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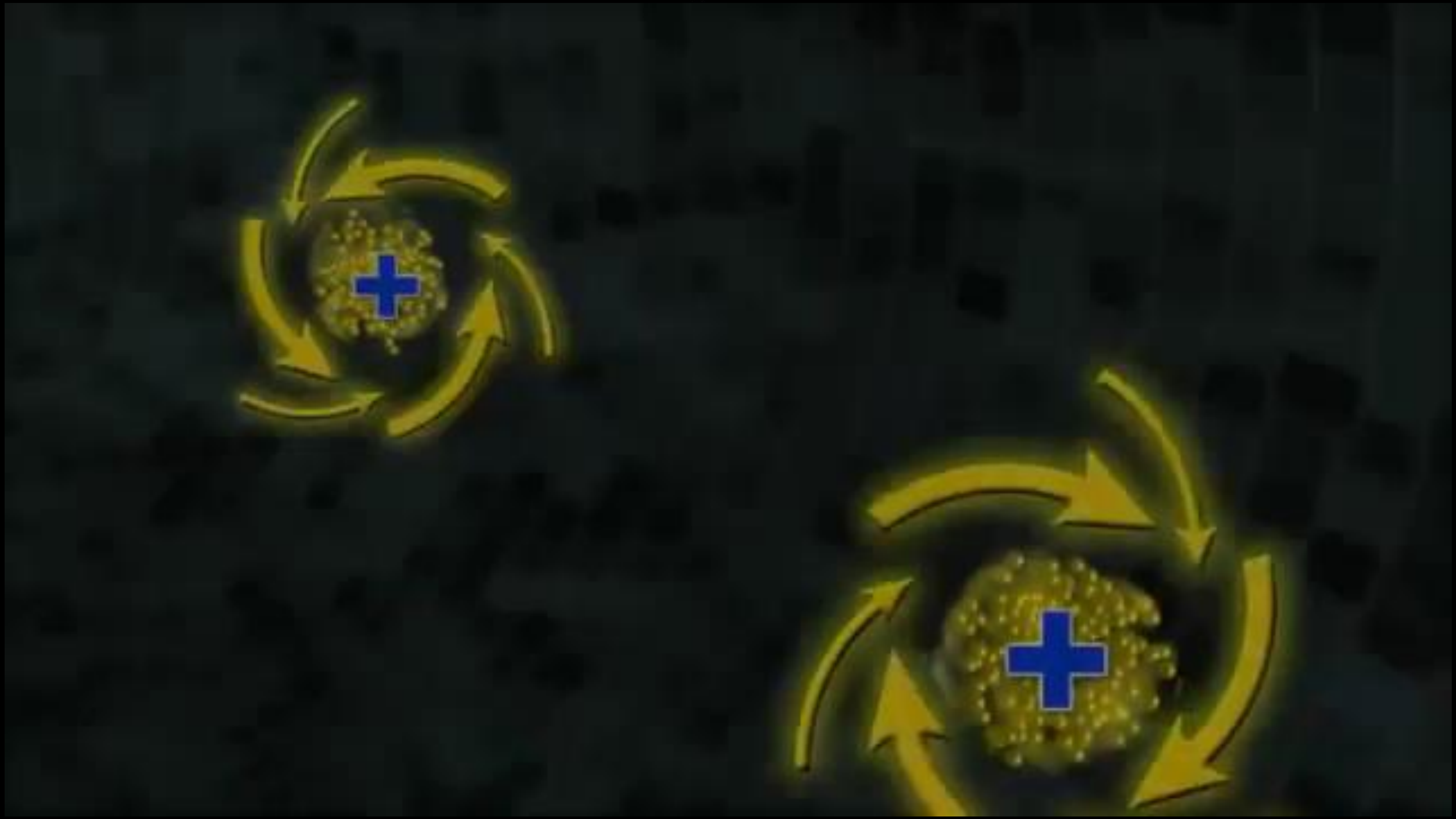
Magnetic monopoles in heavy ion collisions

Arttu Rajantie

Heavy Ions and New Physics, 20 May 2021

Collaborators:
Oliver Gould, David Ho, Cheng Xie
MoEDAL Collaboration

Magnetic Fields in Heavy Ion Collisions



(Video: Brookhaven National Laboratory)

Magnetic Fields in Heavy Ion Collisions

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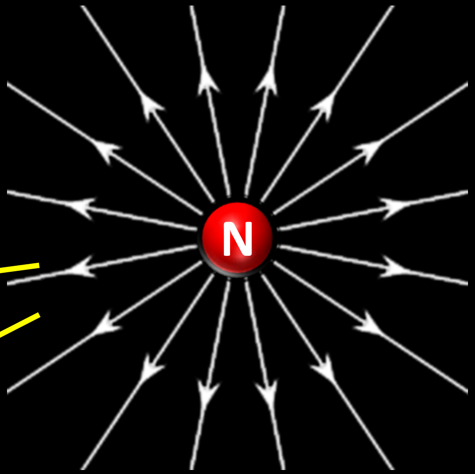
- Magnetic field strengths:
  - LHC magnets $|\vec{B}| \sim 8.3\ \mathrm{T} \sim 1.6 \times 10^{-15}\ \mathrm{GeV}^2$
  - Magnetars $|\vec{B}| \sim 2 \times 10^{11}\ \mathrm{T} \sim 4 \times 10^{-5}\ \mathrm{GeV}^2$
  - Fixed-target Pb collisions at SPS $|\vec{B}| \sim 5 \times 10^{13}\ \mathrm{T} \sim 10^{-2}\ \mathrm{GeV}^2$
  - $5.02\ \mathrm{TeV}$ Pb-Pb collisions at LHC $|\vec{B}| \sim 4 \times 10^{16}\ \mathrm{T} \sim 7\ \mathrm{GeV}^2$

- New physics phenomena?
  - Magnetic monopoles
  - Baryon number violation

Magnetic Charges



$$\vec{\nabla}\cdot\vec{E}=\rho_{\mathrm{E}}$$
$$\vec{\nabla}\cdot\vec{B}=\rho_{\mathrm{M}}$$
$$\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}-\vec{j}_{\mathrm{M}}$$
$$\vec{\nabla}\times\vec{B}=\frac{\partial\vec{E}}{\partial t}+\vec{j}_{\mathrm{E}}$$

- Duality $\vec{E}+i\vec{B}\rightarrow e^{i\theta}\left(\vec{E}+i\vec{B}\right)$
- Dirac quantisation condition $g\in g_D\mathbb{Z},\ g_D=\frac{2\pi}{e_0}$

Monopole Searches in Colliders

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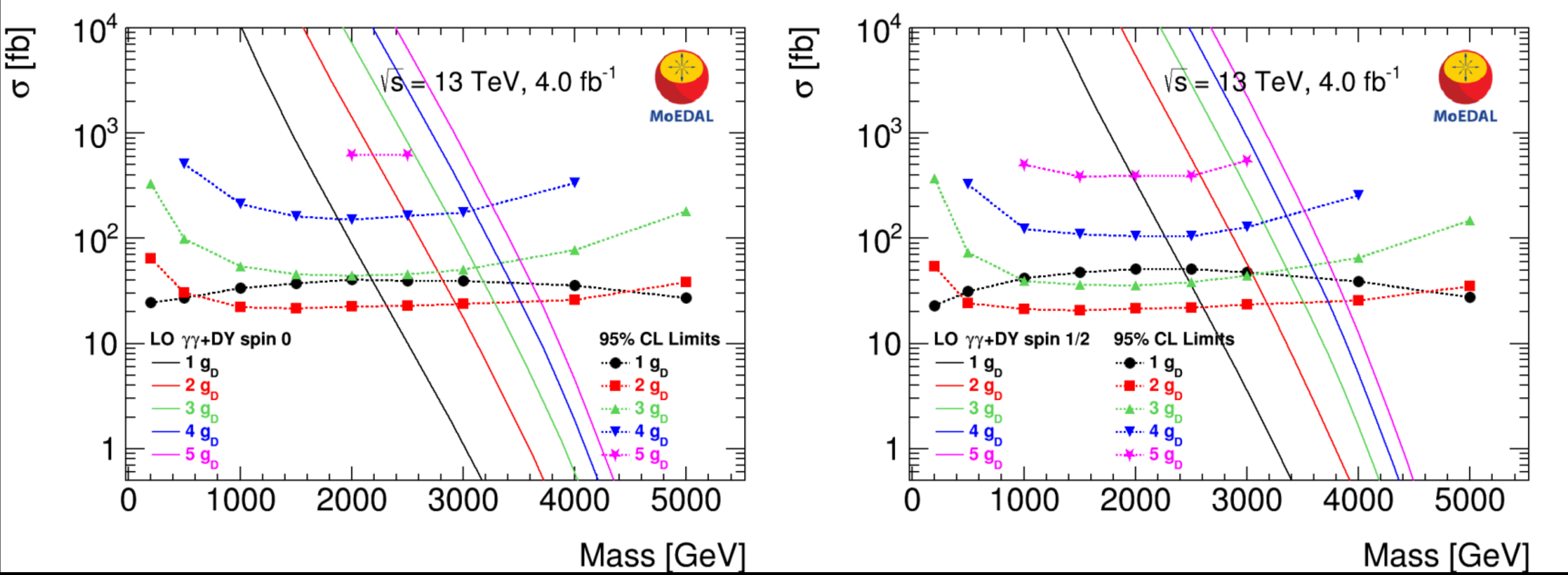
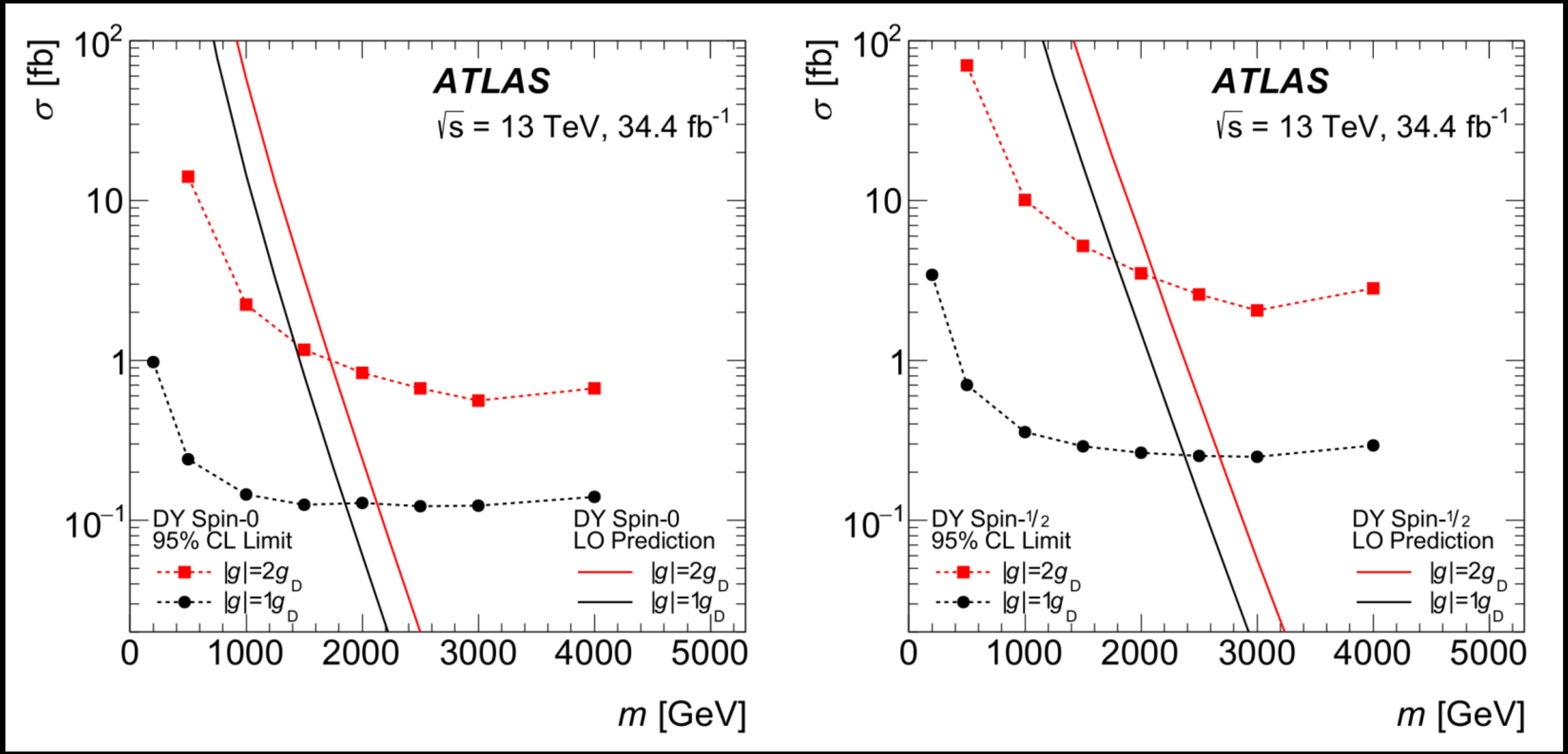


MoEDAL Collaboration, 2019

Monopole Searches in Colliders



ATLAS Collaboration, 2019

- Upper bounds on production cross section
- To obtain a bound on the monopole mass,
  one needs to calculate the cross section from theory

Monopole Searches in Colliders

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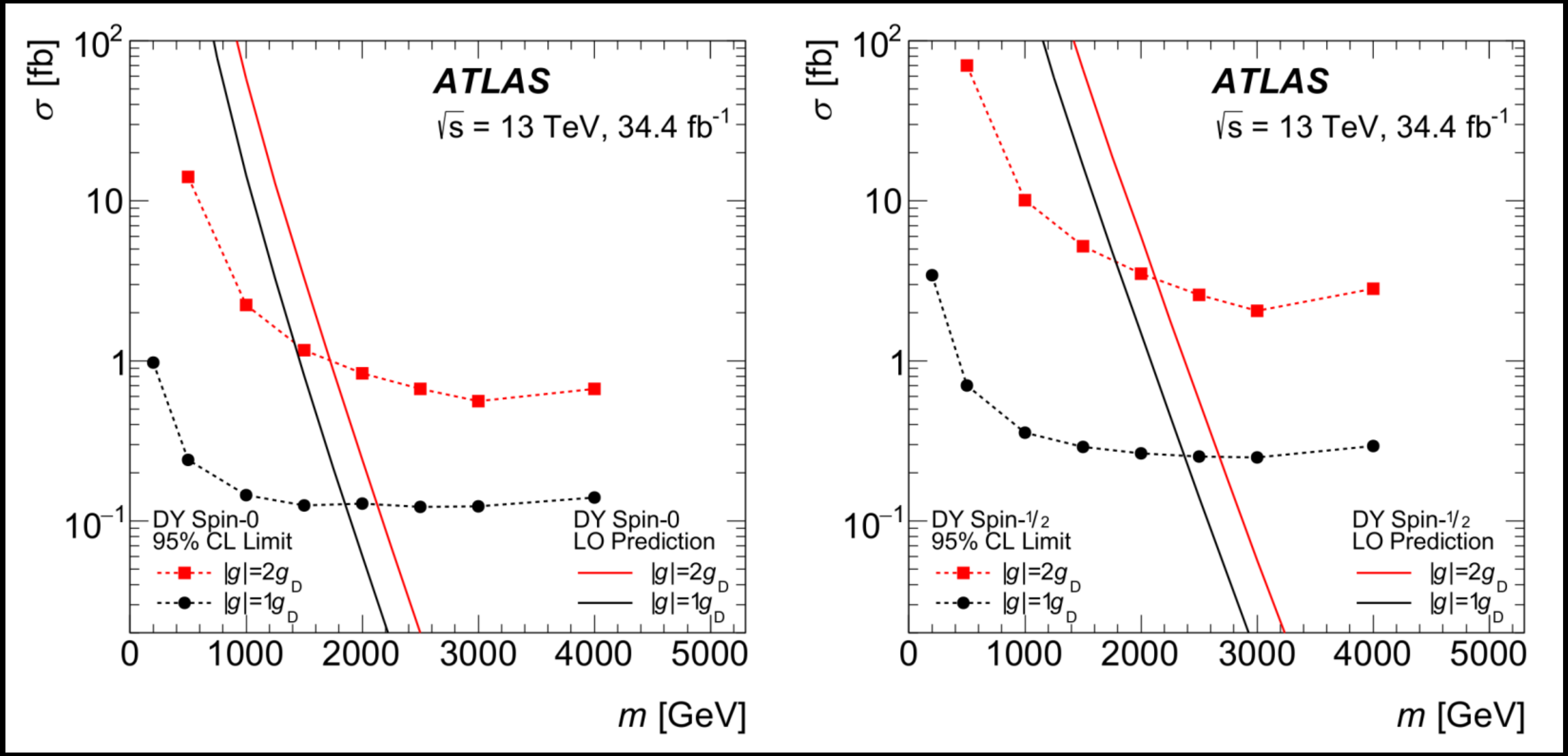
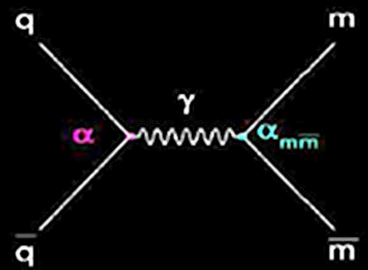


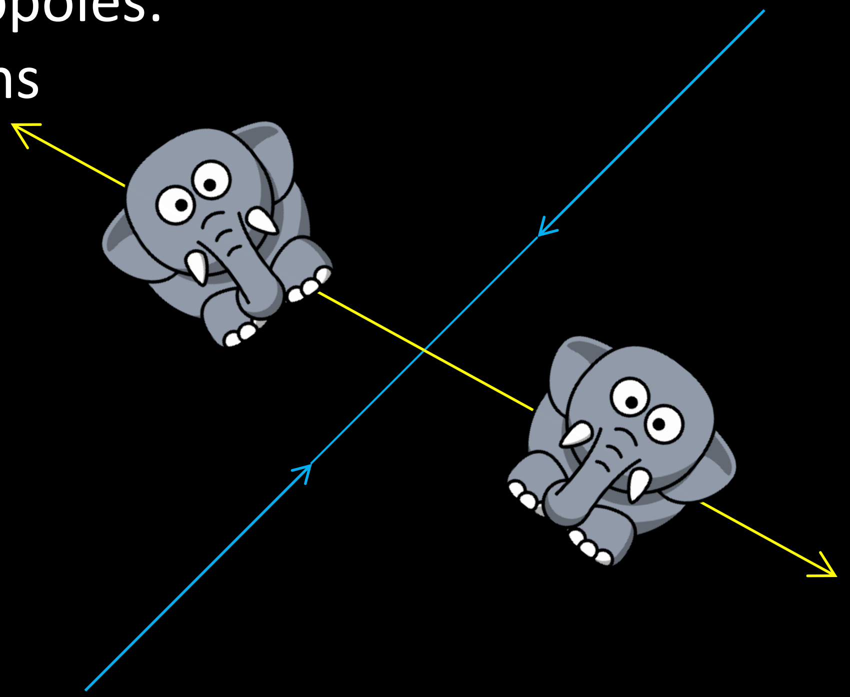
ATLAS Collaboration, 2019

- Searches usually assume tree-level Drell-Yan cross section with $e \to g = 2\pi/e$ (EM duality)
- But $g_D \approx 20.7 \gg 1 \Rightarrow$ Non-perturbative!



Production Amplitude

- Semiclassical argument for solitonic monopoles: pair production from two-particle collisions suppressed by $\sim e^{-4/\alpha} \sim 10^{-238}$ (Witten, Drukier&Nussinov)
- Confirmed numerically for kinks in 1+1D (Demidov&Levkov 2011)
- Production of solitonic monopoles may be practically impossible in two-particle collisions

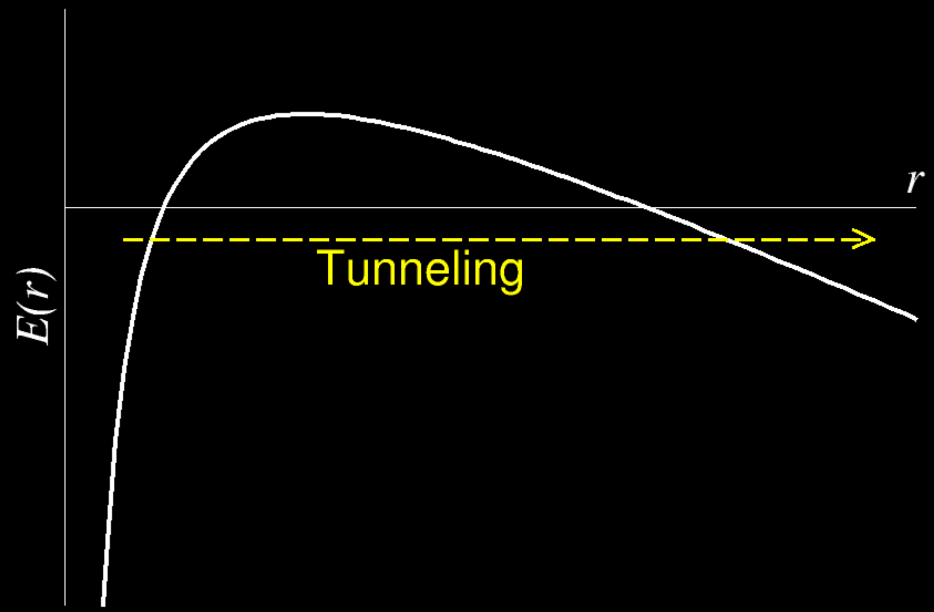


Schwinger Pair Production

- Energy of a charged particle-antiparticle pair in uniform electric field $\vec{E}$:

$$E(\vec{r}) = 2m - \frac{e^2}{4\pi r} - e\vec{E}\cdot\vec{r}$$

- Sauter 1931, Heisenberg&Euler 1936, Schwinger 1951:
  Tunneling through potential barrier
  $\Rightarrow$ pair production from vacuum



Schwinger Rate

- Pair production rate per spacetime volume

$$\Gamma \sim \exp(-S_{\text{inst}})$$

  - S_{inst} is the instanton action
  - Prefactor from functional determinant
- Affleck, Alvarez & Manton 1981:
  - Circular worldline instanton with radius $r = m/e|\vec{E}|$
  - Action $S_{\text{inst}} = \frac{\pi m^2}{e|\vec{E}|} - \frac{e^2}{4}$

$$\Gamma = \frac{e^2|\vec{E}|^2}{8\pi^3} e^{-\frac{\pi m^2}{e|\vec{E}|} + \frac{e^2}{4}}$$

- Arbitrary coupling but weak field $e|\vec{E}| \ll m^2$

Schwinger Production of Monopoles

- Calculation does not require weak coupling: $g \gg 1$ is not a problem
- Pair production rate (constant field, pointline monopoles):

$$\Gamma = \frac{g^2 |\vec{B}|^2}{8\pi^3} e^{-\frac{\pi M^2}{g|\vec{B}|} + \frac{g^2}{4}}$$

- Pair production needs a strong magnetic field $|\vec{B}| \gtrsim M^2/g^3$
- LHC magnets $|\vec{B}| \sim 8.3\ \mathrm{T} \sim 1.6 \times 10^{-15}\ \mathrm{GeV}^2$

$$M \gtrsim 1.5 \left(\frac{g}{g_D}\right)^{3/2} \mathrm{keV}$$

- Magnetars $|\vec{B}| \sim 2 \times 10^{11}\ \mathrm{T} \approx 4 \times 10^{-5}\ \mathrm{GeV}^2$

$$M \gtrsim 0.3 \ldots 0.7\ \mathrm{GeV}$$

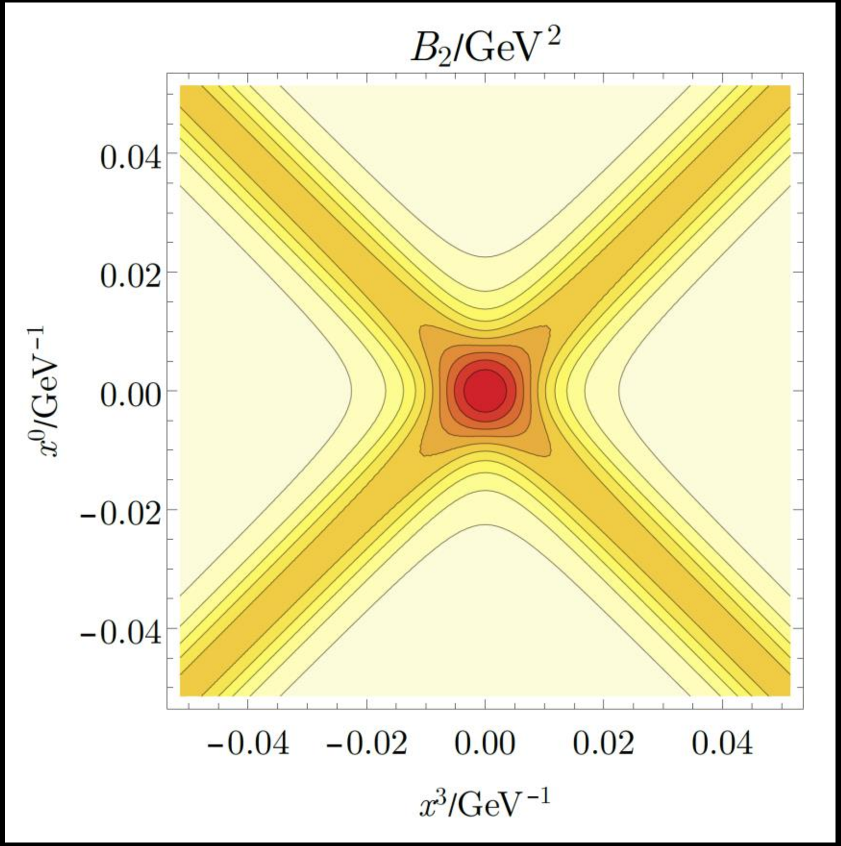
LHC Heavy Ion Collisions

- Pb-Pb collisions in Nov 2018
  - $\sqrt{s_{NN}} = 5.02\ \mathrm{TeV}$,
- Time-dependent field

$$\vec{B} \approx \frac{B\hat{y}}{2}\left(\left(1+\omega^2\left(t-\frac{z}{v}\right)^2\right)^{-3/2} + \left(1+\omega^2\left(t+\frac{z}{v}\right)^2\right)^{-3/2}\right),$$

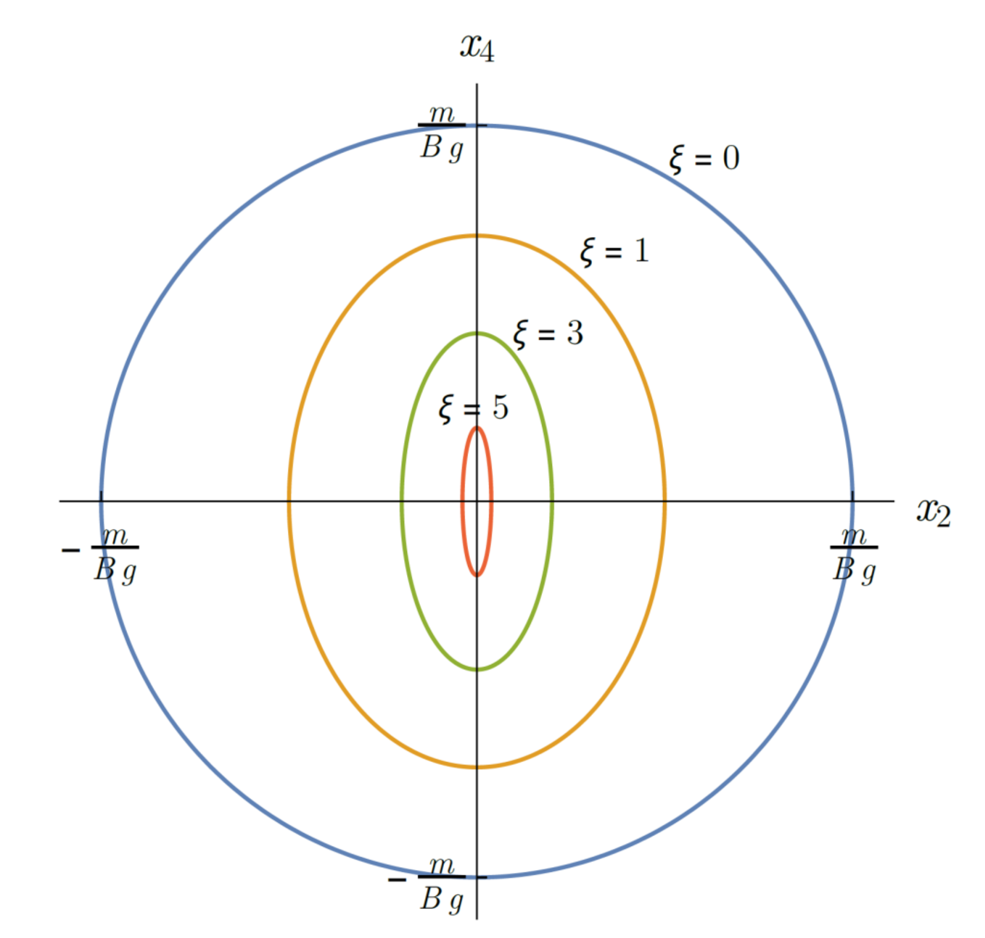
where $B \approx \frac{Ze\gamma}{2\pi R^2} \approx 7.3\ \mathrm{GeV}^2,$

$$\omega \approx \frac{\gamma}{R} \approx 73\ \mathrm{GeV}$$



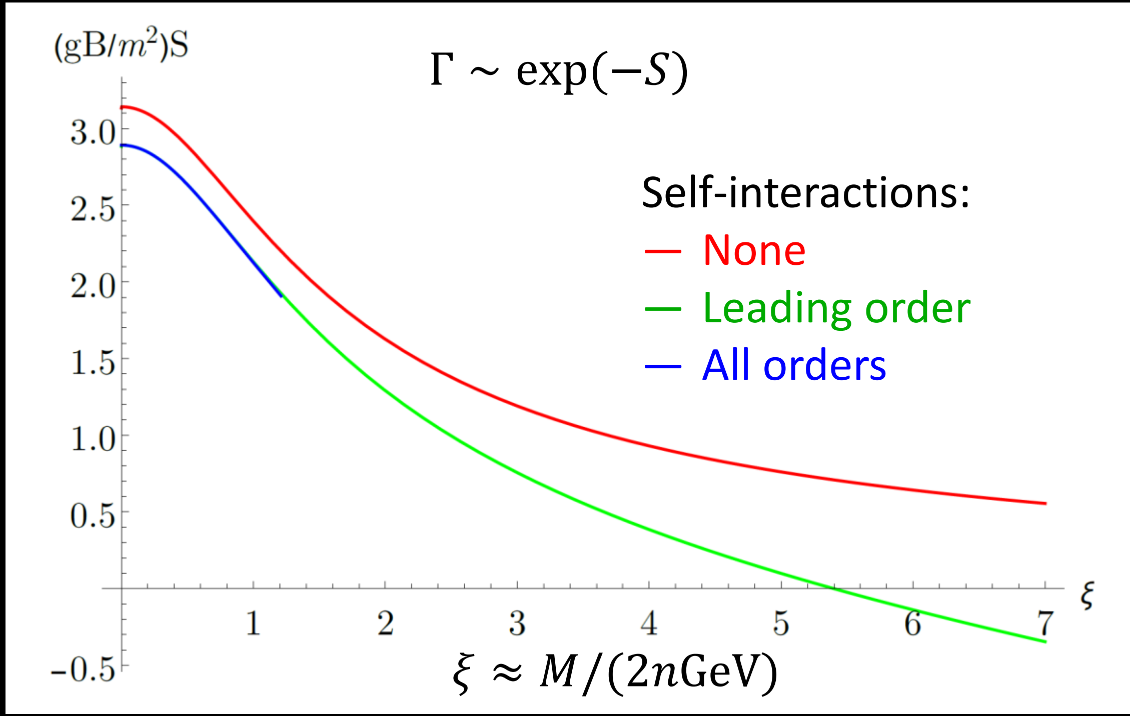
Spacetime Dependence

- Parameterised by
  $$\xi = \frac{M\omega}{gB} \approx M/(2n\ \mathrm{GeV})$$
  - Constant field: $\xi \to 0$
- Find worldline instanton in a time-dependent background (Gould, Ho & AR, PRD2019)
- Zeroth order in self-interaction: Ellipse with axes $\frac{M}{gB\sqrt{1+\xi^2}}, \frac{M}{gB(1+\xi^2)}$
- Production rate at large ξ:
  $$\Gamma \sim \exp(-4M/\omega)$$



Spacetime Dependence

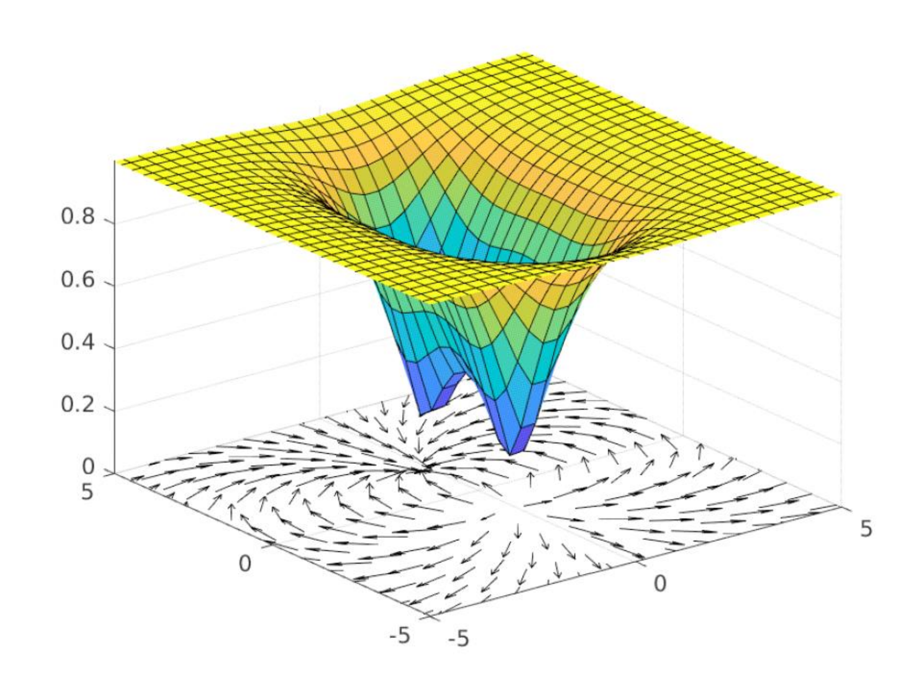
- Space and time dependence enhances the production rate (Gould, Ho & AR, 2019) ⇒ Stronger mass bounds
- Cannot reach LHC parameters yet:
  - Self-interactions
  - Finite monopole size



- Ignoring self-interactions gives the (conservative) estimate $M \gtrsim 80\ \text{GeV}$ if no monopoles found

Sphalerons for Solitonic Monopoles

- Georgi-Glashow model: SU(2)+adjoint scalar, $z = m_{\rm s}/m_{\rm v}$
- Unstable classical sphaleron solutions in external field $B_{\rm ext}$: Barrier height
- Energy determines the thermal rate $\Gamma \propto \exp\left(-E_{\rm sph}/T\right)$: Lower than for pointlike



(Ho and AR, 2020)

- Energy vanishes at Ambjørn-Olesen critical field $B_{\rm crit} = m_{\rm v}^2/e$: Classical monopole production even at zero temperature

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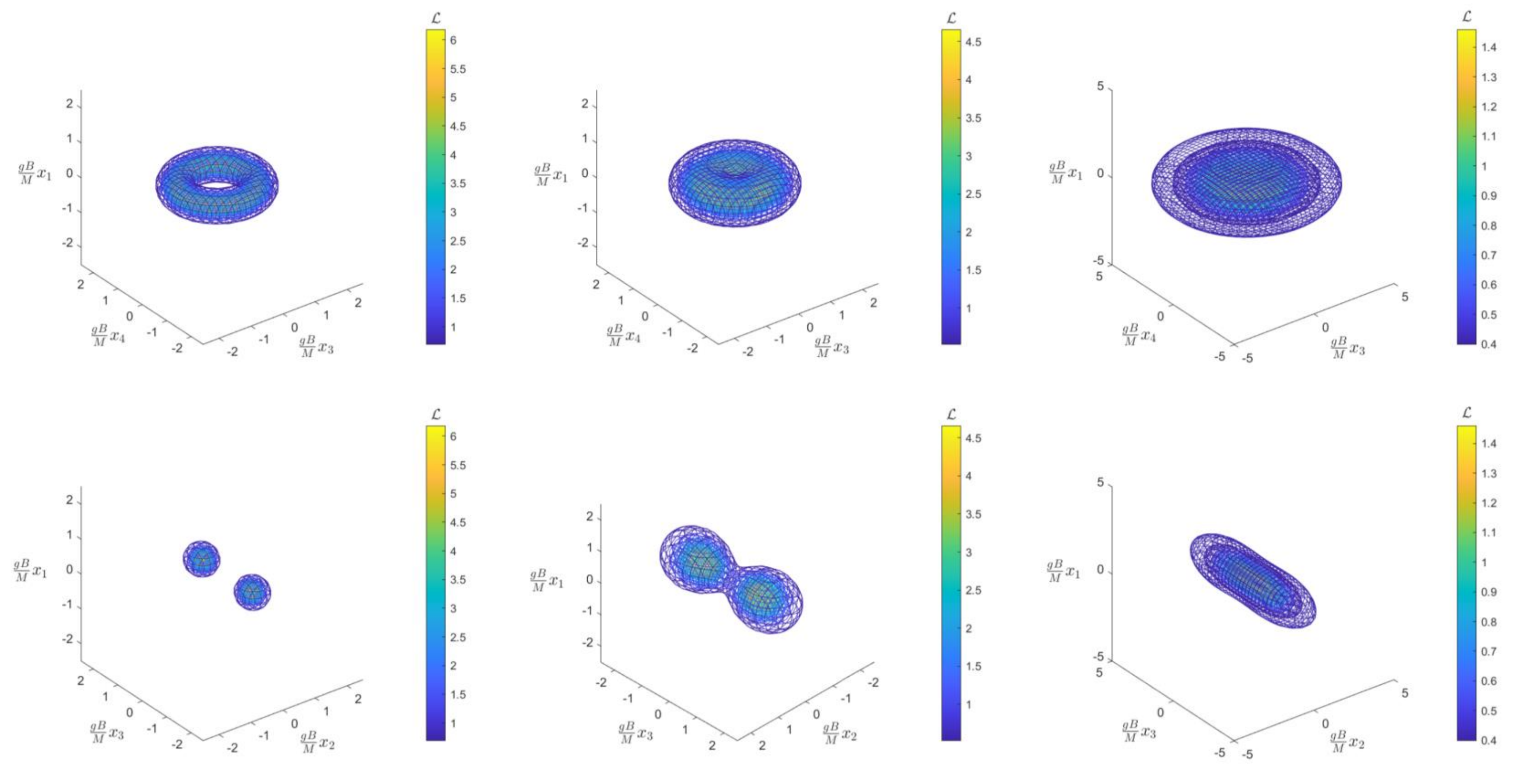
Instantons for Solitonic Monopoles

- Georgi-Glashow model: SU(2)+adjoint scalar , $\beta = m_{\mathrm{s}}/m_{\mathrm{v}}$
- 4D instanton solution
- Action determines the quantum Schwinger rate $\Gamma \propto \exp(-S_{\mathrm{inst}})$


(a) $B/B_{\mathrm{crit}} = 0.3$ (b) $B/B_{\mathrm{crit}} = 0.6$ (c) $B/B_{\mathrm{crit}} = 0.9$

(Ho and AR, 2021)

Instantons for Solitonic Monopoles

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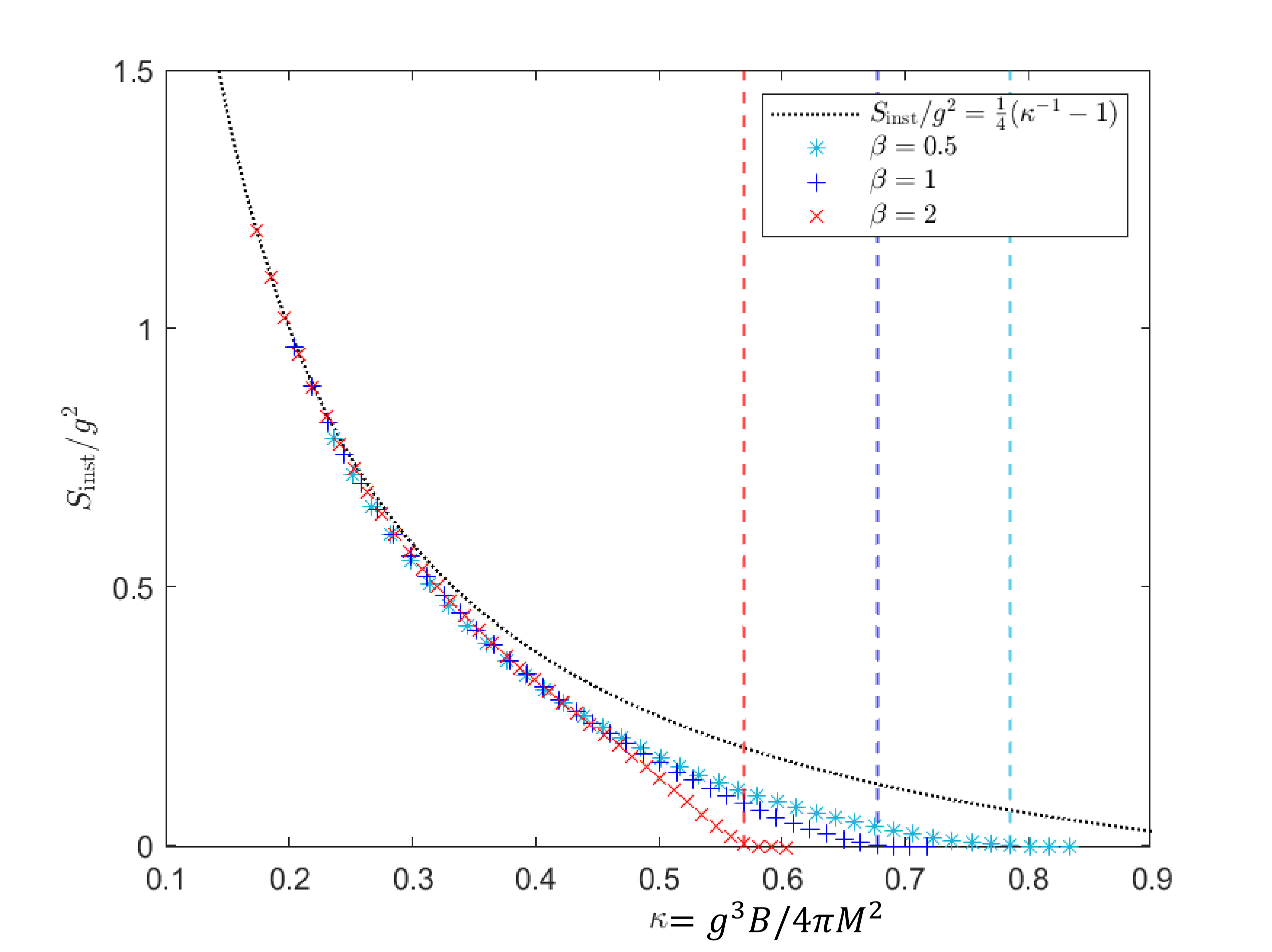


(Ho and AR, 2021)

Monopoles from Magnetic Fields

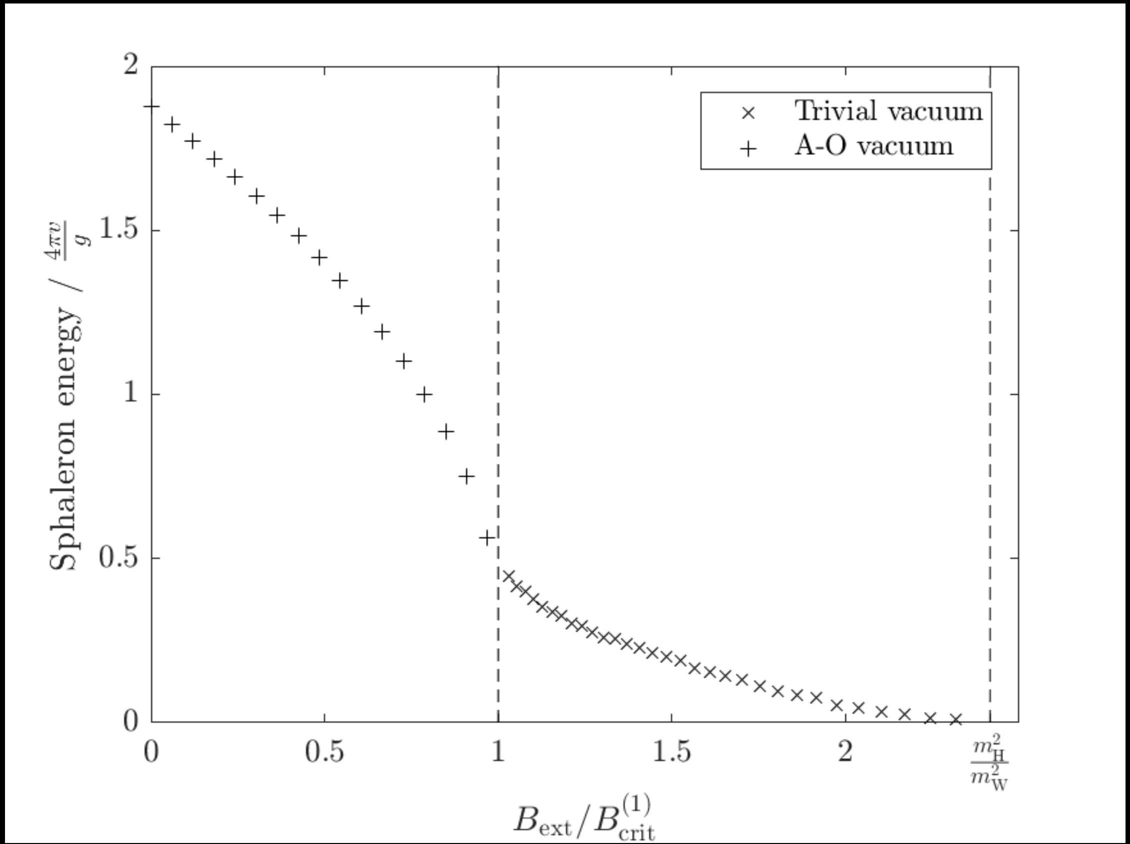
- Constant field $\Gamma = \frac{g^2|\vec{B}|^2}{8\pi^3} e^{-\frac{\pi M^2}{g|\vec{B}|}+\frac{g^2}{4}}$
- Time dependence
  - Enhances production
  - Rapid pulse $\Gamma \sim \exp(-4M/\omega)$
- Spatial inhomogeneity
  - Effect not known
- (Solitonic) monopole size
  - Enhances production
  - Classical instability at $B_{\text{crit}} = m_{\text{v}}^2/e$

Baryon Number Violation

- Chiral anomaly in the Standard Model
  $\Rightarrow$ Electroweak sphaleron transitions change baryon number
- Rate per spacetime volume

$$\Gamma \sim \exp(-E_{\mathrm{sph}}/T)$$

- Zero field $E_{\mathrm{sph}} \approx 1.9 \times \frac{4\pi v}{g} \approx 9\ \mathrm{TeV}$
- Non-zero magnetic field: Numerical calculation (Ho&AR 2020)



Baryon Number Violation

- Unsuppressed B violation when

$$|\vec{B}| > \frac{m_H^2}{e} \approx 5.2 \times 10^4 \mathrm{GeV}^2$$

- Requires

$$\gamma \sim 10^7, \sqrt{s_{NN}} \sim 2 \times 10^4 \text{ TeV}$$

- Effect of time-dependence?