# Physics Assignment 2

PHY411: Do all problems (problems are made easier for you).
PHY 506: Do all problems.

Accept the assignment from github classroom: https://classroom.github.com/a/mIXdDWYC. You will then get a link to your own github area.
You should submit your code through github classroom. Submit your writeup, and a link to your github classroom area where your code is, on UBLearns.

## Problem 1: Helium Atom with LCAO

Similarly to the hydrogen atom we did in class, compute the first two energy eigenvalues for the helium atom with Gaussian Linear Combination of Atomic Orbitals (LCAO).

## Problem 2: Variational MC

Adapt the variational MC we did to the case of a hydrogen atom (undergrads) and a helium atom (grads). Be careful about cusp positions! Compute the energy of the spherically symmetric ground state, and plot the wave function as a function of the radius.

# Problem 3 : Z boson production

Consider the process of di-lepton production at the Large Hadron Collider:

$$pp \to Z \to e^+e^- + X$$

where p denotes the incoming protons, Z is the Z boson,
**Compute the differential cross section as a function of the cosine of the angle (or the pseudorapidity), as well as the total cross section, using the VEGAS algorithm.**

PHY 411: Only consider up quark+antiquark in the initial state.
PHY 506: Consider up, down, and strange quark+antiquark in the initial state.

The center-of-mass energy of the system is:
$s = (p_i + p_j)^2 = (E_i + E_j)^2 - (\mathbf{p_i} + \mathbf{p_j})^\mathbf{2}$

Assume it is equal to 7 TeV (tera electronvolts). The "master formula" for the parton-level cross section is:

Equation 1:

$$\frac{d\sigma}{d\Omega} = \frac{N_c^{\text{final}}}{N_c^{\text{initial}}} \times \frac{1}{256\pi^2 s} \times \frac{s^2}{(s - M_Z^2)^2 + s\Gamma_Z} \times \left[ (L^2 + R^2)(L'^2 + R'^2)(1 + \cos\theta) + (L^2 - R^2)(L'^2 - R'^2) 2\cos\theta \right] ,$$

where $M_Z$ and $\Gamma_Z$ are the mass and width of the Z boson, and Nc is the number of colors (for quarks Nc = 3, for leptons Nc = 1), with L and R denoting the Left and Right handed components:

$$L = \sqrt{\frac{8G_F m_Z^2}{\sqrt{2}}}(T_3 - \sin^2\theta_W Q) , \quad R = -\sqrt{\frac{8G_F m_Z^2}{\sqrt{2}}} \sin^2\theta_W Q ,$$

Then use the MSTW 2009 Parton Distribution Functions and the VEGAS algorithm to compute the cross sections:

Equation 2:

$$\frac{d\sigma}{d\Omega} = \sum_{a_1,a_2} \int_0^1 dx_2 dx_1 f_{a_1}^{h_1}(x_1, M^2) f_{a_2}^{h_2}(x_2, M^2)\, E_Q \frac{d\sigma^{a_1 a_2}}{d^3Q}(x_1 P_1, x_2 P_2, M^2)\,,$$

where P1 and P2 are the momenta of the incoming protons (they are equal and opposite), a1,a2 are indices of the incoming parton species, f(x,$M^2$) are the MSTW parton distribution functions for each particle species, x is the fraction of the proton's momentum carried by that parton, M is the scale of the problem (use M=100 GeV) and $d\sigma/d^3Q$ is the parton-level cross section for particles a1 and a2 and is given by Equation 1.

*Hint: The mass of the Z boson is very small compared to the center-of-mass energy of the system, and the PDFs are strongly peaked at low momentum fractions. Plan your phase space accordingly.*