Real-time lattice simulations of overoccupied gluodynamics

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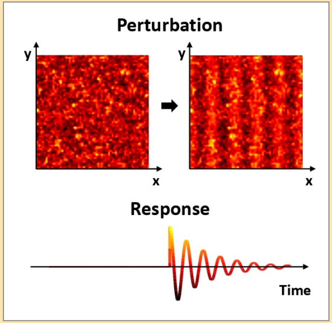
Outline

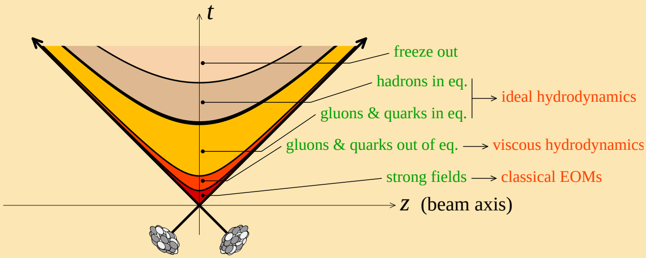
- Why overoccupied, weak coupling gauge theory?
- Method: real time classical lattice
  + linearized fluctuations
- Test case system: isotropic self-similar UV cascade
- Spectral function + comparison HTL
- Heavy quark diffusion
- 2 spatial dimensions



Based on:

- Kurkela, T.L., Peuron, Eur. Phys. J. C **76** (2016) 688, [arXiv:1610.01355 [hep-lat]]
- Spectral function: K. Boguslavski, A. Kurkela, T.L., J. Peuron Phys. Rev. D **98** (2018) 014006, arXiv:1804.01966
- 2d system: K. Boguslavski, A. Kurkela, T.L., J. Peuron arXiv:1907.05892 [hep-ph]
- Heavy quark diffusion: K. Boguslavski, A. Kurkela, T.L., J. Peuron, in preparation

Overoccupied gauge fields



Heavy ion collision: formation and dynamics of Quark-Gluon Plasma

- Initial stage: dynamics dominated by **saturation scale** $Q_s \gg \Lambda_{QCD}$; gluon field nonperturbative: $A_\mu A_\mu \sim 1/\alpha_s$
- Later: $\sim$thermal system, soft fields $p \lesssim gT$ nonperturbative

Want to understand **real time** QCD systems with both

- Perturbative scale $Q \gg \Lambda_{QCD} \implies$ weak coupling $\alpha_s \ll 1$
- Fields (at least at some p) overoccupied

$A_\mu \sim 1/g \gg 1 \implies$ can use **classical field dynamics**, g scales out

Standard method: hard (thermal) loops HTL

- Scale separation: hard $\sim Q$ **(particles)** and soft $\sim m_D$ **(field)** modes
- Initially $Q \sim m_D \sim Q_s \quad \Longrightarrow \quad$ thermal $Q \sim T \gg m_D \sim gT$
- Many numerical implementations with explicit particle+field description: transport, plasma instabilities, sphalerons too many references to list here ...
- Problem: continuum limit; where to put cutoff $m_D \ll 1/a \ll Q$?

  $\Longrightarrow$ cannot go to large m_D/Q

Idea here: all scales on same classical lattice $\Longrightarrow$ do not **need** $m_D \ll Q$

- But **can** also have scale separation (on big, but doable, lattice)
- Hard+hard interactions classical $\Longrightarrow$ thermalize incorrectly,

  but this is slower process (& often neglected anyway)
- Use as generalization of HTL picture?
  - Can vary m_D/Q smoothly
  - Details of hard sector should not matter for HTL

Yang-Mills on a real time lattice

Real-time numerics for classical field: standard Hamiltonian lattice setup

- Gauge potential A_i, cov derivative $D_i = \partial_i + ig[A_i, \cdot]$ $\Longrightarrow$ link $U_i(x) = e^{iagA_i(x)}$
- Canonical conjugate electric field $E^i = \partial_t A_i$
- Temporal gauge $A_0 = 0$; constraint $[D_i, E^i] = 0$ (Gauss' law)

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1st thing to measure: "Statistical function"

$$F^{ab}_{jk}(x, x') = \frac{1}{2}\left\langle\left\{\hat{A}^a_j(x)\,,\,\hat{A}^b_k(x')\right\}\right\rangle$$

- Measures (thermal) fluctuations $\sim$ particles in system $\sim f(p)$
- Now field is classical $A_i \sim 1/g$ $\Longrightarrow$ F is just 2-pt function of classical field

$$F^{ab}_{jk}(x, x') = \left\langle A^b_j(x) A^b_k(x')\right\rangle_{\text{cl}}$$

Linearized fluctuations on a real time lattice

The other independent correlator is the "spectral function"

$$\rho^{ab}_{jk}(x,x') = i\left\langle \left[\hat{A}^a_j(x), \hat{A}^b_k(x')\right]\right\rangle$$

This is "quantum", $\sim \hbar$, but related to retarded propagator

$$G_R(t,t',p) = \theta(t-t')\,\rho(t,t',p).$$

Measure in classical theory: **linear** response

$$\hat{A}^a_i(x) \to \hat{A}^a_i(x) + \hat{a}^a_i(x) \quad , \quad \langle \hat{a}^b_i(x)\rangle = \int \mathrm{d}^4x'\, G_{R,ik}^{\,bc}(x,x')\, j^k_c(x')$$

Algorithm for statistical function

Kurkela, T.L., Peuron, Eur. Phys. J. C **76** (2016) 688

- Perturb system with current $j^k_c(x) = e^{i\mathbf{k}\cdot\mathbf{x}}\delta(t-t_0)$
- Follow linearized equations of motion for $a^a_i(x)$, $e^i_a(x)$
- Correlate field $a^a_i(t)$ with current $j^i_a(t_0)$ $\Longrightarrow$ $\rho(p,t)$

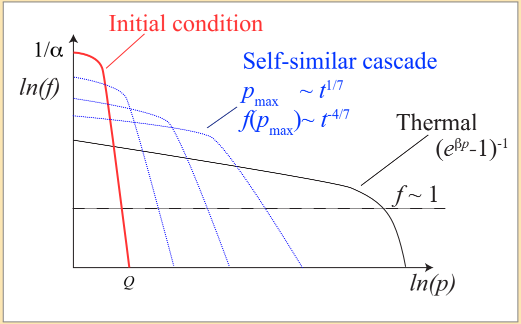
Test case: overoccupied cascade to UV

Extensively studied system:

Berges et al [arXiv:1203.4646 [hep-ph]] + . . . , Kurkela, Moore, [arXiv:1207.1663 [hep-ph]] + . . .

HTL/kinetic theory explains basic properties of numerics

- Start from isotropic $f(p) \sim \frac{n_0}{g^2}\theta(p_0 - p)$
  (actually smoother Gaussian)
- Later p_0, n_0 separately don't matter, only $\varepsilon \sim Q^4/g^2$
- Energy cascades towards UV: largest occupied $p_{\max} \sim t^{1/7}$
- Typical occupation$\sim t^{-4/7}$
  (at hard scale)



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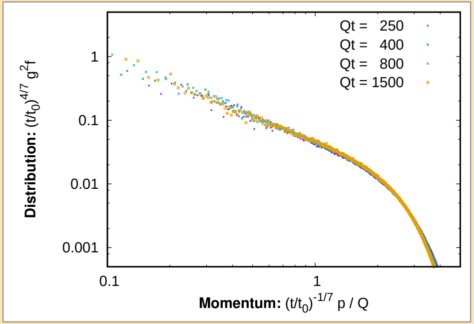


Specifically: define $Q \equiv \sqrt[4]{\varepsilon/g^2}$, (nonexpanding: ε conserved) Plots here: $Qt = 1500$

Debye or plasmon scale

Self-similar scaling

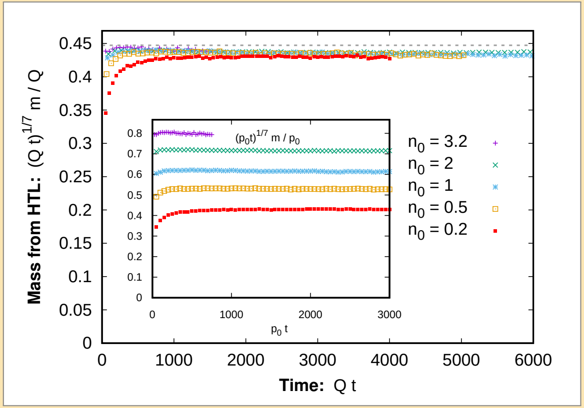
$$f(t,p) = t^{-4/7} f_S(p/t^{1/7})$$

$$m^2 \sim \int \frac{d^3\mathbf{p}}{p} f(p)$$

$\Longrightarrow$ Soft scale goes as

$$m \sim t^{-1/7}$$

- Numerically verified
- Can dial m/Q or m/p_{max} by looking at different t



(Plot: m dependence on $Q \equiv \sqrt[4]{\varepsilon/g^2}$, inset: n_0, p_0 separately)

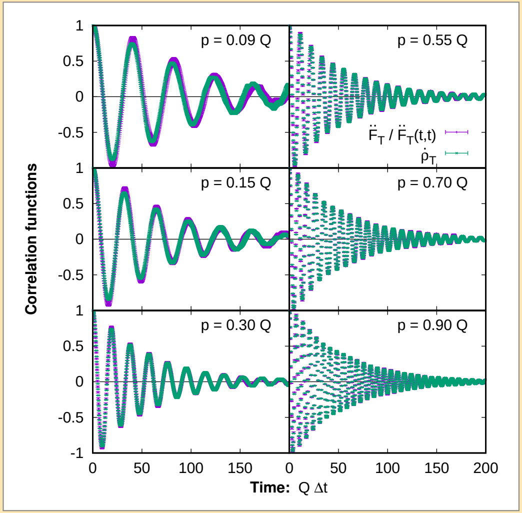
Spectral function: Transversely polarized mode

K. Boguslavski, A. Kurkela, T.L., J. Peuron Phys. Rev. D **98** (2018) 014006

- F and ρ, same quasiparticles?
- For apples-to apples comparison plot

$$\partial_t \rho(t,t') \text{ and } \frac{\partial_t \partial_{t'} F(t,t',p)}{[t \to t']}$$

- Very nice agreement!



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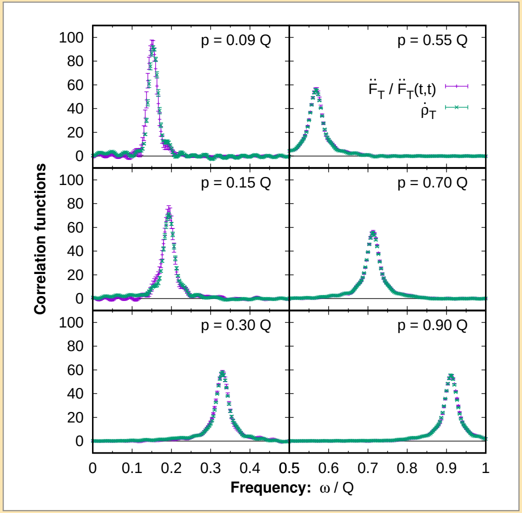
- Same in frequency $t - t' \to \omega$
  $\Longrightarrow$ nice Lorentzian



(This is $\omega\rho(\omega)$, do not see small ω region)

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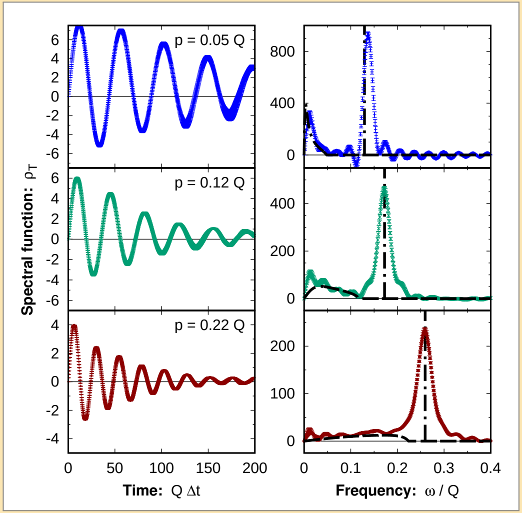
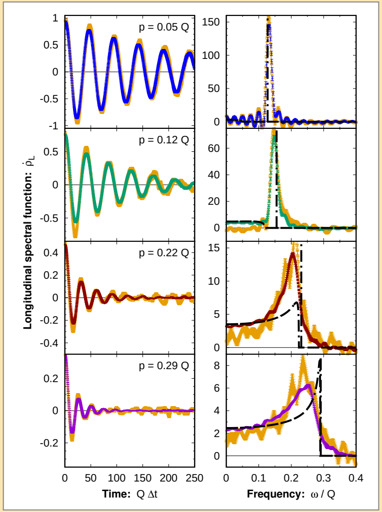
- Even see a Landau cut; line is HTL theory



(This is now $\rho(\omega)$)

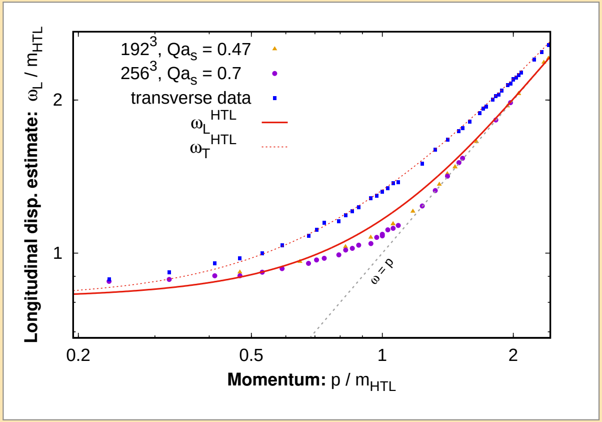
Longitudinally polarization mode

- Story very similar: good agreement between statistical and spectral
- Measurement harder: peak weak at high p
- Linearized fluctuations clearly much cleaner
  Orange: statistical (i.e. bkg field)



Dispersion relations

- Difference between T and L qualitatively as expected from HTL



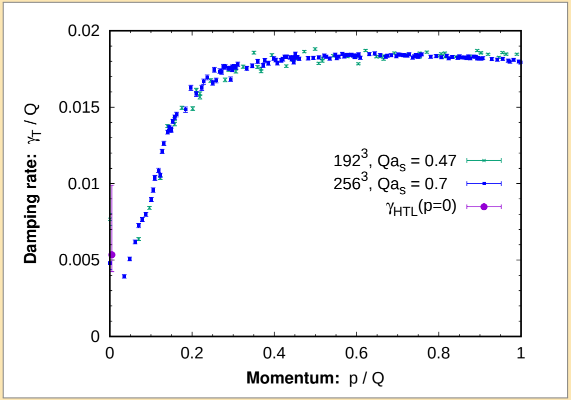
(L peak harder to extract at high p)

Damping rate

Extract damping rate from decay of plasma oscillation

HTL available for only $\gamma(p = 0)$

- Rougly agree
- But extend to higher p



Infrared enhancement?

Equal time correlators of fields

Soft transverse fields:

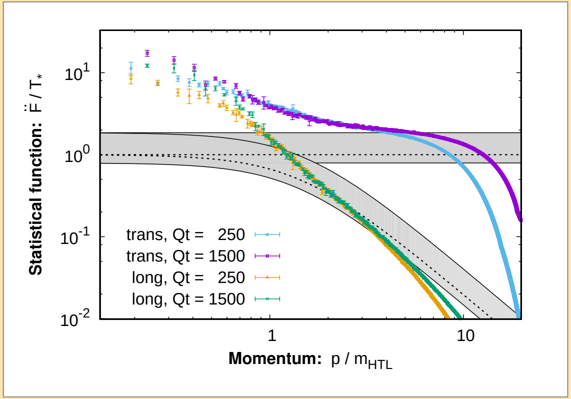
from HTL expect thermal

$$f(p) \sim \frac{T}{\omega_p}$$

with

$$T = T_* \equiv \frac{\frac{1}{2}\int_{\mathbf{p}} f(t,p)\,(f(t,p)+1)}{\int_{\mathbf{p}} \frac{f(t,p)}{\sqrt{m_\infty^2+p^2}}} \sim t^{-3/7}$$

(classical fields: neglect 1 in $(f+1)$)



(HTL estimate grey band)

Numerical result: IR enhanced compared to HTL expectation

Heavy quark diffusion

Preliminary results

Sign of infrared enhancement in **equal-time, Coulomb gauge** statistical function

$$pf(p) \sim \int d^3\mathbf{x}\, d^3\mathbf{y}\, e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})} \left\langle E^i(t,\mathbf{x}) E^i(t,\mathbf{y}) \right\rangle$$

- Interpretation: Magnetic scale physics, condensation, topology ??
- Or just gauge artefact?

Heavy quark diffusion coefficient

Gauge-invariant **unequal-time, equal-space** correlator

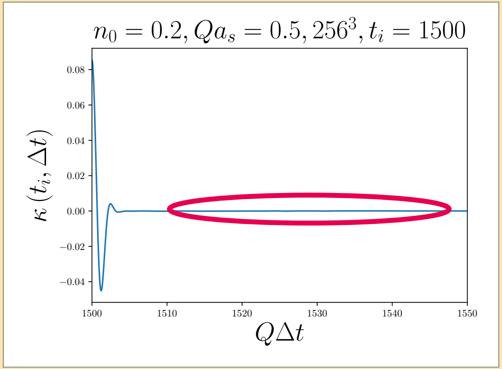
$$\kappa(t_i, \Delta t) \sim \left\langle E^i(t_i,\mathbf{x}) E^i(t_i + \Delta t,\mathbf{x}) \right\rangle \quad \text{(when measured in } A_0 = 0 \text{ gauge)}$$

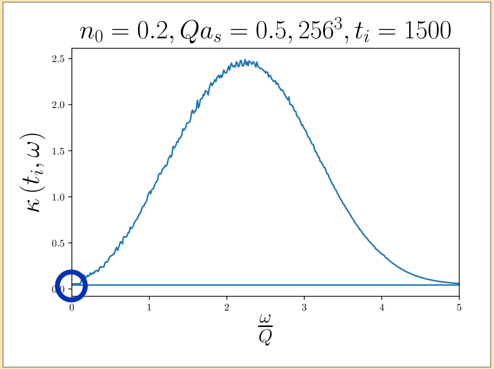
- Does it show some similar IR enhancement?
- By itself of phenomenological interest for . . . heavy quarks

What does it look like?

Rapid initial oscillation, quickly averages to zero

$\Longrightarrow$ Looking for a smaller signal at large Δt, $\omega \to 0$





From now on: plot (excuses for abusive notation)

$$\kappa(t_i, T) \equiv \int_0^T \mathrm{d}\Delta t \kappa(t_i, \Delta t) \quad \kappa \equiv \kappa(T \to \infty)$$

HQ diffusion constant and other physical scales

Time-dependence

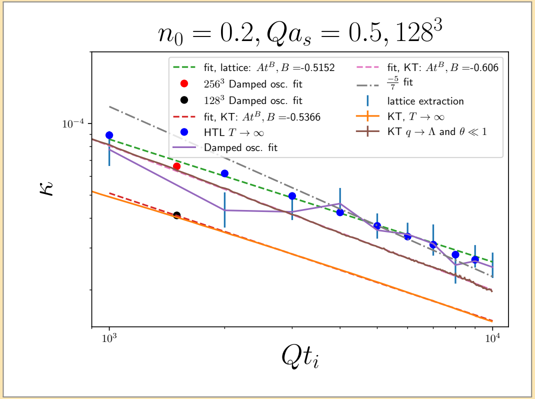
$$\kappa \sim (Qt)^{-5/7}$$

understood as

$$\kappa \sim m_{\mathrm{D}}^2 T_*$$

in terms of

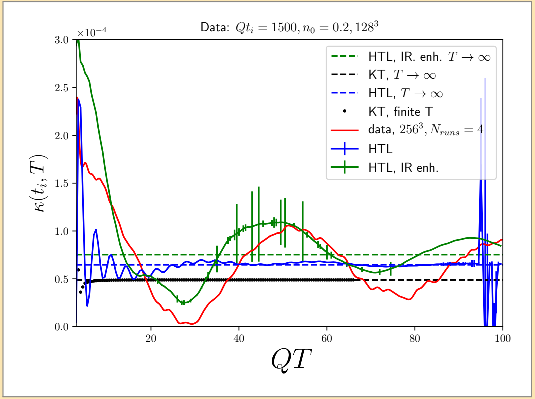
- Hard scale $\Lambda \sim Q(Qt)^{1/7}$
- Debye scale $m_{\mathrm{D}} \sim Q(Qt)^{-1/7}$
- Effective temperature of IR modes $T_* \sim Q(Qt)^{-3/7}$



Zoom in on long time

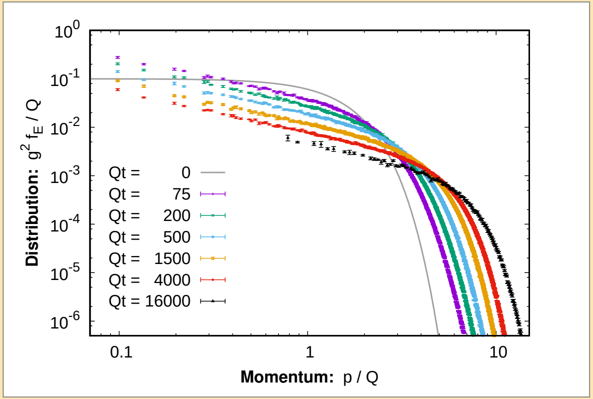
Try to understand not only $\kappa(T=\infty)$ but $\kappa(T)$ vs. upper time limit T

- **Red:** numerical data
- "HTL": model with
  - Numerically extracted gauge-fixed equal-time-correlator (statistical function) $\Delta t = 0, \mathbf{k}$
  - $\omega, \mathbf{k}$ with HTL analytical form
  - **IR enhancement needed to reproduce oscillations**
- "KT" kinetic theory with
  - Numerically extracted $f(p)$
  - m_{D}-screened scattering



2+1 dimensional plasma

- Glasma field is 2-dimensional boost invariant
- Is there a 2-dimensional cascade? $\Longrightarrow$ yes!
- Study 2+1d theory, either
  - Just 2+1d gauge



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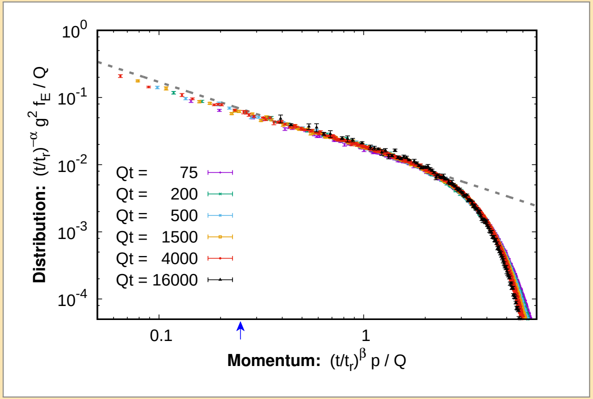

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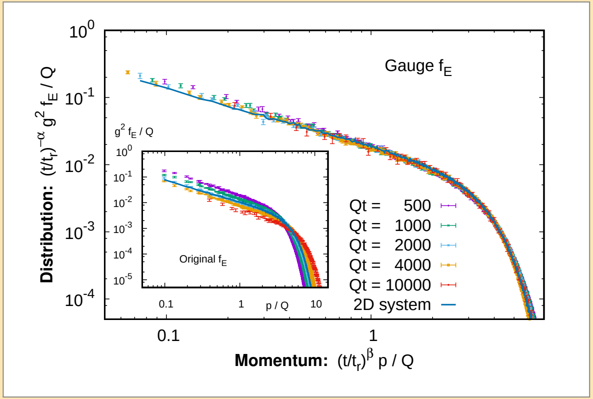
- Both exhibit scaling
  - $p_{\max} \sim t^{1/5}$ (cf 1/7 for 3d)
  - Typical occupation $\sim t^{-3/5}$ (cf $-4/7$ for 3d)



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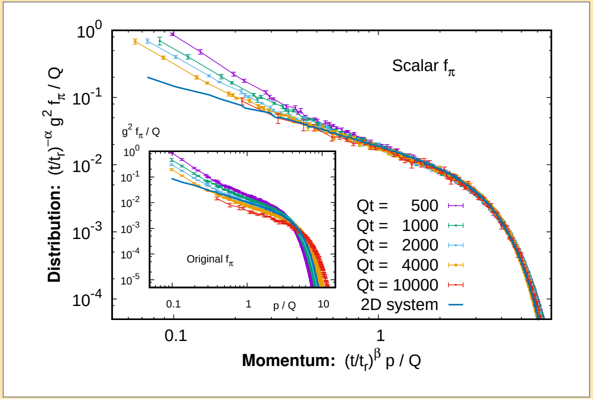
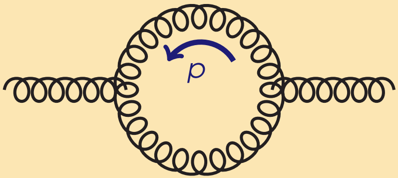


Understand in kinetic theory?

Parametrically yes, but physics very different from 3+1d

Hard and soft modes in $3+1$ and $2+1$ d

For simplicity consider propagator correction



- $$m_D^2 \sim \omega_{\rm pl}^2 \sim \int d^d p \frac{f(p)}{p}$$
- $$\text{Thermal } f(p) \sim \frac{T}{p}\theta(T-p)$$
- In $d=3$ dominated by hard $p \sim T$
- But in $d=2$: log integral, all scales

Similarly for KT $2 \to 2$ collision integral

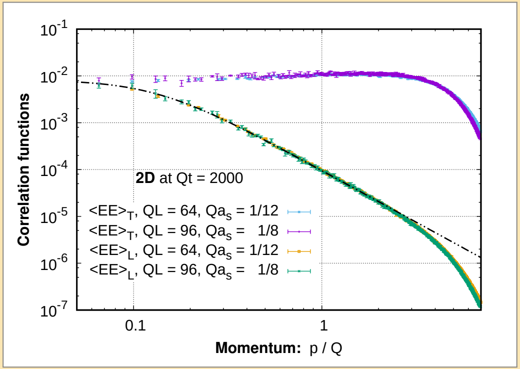
- In 3d hard particles scatter mostly off hard particles
- In 2d hard particles scatter equally often off soft particles

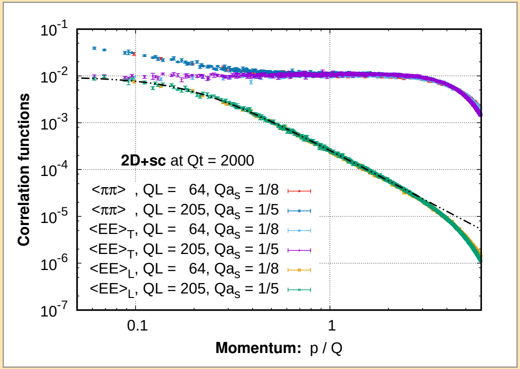
$\Longrightarrow$ Already at LO soft modes are a leading contribution

Thus: can understand power $p_{\rm max} \sim t^{1/5}$ etc — but not use KT quantitatively

2+1 dimensional plasma

IR enhancement seen only in scalars





Conclusions

- Several aspects of a heavy ion collision exhibit overoccupied $f(p) \sim 1/g^2$
  $\Longrightarrow$ classical gauge field
  - Initial glasma fields: one scale problem $p \sim Q_s$
  - Soft fields $p \sim gT$ in thermal system
- For controlled understanding of these fields:
  new numerical algorithm for linearized fluctuations
- First test case: isotropic self-similar UV cascade
  - Here $\exists$ scale separation $\Longrightarrow$ can compare to HTL, and go beyond
  - See enhancement of IR modes over thermal distribution
  - Confirmed by "heavy quark diffusion coefficient"
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  but physics (hard vs soft) very different!

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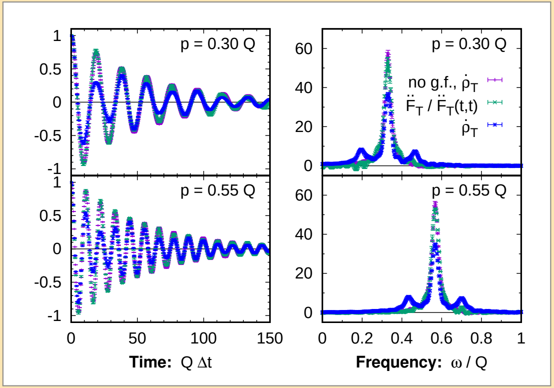
Thank you!

Backup

Gauge fixing

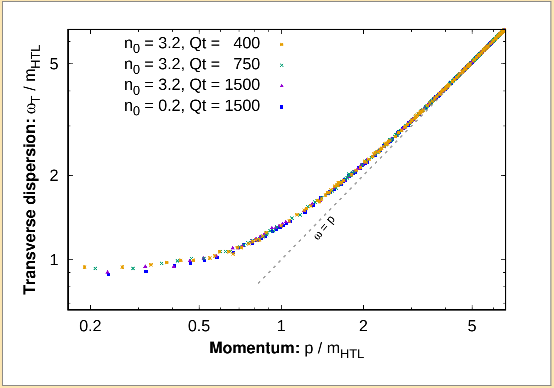
Gauge fixing: equal-time correlators in Coulomb gauge

- For unequal times: fix Coulomb when introducing current j / at first time in statistical function measurement, not later
- Keeping Coulomb gauge condition would introduce gauge artefacts in correlator $\Longrightarrow$ to remove these need to keep track of A_0



Insensitivity to parameters

- Dispersion relation
- Damping rate



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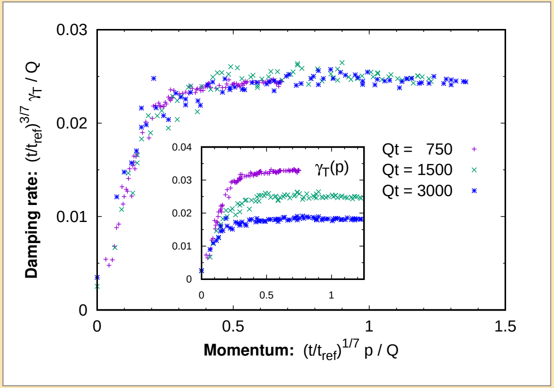
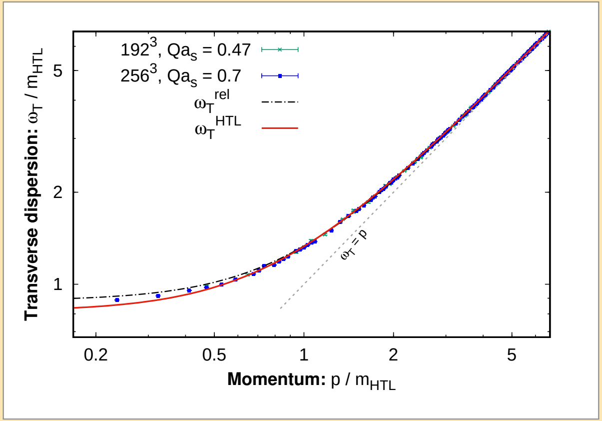


(Inset: without t-scaling from T_*)

Dispersion relation

- Overall shape agrees with HTL



Curve "HTL" uses m_∞ from $f(p)$

(which we estimate using *EE*-correlator)

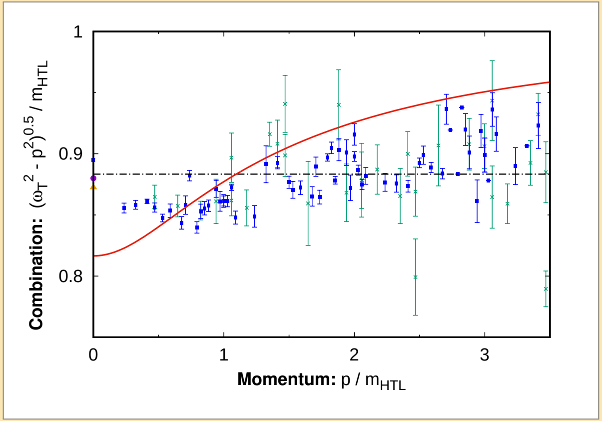
Dispersion relation

- Overall shape agrees with HTL
- More detail: $\sqrt{\omega^2 - p^2}$ between HTL prediction and pure $\omega^2 = m^2 + p^2$
- Characterize by
  - $\omega_{\text{pl}} \equiv \omega(p \to 0)$
  - $m_\infty \equiv$ mass gap at $p \to \infty$
- Numerical estimate:

$$\frac{\omega_{\text{pl}}}{m_\infty} = 0.96$$

where HTL prediction is

$$\frac{\omega_{\text{pl}}}{m_\infty} = \sqrt{2/3} \approx 0.82$$



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