

Quark and gluon contributions to the QCD trace anomaly

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Outline

- Trace anomaly in QCD
- Deconstructing the trace anomaly
- Phenomenology

YH, Abha Rajan, Kazuhiro Tanaka, [JHEP 1812 \(2018\) 008](#)

YH, Di-Lun Yang, [PRD98 \(2018\) 074003](#)

K.Tanaka, [JHEP 1901 \(2019\) 120](#)

Quantum Chromodynamics

$$L = -\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a + \bar{q}(i\gamma^\mu D_\mu - m)q$$

Theory of the strong interaction.

Non-abelian gauge theory based on color SU(3)

Approximate **conformal symmetry** at the classical level.

Energy momentum tensor (EMT)

$$T^{\mu\nu} = -F^{\mu\lambda} F^\nu{}_\lambda + \frac{\eta^{\mu\nu}}{4} F^2 + i\bar{q}\gamma^{(\mu} D^{\nu)} q$$

Vanishing trace in the chiral limit

$$T^\mu{}_\mu = m\bar{q}q$$

The trace anomaly

Conformal symmetry explicitly broken by the quantum effects.

$$T_{\mu}^{\mu} = \frac{\beta(g)}{2g} F^2 + m(1 + \gamma_m(g)) \bar{q}q$$

Collins, Duncan, Joglekar (1977)
N.K. Nielsen (1977)

Fundamentally important in QCD. Trace anomaly is the origin of hadron masses

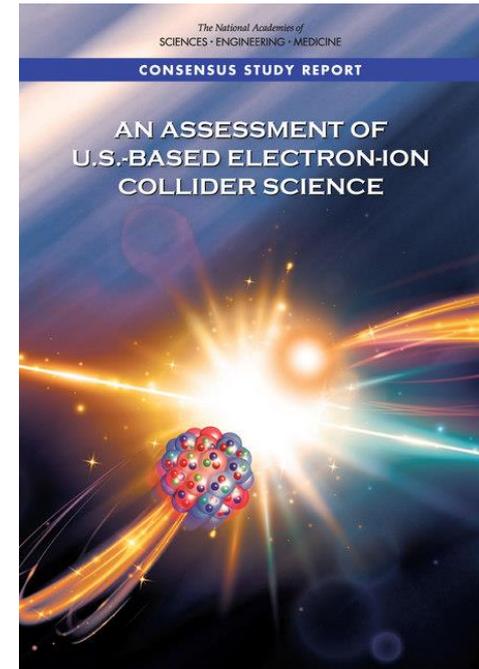
$$\langle P | T^{\mu\nu} | P \rangle = 2P^{\mu} P^{\nu}$$

$$\langle P | T_{\mu}^{\mu} | P \rangle = 2M^2$$

An Assessment of U.S.-Based Electron-Ion Collider Science

Committee on U.S.-Based Electron-Ion Collider Science Assessment

Board on Physics and Astronomy



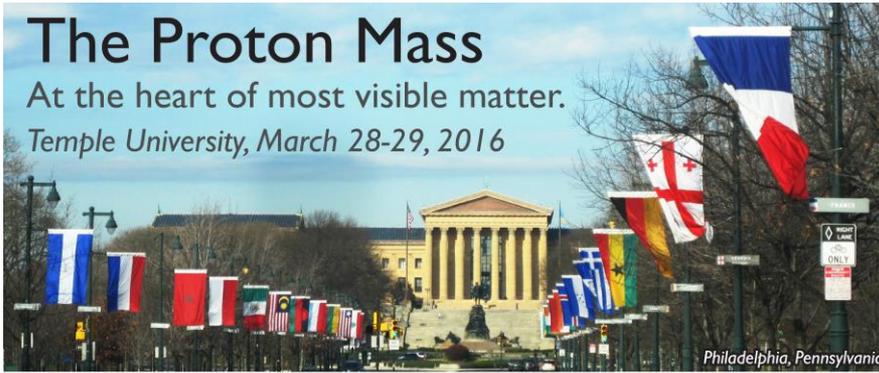
Finding 1: An EIC can uniquely address three profound questions about nucleons—protons—and how they are assembled to form the nuclei of atoms:

- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise?
- What are the emergent properties of dense systems of gluons?

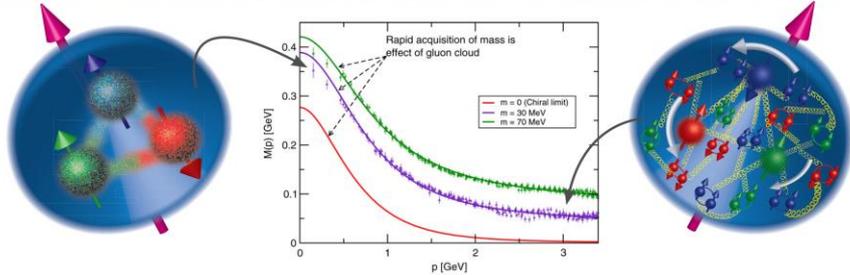
The Proton Mass

At the heart of most visible matter.

Temple University, March 28-29, 2016



Philadelphia, Pennsylvania



$$M_p = 2m_u^{\text{eff}} + m_d^{\text{eff}}$$

Speakers

Stan Brodsky (SLAC)
 Xiandong Ji (Maryland)
 Dima Kharzeev (Stony Brook & BNL)
 Keh-Fei Liu (University of Kentucky)
 David Richards (JLab)
 Craig Roberts (ANL)
 Martin Savage (University of Washington)
 Stepan Stepanyan (JLab)

$$H_{\text{QCD}} = H_q + H_m + H_g + H_a$$

Quark kinetic and potential energy $H_q = \int d^3x \psi^\dagger (-i\mathbf{D} \cdot \boldsymbol{\alpha}) \psi$

Quark masses $H_m = \int d^3x \bar{\psi} m \psi$

Gluon kinetic and potential energy $H_g = \int d^3x \frac{1}{2} (\mathbf{E}^2 + \mathbf{B}^2)$

Trace anomaly $H_a = \int d^3x \frac{9\alpha_s}{16\pi} (\mathbf{E}^2 - \mathbf{B}^2)$

Workshop Topics

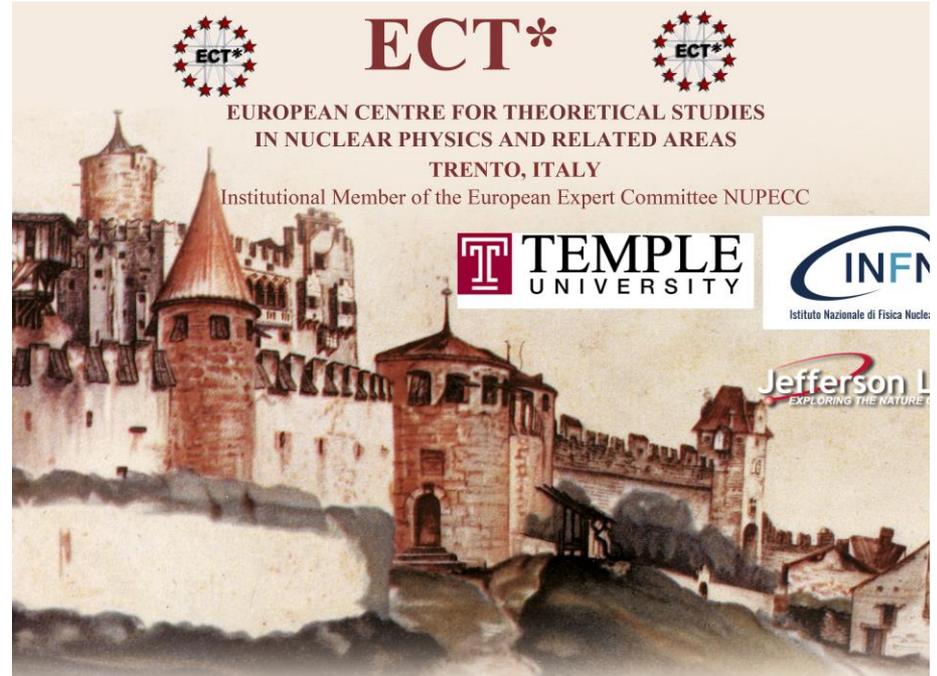


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EUROPEAN CENTRE FOR THEORETICAL STUDIES
 IN NUCLEAR PHYSICS AND RELATED AREAS
 TRENTO, ITALY

Institutional Member of the European Expert Committee NUPECC



Castello di Trento ("Trint"), watercolor 19.8 x 27.7, painted by A. Dürer on his way back from Venice (1495). British Museum.

The Proton Mass: At the Heart of Most Visible Matter

Trento, April 3 - 7, 2017

Main Topics

Hadron mass decomposition in terms of constituents:

Uniqueness of the decomposition, Quark mass, and quark and gluon energy contribution, Anomaly contribution, ...

Hadron mass calculations:

Lattice QCD (total & individual mass components), Approximated analytical methods, Phenomenological model approaches, ...

Experimental access to hadron mass components:

Exclusive heavy quarkonium production at threshold, nuclear gluonometry through polarized nuclear structure function

Proton mass decomposition

Traceless and trace parts of EMT in $d = 4 - 2\epsilon$ dimensions.

$$T^{\mu\nu} = \left(T^{\mu\nu} - \frac{\eta^{\mu\nu}}{d} T^\alpha{}_\alpha \right) + \frac{\eta^{\mu\nu}}{d} T^\alpha{}_\alpha$$

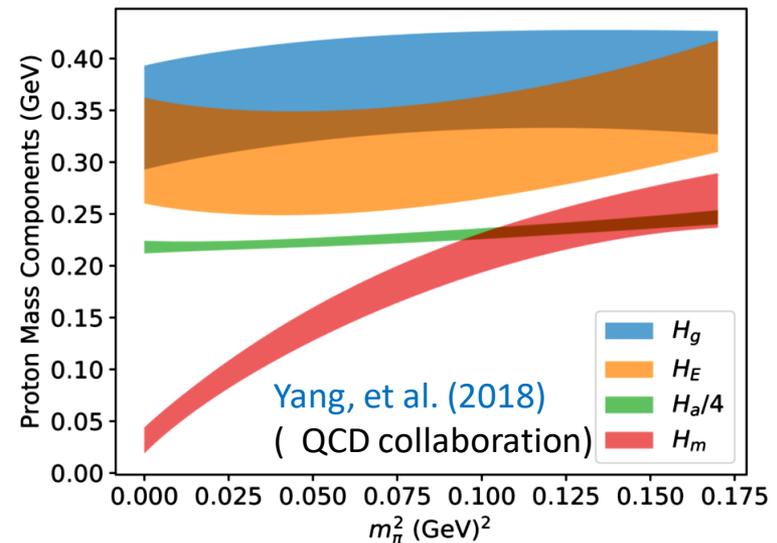
$\frac{\beta(g)}{2g} F^2 + m(1 + \gamma_m(g)) \bar{q}q$

Work in the rest frame.

Mass is the eigenvalue of the Hamiltonian $H = \int d^3x T^{00}$ Ji (1995)

$$M = M_q + M_g + M_a + M_m$$

quark/gluon kinetic energy quark mass
trace anomaly



Lorentz invariant mass decomposition

YH, Rajan, Tanaka (2018)
Tanaka (2018)

$$T^{\mu\nu} = \underbrace{-F^{\mu\lambda} F^\nu{}_\lambda + \frac{\eta^{\mu\nu}}{4} F^2}_{T_g^{\mu\nu}} + \underbrace{i\bar{q}\gamma^{(\mu} D^{\nu)} q}_{T_q^{\mu\nu}}$$

$$T^\mu{}_\mu = (T_q)^\mu{}_\mu + (T_g)^\mu{}_\mu = \frac{\beta}{2g} F^2 + m(1 + \gamma_m)\bar{\psi}\psi$$

 $M^2 = M_q^2 + M_g^2 \quad M_{q,g}^2 \equiv \frac{1}{2} \langle P | (T_{q,g})^\mu{}_\mu | P \rangle$

Exercise: Compute $(T_q)^\mu{}_\mu$ and $(T_g)^\mu{}_\mu$ separately.

Nucleon gravitational form factors

The QCD energy momentum tensor contains rich information about the structure of the nucleon.

Even richer structure if we discuss quark and gluon parts separately
→ partonic decomposition of the nucleon mass and spin

$$\langle P' | T_{q,g}^{\mu\nu} | P \rangle = \bar{u}(P') \left[A_{q,g} \gamma^{(\mu} \bar{P}^{\nu)} + B_{q,g} \frac{\bar{P}^{(\mu} i \sigma^{\nu)\alpha} \Delta_\alpha}{2M} + D_{q,g} \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{4M} + \bar{C}_{q,g} M \eta^{\mu\nu} \right] u(P)$$

$$\Delta^\mu = P'^\mu - P^\mu$$

All the form factors are interesting (next slides).

All the form factors should be renormalized $A_{q,g}(\Delta^2) \rightarrow A_{q,g}^R(\Delta^2, \mu)$, etc.

Physics of $A_{q,g}, B_{q,g}$

Ji sum rule for the nucleon spin $\frac{1}{2} = J_q + J_g$

$$J_{q,g} = \frac{1}{2} (A_{q,g} + B_{q,g})|_{\Delta \rightarrow 0}$$

Related to the second moments of the generalized parton distributions (GPDs)

$$J^q = \frac{1}{2} \int dx x (H_q(x) + E_q(x)) \quad J^g = \frac{1}{2} \int dx x (H_g(x) + E_g(x))$$

H, E measurable in Deeply Virtual Compton Scattering (DVCS) at Jlab, Compass, EIC,...

Physics of $D_{q,g}$: 'D-term', the last global unknown

$D_{q+g}(\Delta = 0)$ is a fundamental constant of the nucleon,
just like mass and spin!

Fourier transform of the D-term can be interpreted
as the radial pressure distribution inside a nucleon

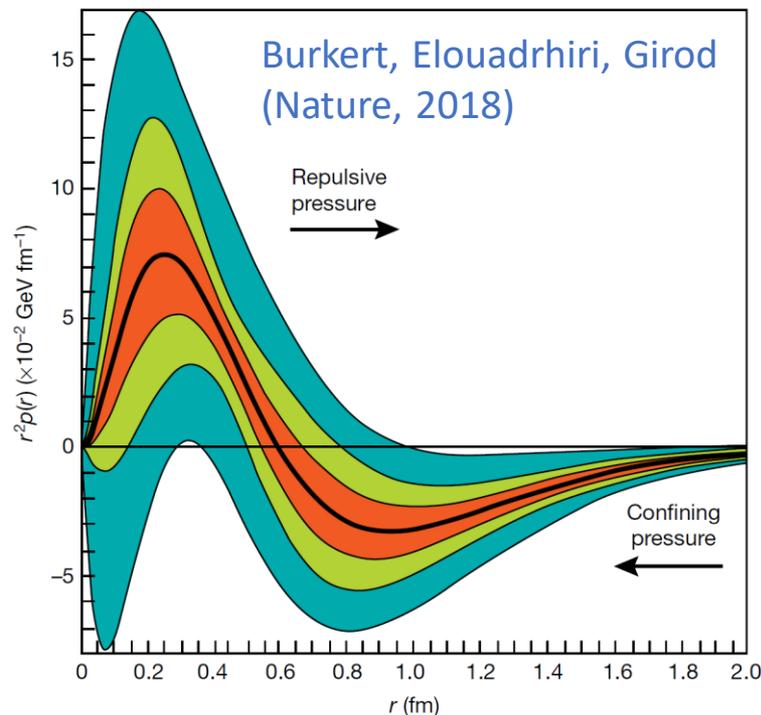
[Polyakov, Schweitzer,...](#)

$$\langle P' | T^{ij} | P \rangle \sim (\Delta^i \Delta^k - \delta^{ik} \Delta^2) D(t)$$

$$T^{ij}(r) = \left(\frac{r^i r^j}{r^2} - \frac{1}{3} \delta^{ij} \right) s(r) + \delta^{ij} p(r)$$

Measurable in DVCS

Computable in lattice QCD [Hagler et al. \(2007\)](#); [Shanahan, Detmold \(2018\)](#)



Physics of $\bar{C}_{q,g}$?

Non-conservation of the quark/gluon parts

$$\langle \partial_\mu T_{q,g}^{\mu\nu} \rangle \sim \Delta^\nu \bar{C}_{q,g}$$

$$\bar{C}_q + \bar{C}_g = 0 \quad \text{because the total EMT is conserved.}$$

Related to the quark and gluon parts of the trace anomaly

$$\langle P | (T_{q,g})^\mu_\mu | P \rangle = 2M^2 (A_{q,g} + 4\bar{C}_{q,g})$$

Renormalization of $\bar{C}_{q,g}$: a first look

Polyakov, Son, arXiv.1808.00155

Because of the relation $\langle \partial_\mu T_{q,g}^{\mu\nu} \rangle \sim \Delta^\nu \bar{C}_{q,g}$, the scale dependence of $\bar{C}_{q,g}$ is governed by the second anomalous dimension of the energy momentum tensor.

$$\frac{\partial}{\partial \ln \mu} \bar{C}_q^R = -\frac{\alpha_s}{4\pi} \left(\frac{16}{3} C_F + \frac{4n_f}{3} \right) \bar{C}_q^R + \mathcal{O}(m) + \mathcal{O}(\alpha_s^2)$$

This implies $\bar{C}_{q,g}^R(\mu \rightarrow \infty) \rightarrow 0$ (in the chiral limit)

Trace anomaly in perturbation theory

First choose a regularization scheme.

e.g., [dimensional regularization \(DR\)](#), [Pauli-Villars](#), etc.

Trace anomaly shows up by exploiting the pathologies of the chosen scheme.

→ The decomposition $T_{\mu}^{\mu} = (T_g)_{\mu}^{\mu} + (T_q)_{\mu}^{\mu}$ is **scheme dependent**.

In the following, I consider only DR.

In $4 - 2\epsilon$ dimensions,

$$T_{\mu}^{\mu} = -2\epsilon \frac{F^2}{4} + m\bar{q}q$$


from $(T_g)_{\mu}^{\mu}$

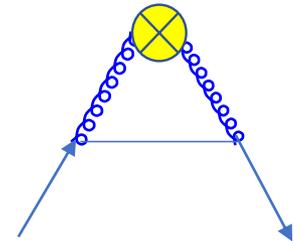
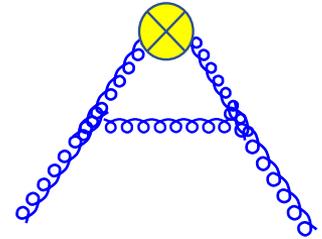
from $(T_q)_{\mu}^{\mu}$

Operator renormalization and mixing

The bare operator F^2 is divergent [c.f. Tarrach \(1982\)](#)

$$F^2 = \left(1 + \beta_0 \frac{\alpha_s}{4\pi\epsilon}\right) (F^2)_R - \frac{2\gamma_m^0}{\epsilon} (m\bar{q}q)_R + \dots$$

$$\begin{aligned}
 T_\mu^\mu &= -2\epsilon \frac{F^2}{4} + m\bar{q}q \\
 &= \underbrace{\frac{\beta(g)}{2g} (F^2)_R + \gamma_m (m\bar{q}q)_R}_{\text{from } (T_g)^\mu_\mu} + \underbrace{(m\bar{q}q)_R}_{\text{from } (T_q)^\mu_\mu}
 \end{aligned}$$



For the **bare** EMT, the anomaly entirely comes from the gluon part $T_g^{\mu\nu}$

Renormalized trace $(T_{q,g}^R)_\alpha^\alpha$: my three mistakes (1/3)

For the **bare** operators,

$$(T_q)_\mu^\mu = (m\bar{q}q)_R = m\bar{q}q$$

$$(T_g)_\mu^\mu = \frac{\beta}{2g}(F^2)_R + \gamma_m(m\bar{q}q)_m = \frac{\beta}{2g}F^2 + \gamma_m m\bar{q}q$$

RG-invariant in DR

Does this mean

$$(T_q^R(\mu))_\alpha^\alpha = (T_q)_\alpha^\alpha$$
$$(T_g^R(\mu))_\alpha^\alpha = (T_g)_\alpha^\alpha$$

??

Renormalized trace $(T_{q,g}^R)_\alpha^\alpha$: my three mistakes (2/3)

Write down the renormalized operators in dimensional regularization.

$$T_{gR}^{\mu\nu} = -(F^{\mu\lambda} F^\nu{}_\lambda)_R + \frac{\eta^{\mu\nu}}{4} (F^2)_R$$

$$T_{qR}^{\mu\nu} = i(\bar{\psi}\gamma^\mu D^\nu\psi)_R$$

$(F^2)_R$ is now a finite operator. $d \rightarrow 4$ limit can be safely taken

$$(T_g^R)_\mu^\mu = 0$$

$$(T_q^R)_\mu^\mu = (m\bar{\psi}\psi)_R$$

??

Renormalized trace $(T_{q,g}^R)_\alpha^\alpha$: my three mistakes (3/3)

By definition, $\langle P | (T_{q,g}^R)_\mu^\mu | P \rangle = 2M^2 (A_{q,g}^R + 4\bar{C}_{q,g}^R)$

Consider the $\mu \rightarrow \infty$ limit of this.

From Peskin's Eq.(18.193), $A_q^R(\infty) = \frac{3n_f}{16+3n_f}$ $A_g^R(\infty) = \frac{16}{16+3n_f}$

According to [Polyakov, Son \(2018\)](#), $\bar{C}_{q,g}(\infty) = 0$ in the chiral limit

Therefore, in the chiral limit,

$$(T_q)_\alpha^\alpha = (T_q^R(\infty))_\alpha^\alpha \sim \frac{3n_f}{16+3n_f}$$

$$(T_g)_\alpha^\alpha = (T_g^R(\infty))_\alpha^\alpha \sim \frac{16}{16+3n_f}$$

??

Renormalized trace $(T_{q,g}^R)_\alpha$: the correct result

Choose the basis of operators

$$O_1 = -F^{\mu\lambda} F^\nu{}_\lambda,$$

$$O_2 = \eta^{\mu\nu} F^2,$$

$$O_3 = i\bar{\psi}\gamma^{(\mu}\overleftrightarrow{D}^{\nu)}\psi,$$

$$O_4 = \eta^{\mu\nu} m\bar{\psi}\psi.$$

$$T^{\mu\nu} = O_1 + \frac{O_2}{4} + O_3.$$

Mixing under renormalization

$$O_1^R = Z_T O_1 + Z_M O_2 + Z_L O_3 + Z_S O_4,$$

$$O_2^R = Z_F O_2 + Z_C O_4,$$

$$O_3^R = Z_\psi O_3 + Z_K O_4 + Z_Q O_1 + Z_B O_2,$$

$$O_4^R = O_4.$$

Impose two conditions. **First condition** is simply $T^{\mu\nu} = T_R^{\mu\nu}$

Second condition:

O_1^R – (trace) and O_3^R – (trace) satisfy the twist-2, spin-2 DGLAP equation

$$\frac{\partial}{\partial \ln \mu} \begin{pmatrix} A_q^R \\ A_g^R \end{pmatrix} = \frac{\alpha_s}{4\pi} \begin{pmatrix} -\frac{16}{3}C_F & \frac{4n_f}{3} \\ \frac{16}{3}C_F & -\frac{4n_f}{3} \end{pmatrix} \begin{pmatrix} A_q^R \\ A_g^R \end{pmatrix},$$

$$\begin{aligned} O_1 &= -F^{\mu\lambda} F^\nu{}_\lambda, \\ O_2 &= \eta^{\mu\nu} F^2, \\ O_3 &= i\bar{\psi}\gamma^{(\mu}\overleftrightarrow{D}^{\nu)}\psi, \\ O_4 &= \eta^{\mu\nu} m\bar{\psi}\psi. \end{aligned}$$

Beware, in DR, trace operation and renormalization do **not** commute

$$\eta_{\mu\nu}(F^{\mu\lambda}F^\nu{}_\lambda)_R \neq (F^{\mu\lambda}F_{\mu\lambda})_R \quad \eta_{\mu\nu}(\bar{\psi}\gamma^\mu D^\nu\psi)_R \neq (\bar{\psi}\not{D}\psi)_R$$

Introduce more unknown constants

$$\eta_{\mu\nu}(F^{\mu\lambda}F^\nu{}_\lambda)_R = (1+a)(F^2)_R + b(m\bar{\psi}\psi)_R \quad a, b = O(\alpha_s)$$

Renormalization \rightarrow trace

$$\begin{aligned}\int \frac{d^d p}{(2\pi)^d} \frac{p^\mu p^\nu}{(p^2 + \Delta)^2} &= \frac{g^{\mu\nu}}{d} \int \frac{d^d p}{(2\pi)^d} \frac{p^2}{(p^2 + \Delta)^2} \\ &= \frac{g^{\mu\nu}}{2(4\pi)^2} \left(\frac{1}{\epsilon} - \gamma_E + \ln 4\pi - \ln \Delta \right) \Delta \\ &\xrightarrow{\overline{\text{MS}}} \frac{g^{\mu\nu}}{2(4\pi)^2} (-\ln \Delta) \Delta \\ &\xrightarrow{\text{trace}} \frac{4}{2(4\pi)^2} (-\ln \Delta) \Delta\end{aligned}$$

Trace \rightarrow Renormalization

$$\begin{aligned}\int \frac{d^d p}{(2\pi)^d} \frac{p^2}{(p^2 + \Delta)^2} &= \frac{d}{2(4\pi)^2} \left(\frac{1}{\epsilon} - \gamma_E + \ln 4\pi - \ln \Delta \right) \Delta \\ &= \frac{4}{2(4\pi)^2} \left(\frac{1}{\epsilon} - \frac{1}{2} - \gamma_E + \ln 4\pi - \ln \Delta \right) \Delta \\ &\xrightarrow{\overline{\text{MS}}} \frac{4}{2(4\pi)^2} \left(-\frac{1}{2} - \ln \Delta \right) \Delta\end{aligned}$$

Result in $\overline{\text{MS}}$ at one-loop

YH, Rajan, Tanaka, JHEP 1812 (2018) 008

$$\eta_{\mu\nu} T_{gR}^{\mu\nu} = \frac{\alpha_s}{4\pi} \left(-\frac{11C_A}{6} (F^2)_R + \frac{14C_F}{3} (m\bar{\psi}\psi)_R \right),$$

$$\eta_{\mu\nu} T_{qR}^{\mu\nu} = (m\bar{\psi}\psi)_R + \frac{\alpha_s}{4\pi} \left(\frac{n_f}{3} (F^2)_R + \frac{4C_F}{3} (m\bar{\psi}\psi)_R \right)$$

 n_f term in the 1-loop beta function

$$\lim_{\mu \rightarrow \infty} (T_q^R(\mu))_\alpha^\alpha \neq (T_q)_\alpha^\alpha$$

$$\lim_{\mu \rightarrow \infty} (T_g^R(\mu))_\alpha^\alpha \neq (T_g)_\alpha^\alpha$$

Finite renormalization

Result in $\overline{\text{MS}}$ at two-loops

YH, Rajan, Tanaka, JHEP 1812 (2018) 008

$$\eta_{\mu\nu} (T_g^{\mu\nu})_R = \frac{\alpha_s}{4\pi} \left(\frac{14}{3} C_F (m\bar{\psi}\psi)_R - \frac{11}{6} C_A (F^2)_R \right) + \left(\frac{\alpha_s}{4\pi} \right)^2$$

$$\times \left[\left(C_F \left(\frac{812C_A}{27} - \frac{22n_f}{27} \right) + \frac{85C_F^2}{27} \right) (m\bar{\psi}\psi)_R + \left(\frac{28C_A n_f}{27} - \frac{17C_A^2}{3} + \frac{5C_F n_f}{54} \right) (F^2)_R \right]$$

n_f terms contribute to
the gluon part

$$\eta_{\mu\nu} (T_q^{\mu\nu})_R = (m\bar{\psi}\psi)_R + \frac{\alpha_s}{4\pi} \left(\frac{4}{3} C_F (m\bar{\psi}\psi)_R + \frac{1}{3} n_f (F^2)_R \right) + \left(\frac{\alpha_s}{4\pi} \right)^2$$

$$\times \left[\left(C_F \left(\frac{61C_A}{27} - \frac{68n_f}{27} \right) - \frac{4C_F^2}{27} \right) (m\bar{\psi}\psi)_R + \left(\frac{17C_A n_f}{27} + \frac{49C_F n_f}{54} \right) (F^2)_R \right]$$

Result in $\overline{\text{MS}}$ at three-loops

Tanaka, JHEP 1901 (2019) 120

$$\begin{aligned}
 \eta_{\mu\nu} (T_g^{\mu\nu})_R &= \frac{\alpha_s}{4\pi} \left(\frac{14}{3} C_F (m\bar{\psi}\psi)_R - \frac{11}{6} C_A (F^2)_R \right) + \left(\frac{\alpha_s}{4\pi} \right)^2 \\
 &\quad \times \left[\left(C_F \left(\frac{812C_A}{27} - \frac{22n_f}{27} \right) + \frac{85C_F^2}{27} \right) (m\bar{\psi}\psi)_R + \left(\frac{28C_A n_f}{27} - \frac{17C_A^2}{3} + \frac{5C_F n_f}{54} \right) (F^2)_R \right] \\
 &+ \left(\frac{\alpha_s}{4\pi} \right)^3 \left[\left\{ n_f \left(\left(\frac{368\zeta(3)}{9} - \frac{25229}{729} \right) C_F^2 - \frac{2}{243} (4968\zeta(3) + 1423) C_A C_F \right) \right. \right. \\
 &+ \left(\frac{32\zeta(3)}{3} - \frac{91753}{1458} \right) C_A C_F^2 + \left(\frac{294929}{1458} - \frac{32\zeta(3)}{9} \right) C_A^2 C_F - \frac{554}{243} C_F n_f^2 \\
 &+ \left. \left(\frac{95041}{729} - \frac{64\zeta(3)}{9} \right) C_F^3 \right\} (m\bar{\psi}\psi)_R \\
 &+ \left. \left\{ n_f \left(\left(\frac{1123}{162} - \frac{52\zeta(3)}{9} \right) C_A C_F + \left(4\zeta(3) + \frac{293}{36} \right) C_A^2 + \frac{16}{729} (81\zeta(3) - 98) C_F^2 \right) + n_f^2 \left(\frac{655C_A}{2916} - \frac{361C_F}{729} \right) - \frac{2857C_A^3}{108} \right\} (F^2)_R \right]
 \end{aligned}$$

$$\begin{aligned}
 \eta_{\mu\nu} (T_q^{\mu\nu})_R &= (m\bar{\psi}\psi)_R + \frac{\alpha_s}{4\pi} \left(\frac{4}{3} C_F (m\bar{\psi}\psi)_R + \frac{1}{3} n_f (F^2)_R \right) + \left(\frac{\alpha_s}{4\pi} \right)^2 \\
 &\quad \times \left[\left(C_F \left(\frac{61C_A}{27} - \frac{68n_f}{27} \right) - \frac{4C_F^2}{27} \right) (m\bar{\psi}\psi)_R + \left(\frac{17C_A n_f}{27} + \frac{49C_F n_f}{54} \right) (F^2)_R \right] \\
 &+ \left(\frac{\alpha_s}{4\pi} \right)^3 \left[\left\{ n_f \left(\left(\frac{64\zeta(3)}{9} - \frac{8305}{729} \right) C_F^2 - \frac{2}{243} (864\zeta(3) + 1079) C_A C_F \right) \right. \right. \\
 &- \frac{8}{729} (972\zeta(3) + 143) C_A C_F^2 + \left(\frac{32\zeta(3)}{9} + \frac{6611}{729} \right) C_A^2 C_F - \frac{76}{243} C_F n_f^2 \\
 &+ \left. \frac{8}{729} (648\zeta(3) - 125) C_F^3 \right\} (m\bar{\psi}\psi)_R \\
 &+ \left\{ n_f \left(\left(\frac{52\zeta(3)}{9} - \frac{401}{324} \right) C_A C_F + \left(\frac{134}{27} - 4\zeta(3) \right) C_A^2 + \left(\frac{2407}{1458} - \frac{16\zeta(3)}{9} \right) C_F^2 \right) \right. \\
 &+ \left. n_f^2 \left(-\frac{697C_A}{729} - \frac{169C_F}{1458} \right) \right\} (F^2)_R,
 \end{aligned}$$

Renormalization of $\bar{C}_{q,g}$ revisited

Return to

$$\frac{\partial}{\partial \ln \mu} \bar{C}_q^R = -\frac{\alpha_s}{4\pi} \left(\frac{16}{3} C_F + \frac{4n_f}{3} \right) \bar{C}_q^R + \mathcal{O}(m) + \mathcal{O}(\alpha_s^2)$$

Correct result at $\mathcal{O}(\alpha_s)$

$$\begin{aligned} \frac{\partial \bar{C}_q^R}{\partial \ln \mu} = & -\frac{\alpha_s}{4\pi} \left(\frac{16C_F}{3} + \frac{4n_f}{3} \right) \bar{C}_q^R \\ & + \frac{\alpha_s}{4\pi} \left[\frac{4C_F}{3} \frac{\langle P | (m\bar{\psi}\psi)_R | P \rangle}{2M^2} + \frac{n_f}{3} \left(\frac{\langle P | (m\bar{\psi}\psi)_R | P \rangle}{2M^2} - 1 \right) \right] \end{aligned}$$

\uparrow
 $\mathcal{O}(m)$

\uparrow
 $\mathcal{O}(\alpha_s^2)$, but promoted to $\mathcal{O}(\alpha_s)$
due to trace anomaly! $\alpha_s F^2 \sim \mathcal{O}(1)$

$$\begin{aligned}
\bar{C}_q^R(\mu) = & -\frac{1}{4} \left(\frac{n_f}{4C_F + n_f} + \frac{2n_f}{3\beta_0} \right) + \frac{1}{4} \left(\frac{2n_f}{3\beta_0} + 1 \right) \frac{\langle P | (m\bar{\psi}\psi)_R | P \rangle}{2M^2} \\
& - \frac{4C_F A_q^R(\mu_0) + n_f (A_q^R(\mu_0) - 1)}{4(4C_F + n_f)} \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\frac{8C_F + 2n_f}{3\beta_0}} \\
& + \frac{\alpha_s(\mu)}{4\pi} \left[\frac{n_f \left(-\frac{34C_A}{27} - \frac{49C_F}{27} \right)}{4\beta_0} + \frac{\beta_1 n_f}{6\beta_0^2} \right. \\
& \left. + \frac{1}{4} \left(\frac{n_f \left(\frac{34C_A}{27} + \frac{157C_F}{27} \right)}{\beta_0} + \frac{4C_F}{3} - \frac{2\beta_1 n_f}{3\beta_0^2} \right) \frac{\langle P | (m\bar{\psi}\psi)_R | P \rangle}{2M^2} \right] + \dots, \\
\approx & -0.146 - 0.25 (A_q^R(\mu_0) - 0.36) \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\frac{50}{81}} - 0.01\alpha_s(\mu) \\
& + (0.306 + 0.08\alpha_s(\mu)) \frac{\langle P | (m\bar{\psi}\psi)_R | P \rangle}{2M^2},
\end{aligned}$$

Asymptotic value
in the chiral limit

($n_f = 3$)

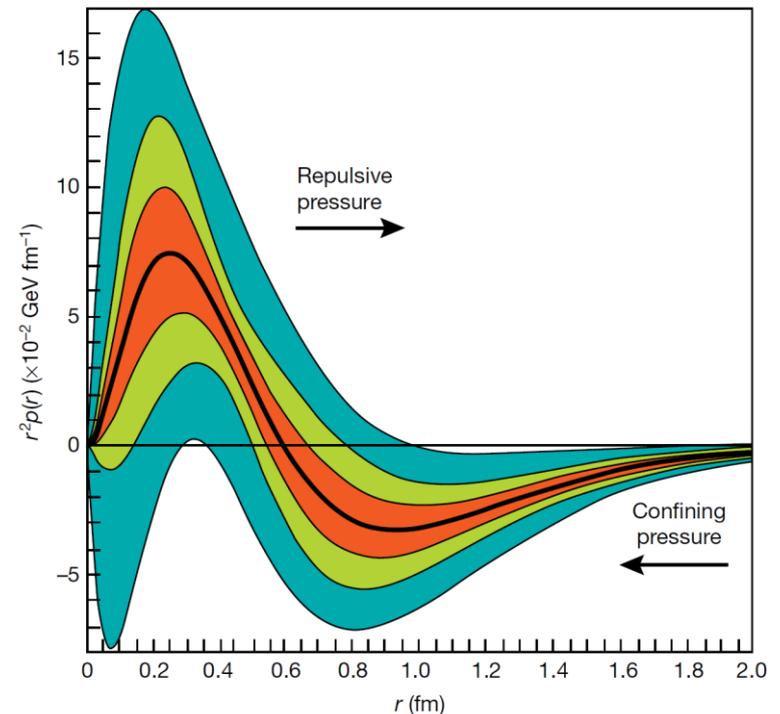
cf. $\bar{C}_q^R(\mu \sim 0) \approx 0.014$ from instantons [Polyakov, Son \(2018\)](#)

Pressure inside the proton from quark and gluon subsystems

$$p_{q,g}(r) = \frac{1}{6Mr^2} \frac{d}{dr} r^2 \frac{d}{dr} D_{q,g}(r) - M\bar{C}_{q,g}(r)$$

$$\bar{C}_q = -\bar{C}_g < 0$$

Discussed in [Polyakov, Son \(2018\)](#), but should be revisited since our result is 10 times larger and has an opposite sign.



Hadron mass decomposition

From the three-loop formula,

$$\eta_{\lambda\nu} (T_g^{\lambda\nu})_R \Big|_{\mu=1 \text{ GeV}} = -0.285452 (F^2)_R \Big|_{\mu=1 \text{ GeV}} + 0.500593 (m\bar{\psi}\psi)_R ,$$
$$\eta_{\lambda\nu} (T_q^{\lambda\nu})_R \Big|_{\mu=1 \text{ GeV}} = 0.0531842 (F^2)_R \Big|_{\mu=1 \text{ GeV}} + 1.05832 (m\bar{\psi}\psi)_R .$$

$$m_\pi^2 = \underbrace{0.521 m_\pi^2}_{\text{from gluons}} + \underbrace{0.479 m_\pi^2}_{\text{from quarks}} \quad \text{Tanaka (2018)}$$

For the proton, the quark contribution is negative

$$m_p^2 \approx 1.1 m_p^2 - 0.1 m_p^2$$

Can we measure the gluon condensate $\langle P|F^{\mu\nu}F_{\mu\nu}|P\rangle$?

The operator $F^{\mu\nu}F_{\mu\nu}$ is twist-**four**,
highly suppressed in high energy scattering.

Purely gluonic operator, very difficult to compute in lattice QCD

Instead, we should look at **low**-energy scattering.

Purely gluonic operator. Use **quarkonium** as a probe.

Luke-Manohar-Savage (1992)

→ J/ψ photo-production near threshold.

Kharzeev, Satz, Syamtomov, Zinovjev (1998)

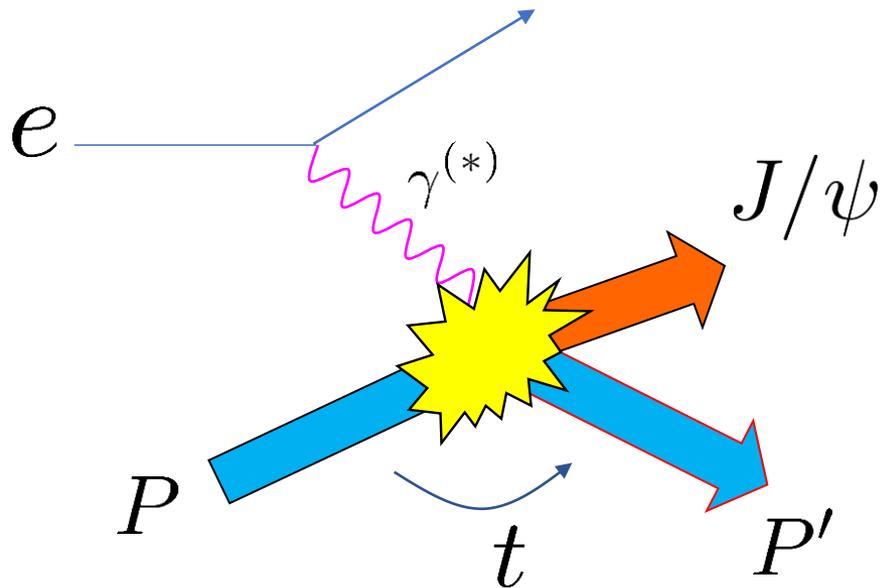
Photo-production of J/ψ near threshold

Kharzeev, Satz, Syamtomov, Zinovjev (1998)

Brodsky, Chudakov, Hoyer, Laget (2000)

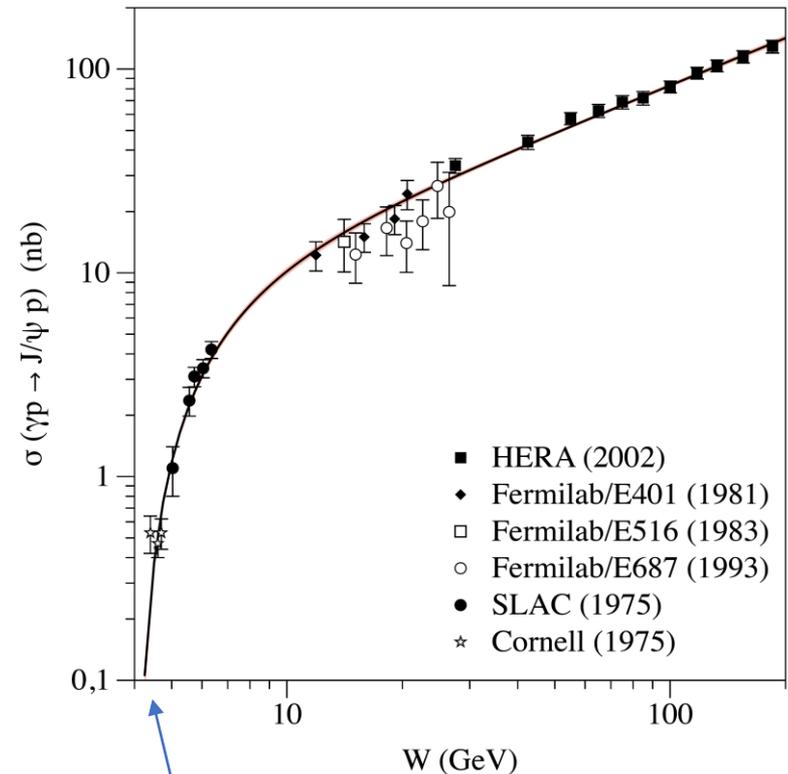
Sensitive to the **non-forward**
matrix element

$$\langle P' | F^{\mu\nu} F_{\mu\nu} | P \rangle$$



Straightforward to measure.
Ongoing experiments at Jlab.

Difficult to compute from first principles
(need nonperturbative approaches)



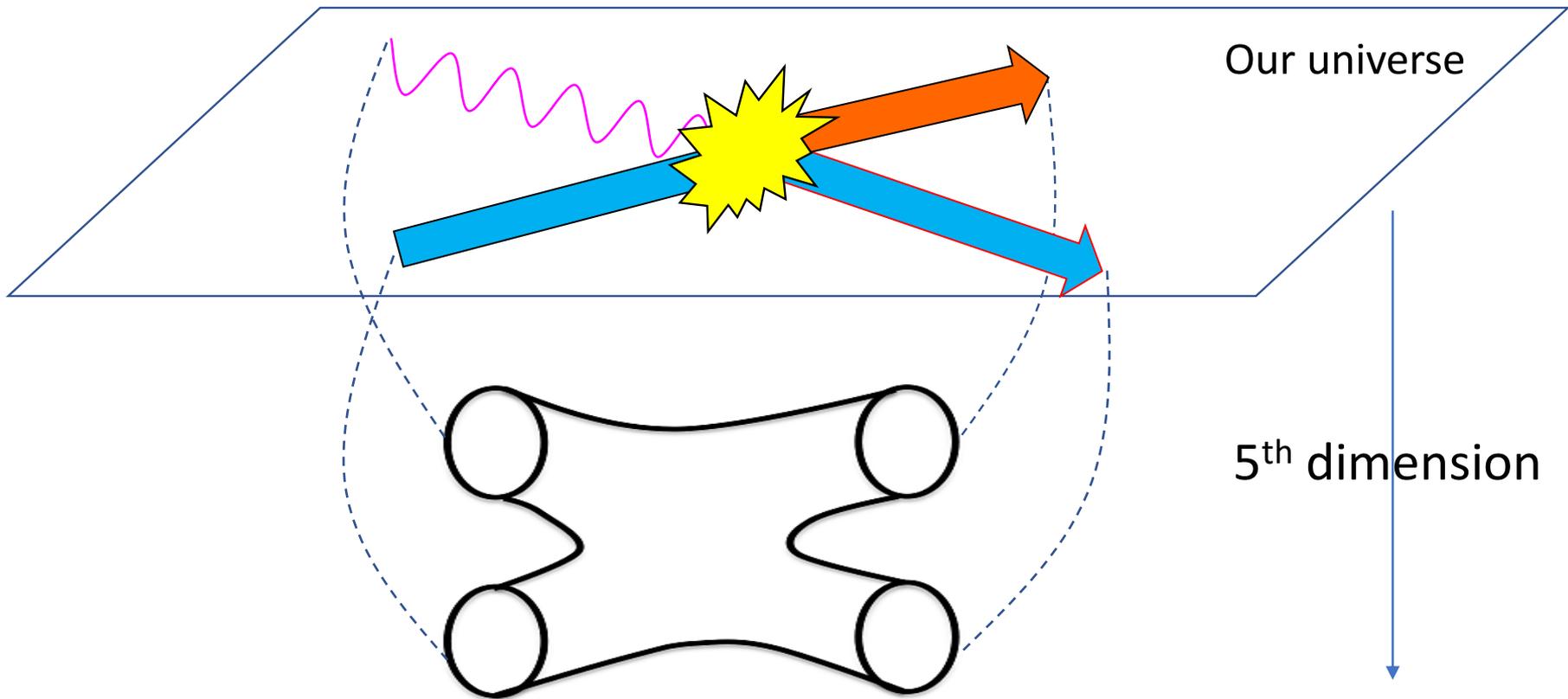
$$W_{th} \approx 4.04 \text{ GeV}$$

Holographic approach

YH, Yang (2018)

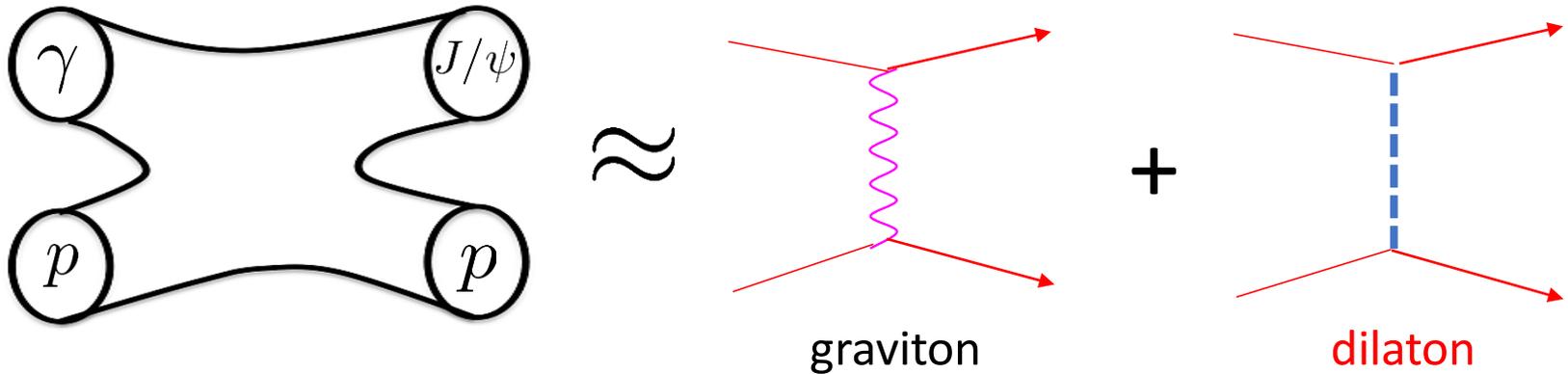
Need nonperturbative methods, e.g., AdS/CFT

QCD amplitude \approx string amplitude in asymptotically AdS_5 .



The dilaton

The operator $F^{\mu\nu} F_{\mu\nu}$ is dual to a massless string called **dilaton**



Suppressed compared to graviton exchange at high energy, but **not** at very low energy!

$$\langle P | \epsilon \cdot J(0) | P' k \rangle \approx -\frac{2\kappa^2}{f_\psi R^3} \int_0^{z_m} dz \frac{\delta S_{D7}(q, k, z)}{\delta g_{\mu\nu}} \frac{z^2 R^2}{4} \langle P | T_{\mu\nu}^{gTT} | P' \rangle$$

$$+ \frac{2\kappa^2}{f_\psi R^3} \frac{3}{8} \int_0^{z_m} dz \frac{\delta S_{D7}(q, k, z)}{\delta \phi} \frac{z^4}{4} \langle P | \frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a | P' \rangle$$

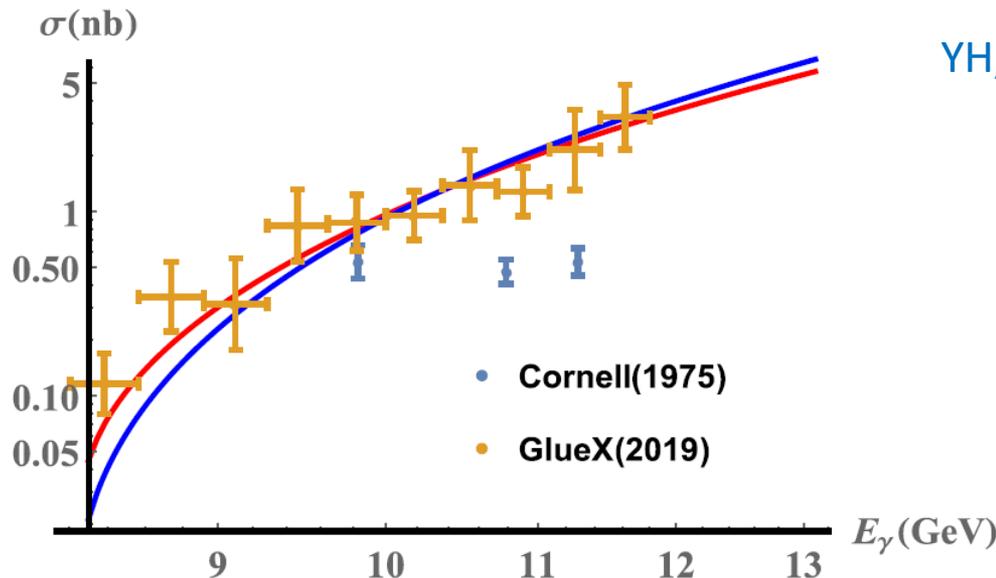
Gluon condensate (nonforward version)

Fitting the latest Jlab data

GlueX collaboration 1905.10811

$$M_m = \frac{1}{4} \frac{\langle P | m(1 + \gamma_m) \bar{\psi} \psi | P \rangle}{2M} \equiv \frac{b}{4} M$$

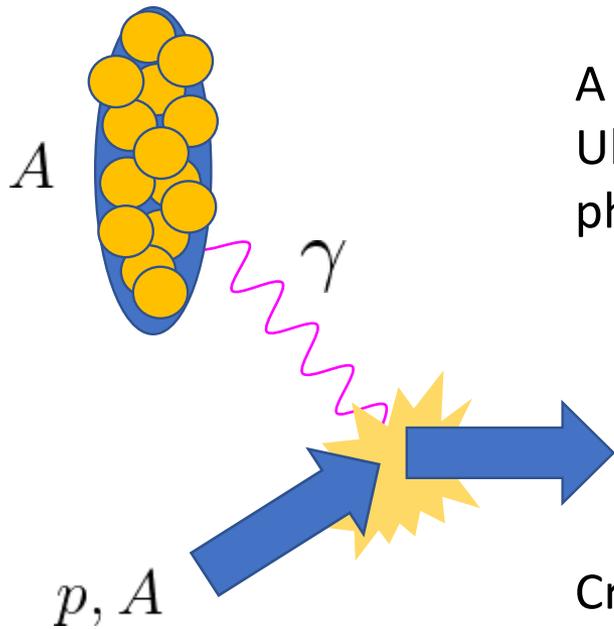
$$M_a = \frac{1}{4} \frac{\langle P | \frac{\beta}{2g} F^2 | P \rangle}{2M} \equiv \frac{1-b}{4} M$$



Small-b favored, b=1 disfavored

→ Gluon condensate important for the proton mass!

Ultraperipheral collisions at RHIC



A heavy ion is an abundant source of real photons. Ultraperipheral collisions (UPC) closely mimic the photoproduction limit of DIS.

Cross section enhanced by $Z^2 = 6241$ (for gold nucleus)
Can study Υ photo-production

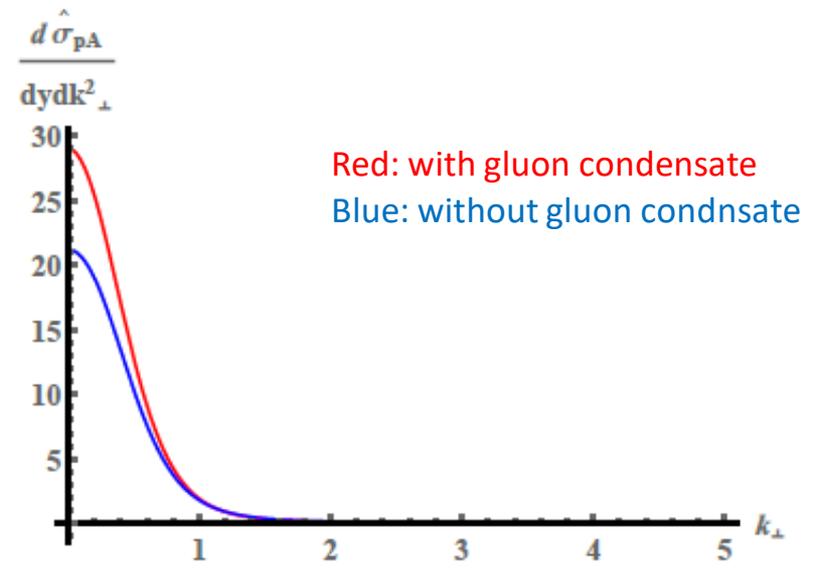
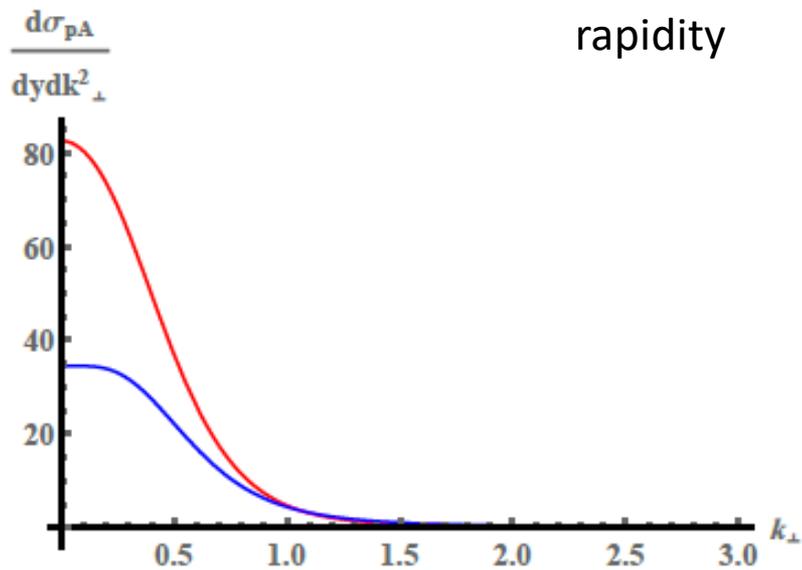
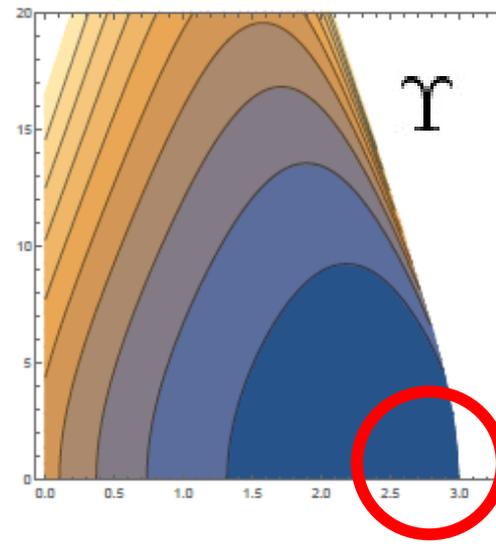
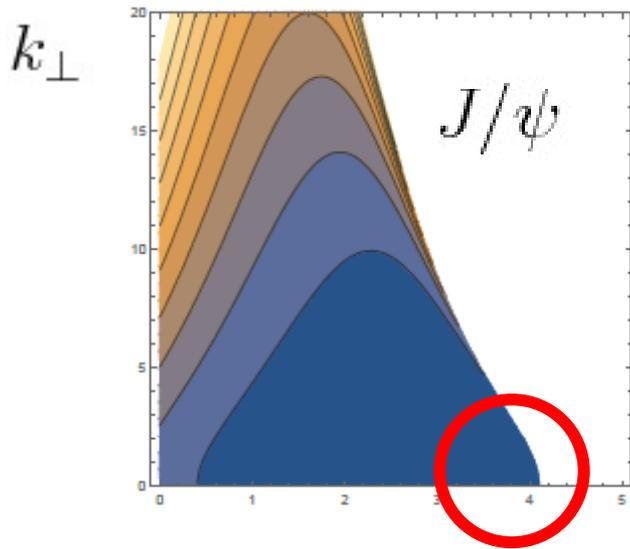
Typical photon energy at 200GeV runs $\omega = 2 \sim 3 \text{ GeV}$, $\sqrt{(P+q)^2} \sim 28 \text{ GeV}$

Enough to produce Υ , but they are produced in the **forward** rapidity region

→ Measurable after the completion of **STAR forward upgrades**.

Near threshold production in UPC pA collisions at RHIC

YH, Rajan, Yang, 1906.00894



Conclusion

- Understanding the proton's mass, spin, pressure etc. from partonic degrees' of freedom is a major goal of the EIC.
- New developments in QCD trace anomaly.
Connection to experiments at Jlab, RHIC, EIC.