Manifold reconstruction using linear approximations

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Tracks after Track finding, a few data points per particle

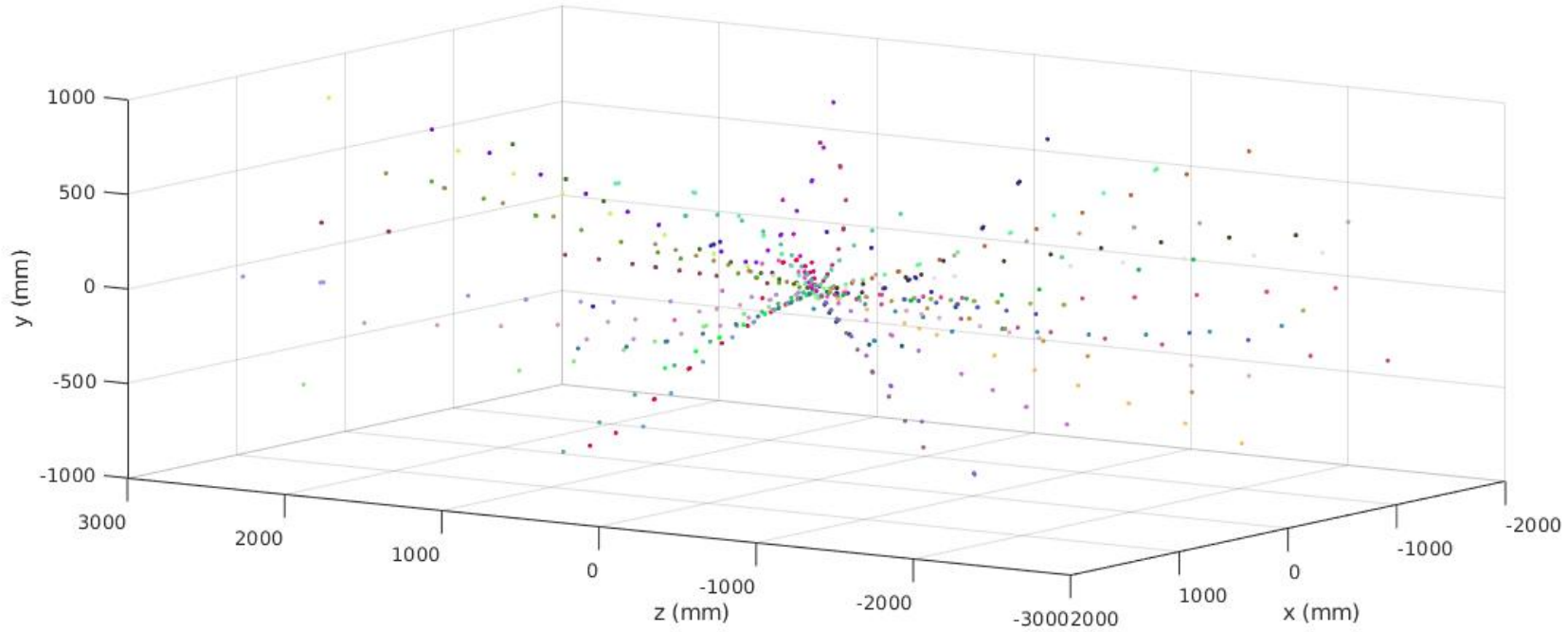


Figure: Particle trajectories from the TrackML dataset assuming track finding has been performed, different colors indicate different particles

Linear Track Fitting

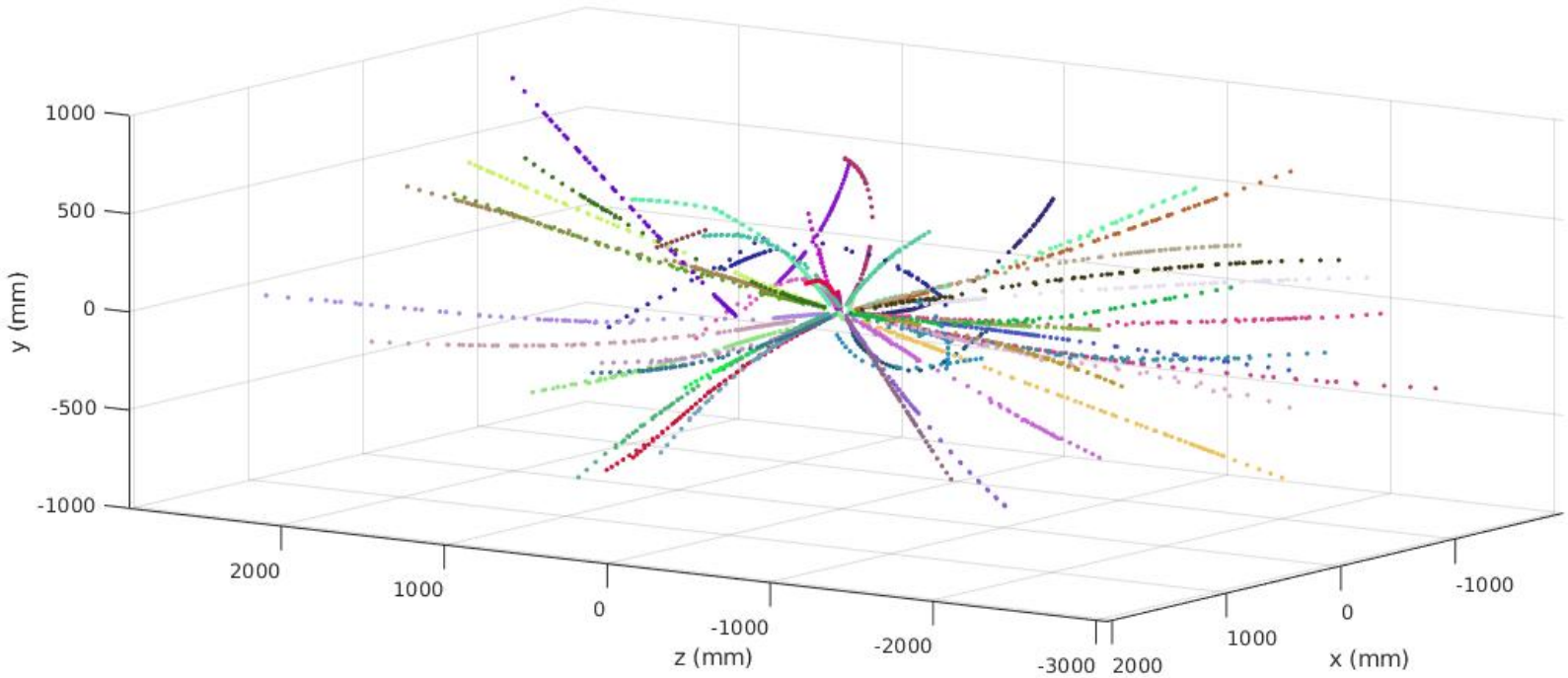


Figure: The same particle trajectories reconstructed from linear fit

Toy Example:

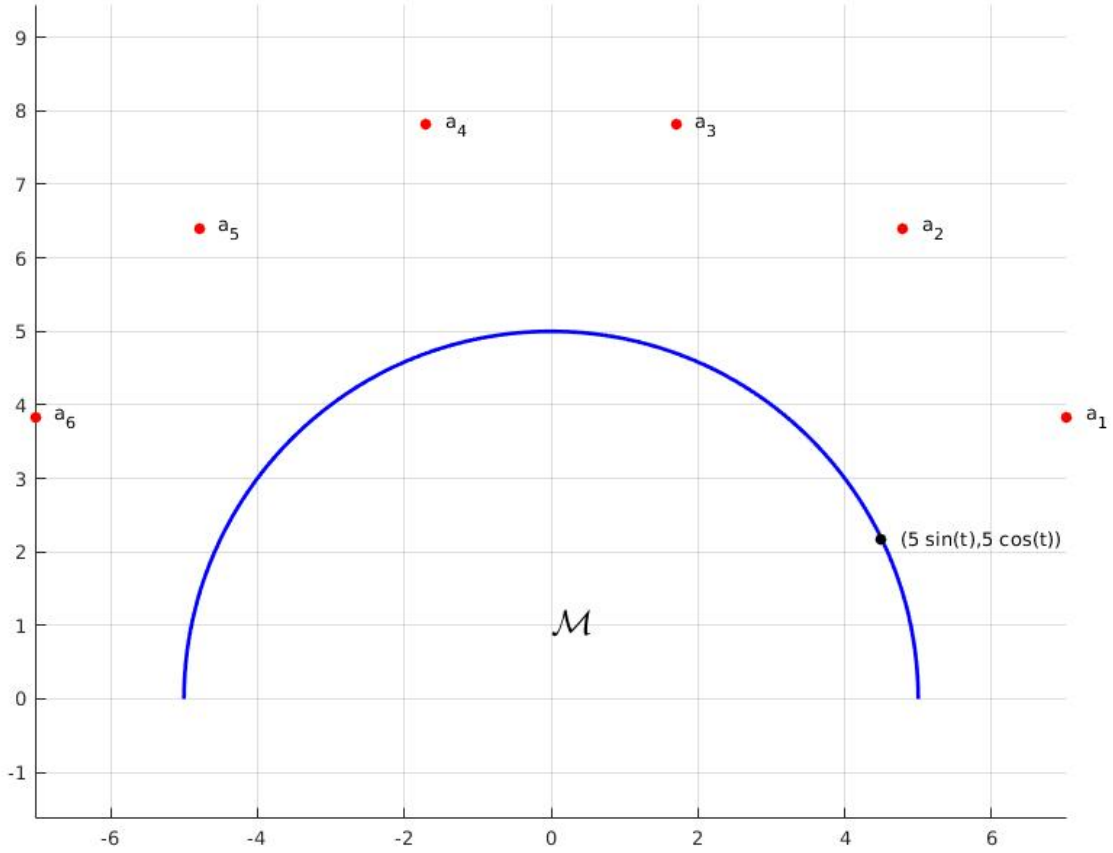


Figure: Semicircle and anchor points a_i

Toy Example:

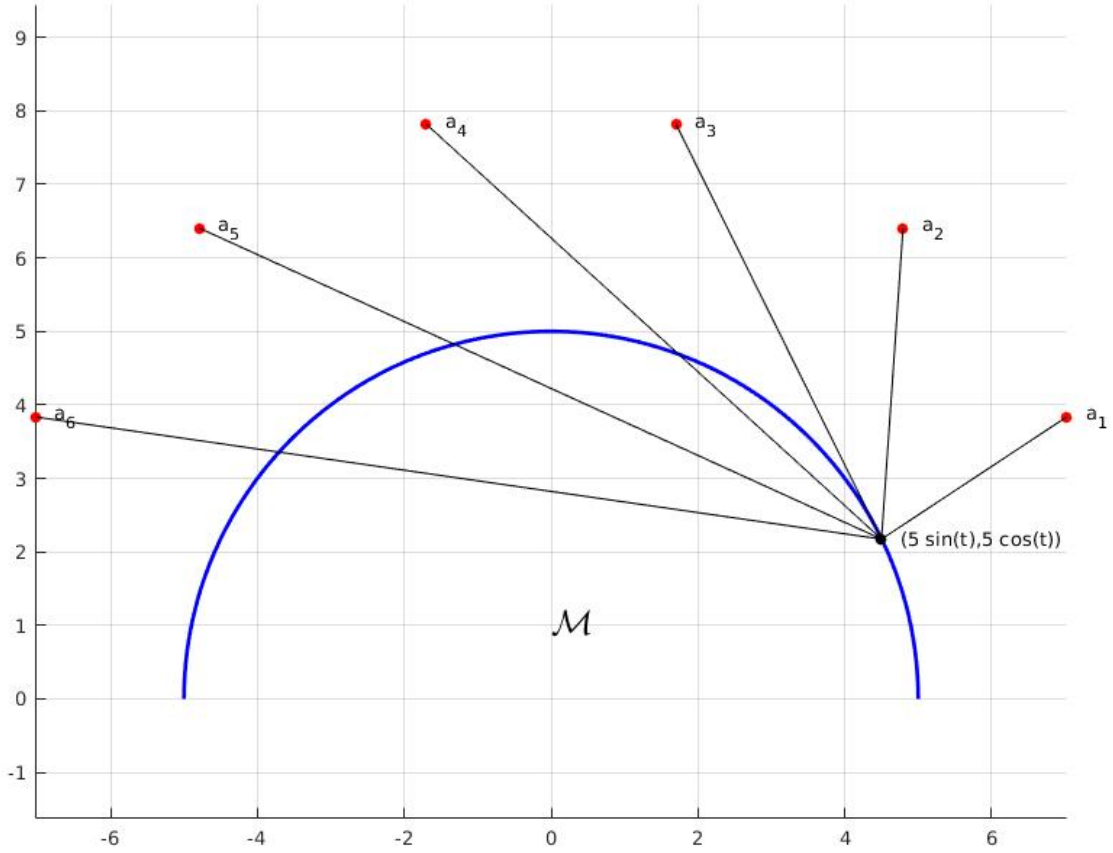
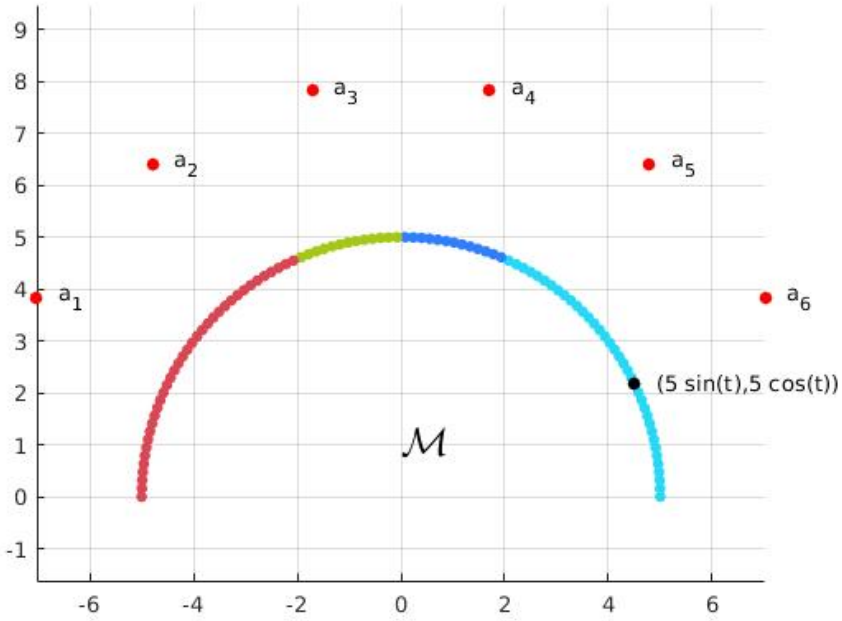
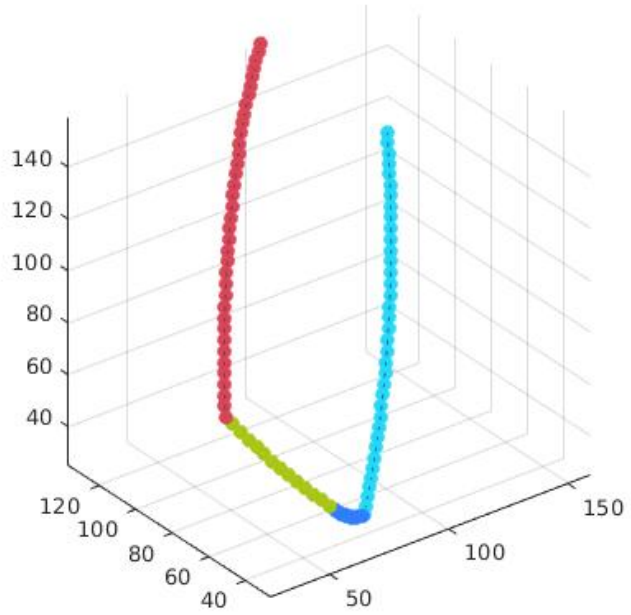


Figure: Compute the distances d_i





consider the map
$r \mapsto \left((d_1)^2, ..., (d_6)^2\right)$

Embedding of the semicircle by three max values of d_i

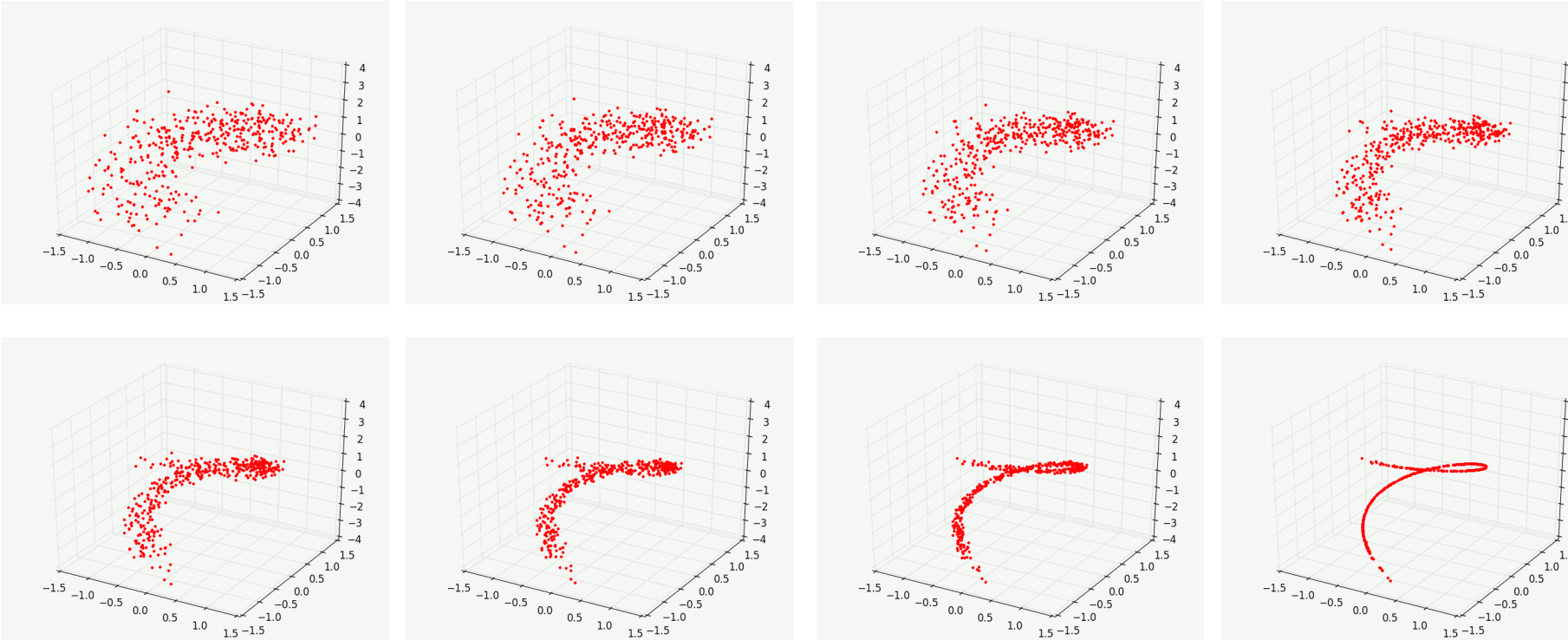
Some background of the theory

The linearity is established by proving that

1. curves or manifold embedded in euclidean space when mapped to higher dimension by a function of the squared distance information with assumptions on the locations of a constellation of anchor points and assumptions on sectional curvature of $\mathcal{M}$, can have their curvature lowered
2. under certain sampling assumptions, most of the data obtained from the warped representation of the manifold can be well approximated by degree-1 regression that is by approximating the tangent (or tangent space)
3. the inverse of the map is unique and its output is smooth

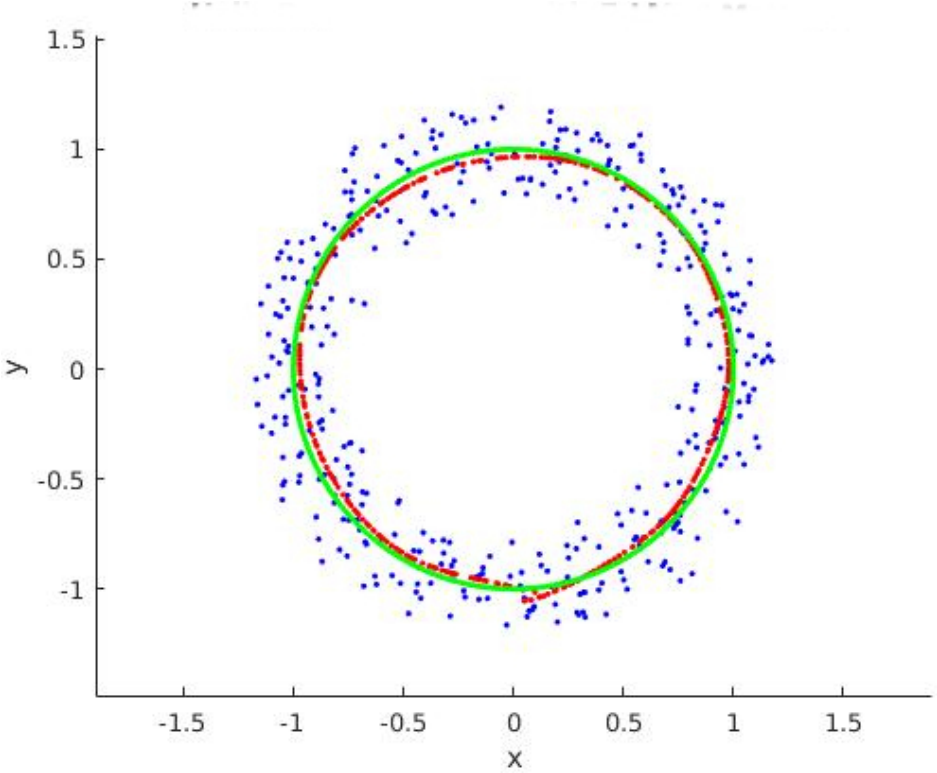
Manifold Moving Least Squares

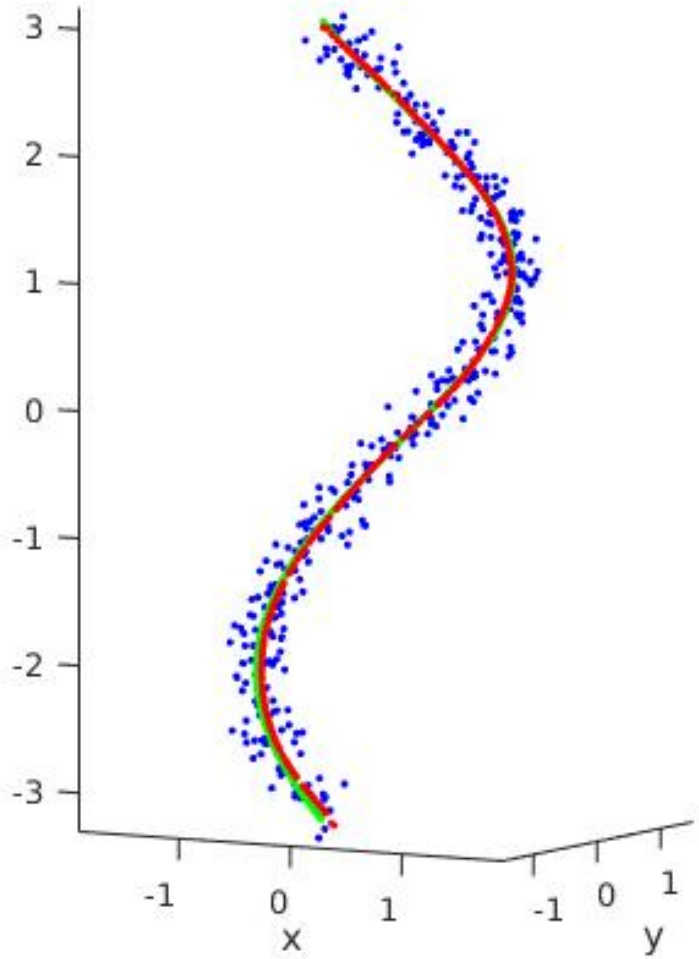
- If the data is noisy or a smoothening step is required, there is a (polynomial regression) manifold learning algorithm (MMLS) by Barak Sober and David Levin.



- We can use the flattening strategy with degree-1 regression of MMLS to denoise

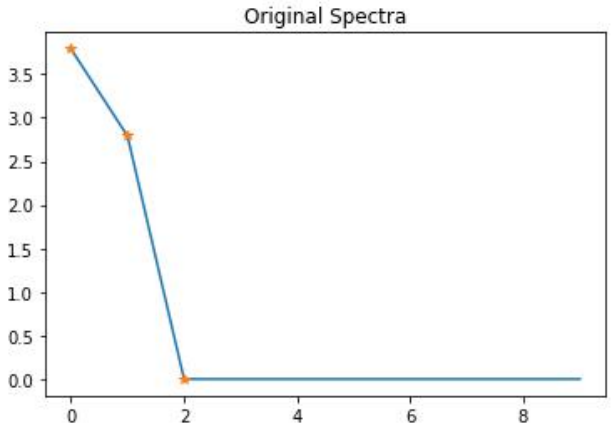
Denoising through linear regression (1 dim curve in 3 dim)

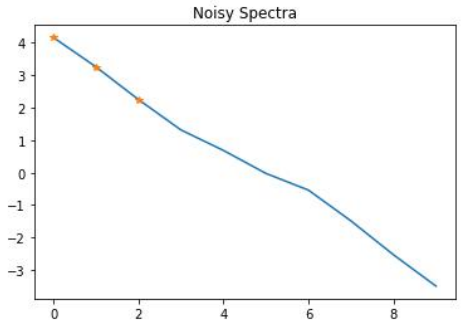


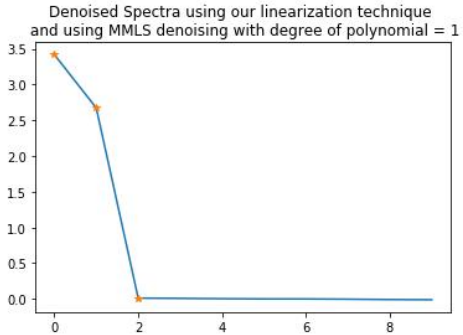


Denoising through linear regression (2 dim manifold in 100 dim)

- 10-dim symmetric matrix UDU^T, where $UU^T = 1$ with 2 leading eigenvalues $\lambda_1 \in [2, 3]$, $\lambda_2 \in [3, 4]$
- Each matrix is column stacked so that each matrix becomes an element in 10^2-dim space
- Each coordinate is cubed
- We have a 2-dimensional non-linear (cubic) submanifold in $\mathbb{R}^{100}$. With zero mean additive uniform noise of 0.5, below is the spectrum reconstruction







Tail Energy $\sqrt{\frac{\lambda_3^2+..+\lambda_{10}^2}{\lambda_1^2+..+\lambda_{10}^2}} = 0.00028$

Key points

1. For certain collections of anchor points $\mathbf{a}_i$, the image under the map

$$\mathbf{r} \mapsto \left(f(|\mathbf{r} - \mathbf{a}_1|)^2, ..., f(|\mathbf{r} - \mathbf{a}_\Gamma|)^2 \right) \tag{1}$$

   can have low curvature
2. Can find applications in denoising, reconstruction of curves and manifolds and generally to problems where the expression of the polynomial is not well known

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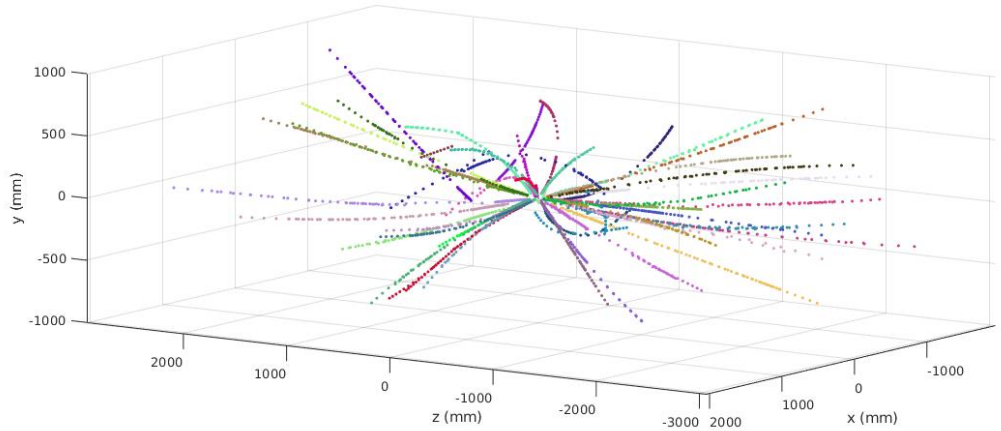

References

[1] Nag, Panchali. "Manifold Denoising Using Distance Functions." 2019 New York Scientific Data Summit (NYSDS). IEEE, 2019.
[2] Sober, Barak, and David Levin. "Manifold approximation by moving least-squares projection (mmls)." Constructive Approximation (2019): 1-46, Sober, Barak PhD Thesis 2018
[3] Nag, Panchali. "Manifold Reconstruction using Linear Approximations" (in preparation)

Thank You