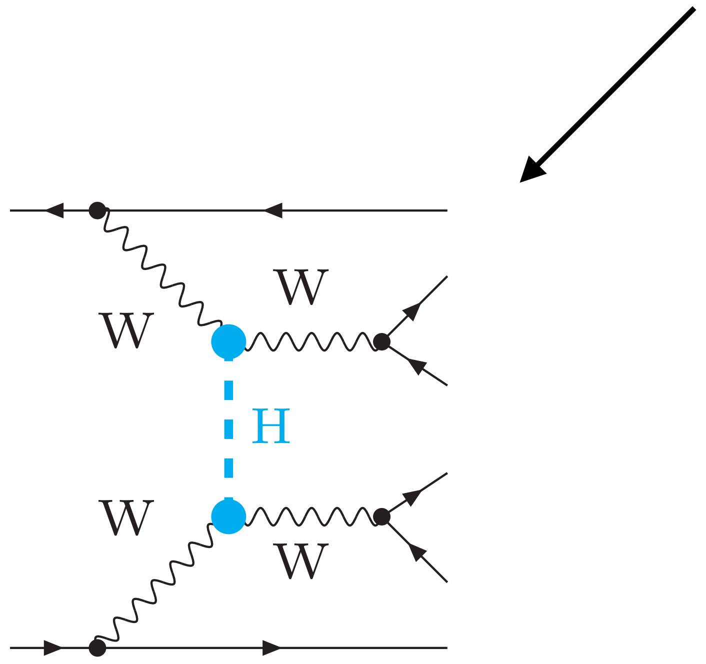
Example: dijet production at the LHC

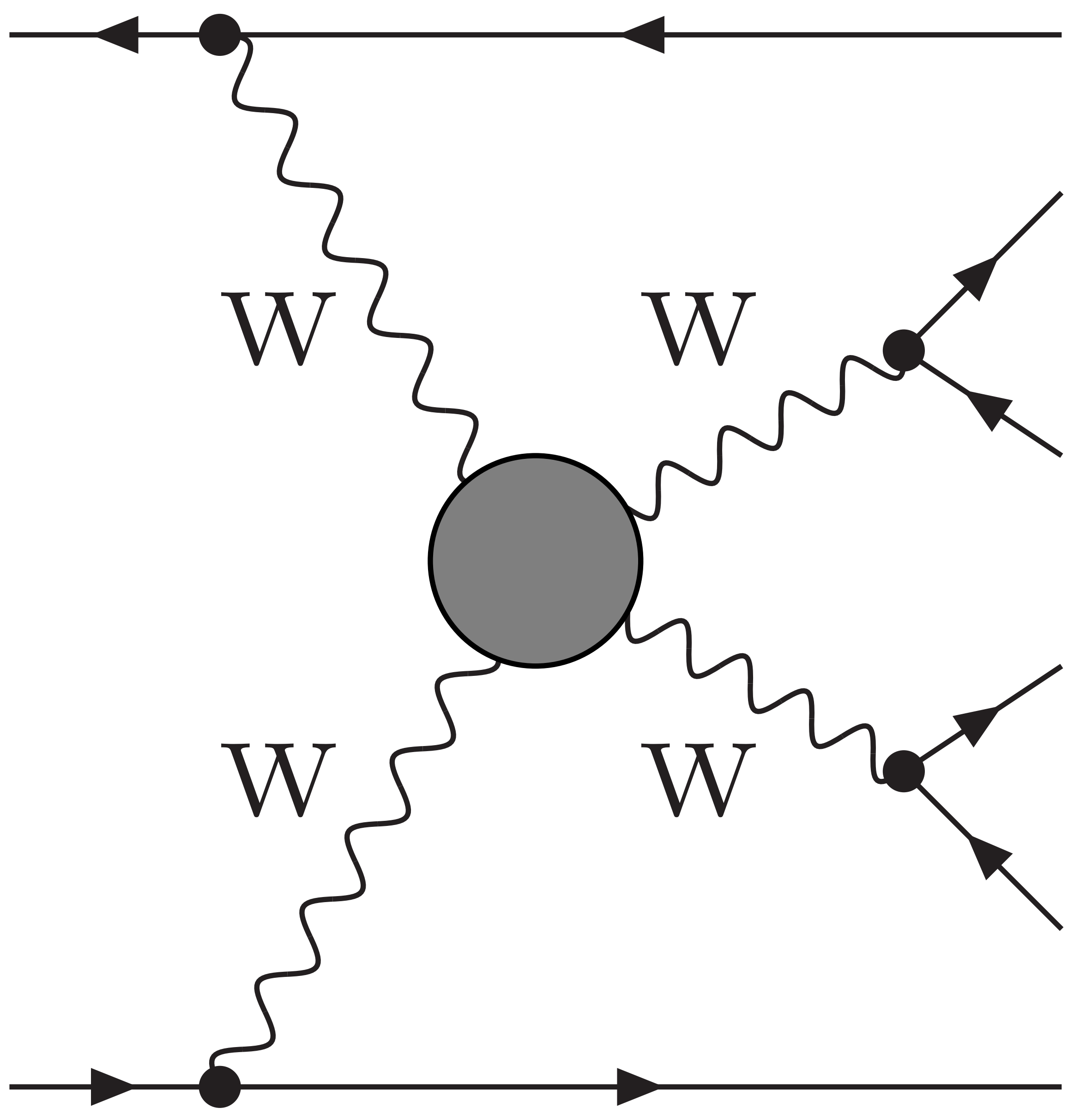


Be aware of double counting: LO3 = DY with hadronic decays

Interpretation of multi-particle processes: case study VBS

- direct access to quartic EW gauge couplings
- VBS: longitudinal gauge bosons at high energies
- VBS key process to investigate electroweak symmetry breaking (off-shell Higgs exchange ensures unitarity)

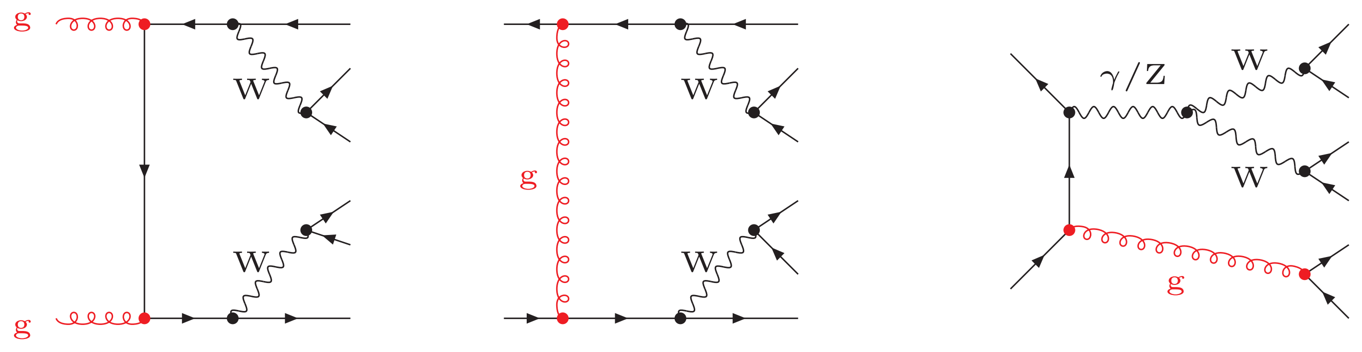




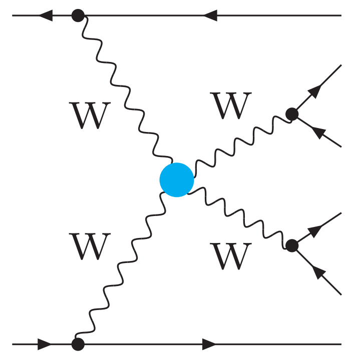
Signature: 2l + 2n + 2 jets

VV+2jets production

Note: severe QCD background to VBS signatures + interference:



vs.



$$\mathrm{d}\sigma = \mathrm{d}\sigma(\alpha_S^2\alpha^4) + \mathrm{d}\sigma(\alpha_S\alpha^5) + \mathrm{d}\sigma(\alpha^6) + \ldots$$ LO

QCD-background $\mathcal{O}(\alpha_s)$ $\mathcal{O}(\alpha)$ interference $\mathcal{O}(\alpha_s)$ $\mathcal{O}(\alpha)$ VBS-signal

$$\cdots + \mathrm{d}\sigma(\alpha_S^3\alpha^4) + \mathrm{d}\sigma(\alpha_S^2\alpha^5) + \mathrm{d}\sigma(\alpha_S\alpha^6) + \sigma(\alpha^7)$$ NLO

"NLO QCD" "NLO EW" "NLO QCD" "NLO EW"

VV+2jets production

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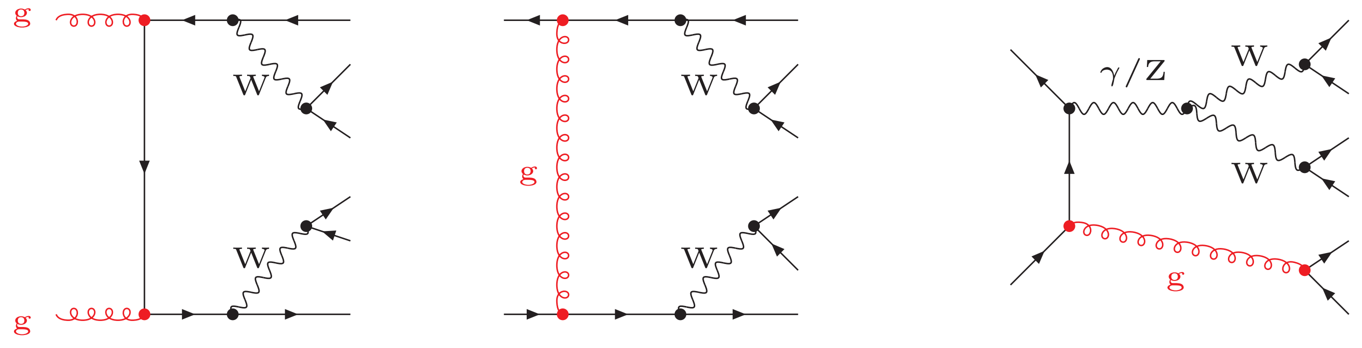

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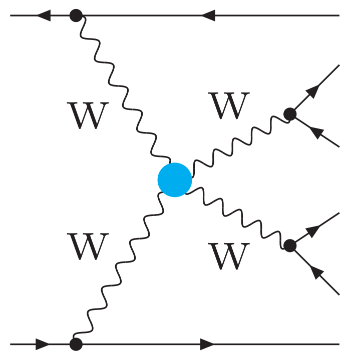

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QCD-background | interference | VBS-signal

$\mathcal{O}(\alpha_s)$ $\mathcal{O}(\alpha)$

$$\cdots + \mathrm{d}\sigma(\alpha_S^3\alpha^4) + \mathrm{d}\sigma(\alpha_S^2\alpha^5) + \mathrm{d}\sigma(\alpha_S\alpha^6) + \sigma(\alpha^7)$$ NLO

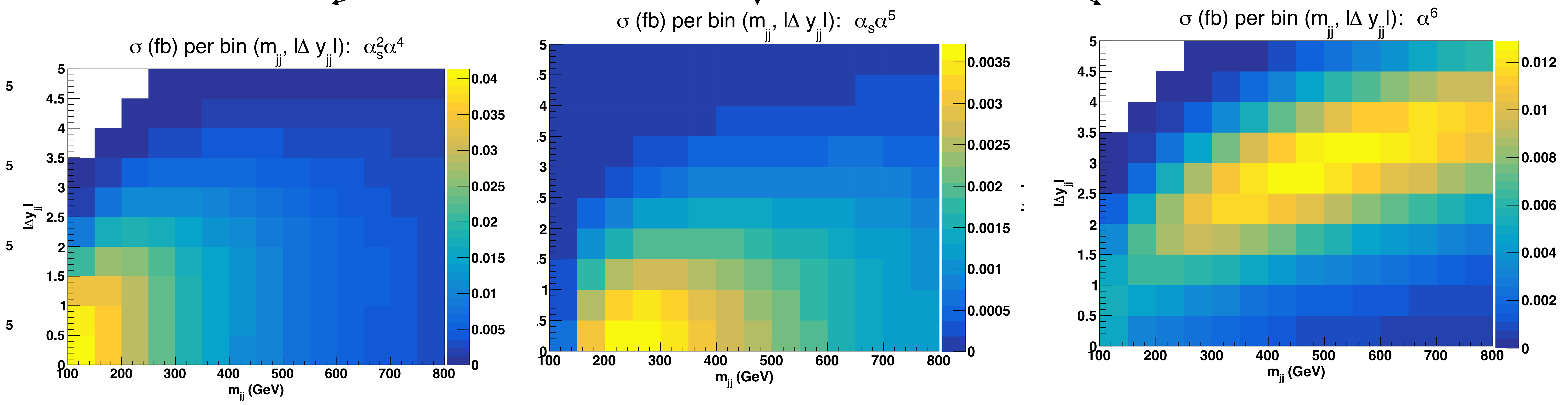
"NLO QCD" "NLO EW" "NLO QCD" "NLO EW"

➡ separation meaningless at NLO
➡ only well defined measurements: fiducial cross sections

VBS

$$d\sigma = d\sigma(\alpha_S^2\alpha^4) + d\sigma(\alpha_S\alpha^5) + d\sigma(\alpha^6) + \ldots$$

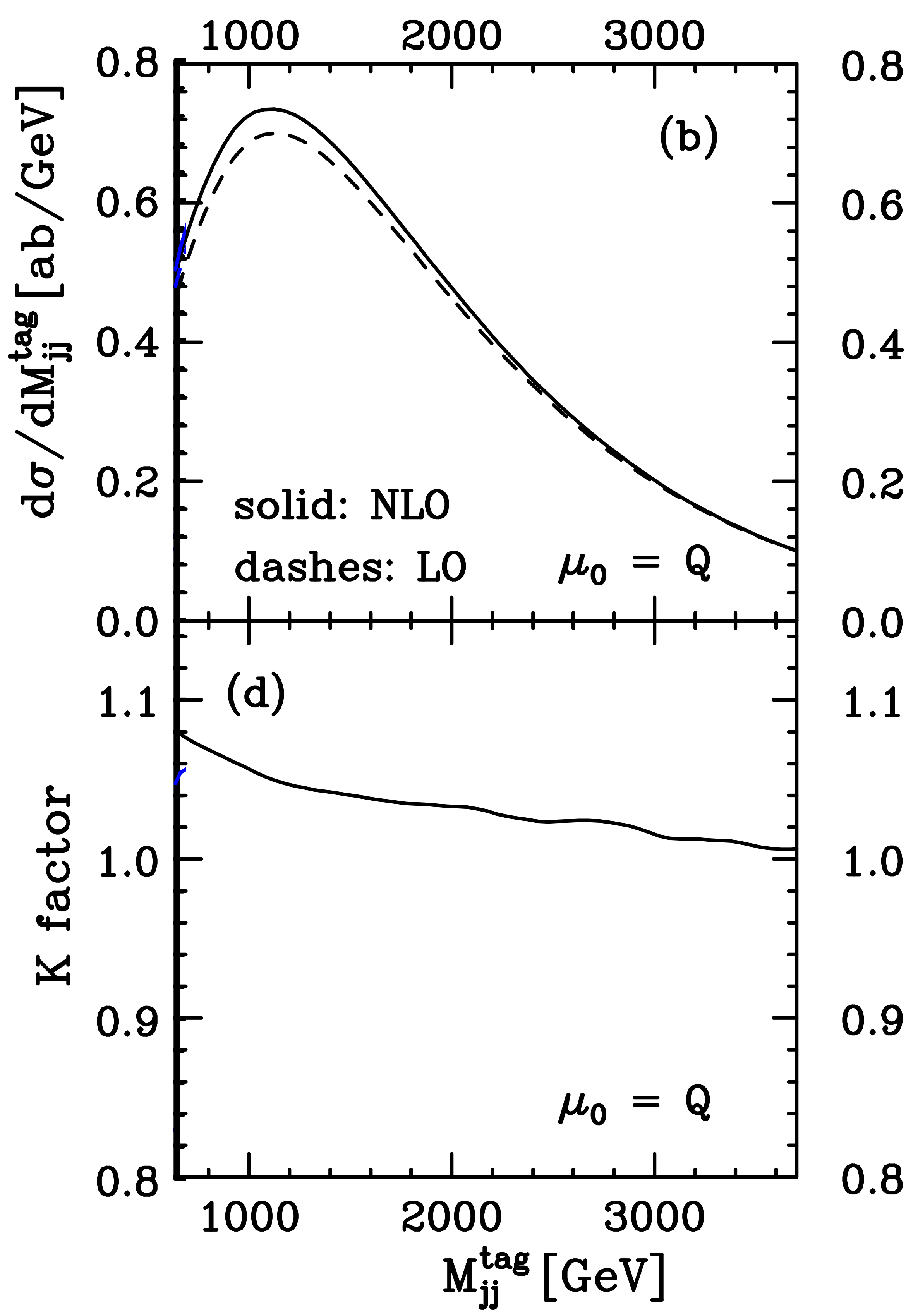
[Ballestrero, et al.; 1803.07943]



- The three contributions have very different kinematics
- typical "VBF" cuts to enhance EW mode: **Δ**yjj > 2.5 and mjj ~> 500 GeV

VBS production @ NLO QCD (in VBF-approx)

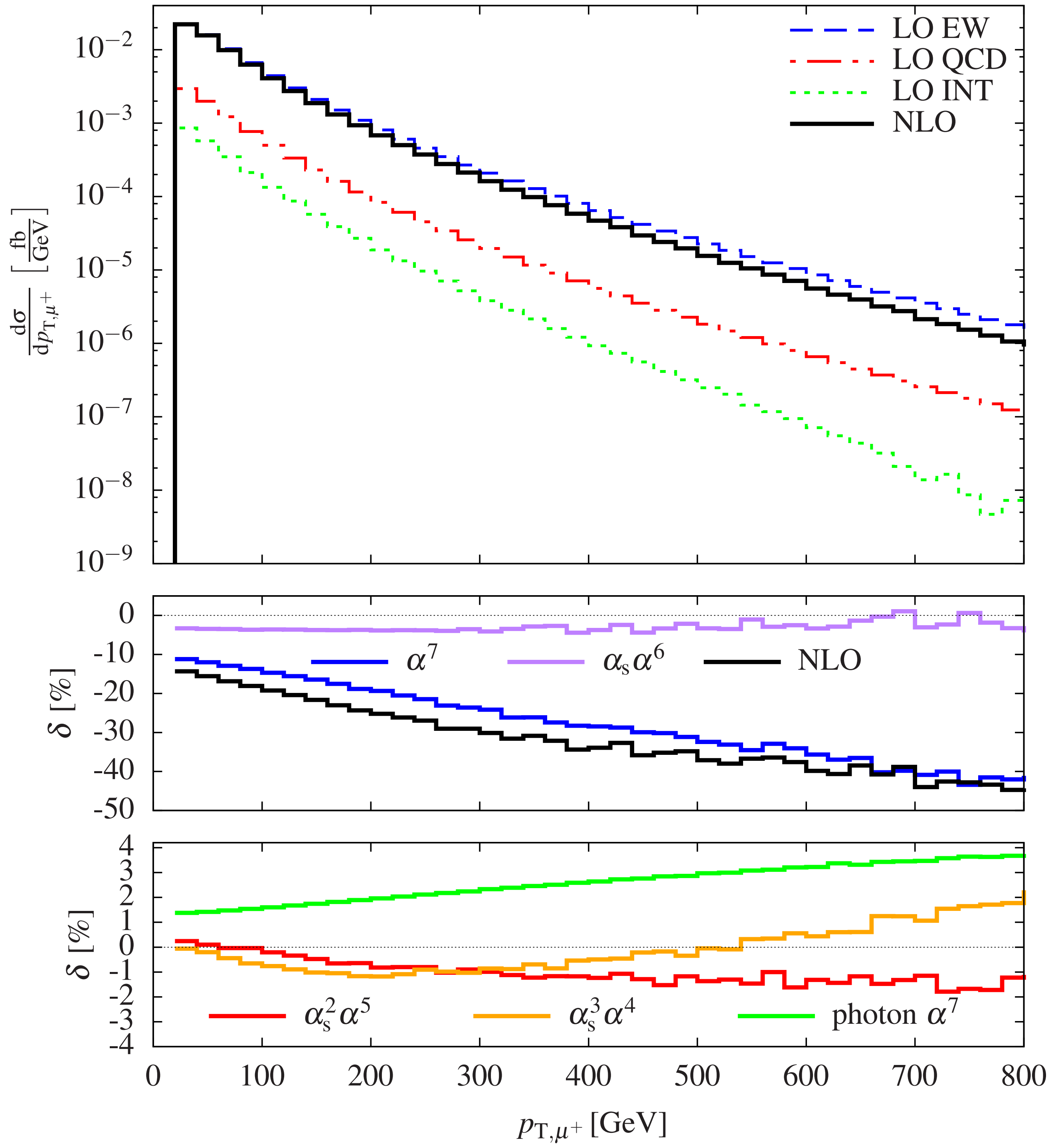
[Jäger, Oleari, Zeppenfeld, '09]

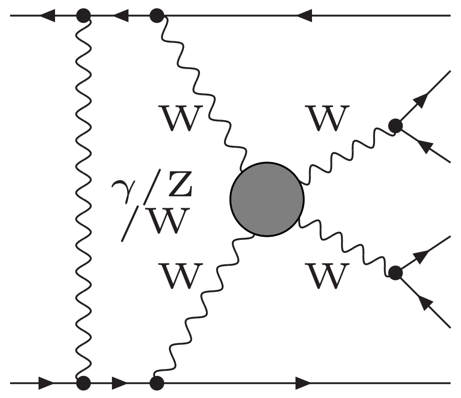


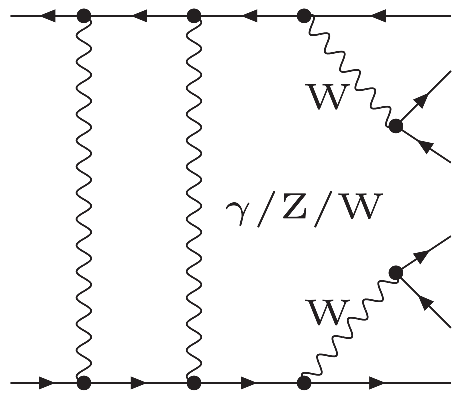
- very small QCD corrections (as for all VBF-type processes)

VBS-W^+W^+ @ full NLO

[Biedermann, Denner, Pellen '16+'17]







- 2 → 6 particles at NLO EW !
- highly challenging computation!

- NLO corrections dominated by α^7 :

Order	$\mathcal{O}(\alpha^7)$	$\mathcal{O}(\alpha_s\alpha^6)$	$\mathcal{O}(\alpha_s^2\alpha^5)$	$\mathcal{O}(\alpha_s^3\alpha^4)$	Sum
$\delta\sigma_{\mathrm{NLO}}$ [fb]	$-0.2169(3)$	$-0.0568(5)$	$-0.00032(13)$	$-0.0063(4)$	$-0.2804(7)$
$\delta\sigma_{\mathrm{NLO}}/\sigma_{\mathrm{LO}}$ [%]	-13.2	-3.5	0.0	-0.4	-17.1

with $M_{jj} > 500\,\mathrm{GeV},\ p_{T,j} > 30\,\mathrm{GeV},\ p_{T,\ell} > 20\,\mathrm{GeV},$

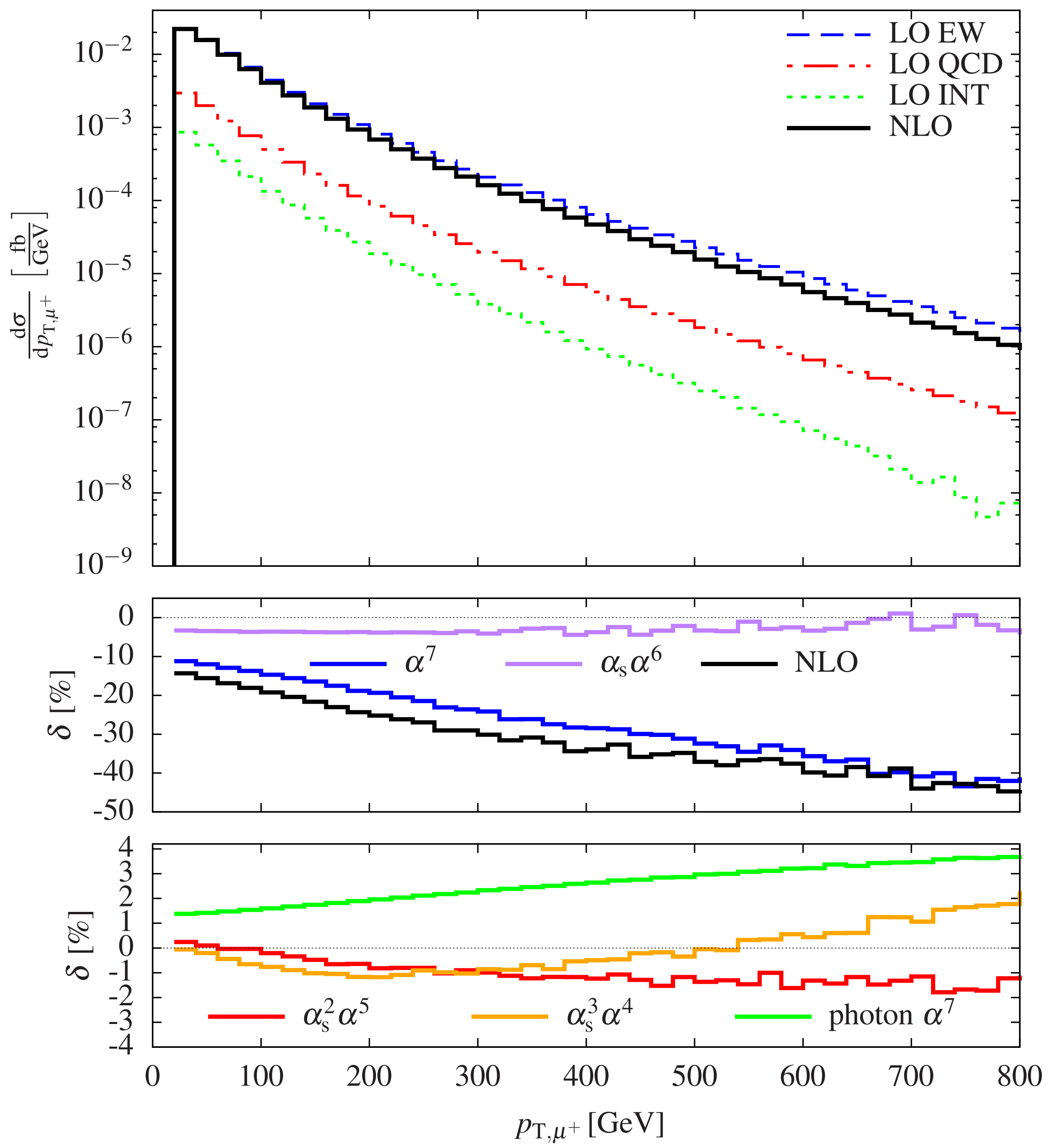
LO: $\mathcal{O}(\alpha^6)$
NLO: $\mathcal{O}(\alpha^7)$

σ^{LO} [fb]	$\sigma^{\mathrm{NLO}}_{\mathrm{EW}}$ [fb]	δ_{EW} [%]
1.5348(2)	1.2895(6)	−16.0

- VERY large inclusive EW corrections (dominated by Sudakov logs)

VBS-W⁺W⁺ @ full NLO

[Biedermann, Denner, Pellen '16+'17]



- VERY large EW corrections (dominated by Sudakov logs)

LO: $\mathcal{O}(\alpha^6)$
NLO: $\mathcal{O}(\alpha^7)$

σ^{LO} [fb]	$\sigma^{\mathrm{NLO}}_{\mathrm{EW}}$ [fb]	δ_{EW} [%]
1.5348(2)	1.2895(6)	−16.0

Leading logarithm approximation [Denner, Pozzorini; hep-ph/0010201]

$$\sigma_{\mathrm{LL}} = \sigma_{\mathrm{LO}}\left[1 - \frac{\alpha}{4\pi}4C_{\mathrm{W}}^{\mathrm{ew}}\log^2\left(\frac{Q^2}{M_{\mathrm{W}}^2}\right) + \frac{\alpha}{4\pi}2b_{\mathrm{W}}^{\mathrm{ew}}\log\left(\frac{Q^2}{M_{\mathrm{W}}^2}\right)\right]$$

$= -16\%$ (!)

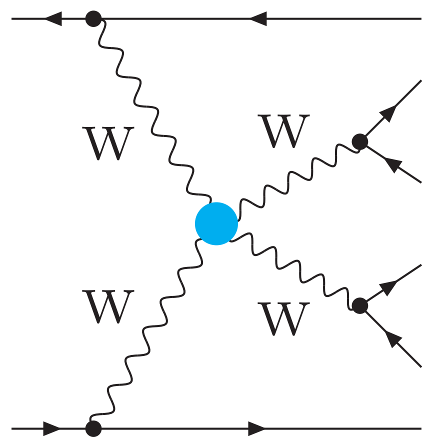
For $Q = \langle m_{4\ell}\rangle \sim 390$ GeV

$\langle m_{4\ell}\rangle$ larger for VBS (massive t-channel
NB: $\langle m_{4\ell}\rangle \sim 250$ GeV for $q\bar{q} \to \mathrm{W^+W^+}$

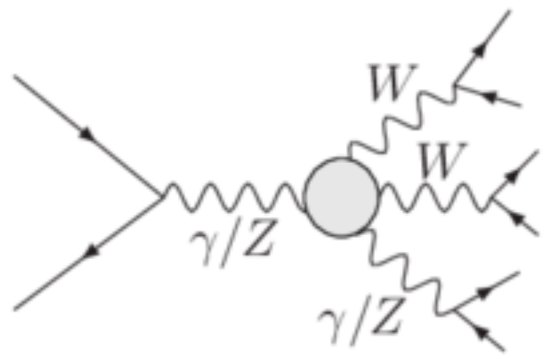
➡Large NLO EW corrections: intrinsic feature of VBS at the LHC

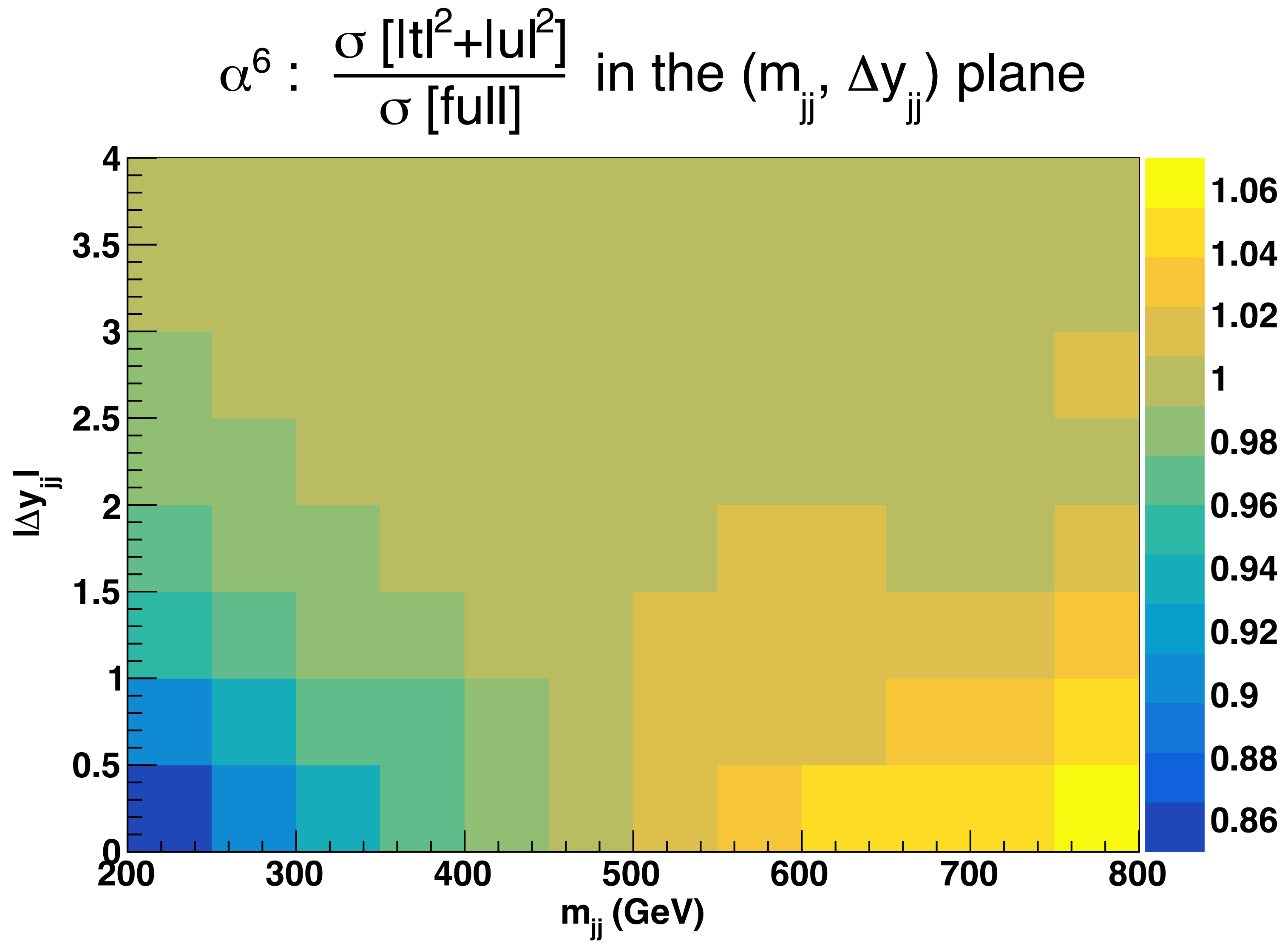
Quality of VBF approximation @ LO

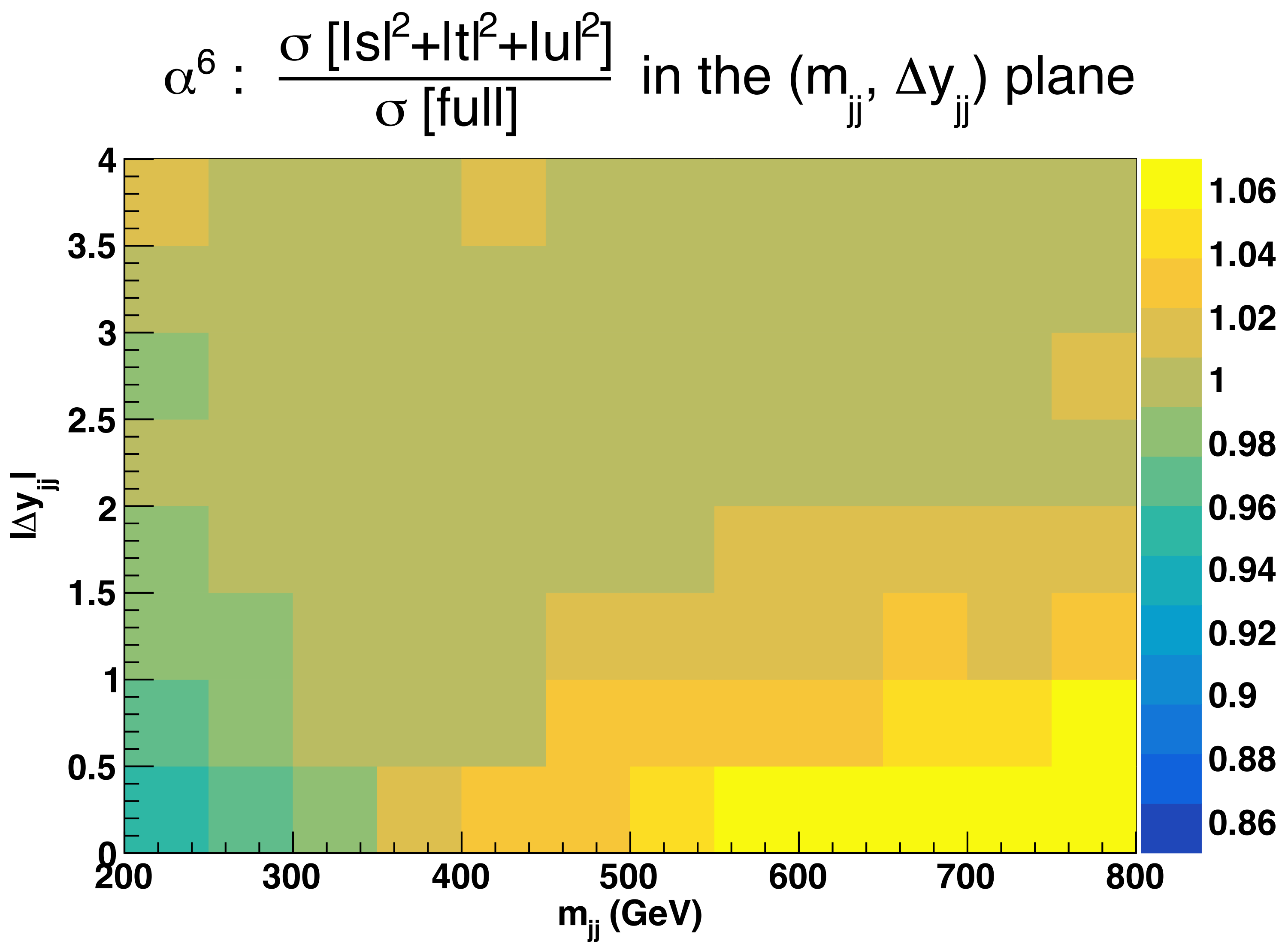
t/u:



s:







[Ballestrero, MP et al.; 1803.07943]

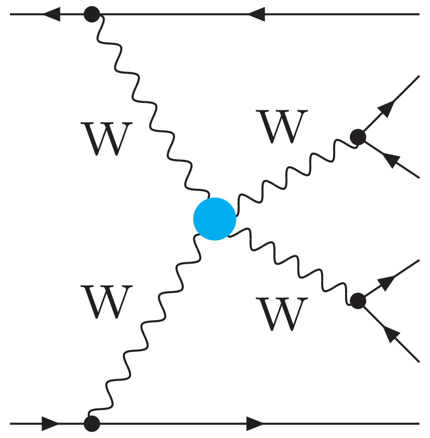
- For low m_{jj} and low Δy_{jj}, significant s-channel contributions
  $\rightarrow$ tri-boson contributions with resonant W-boson
- Good approximation in fiducial region for $\mathrm{W^+W^+}$
  $\rightarrow$ confirmed for $\mathrm{W^\pm Z}$ [Andersen, MP et al.; 1803.07977]

Common feature of all VBS signatures

[Slide: M. Pellen]

Quality of VBF approximation @ NLO

t/u:



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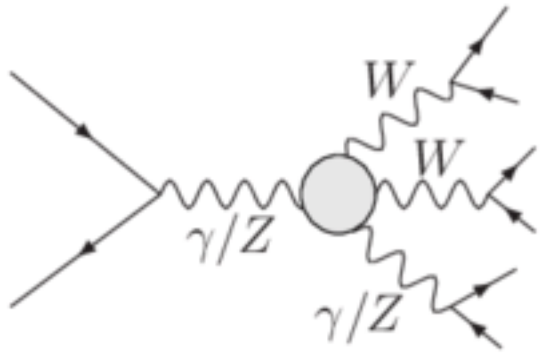
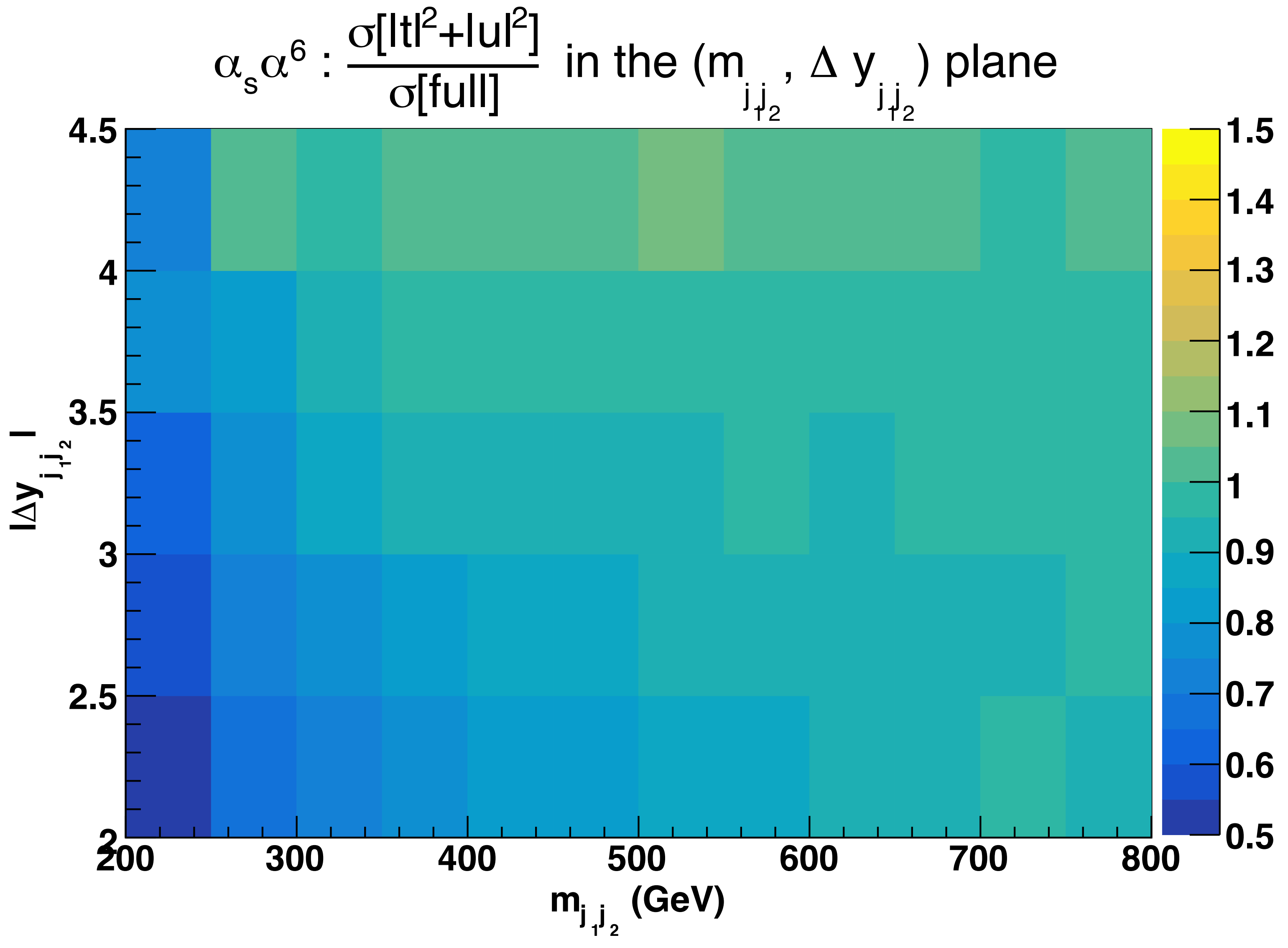
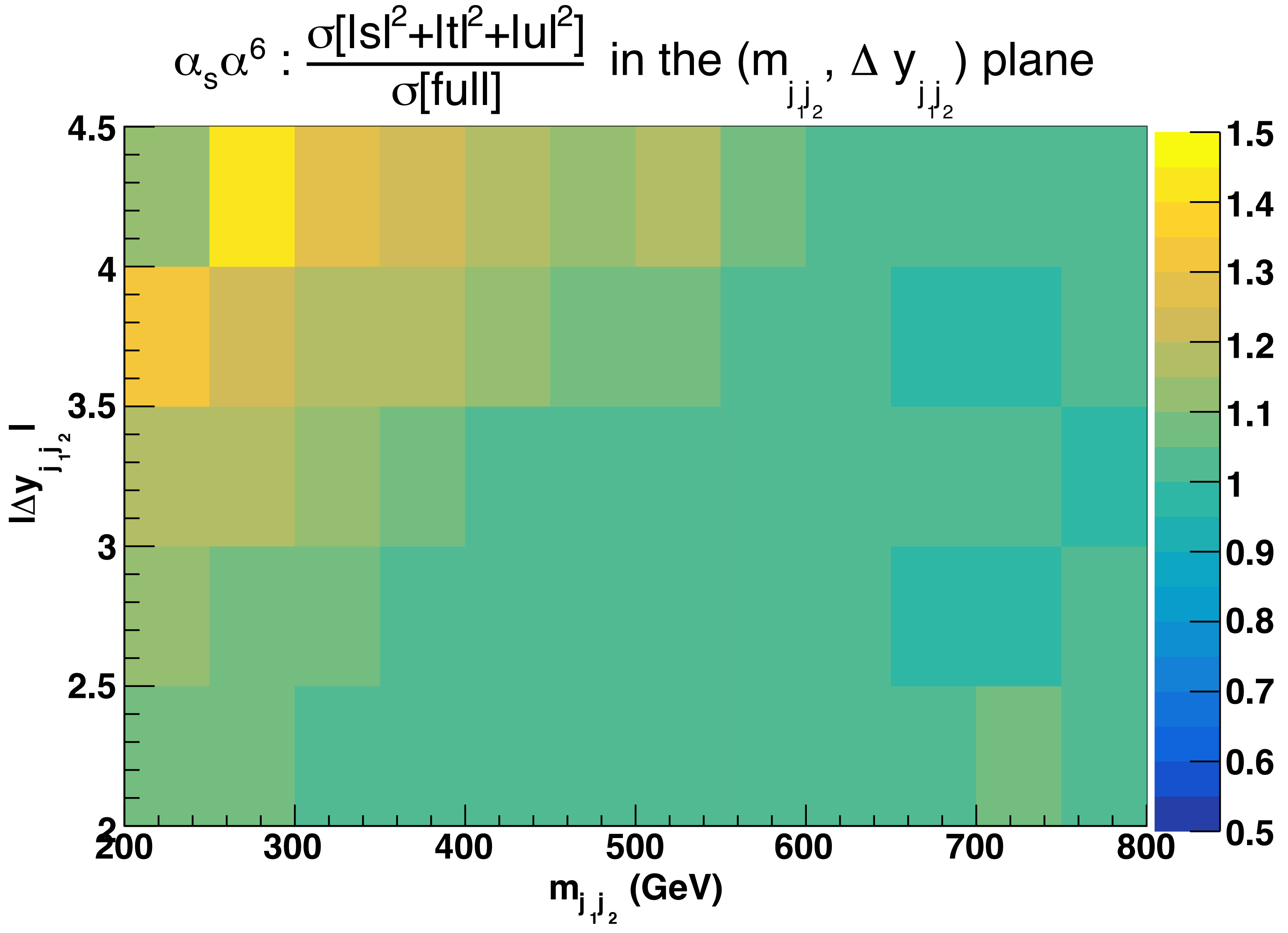






[Ballestrero, MP et al.; 1803.07943]

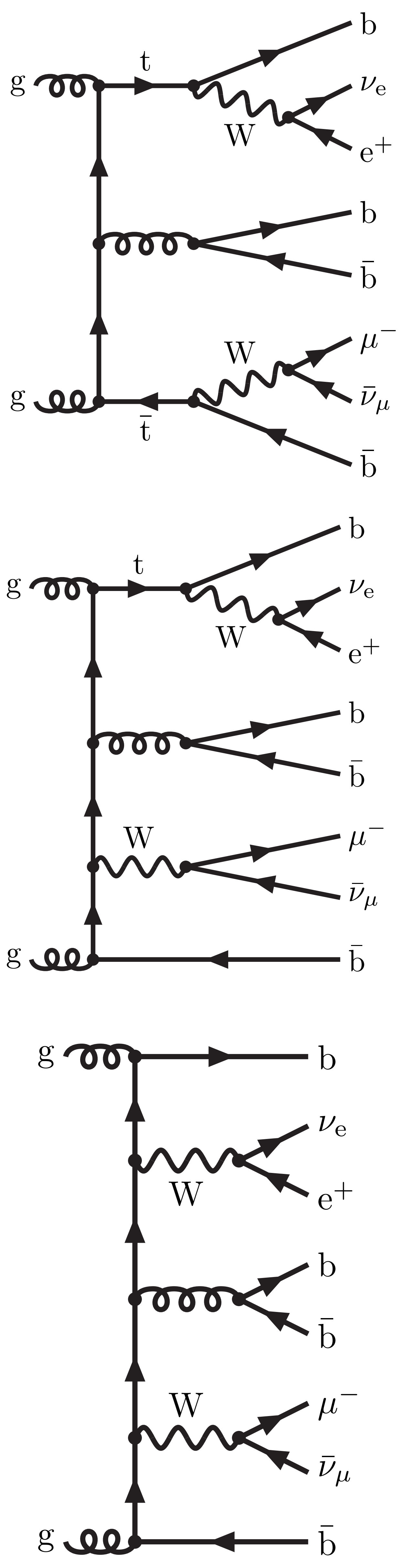
- The approximations are in general worse at NLO
- Approximation can fail by up to 20% even in fiducial region
  → OK now for current experimental precision but might be important in the future

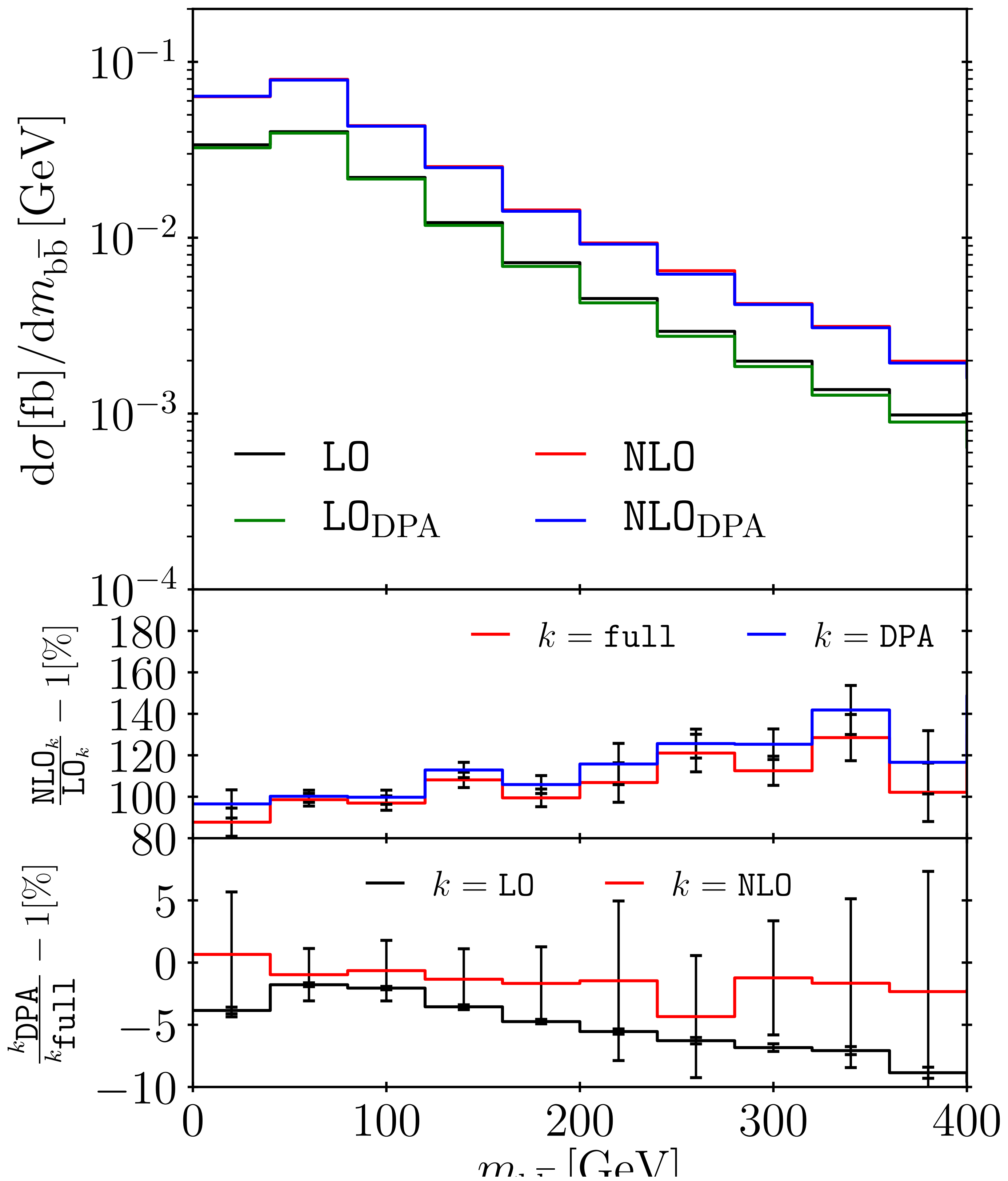
[Slide: M. Pellen]

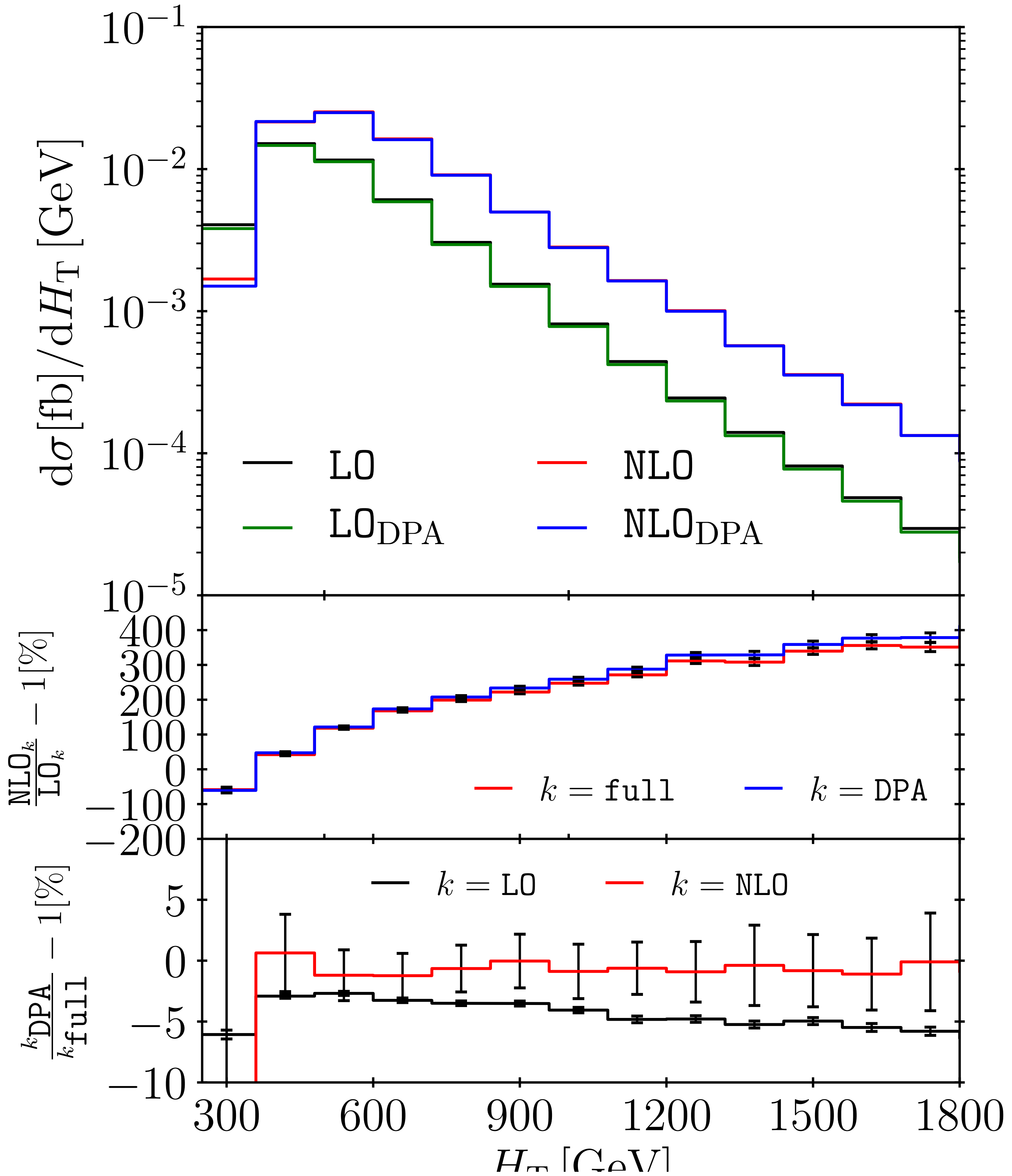
Precision for the highest multiplicities

[Denner, Lang, Pellen `20]

$pp \to 2\ell 2\nu b\bar{b}b\bar{b}$ (2->6) @ NLO QCD







- Thorough understanding of theory systematics in this channel crucial for ttH measurements where H->bb
- ttbb receives sizeable QCD corrections
- Very important confirmation of (ttbb) double pole approximation

Back to the perturbative expansion…

$$d\sigma = d\sigma_{\mathrm{LO}} + \alpha_S\, d\sigma_{\mathrm{NLO}} + \alpha_{\mathrm{EW}}\, d\sigma_{\mathrm{NLO\,EW}}$$

NLO QCD   NLO EW

$$+\alpha_S^2\, d\sigma_{\mathrm{NNLO}} + \alpha_{\mathrm{EW}}^2\, d\sigma_{\mathrm{NNLO\,EW}} + \alpha_S \alpha_{\mathrm{EW}}\, d\sigma_{\mathrm{NNLO\,QCDxEW}}$$

NNLO QCD   NNLO EW   NNLO QCD-EW

In order to match experimental precision NNLO QCD is becoming mandatory for many processes

NNLO techniques

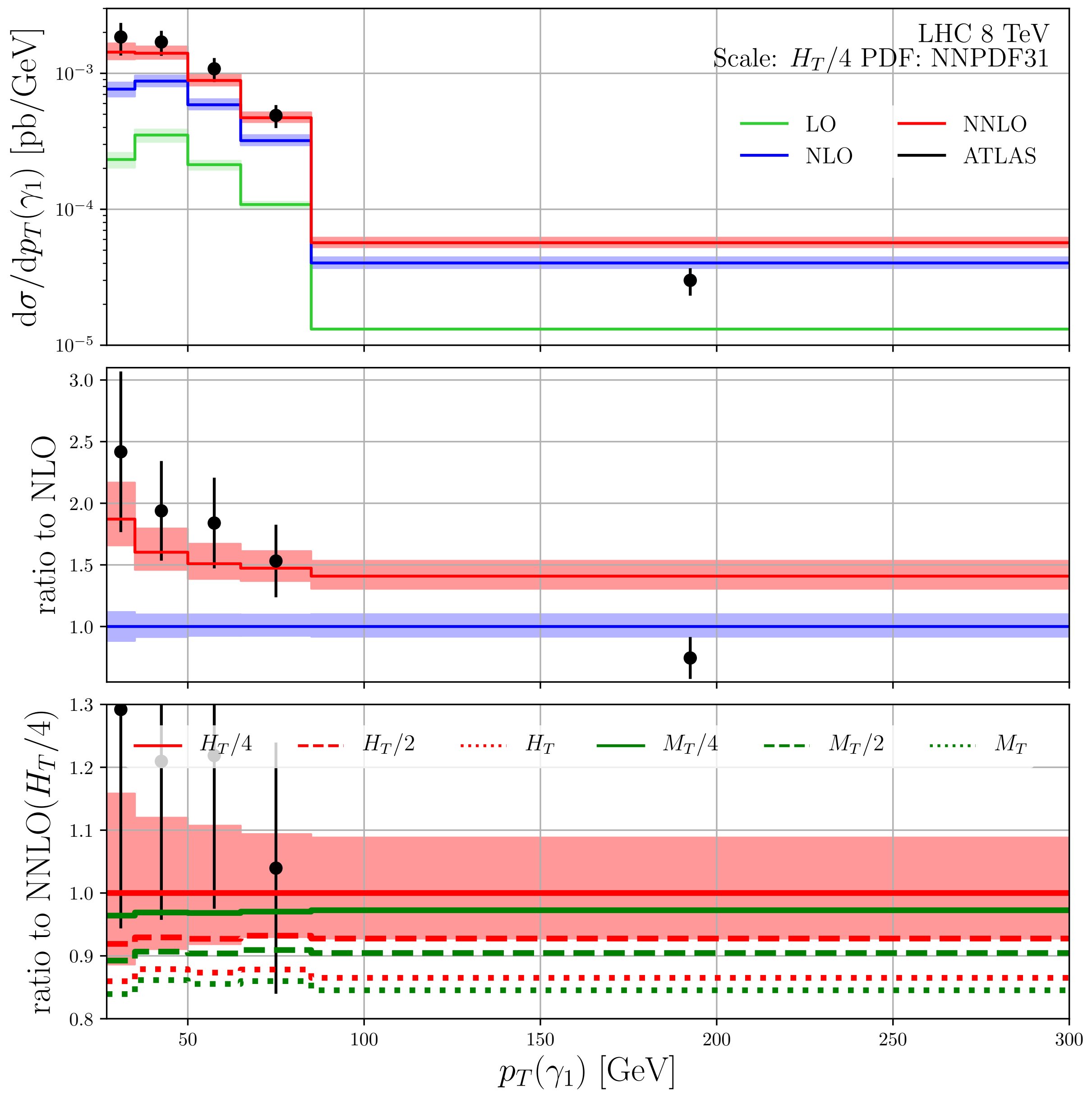
"Slicing"

- qT-subtraction: pp➞V, pp➞H, pp➞VV/4l, pp➞HH
- N-jettiness subtraction: pp➞V+jet
- Projection to Born: pp➞H+2jet (VBF)

Subtraction

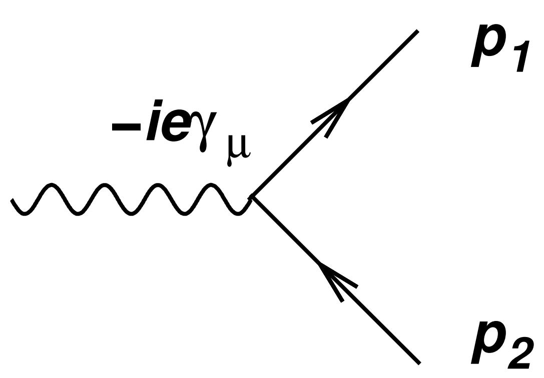
- Antenna subtraction: pp➞V+jet, pp➞2jets, …
- Sector decomposition: pp➞tt, pp➞AAA
- "colourful" subtraction: e+e-➞3jets
- join subtraction and sector decomposition: pp➞VH,…

➡ Pretty much all 2➞2 processes are known at NNLO QCD
➡ First steps towards 2➞3 (pp➞AAA down since 2019)
➡ Bottleneck: two-loop virtual amplitudes (can not cover here)



qT subtraction: Idea

Consider again $\gamma^* \to qq$ (dijet production in e+e-)

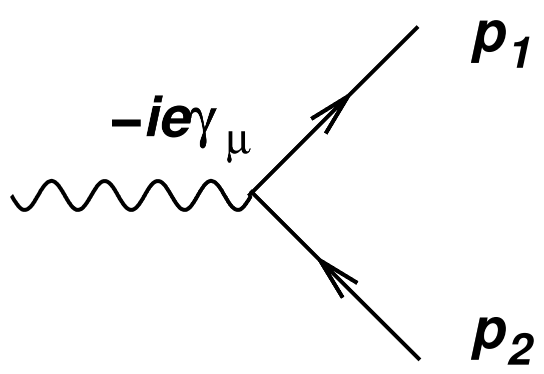


For this process we can define n-jet rates: $R_{n-\mathrm{jets}} = \sigma(> n - \mathrm{jets})/\sigma(\mathrm{tot})$

$$R_{2\mathrm{jets}}^{LO} = \; ?$$

qT subtraction: Idea

Consider again $\gamma^* \to qq$ (dijet production in e+e-)



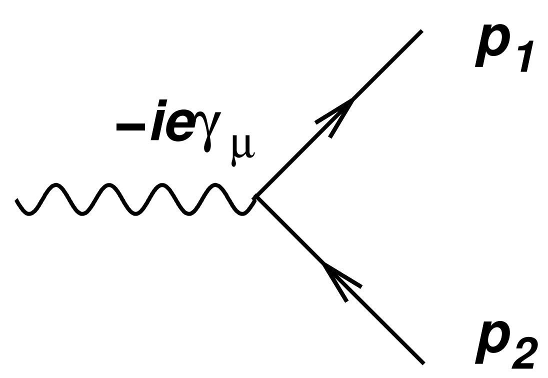
For this process we can define n-jet rates: $R_{n-\mathrm{jets}} = \sigma(> n - \mathrm{jets})/\sigma(\mathrm{tot})$

$$R^{LO}_{2\mathrm{jets}} = 1$$

All LO all events are two-jet like

qT subtraction: Idea

Consider again $\gamma^* \rightarrow qq$ (dijet production in e+e-)



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$$R^{NLO}_{\mathrm{2jets}} = ?$$

qT subtraction: Idea

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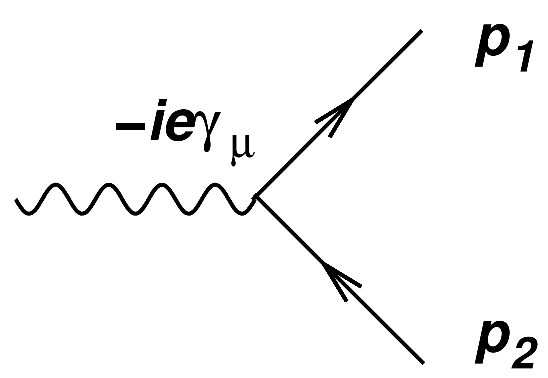

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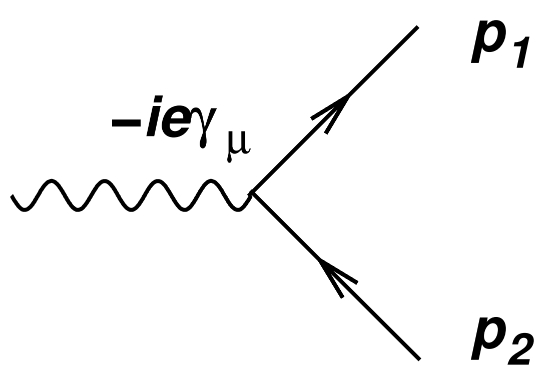
$R^{LO}_{\mathrm{2jets}} = 1$ All LO all events are two-jet like

$R^{NLO}_{\mathrm{2jets}} = 1 - R^{LO}_{3jets}$ At NLO all events are two-jet like except those that contribute to the LO three-jet rate

qT subtraction: Idea

Consider again $\gamma^* \to qq$ (dijet production in e+e-)



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$$R^{LO}_{\mathrm{2jets}} = 1$$

All LO all events are two-jet like

$$R^{NLO}_{\mathrm{2jets}} = 1 - R^{LO}_{3jets}$$

At NLO all events are two-jet like except those that contribute to the LO three-jet rate

$$R^{NNLO}_{\mathrm{2jets}} = 1 - R^{NLO}_{3jets} - R^{LO}_{4jets}$$

At NNLO all events are two-jet like except those that contribute to the NLO three-jet rate and to the LO four jet rate

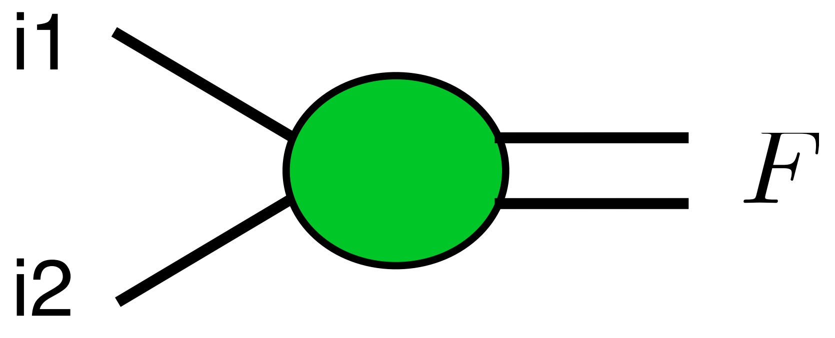
➡Use NLO (LO) information to construct NNLO objects

➡In general: R^{NnLO}_{2jet} can be obtained from an $N^{n-1}LO$ computation

qT subtraction

- This method can be applied to compute NNLO QCD corrections to any pp→F processes, where F is a colourless system (F may consist of lepton-pairs, vector bosons, Higgs Bosons,…)

LO



qT=pT(F)=0

NLO $$d\sigma^F_{NLO}|_{q_T\neq 0} = d\sigma^{F+\text{jets}}_{LO}$$

NNLO $$d\sigma^F_{NNLO}|_{q_T\neq 0} = d\sigma^{F+\text{jets}}_{NLO}$$

- For which standard NLO techniques can be used
- However, $d\sigma^F_{NLO}|_{q_T\to 0} \to \infty$ (IR singular)

Missing contribution: $d\sigma^F_{NNLO}|_{q_T\to 0}$

qT subtraction

Solution: construct counterterm which subtracts the singularity for qT→0 based on eikonal

$$d\sigma^{CT} \sim d\sigma^{(LO)} \otimes \Sigma^F(q_T/Q)$$

Factorisation from Born!

where: $\Sigma^F(q_T/Q) \sim \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n \sum_{k=1}^{2n} \Sigma^{F(n;k)} \frac{Q^2}{q_T^2} \ln^{k-1} \frac{Q^2}{q_T^2}$

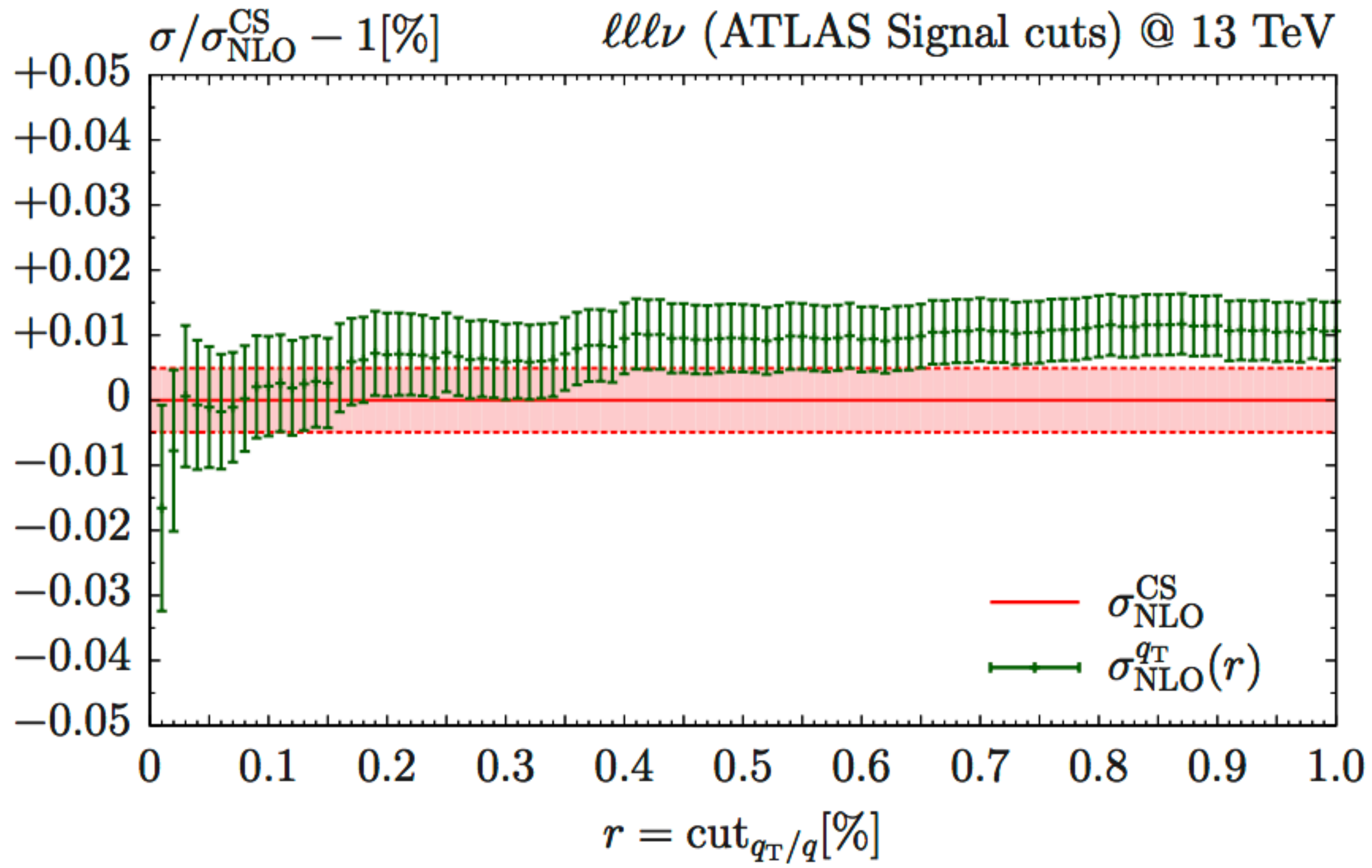
NNLO cross section:

$$d\sigma^F_{(N)NLO} = \mathcal{H}^F_{(N)NLO} \otimes d\sigma^F_{LO} + \left[d\sigma^{F+\text{jets}}_{(N)LO} - d\sigma^{CT}_{(N)LO}\right]$$

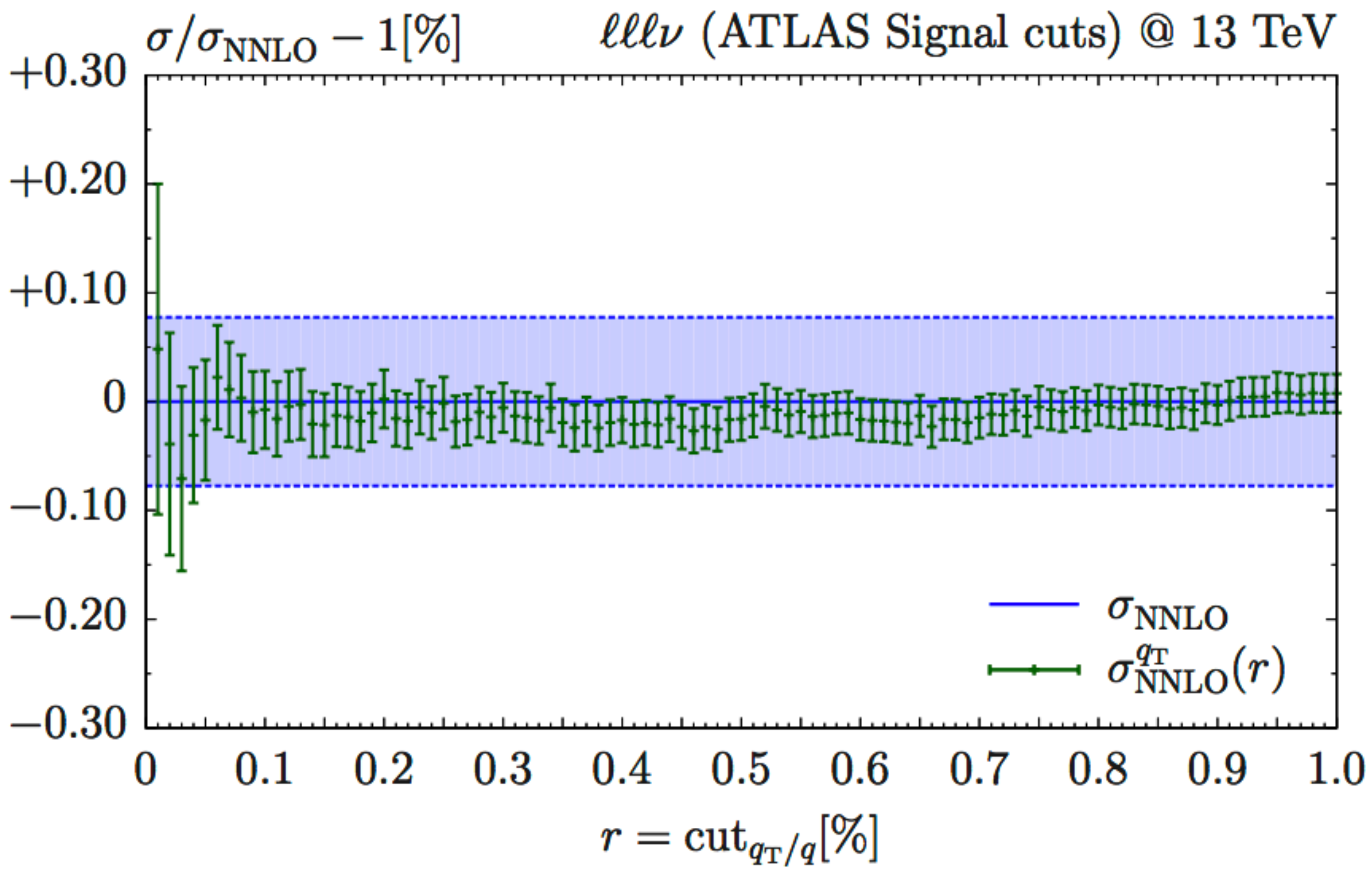
$\mathcal{H}^F$ can be computed via perturbative expansion and involves the two-loop virtual contributions.

qT subtraction: independence of qTcut

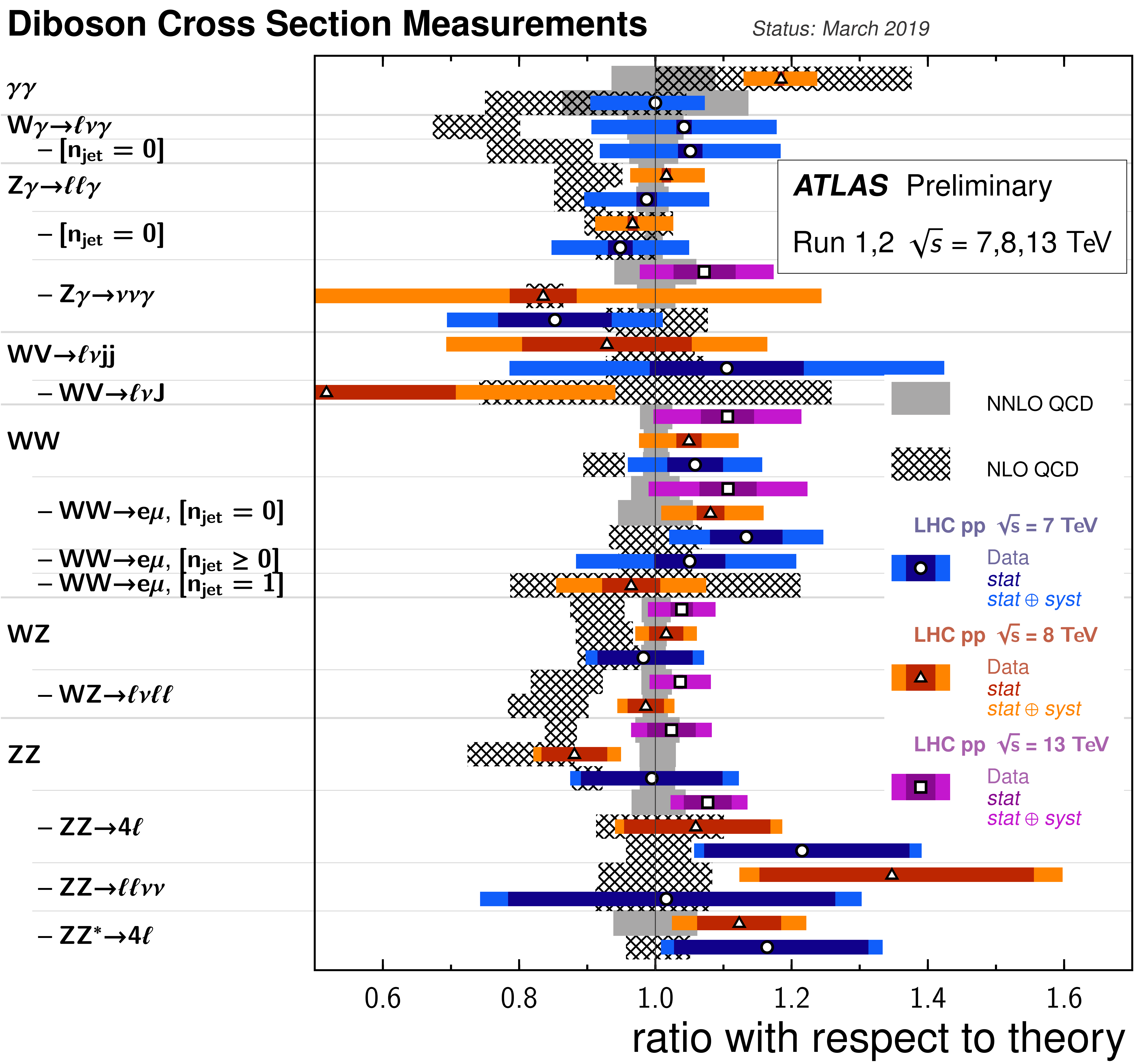
NLO



NNLO



NNLO for diboson processes



Remarkable agreement of inclusive diboson cross sections with **NNLO QCD**

Allows for stringent SM tests

Dibosons important background for Higgs and BSM searches

Back to the perturbative expansion…

$$d\sigma = d\sigma_{\rm LO} + \alpha_S \, d\sigma_{\rm NLO} + \alpha_{\rm EW} \, d\sigma_{\rm NLO\,EW}$$

NLO QCD NLO EW

$$+\alpha_S^2 \, d\sigma_{\rm NNLO} + \alpha_{\rm EW}^2 \, d\sigma_{\rm NNLO\,EW} + \alpha_S \alpha_{\rm EW} \, d\sigma_{\rm NNLO\,QCDxEW}$$

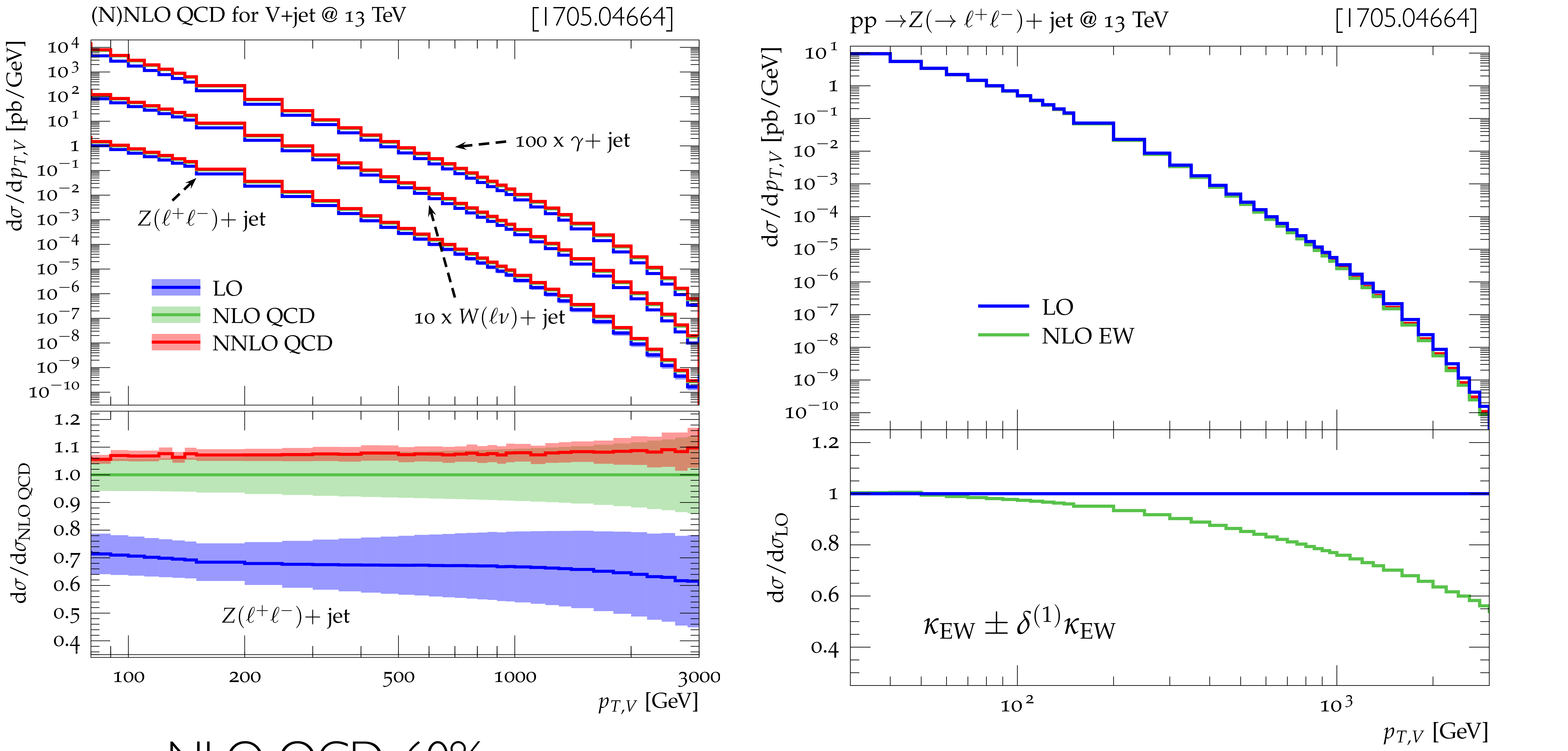
NNLO QCD NNLO EW NNLO QCD-EW

For cases where QCD and EW corrections are sizeable, also mixed QCD-EW corrections of relative $\mathcal{O}(\alpha\alpha_s)$ have to be considered.

Combination of QCD and EW corrections

Example: V+jets

(N)NLO QCD | NLO EW

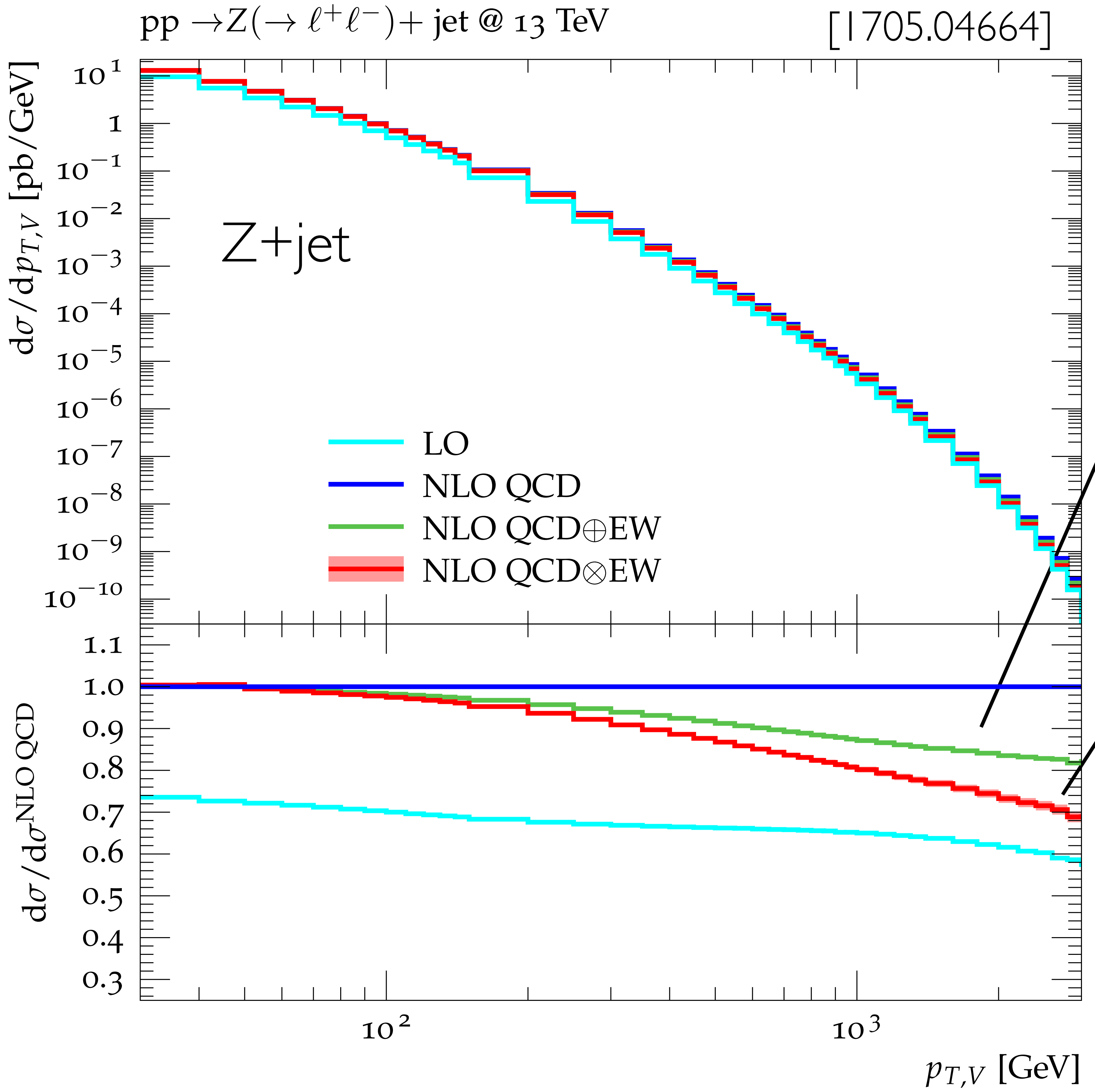


- NLO QCD: 60%
- NNLO QCD: 10%

- NLO EW: up to -25(40)% at 1(2) TeV

➡Very naive estimate: NNLO QCD-EW=NLO QCD x NLO EW=15% at 1 TeV

Combination of QCD and EW corrections



Additive combination

$$\sigma^{\rm NLO}_{\rm QCD+EW} = \sigma^{\rm LO} + \delta\sigma^{\rm NLO}_{\rm QCD} + \delta\sigma^{\rm NLO}_{\rm EW}$$

Multiplicative combination

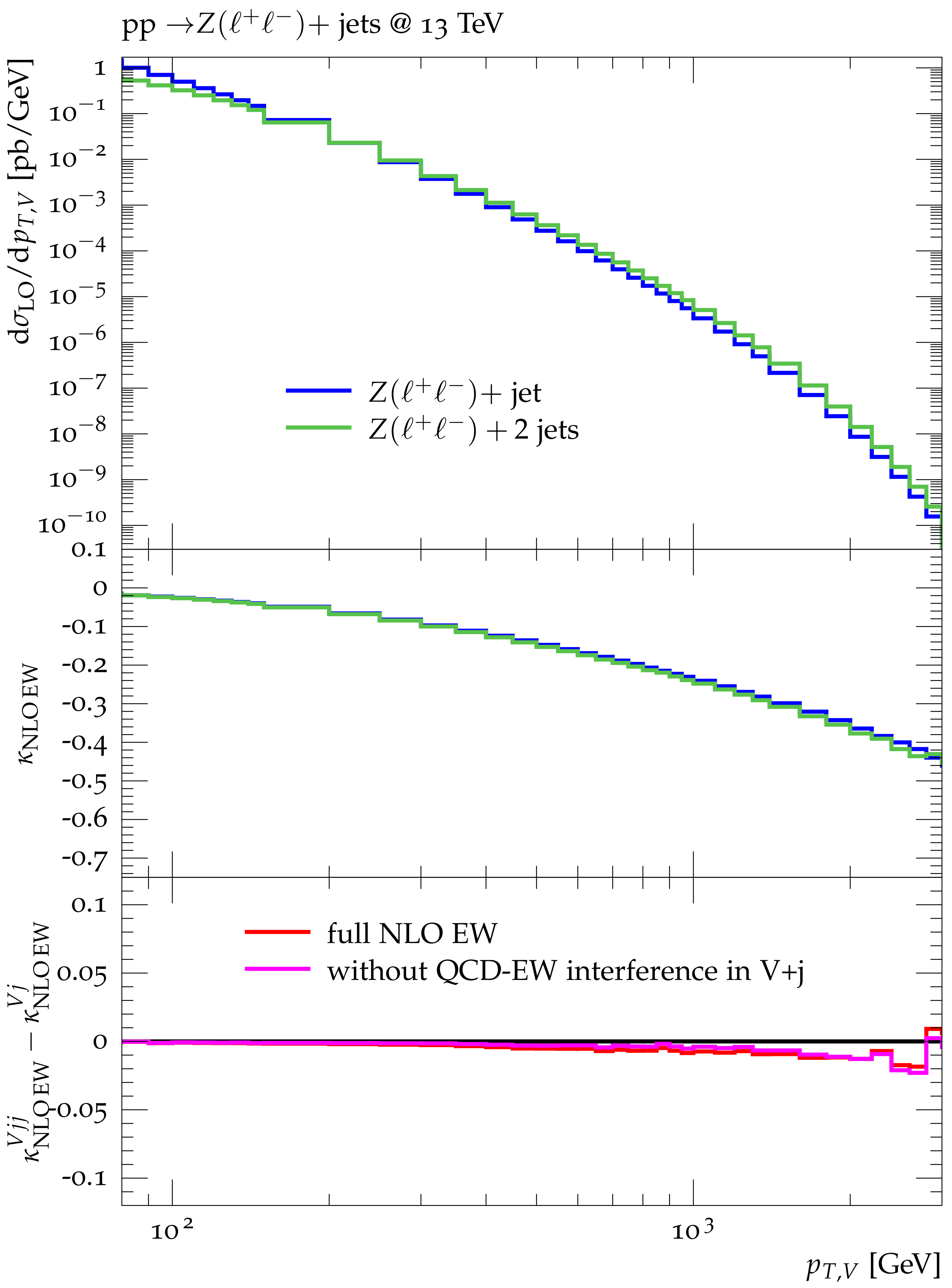
$$\sigma^{\rm NLO}_{\rm QCD\times EW} = \sigma^{\rm NLO}_{\rm QCD}\left(1 + \frac{\delta\sigma^{\rm NLO}_{\rm EW}}{\sigma^{\rm LO}}\right)$$

(try to capture some $\mathcal{O}(\alpha\alpha_s)$ contributions, e.g. EW Sudakov logs × soft QCD)

Difference between these two approaches indicates size of missing mixed EW-QCD corrections.

$K_{\rm QCD\otimes EW} - K_{\rm QCD\oplus EW} \sim 10\%$ at 1 TeV

Mixed QCD-EW uncertainties



pT$_{j,2}$ > 30 GeV

Bold estimate:

Consider real $\mathcal{O}(\alpha\alpha_s)$ correction to V+jet
$\simeq$ NLO EW to V+2jets

and we observe

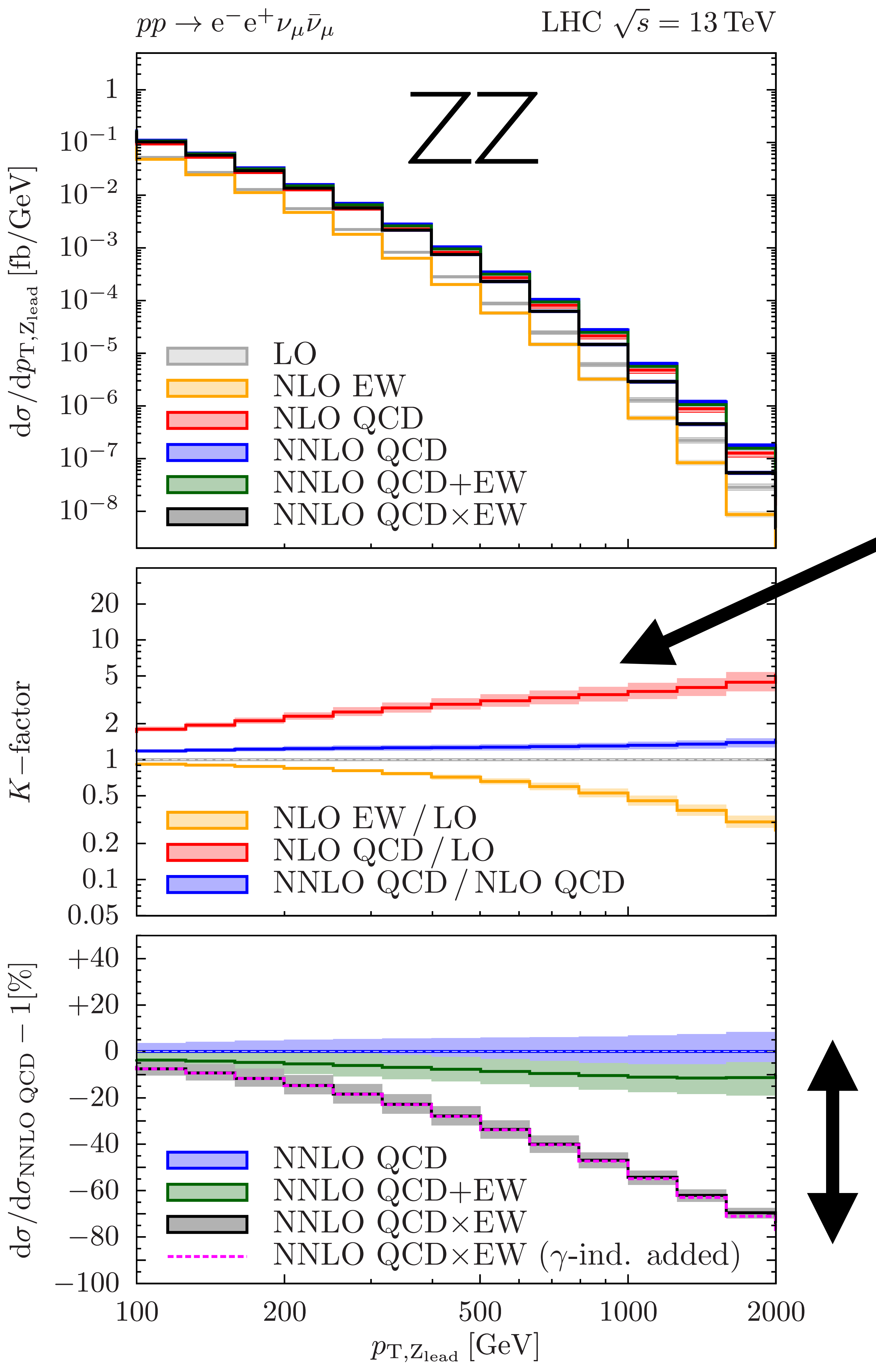
$$\left.\frac{d\sigma_{\mathrm{NLO\,EW}}}{d\sigma_{\mathrm{LO}}}\right|_{V+2\mathrm{jet}} - \left.\frac{d\sigma_{\mathrm{NLO\,EW}}}{d\sigma_{\mathrm{LO}}}\right|_{V+1\mathrm{jet}} \lesssim 1\%$$

strong support for

- factorization
- multiplicative QCD x EW combination

Combination of QCD and EW corrections

Example: VV



- NLO/LO=2-5! ("giant K-factor")
- at large pTV1: VV is dominated by V+jets (w/ soft V radiation)
- NNLO uncertainty: 5-10%
- O(1) difference in

$$d\sigma^{(N)NLO}_{QCD+EW} \quad \text{vs.} \quad d\sigma^{(N)NLO}_{QCD\times EW}$$

Combination of QCD and EW corrections

Example: VV

jet veto

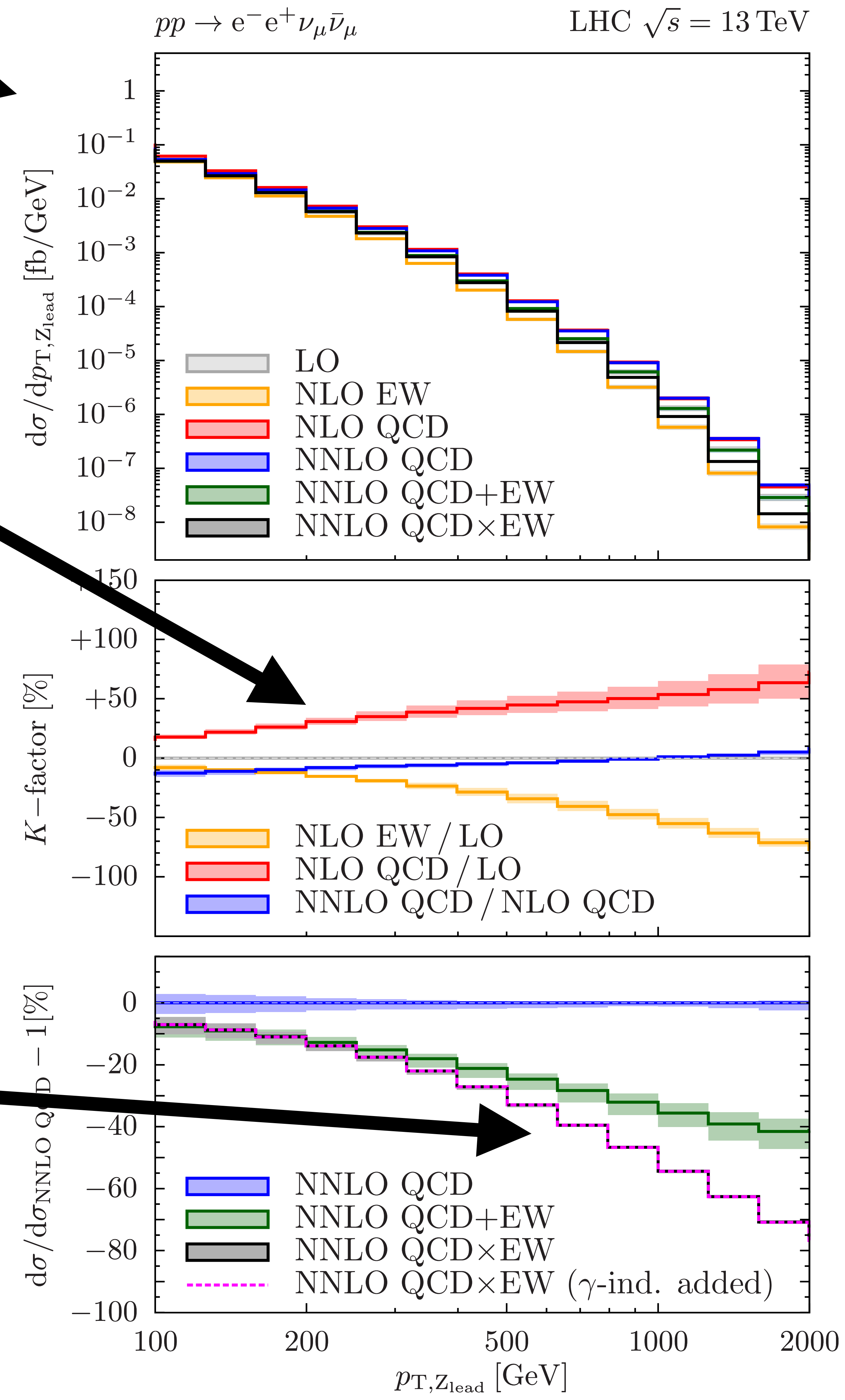
- NLO/LO=~<1.5 ("normal K-factor")

- Reliable estimate of O(as a) from

$$\mathbf{d}\boldsymbol{\sigma}^{\mathbf{(N)NLO}}_{\mathbf{QCD+EW}} \text{ vs. } \mathbf{d}\boldsymbol{\sigma}^{\mathbf{(N)NLO}}_{\mathbf{QCD\times EW}}$$

Here: 10-20% in TeV range

- However:
additional uncertainty due to efficiency of jet veto



And again…

$$d\sigma = d\sigma_{\rm LO} + \alpha_S\, d\sigma_{\rm NLO} + \alpha_{\rm EW}\, d\sigma_{\rm NLO\,EW}$$

NLO QCD   NLO EW

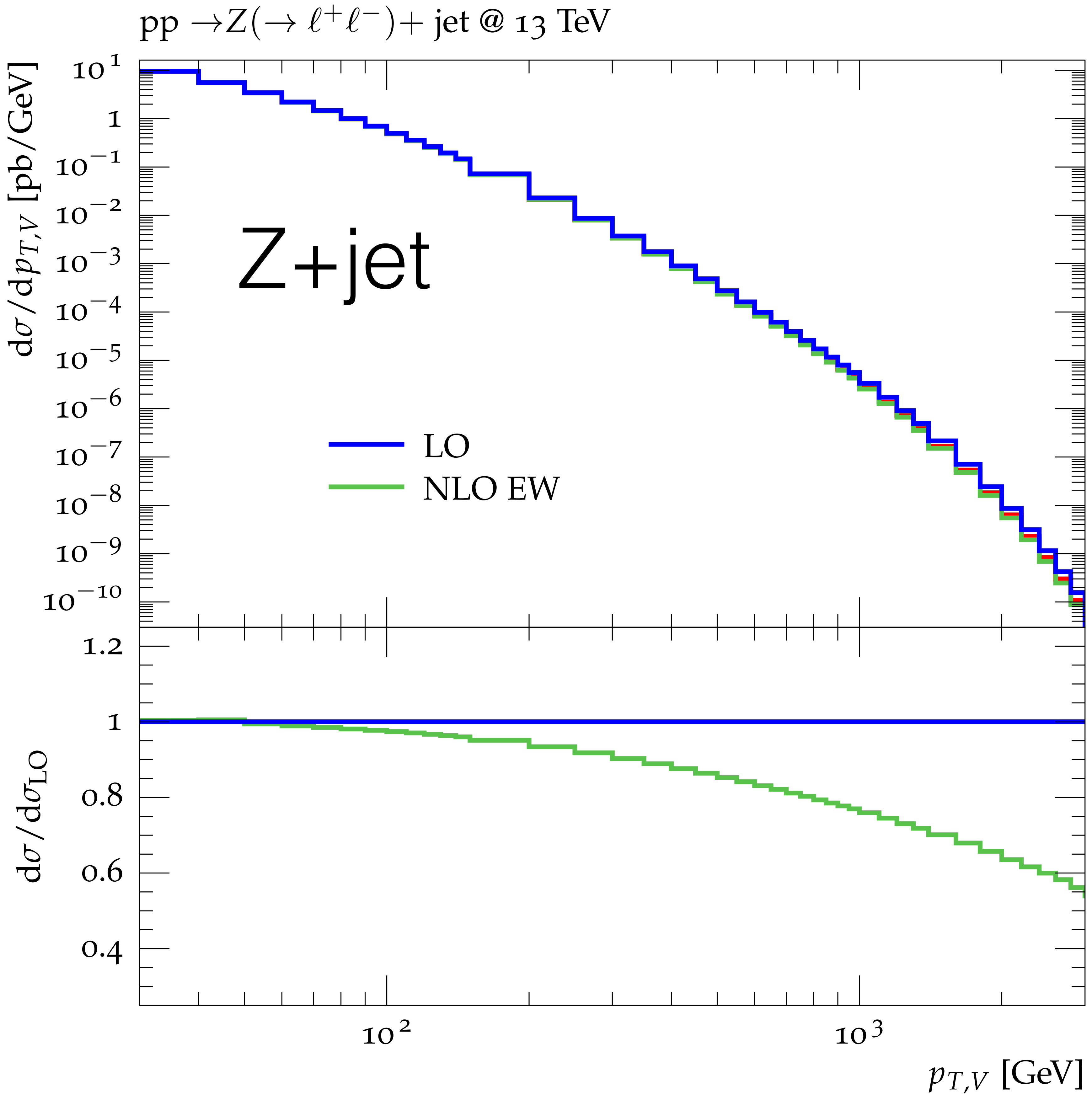
$$+\alpha_S^2\, d\sigma_{\rm NNLO} + \alpha_{\rm EW}^2\, d\sigma_{\rm NNLO\,EW} + \alpha_S\alpha_{\rm EW}\, d\sigma_{\rm NNLO\,QCDxEW}$$

NNLO QCD   NNLO EW   NNLO QCD-EW

What about this contribution?

- Explicit calculation for most processes out of reach
- Uncertainty estimates?

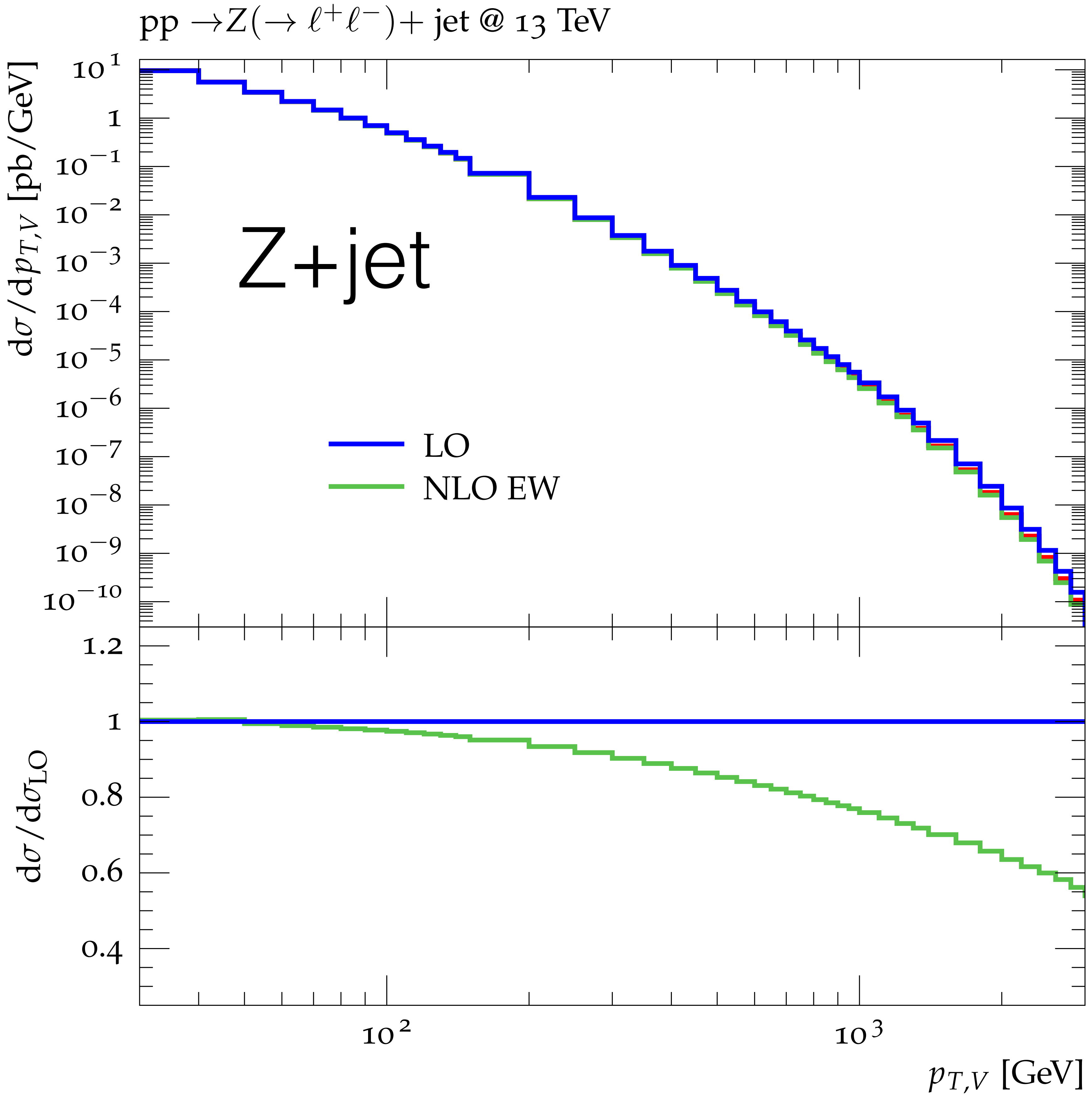
Pure EW uncertainties



NLO EW corrections are negative & sizeable at large $p_{T,V}$

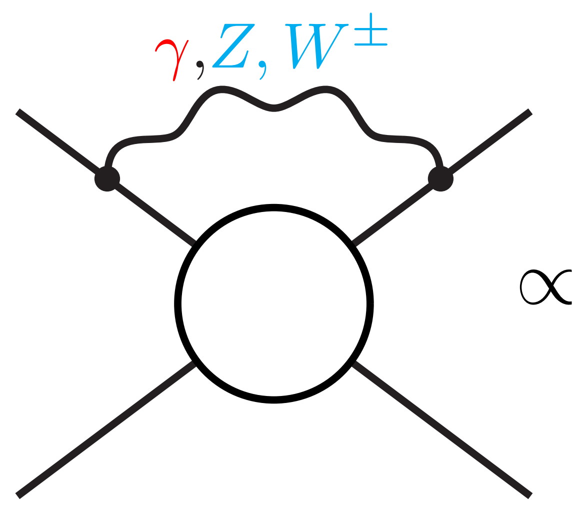
How to estimate corresponding pure EW uncertainties of relative $\mathcal{O}(\alpha^2)$?

Pure EW uncertainties



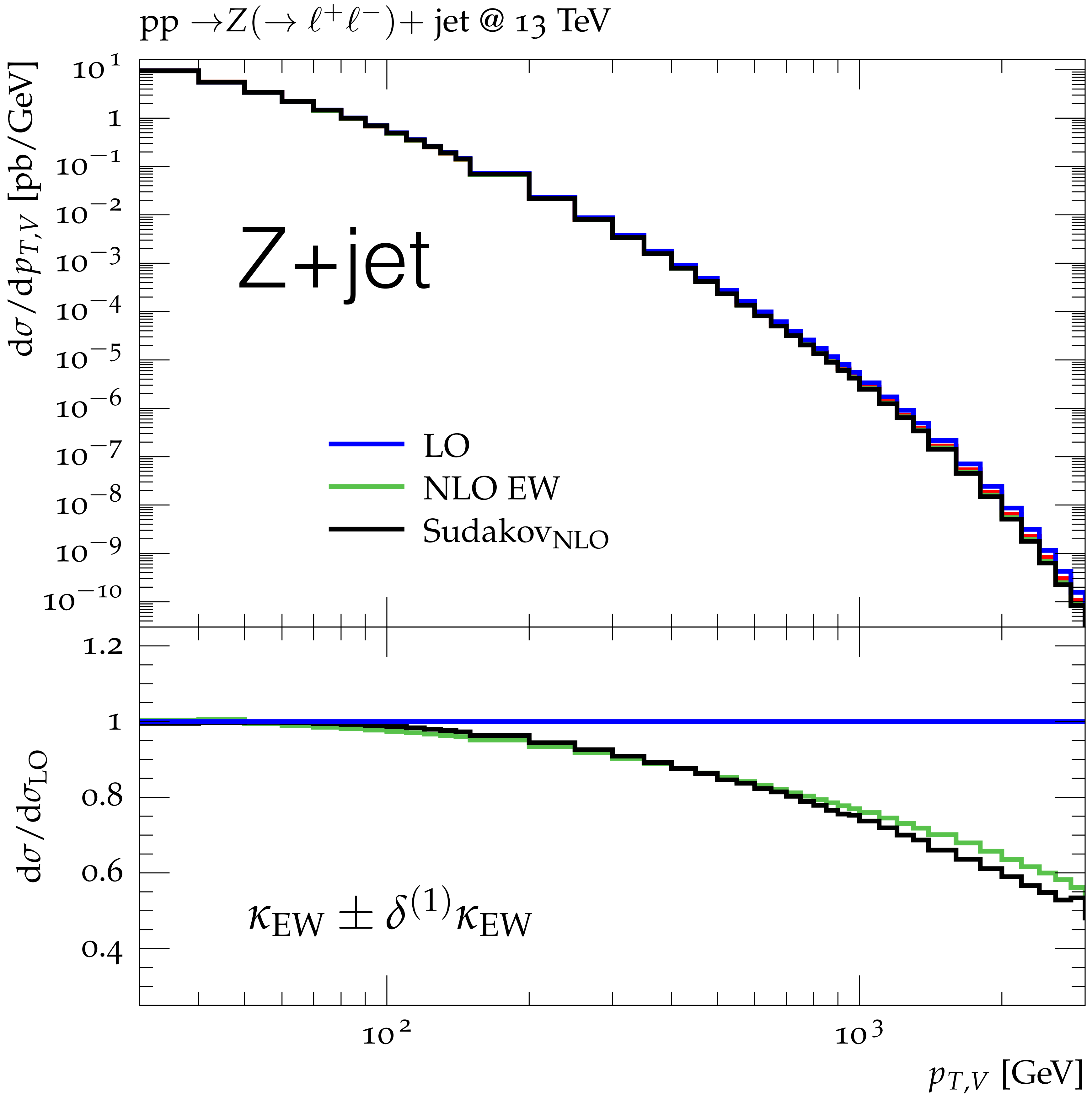
NLO EW corrections are negative & sizeable at large $p_{T,V}$

Origin: soft/collinear logs from virtual EW gauge boson (EW **Sudakov logarithms**)



$$\propto \ln^2\left(\frac{\hat{s}}{M_W^2}\right)\mathcal{M}_0$$

Pure EW uncertainties



NLO EW corrections are negative & sizeable at large $p_{T,V}$

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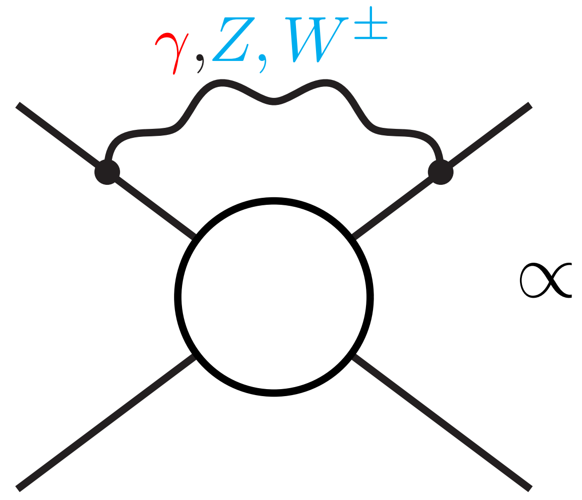
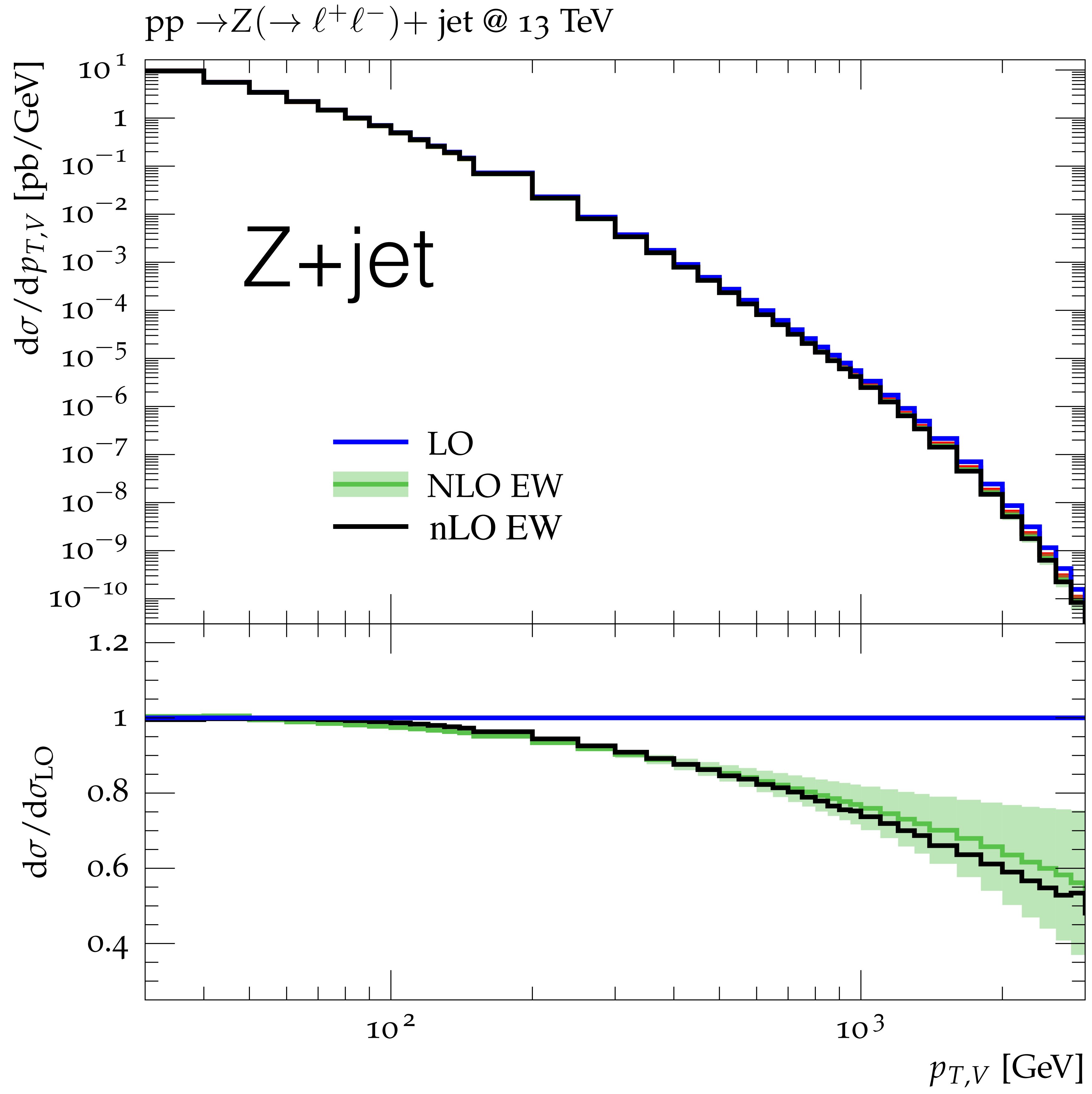


$$\propto \ln^2\left(\frac{\hat{s}}{M_W^2}\right)\mathcal{M}_0$$

Large EW corrections dominated by Sudakov logs!

Pure EW uncertainties



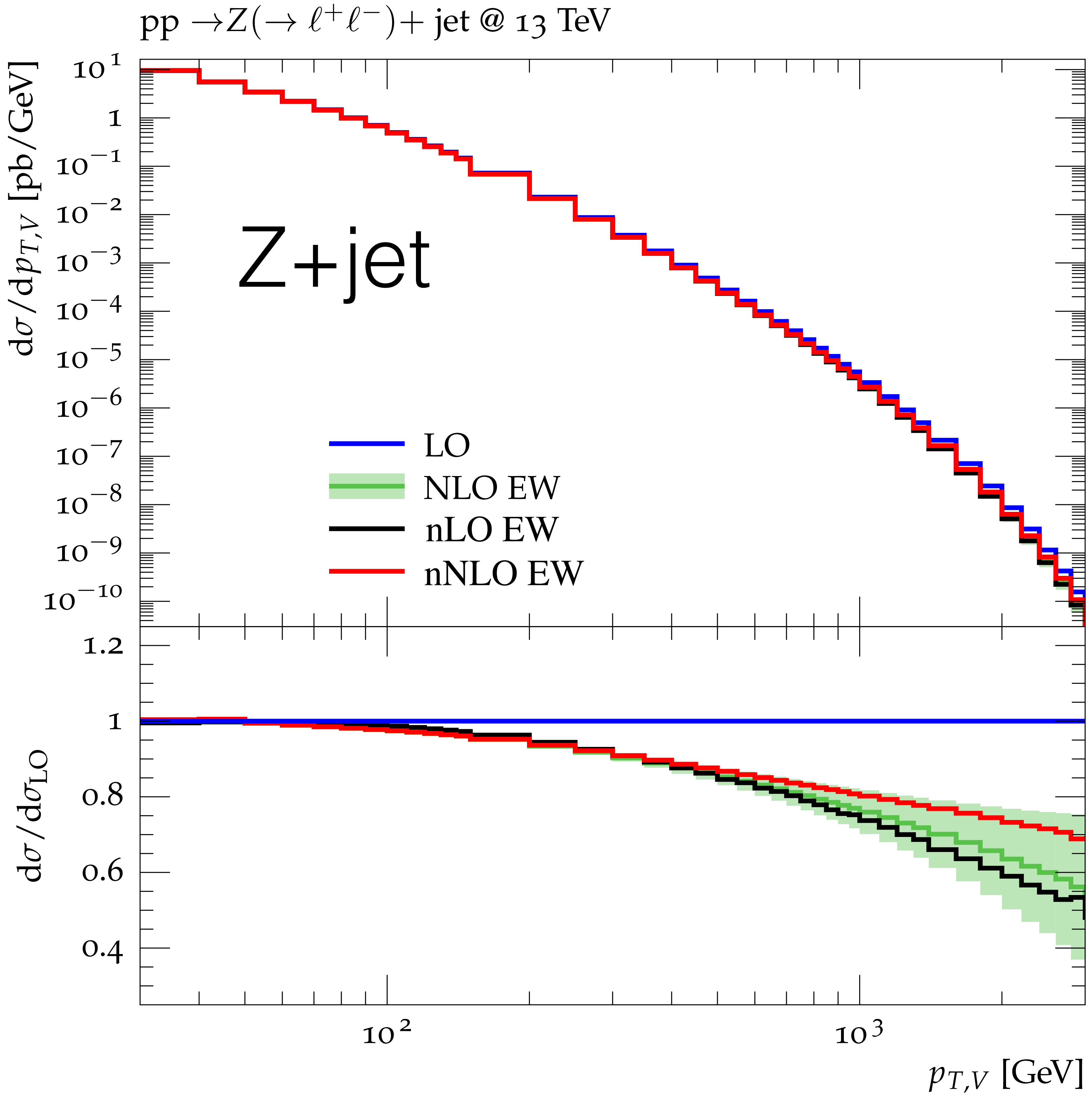
Large EW corrections dominated by Sudakov logs!

↓

Uncertainty estimate of NLO EW from naive exponentiation

$$\delta^{(1)}_{\mathrm{N^kLO\,Sud}} \simeq \frac{1}{k!}\left[\delta^{(k-1)}_{\mathrm{Sud}}\right]^k$$

Pure EW uncertainties



Large EW corrections dominated by Sudakov logs

↓

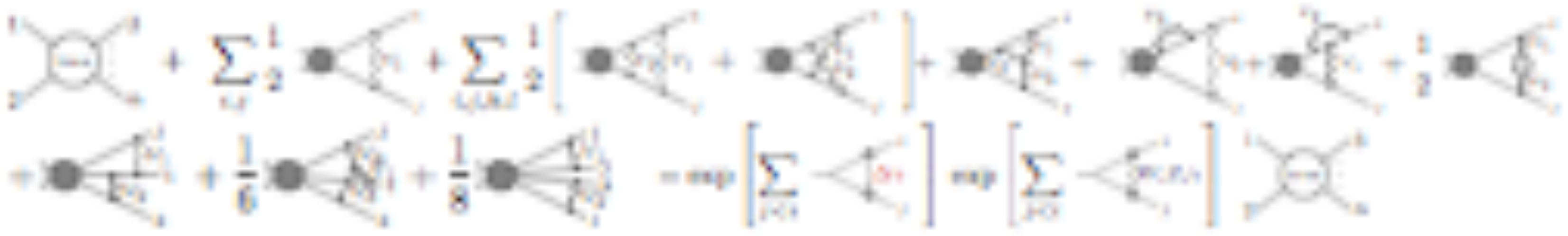
Uncertainty estimate of NLO EW from naive exponentiation x 2:

$$\delta^{(1)}\kappa^{(V)}_{\mathrm{NLO\,EW}}(x) = \frac{2}{2}\left[\kappa^{(V)}_{\mathrm{NLO\,EW}}(x)\right]^2$$

↓

check against two-loop Sudakov logs

[Kühn, Kulesza, Pozzorini, Schulze; 05-07]



Recap

- NLO QCD+EW corrections fully automated in several tools
- Based on efficient evaluation of tree and one-loop amplitudes via numerical methods
- ..and automated subtraction methods (CS and FKS)
- QCD and EW corrections overlap and are not unambiguously defined for processes involving four quarks at LO
- NLO results are available up to very high multiplicities
- Remaining perturbative uncertainties from NNLO QCD, EW, and QCD-EW are often becoming relevant

References

These Lectures are partly based on:

- Stefan Weinzierl, DESY Monte Carlo school, 2012
- Ansgar Denner, DESY Monte Carlo school, 2014
- Andreas van Hameren, DESY Monte Carlo school, 2017
- Giulia Zanderighi, Graduate Course on QCD, 2013
- Rikkert Frederix, MCnet Summer School, 2015
- Gavin Salam, Basics of QCD, ICTP–SAIFR school on QCD and LHC physics, 2015