Continuity from nuclear matter to two-flavor quark matter with $^1S_0$ and $^3P_2$ superfluid

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— Quo vadis QCD theory: heavy-ion collion perspectives and beyond —

I am talking about...

Neutron Star EoS

- □ Machine learning with mock data
- □ Machine learning with real data ~ Quarkyonic?

Continuity from nuclear to three-flavor CFL state

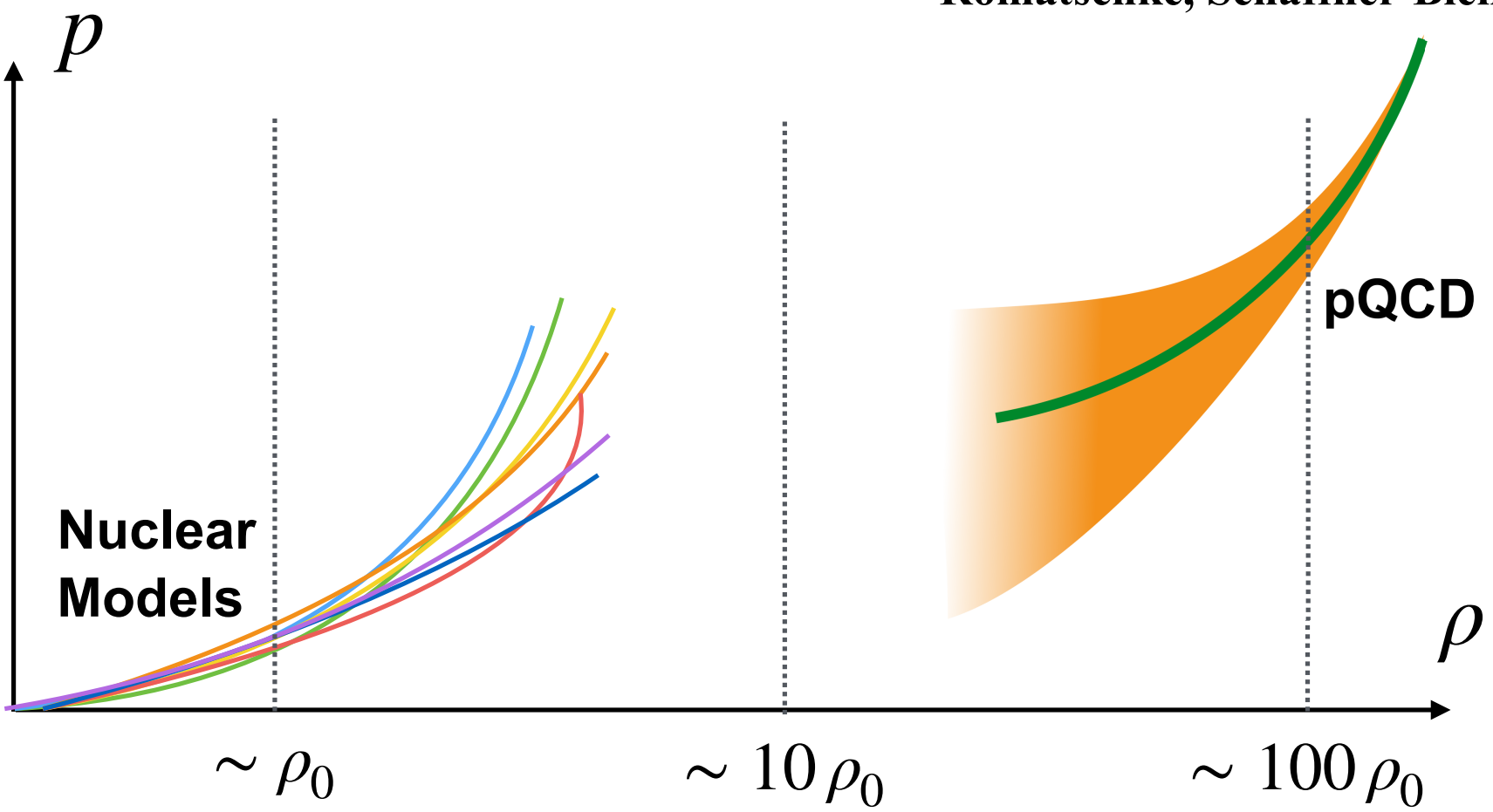
- □ Quark-hadron continuity
- □ Any interface? Vortex controversies

Toward more realistic matter with two flavors

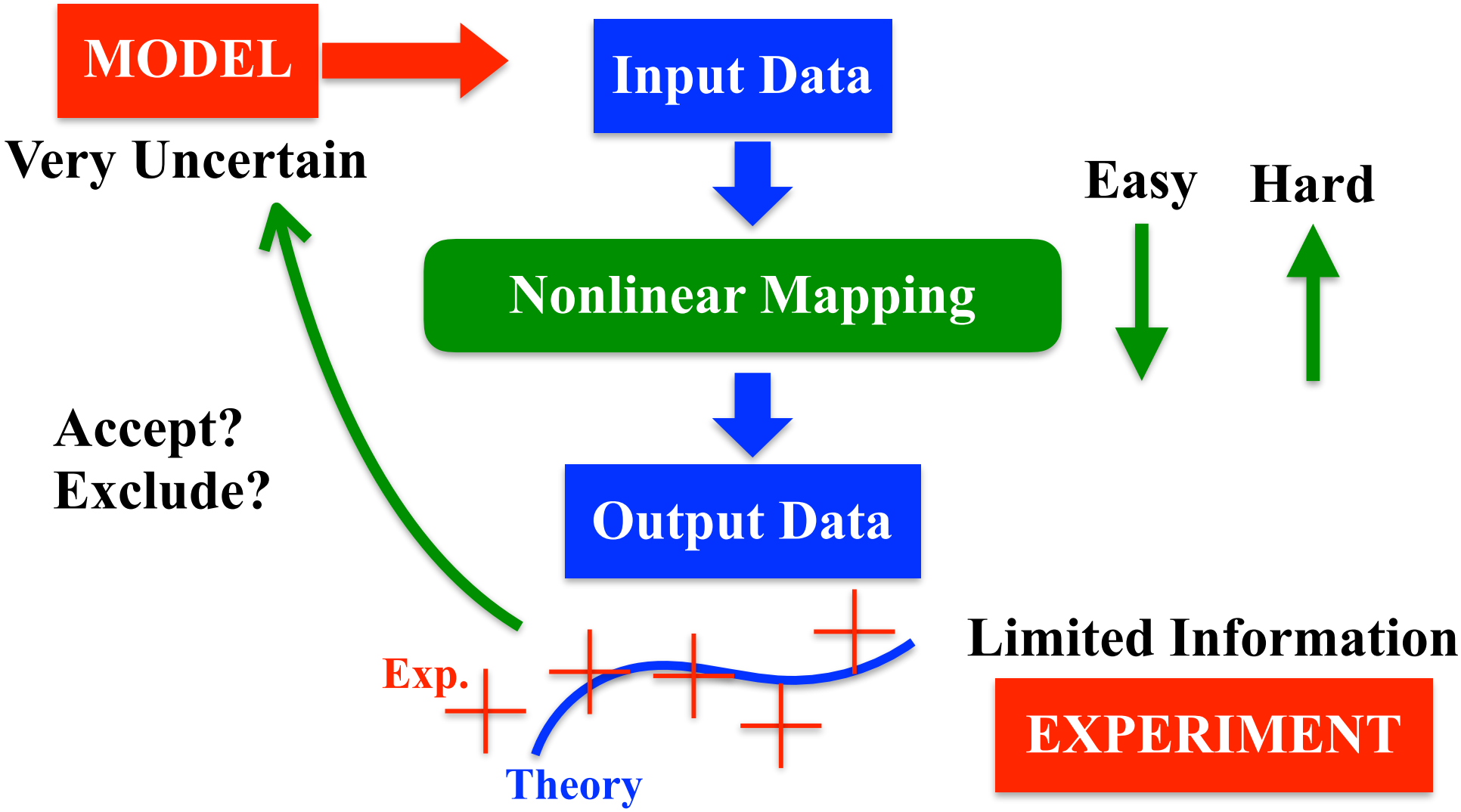
- □ Two-flavor case with only *S*-wave superfluidity
- □ *P*-wave superfluidity at high density

QCD EoS from NS Observation

Kurkela, Vuorinen, Fraga, Romatschke, Schaffner-Bielich, …

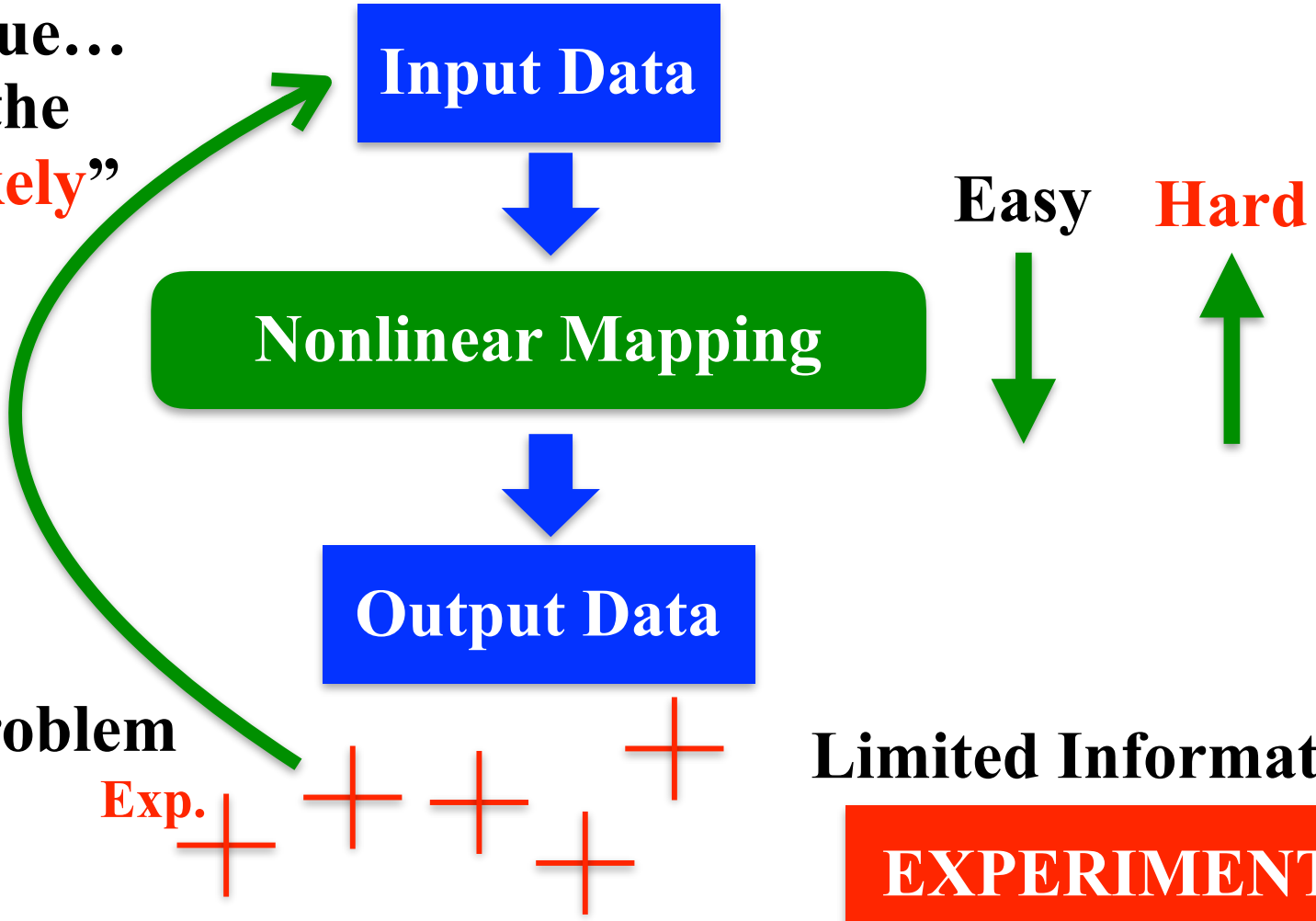


QCD EoS from NS Observation



QCD EoS from NS Observation

Not unique…
What is the
"most likely"
one?



Inverse Problem

Limited Information

EXPERIMENT

QCD EoS from NS Observation

- **Bayesian Analysis**
- **Supervised Learning**

Several *M-R* observation points with errors

Nonlinear Mapping

Several parameters to characterize EoS

$$\{M_i, R_i\} \qquad \{P_i\} = F(\{M_i, R_i\}) \qquad \{P_i\}$$

~ 15 Points

observation data hopefully available in the future

~ 5 Points

corresponding to 5 polytropes (your choice)

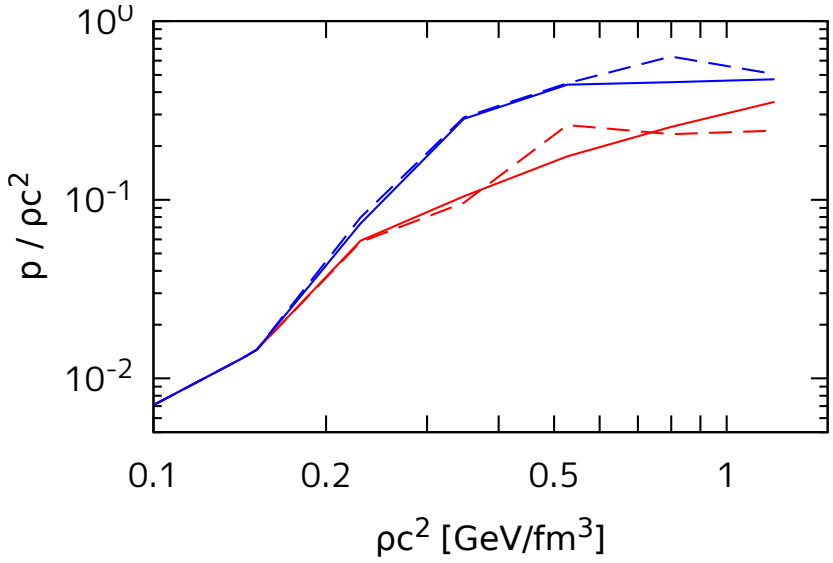
Machine Learning

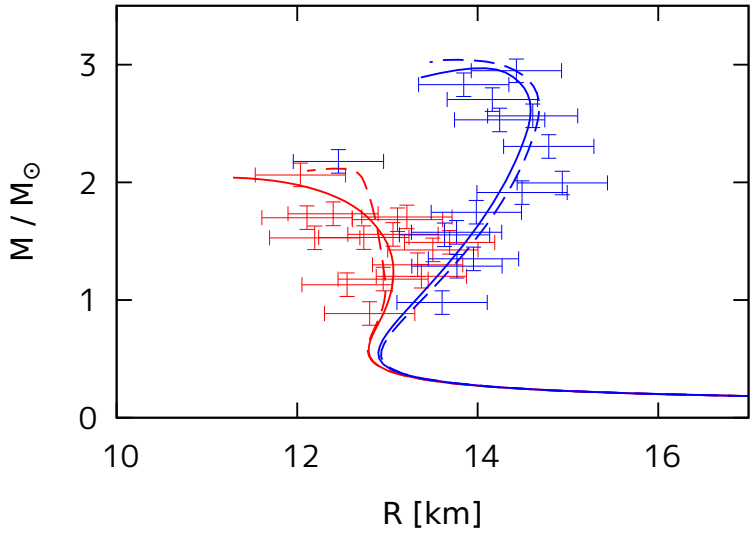
Test with mock data

Fujimoto-Fukushima-Murase, PRD(2018)

(not fitted results but reconstructed!)

Two Typical Examples (not biased choice)





: randomly generated original EoS

: reconstructed EoS and guessed *M-R*

Machine Learning

Overall performance test

Fujimoto-Fukushima-Murase, PRD(2018)

Mass ($M_\odot$)	0.6	0.8	1.0	1.2	1.4	1.6	1.8
RMS (km)	0.16	0.12	0.10	0.099	0.11	0.11	0.12

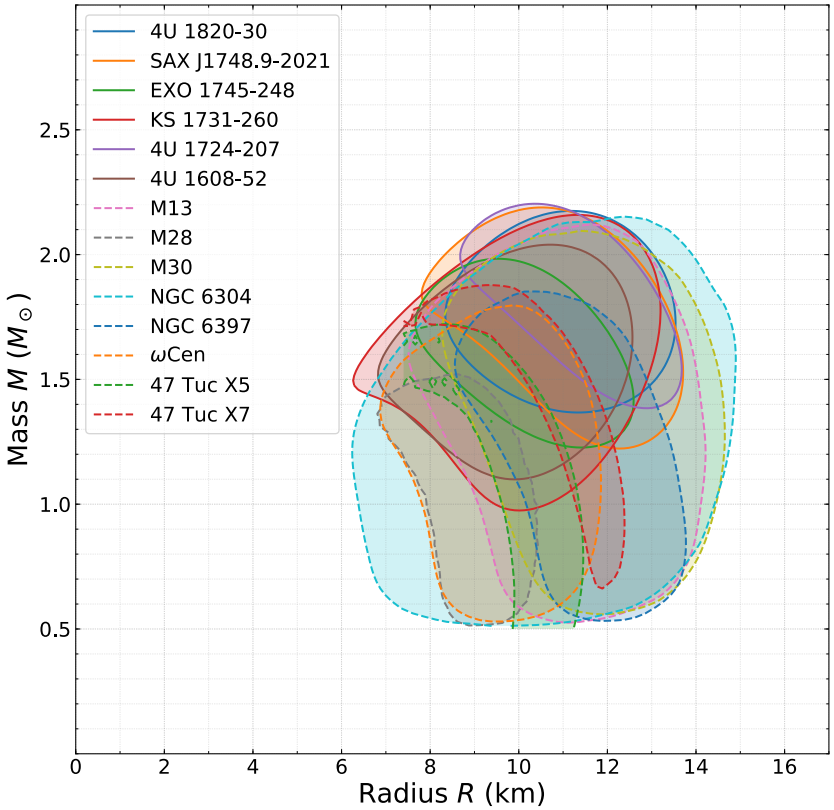
$$(\text{with } \Delta M = 0.1 M_\odot,\ \Delta R = 0.5\,\text{km})$$

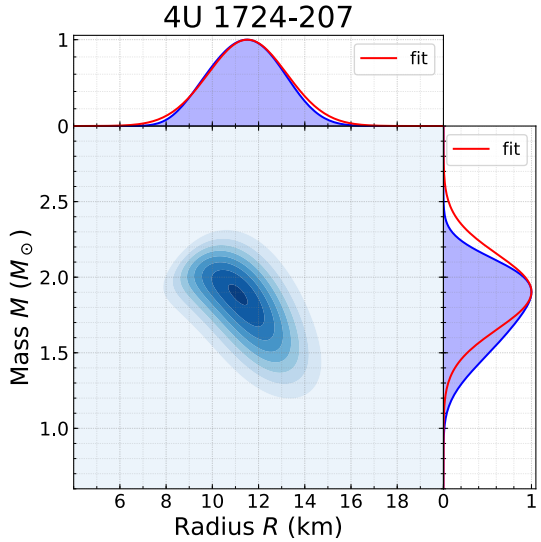
Excellent performance!

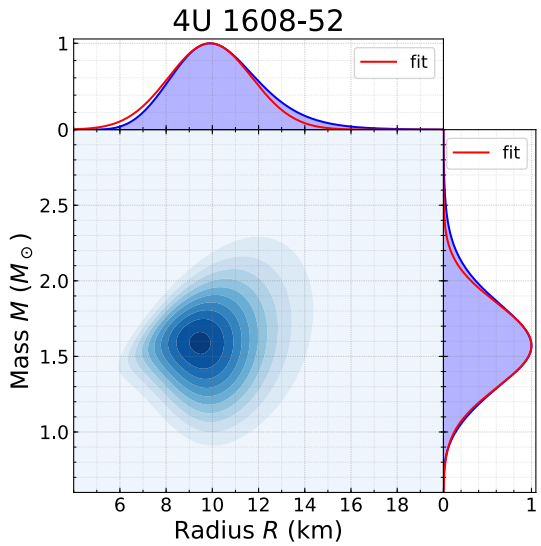
For real data, we should take account of "error bars"

Machine Learning

Fujimoto-Fukushima-Murase
1903.03400 [nucl-th]

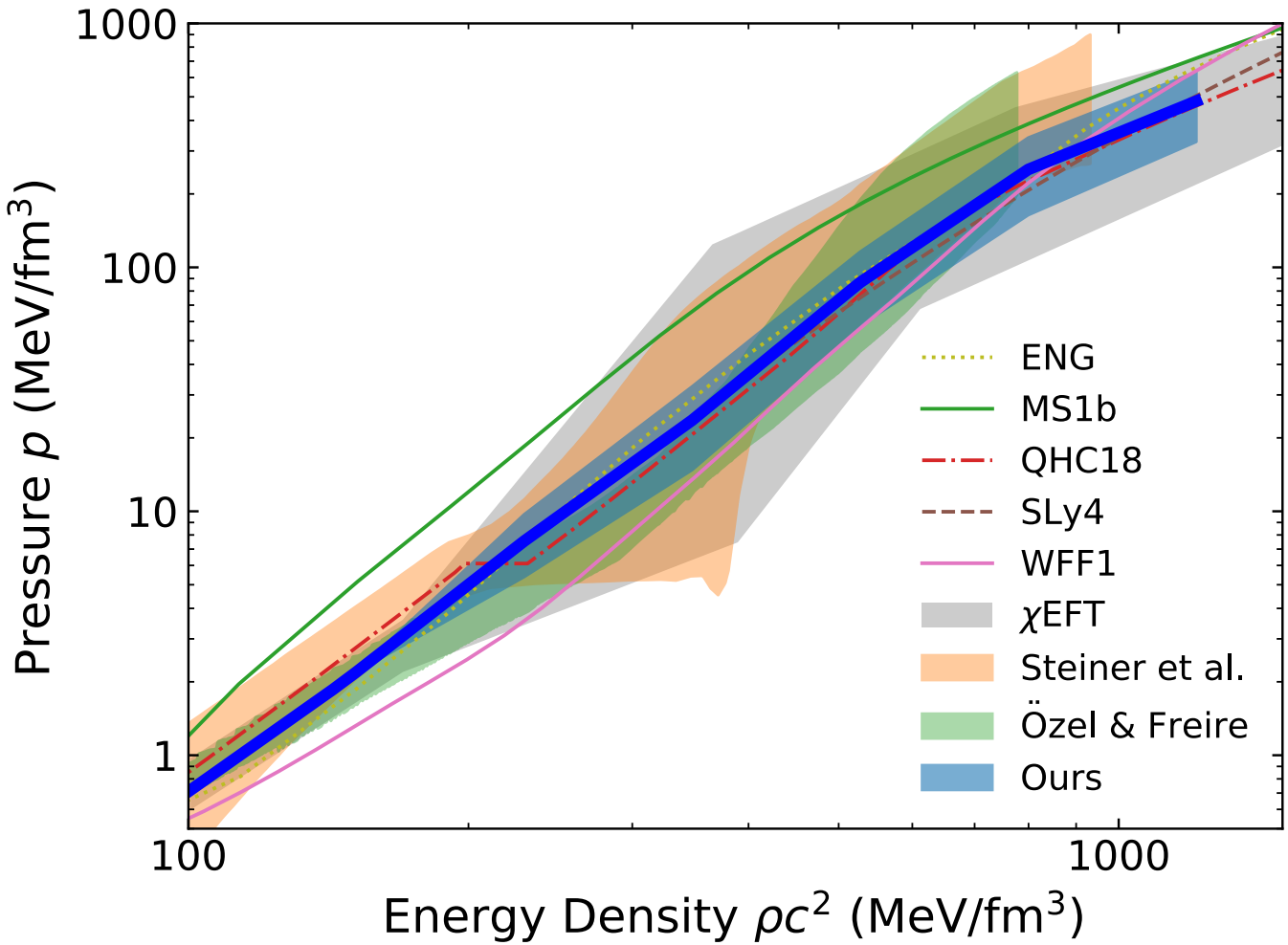






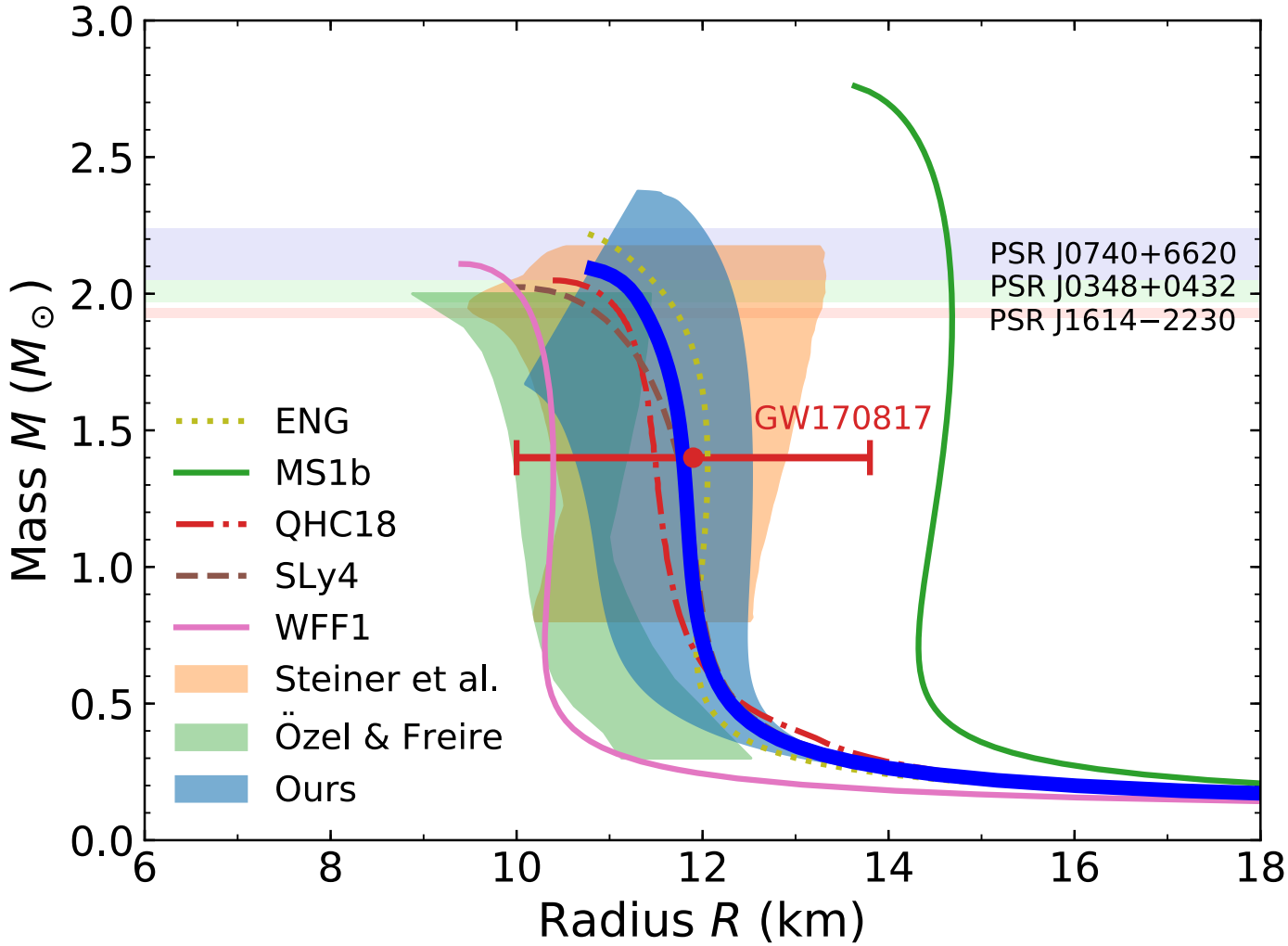
Machine Learning

Fujimoto-Fukushima-Murase, 1903.03400 [nucl-th]



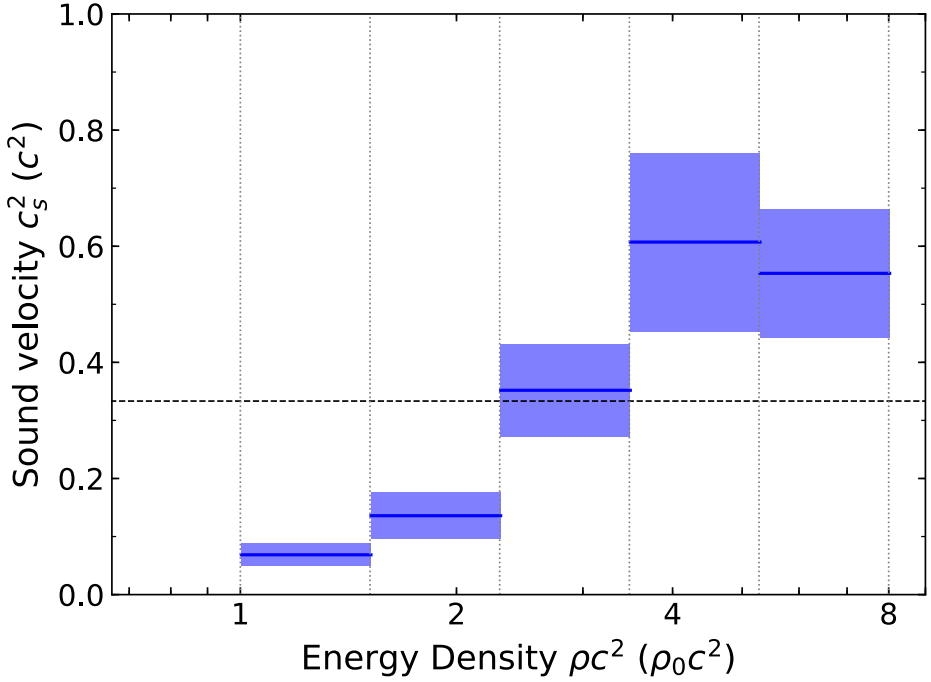
Machine Learning

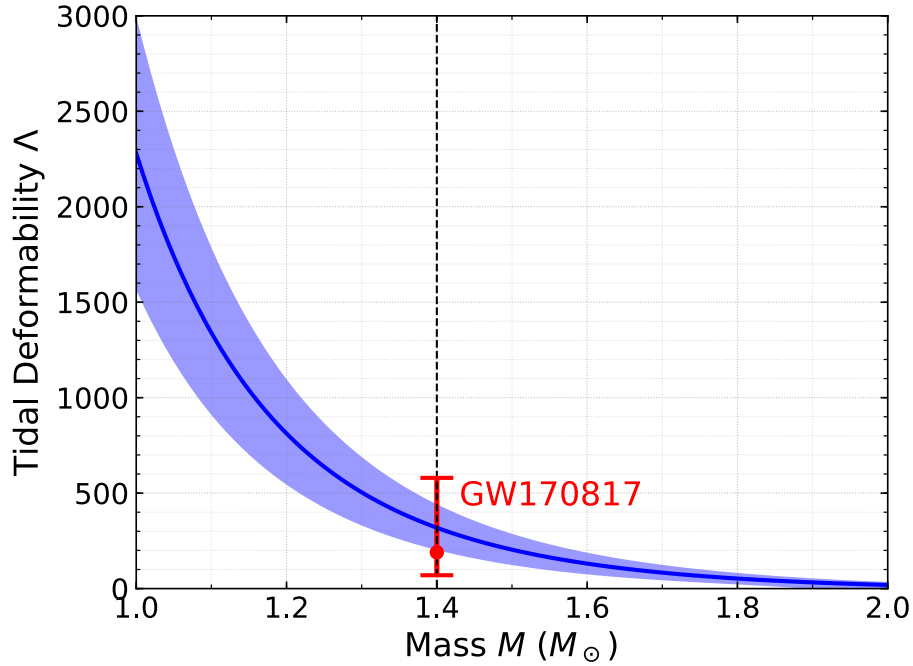
Fujimoto-Fukushima-Murase, 1903.03400 [nucl-th]



Machine Learning

Fujimoto-Fukushima-Murase, 1903.03400 [nucl-th]



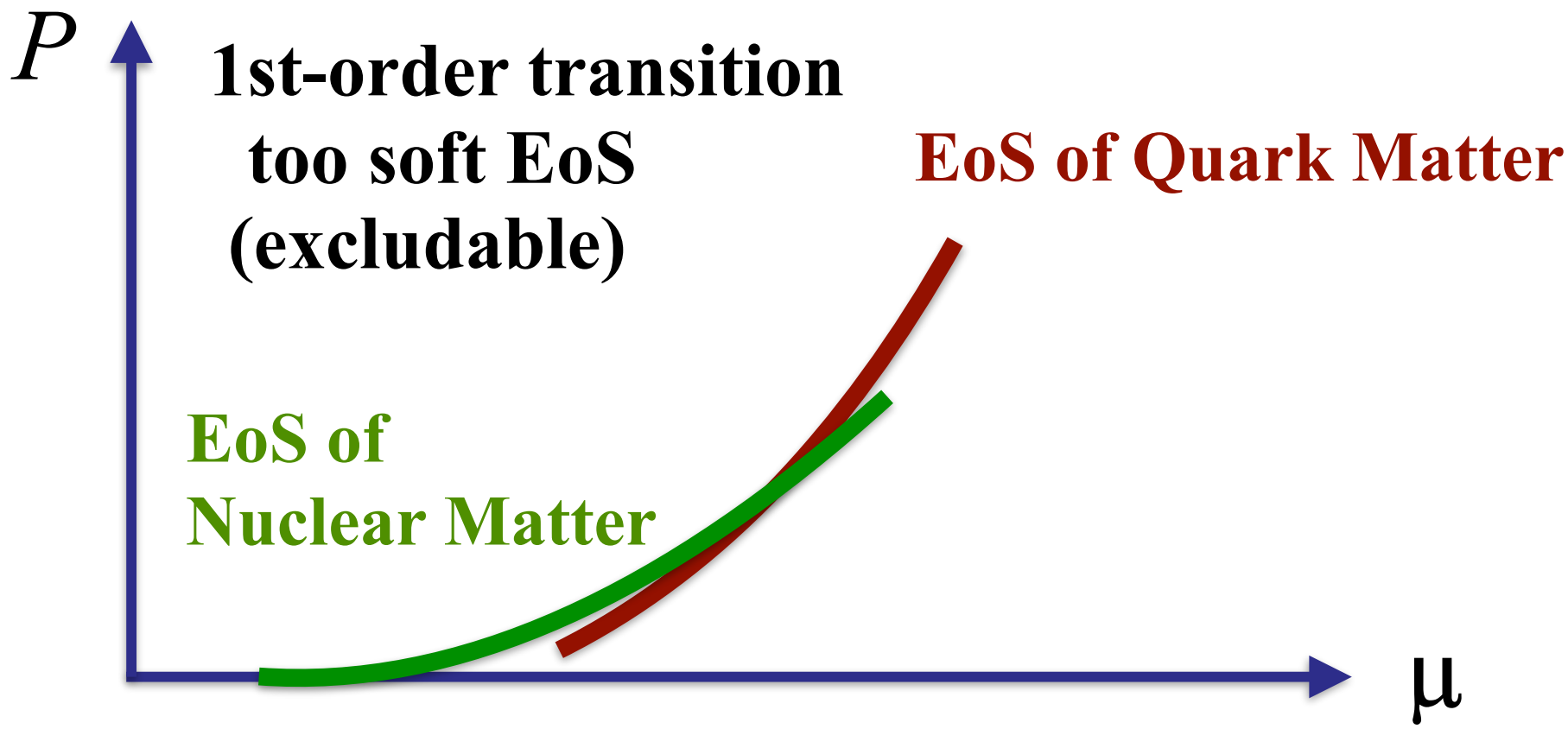


Looks consistent with "quarkyonic scenario"

McLerran-Reddy (2018), Jeong-McLerran-Sen (2019)

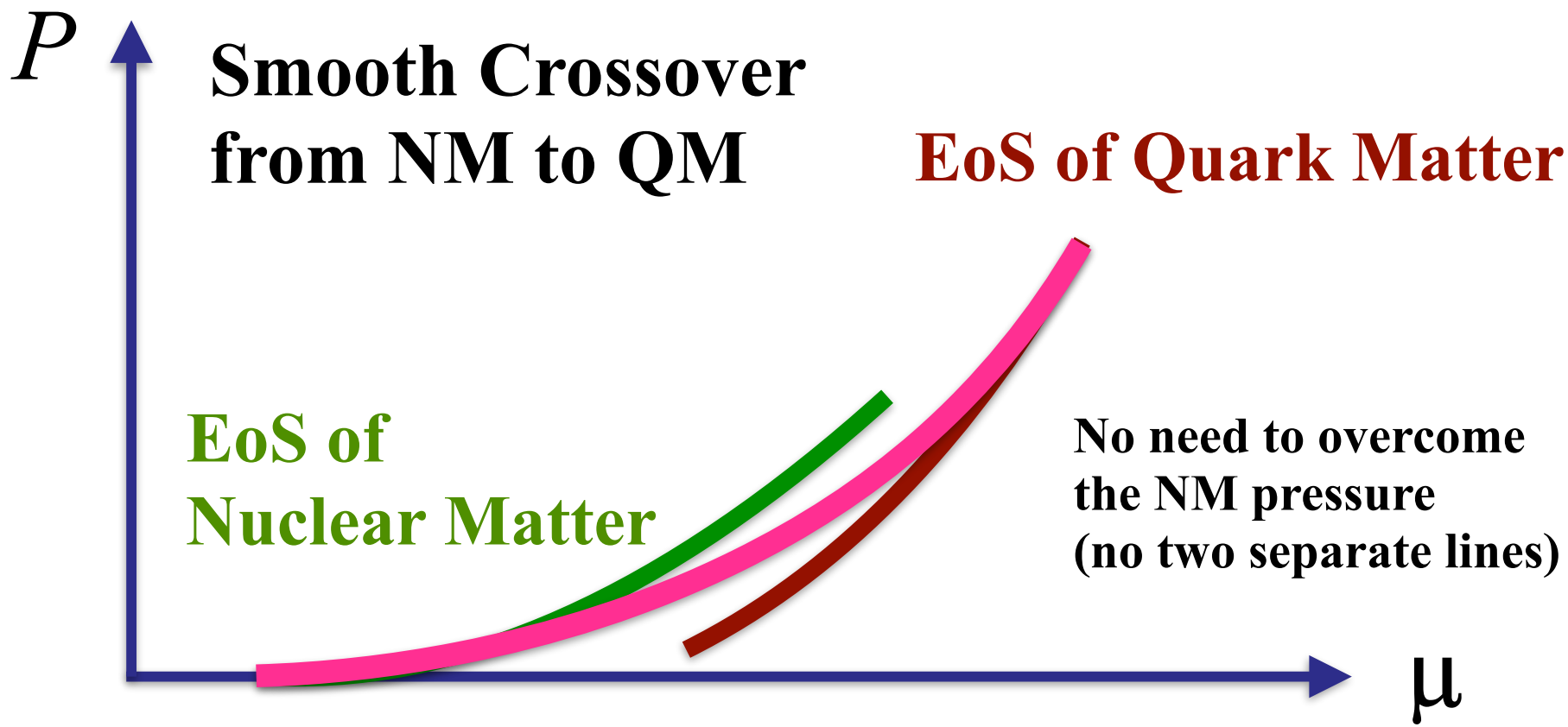
EoS Looks Continuous

Old View



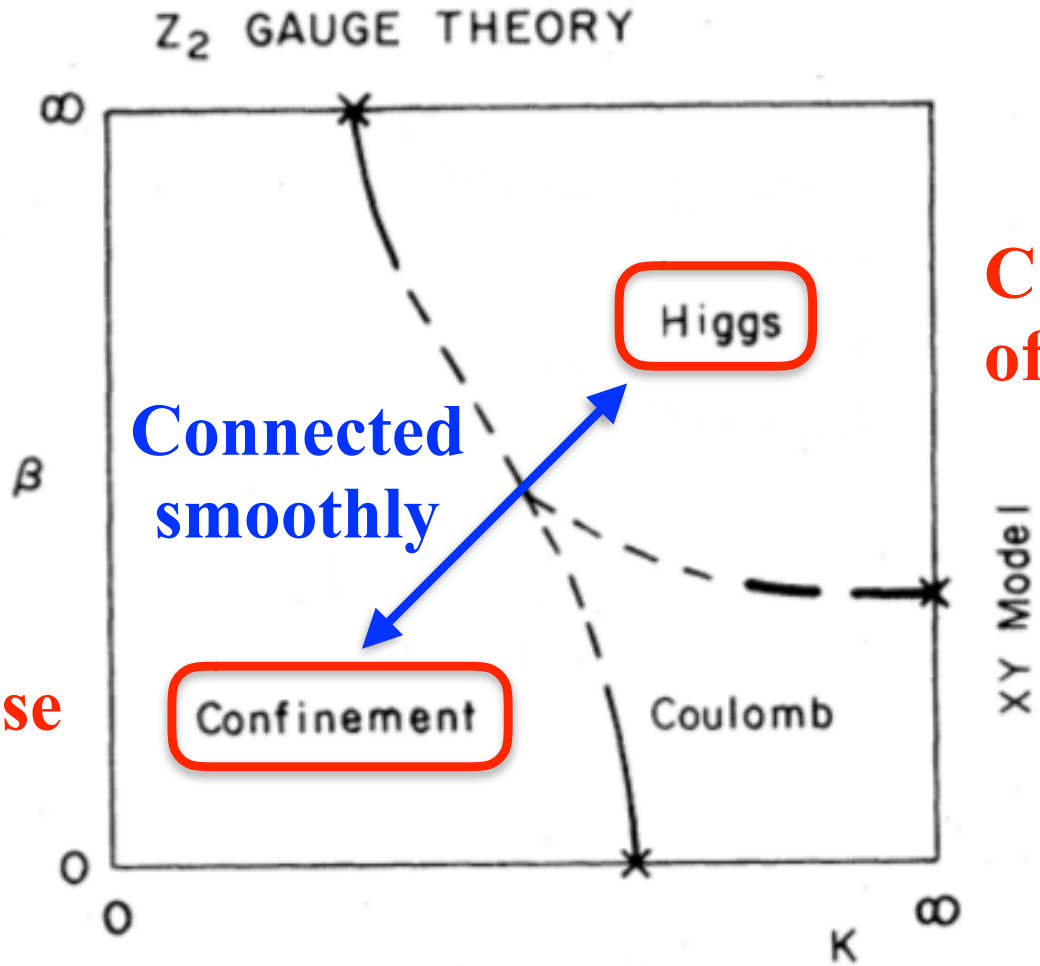
EoS Looks Continuous

Another Possible View



Quark-hadron Continuity

Fradkin-Shenker (1979)



Connected smoothly

CSC phase of QCD

Hadronic phase of QCD

Quark-hadron Continuity

Schaefer-Wilczek (1998)

Left and right diquarks break flavor and color
Flavor rotation can be canceled by color rotation

$$\mathcal{G} = [\mathrm{SU}(3)_\mathrm{C}] \times \mathrm{SU}(3)_\mathrm{L} \times \mathrm{SU}(3)_\mathrm{R} \times \mathrm{U}(1)_\mathrm{B}$$

$$\mathcal{H} = \mathrm{SU}(3)_\mathrm{C+L+R}$$

Three-flavor symmetric color-superconductor (CFL) has symmetry breaking identical to the hadronic one.

Baryons 8+1 (low-lying) **Quarks 3color × 3flavor = 9**

Quark-hadron Continuity

Order Parameter — Three flavor case

Diquarks: $\hat{\Phi}^{\alpha A} \equiv \mathcal{N}\, \epsilon^{\alpha\beta\gamma} \epsilon^{ABC}\, \hat{q}^{\top}_{\beta B}\, \mathcal{C}\gamma_5\, \hat{q}_{\gamma C}$

Baryons: $\hat{\mathcal{B}}^{AB}_{\sigma} = \hat{\Phi}^{\alpha A} \hat{q}^{B}_{\alpha\sigma}$

Gauge invariant order parameter for a U(1) superfluid

$$\Upsilon^{ABC} = \langle \hat{\Upsilon}^{ABC}_{\rm CFL} \rangle = \langle \epsilon^{\alpha\beta\gamma} \hat{\Phi}^{\alpha A} \hat{\Phi}^{\beta B} \hat{\Phi}^{\gamma C} \rangle$$
$$= 2\epsilon^{AMN} \hat{\mathcal{B}}^{BM}_{\sigma} (\mathcal{C}\gamma_5)_{\sigma\sigma'} \hat{\mathcal{B}}^{CN}_{\sigma'}$$

$$\mathcal{B}^{AB}_{\mathbf{8}} = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}_{AB}$$

3 Diquarks
~ 2 Baryons

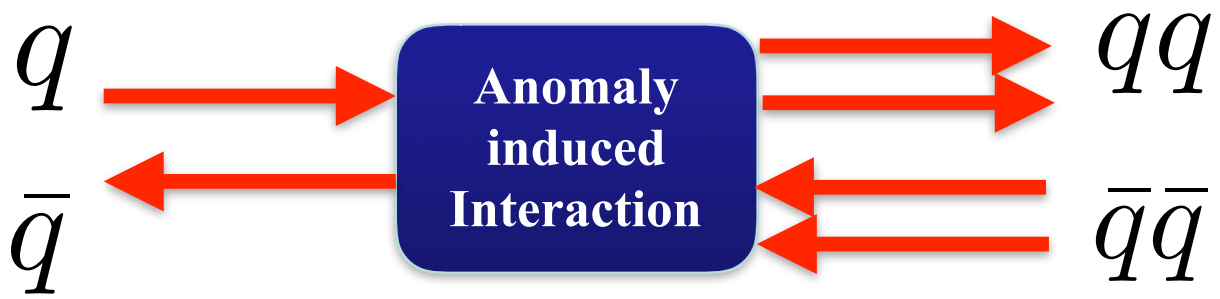
Quark-hadron Continuity

Another view to understand the same physics

Hadronic Phase : Chiral broken by the chiral condensate.

Quark Phase : Chiral broken by the diquark condensate.

Breaking patters is indistinguishable (due to anomaly)

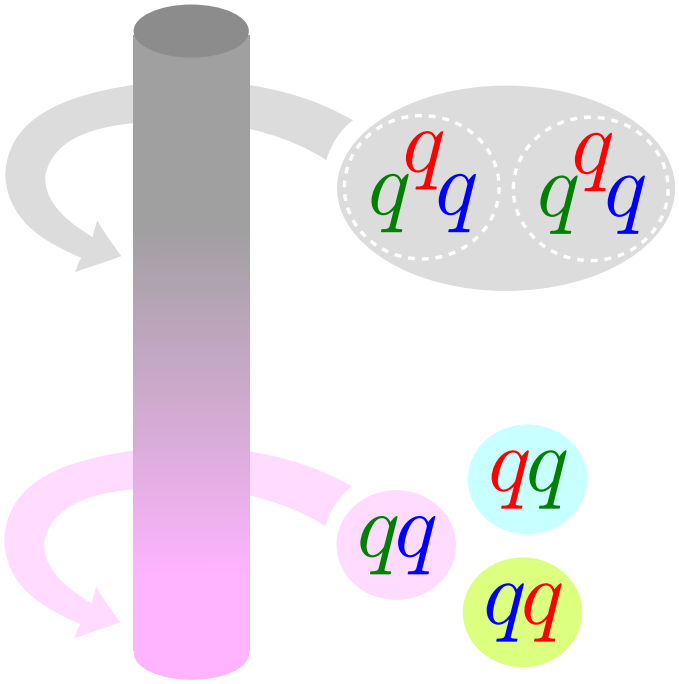


Ordinary meson-like **Tetra-quark meson-like**

Quark-hadron Continuity

Hadronic and color-superconducting matter indistinguishable

Continuity of superfluid vortices



Alford-Baym-Fukushima-
-Hatsuda-Tachibana (2018)

Quark-hadron Continuity

Hadronic and color-superconducting matter indistinguishable

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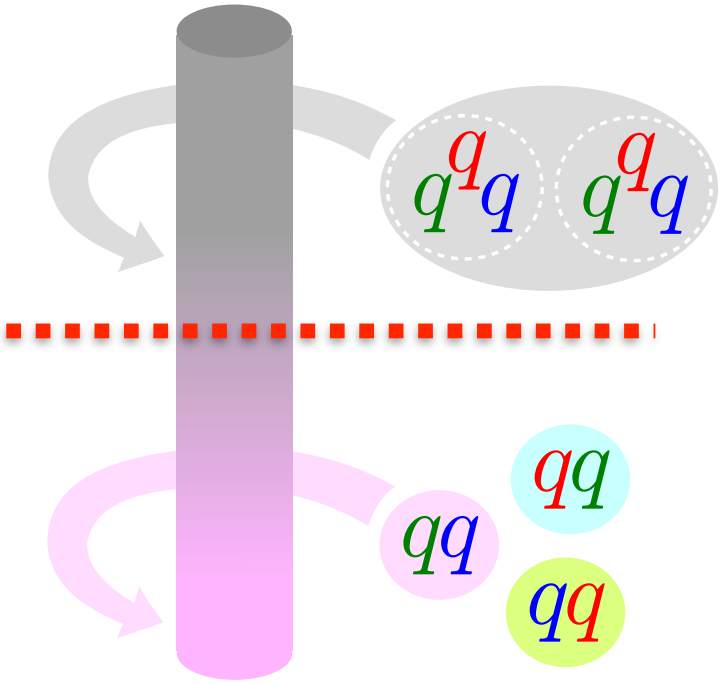


Alford-Baym-Fukushima-
-Hatsuda-Tachibana (2018)

There should be an interface

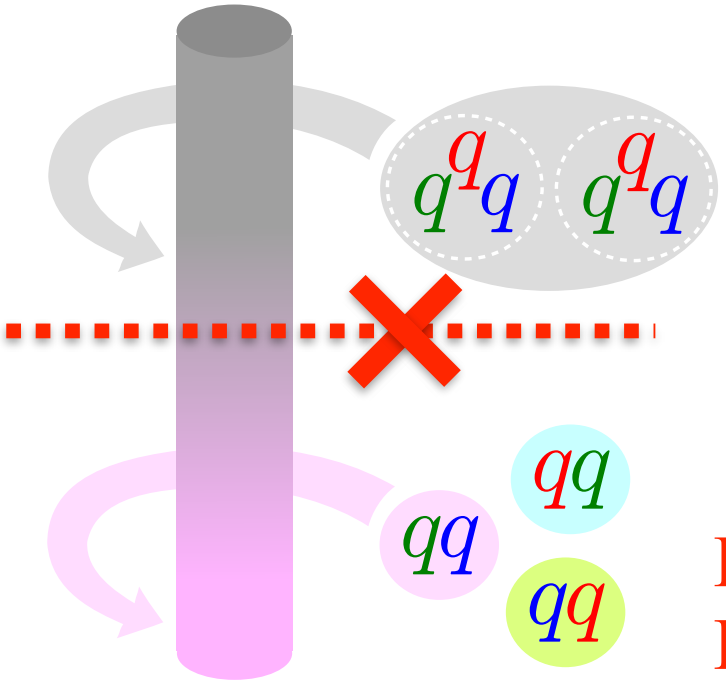
Cherman-Sen-Yaffe (2018)

Braiding phase ~ Topological?

Quark-hadron Continuity

Hadronic and color-superconducting matter indistinguishable

Continuity of superfluid vortices



Alford-Baym-Fukushima-
-Hatsuda-Tachibana (2018)

~~There should be an interface~~

Hirono-Tanizaki (2018)

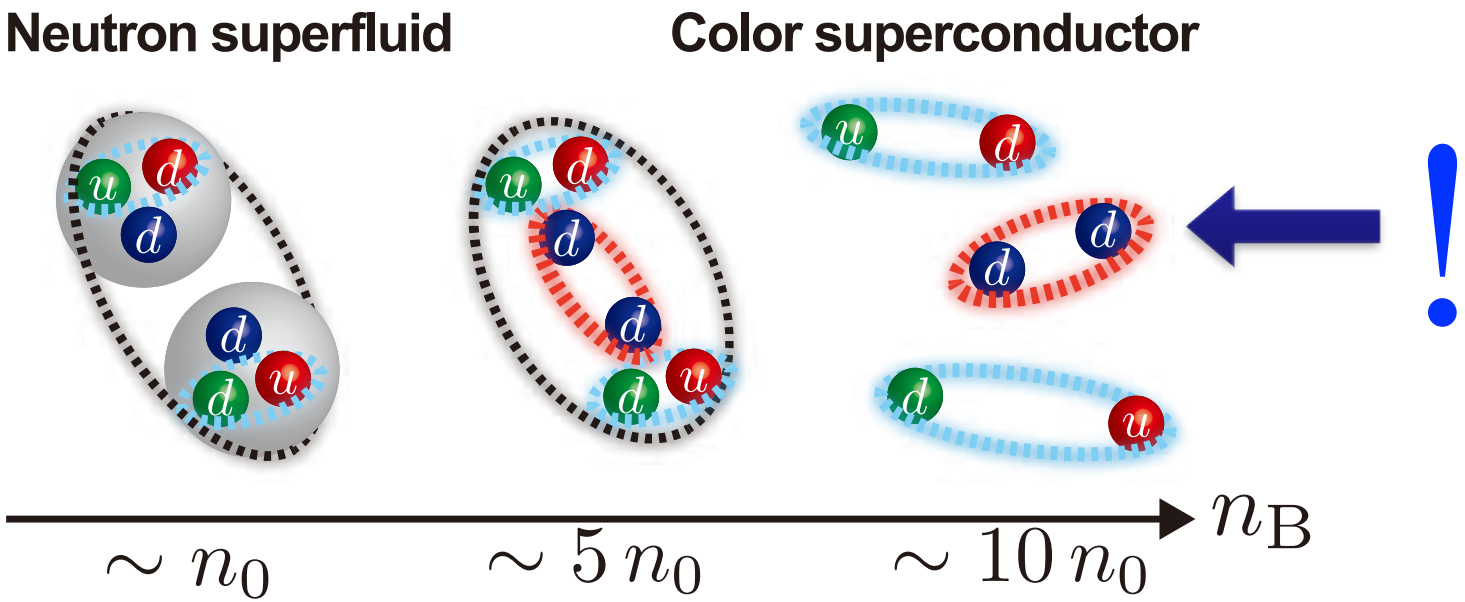
Higher-form symmetry not broken
Braiding phase unseen

Quark-hadron Continuity

$\langle ud \rangle$ **alone does not break any symmetry!**

2SC (*two-flavor color-superconductor*) may be connected to a QGP, but not to hadronic matter…?

How "neutron matter" melts into "quark matter" ?



Quark-hadron Continuity

Order Parameter — Two flavor case

Neutrons: $\hat{n}_\sigma = \epsilon^{\alpha\beta\gamma}\,(\hat{u}_\alpha^\top \mathcal{C}\gamma_5 \hat{d}_\beta)\hat{d}_{\gamma\sigma} = \hat{\Phi}^\gamma_{ud}\hat{d}_{\gamma\sigma}$

$$\Upsilon_S = \langle \hat{\Phi}^\alpha_{ud}\hat{\Phi}^\beta_{ud}\,\hat{d}_\alpha^\top \mathcal{C}\gamma_5 \hat{d}_\beta\rangle \approx \Phi^\alpha_{ud}\,\Phi^\beta_{ud}\,\langle \hat{d}_\alpha^\top \mathcal{C}\gamma_5 \hat{d}_\beta\rangle$$

$\langle dd \rangle$ **mixture breaks U(1) leading to a superfluid!**

Flavor symmetric / Spin antisymmetric
Color should be symmetric (6-tet channel)

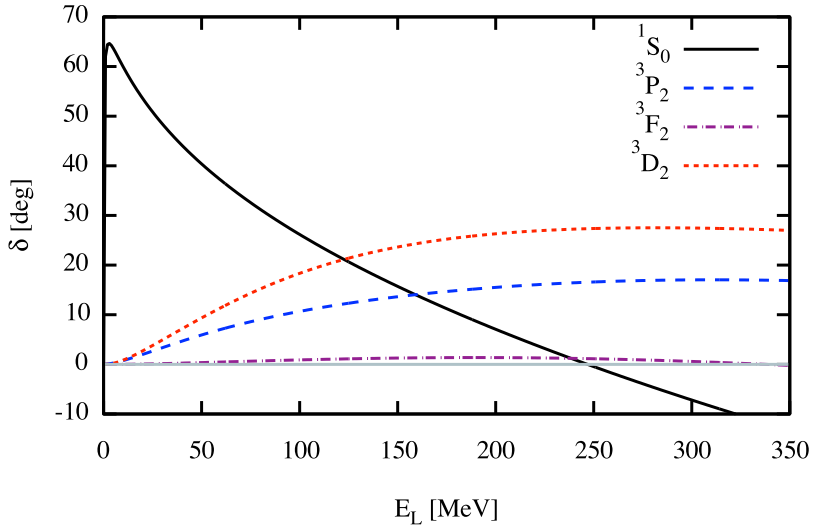
One-gluon exchange (short-range) repulsive !?

Quark-hadron Continuity

Fujimoto-Fukushima-Weise (2019)

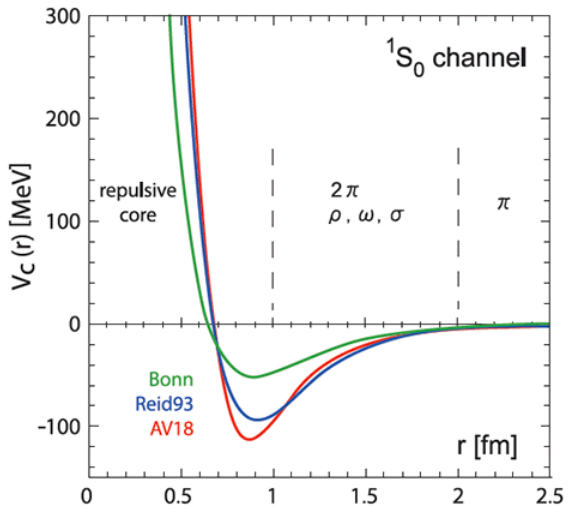
Very natural for continuity to neutron matter!

NN scattering (theory)



Haskell-Sedrakian (2017)
Originally Kyoto group (Tamagaki et al.)

***S*-wave is suppressed at higher density because of short-range repulsive core!**

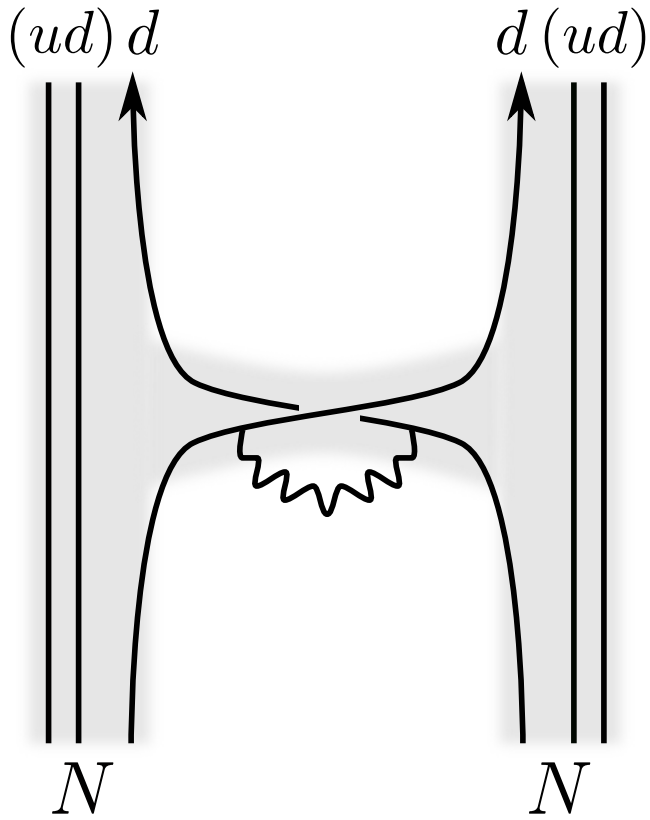


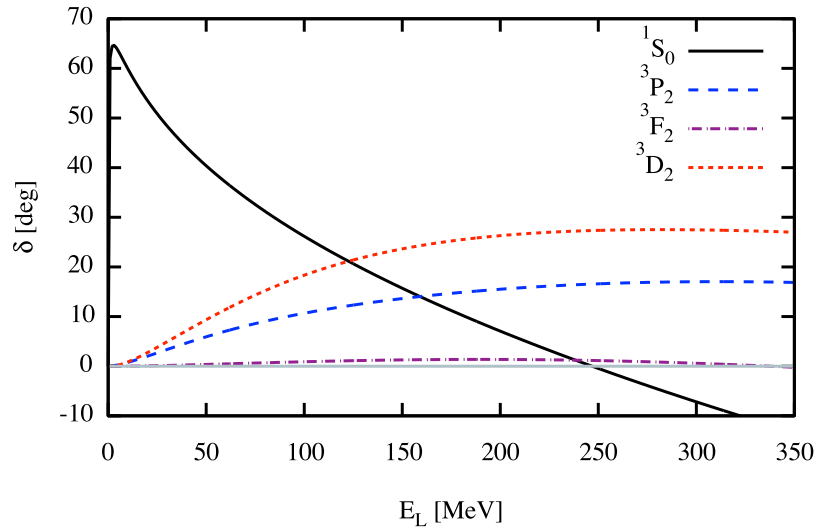
Ishii-Aoki-Hatsuda (2007)

Quark-hadron Continuity

Short-range repulsive core arises from quark exchanges

Oka-Yazaki (1980)

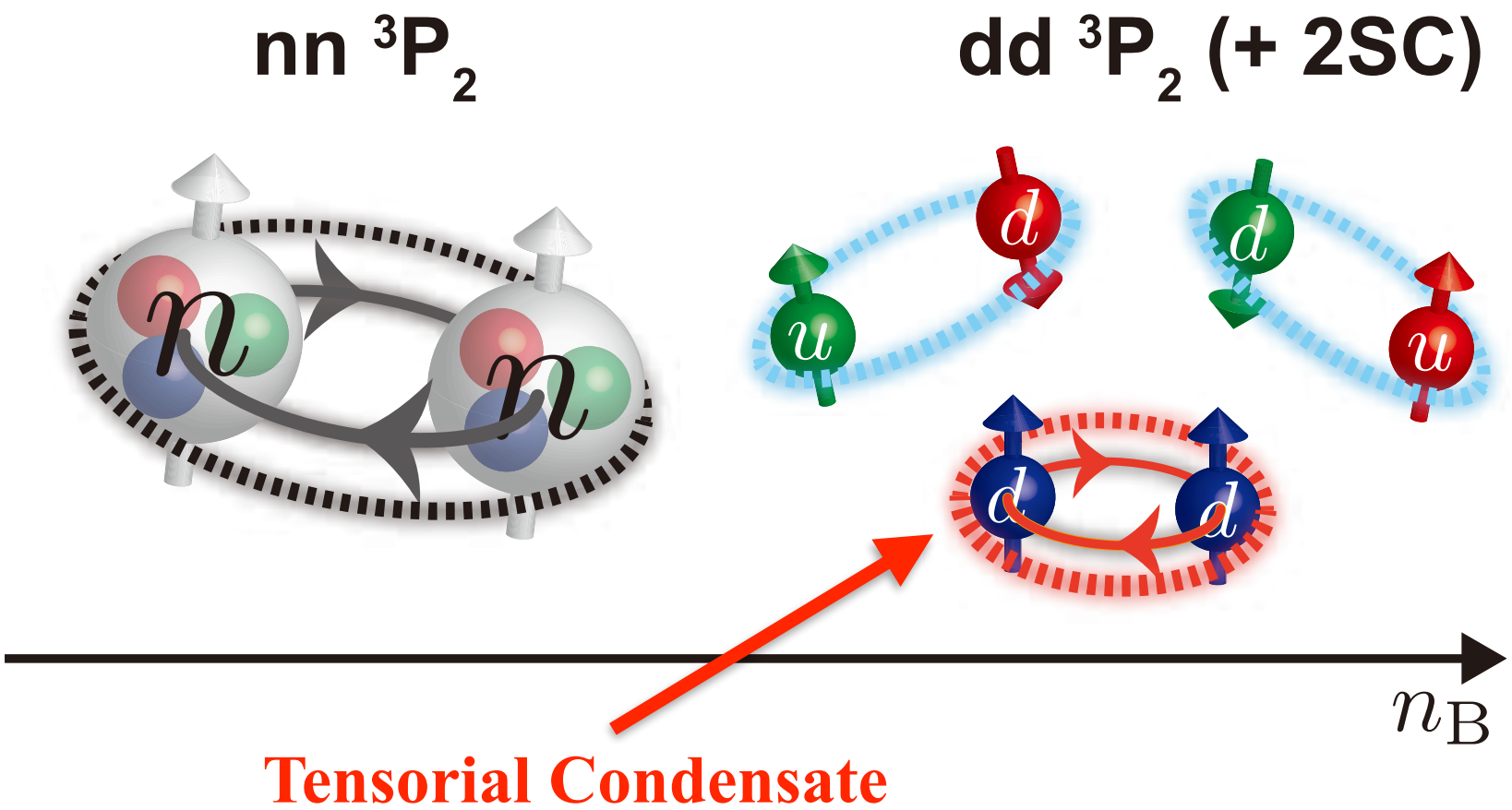




$^3P_2$ overcomes $^1S_0$ to avoid short-range repulsion!

Quark-hadron Continuity

Continuity with $^3P_2$ superfluidity?



Quark-hadron Continuity

Continuity with $^3P_2$ superfluidity?

Gauge invariant order parameter for tensorial diquarks

$$\hat{\mathcal{I}}_P = (\bar{\psi}\gamma^i\nabla^j\mathcal{C}\bar{\psi}^\top)(\psi^\top\mathcal{C}\gamma_i\nabla_j\psi)$$

Fierz transformed to the energy-momentum tensor squared

$$\langle\hat{\mathcal{I}}_P\rangle \approx \frac{3}{4}p^2$$ if other channels negligible

The order parameter always takes a macroscopic value!

Summary

EoS construction

- □ Machine learning more adaptive than Bayesian
- □ More experimental data

Quark-hadron continuity

- □ Vortex (higher-form symmetry) controversies
- □ Quarkyonic scenario

Physics of $^3P_2$ superfluidity

- □ Implication to NS cooling — 2SC+X scenario?