Most galaxies contain supermassive black holes at their centers, surrounded by dense clusters of stars. Many exotic astrophysical phenomena arise within these clusters, including flares arising from the tidal disruption of stars that pass close to the black hole, gravitational-wave bursts from stars spiraling into the black hole, and mergers of binary black holes. I will review our observational understanding of galactic nuclei as well as the novel dynamical processes that operate within them. I will also discuss the application of classical statistical mechanics to galactic nuclei, and present evidence that a phase transition can occur between a spherical disordered state and a lopsided ordered state.