QCD in extreme conditions on the lattice

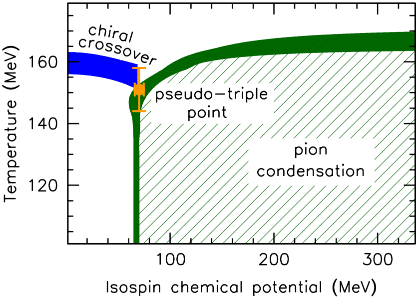
Gergely Endrődi

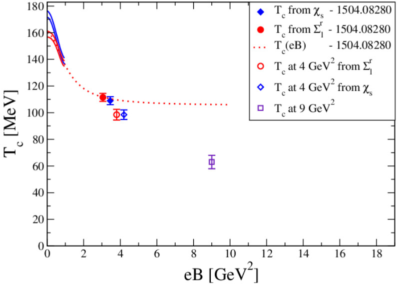
University of Bielefeld

18th International Conference on QCD in Extreme Conditions
NTNU Trondheim, July 27, 2022

Appetizer

fundamental phase diagrams of QCD
with possible phenomenological implications





Brandt, Endrődi, Schmalzbauer '18

Brandt, Endrődi '19

D'Elia, Maio, Sanfilippo, Stanzione '21

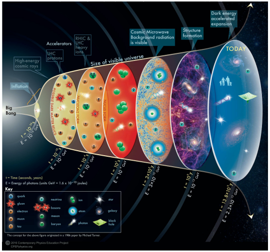
Outline

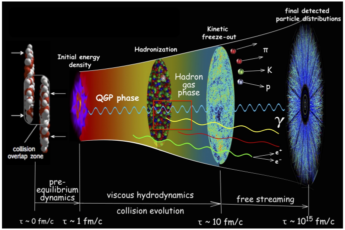
- introduction: strongly interacting matter in
  - strong electromagnetic fields
  - nonzero isospin density
- lattice simulation techniques
- phase diagram: current status
- application: cosmic trajectory
- beyond constant magnetic fields
- summary

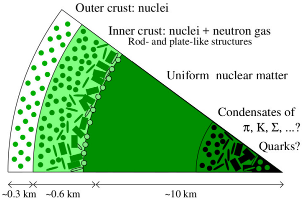
Introduction

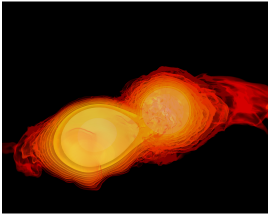
Extreme environments

- hot and/or dense strongly interacting matter in
  - QCD epoch of early Universe
  - heavy-ion collisions
  - neutron stars and their mergers



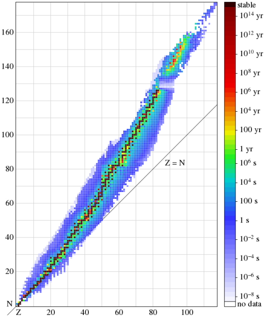


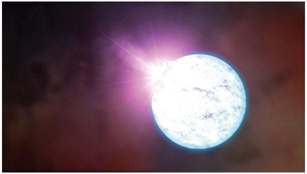




Isospin asymmetry: nuclei and neutron stars

- isospin asymmetry: $n_I \propto n_u - n_d$
  creates p^+-n asymmetry, excites π^+

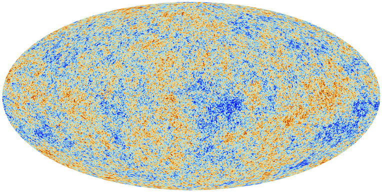




- proton to nucleon ratio in nuclei $\frac{Z}{A} \approx 0.4$
  but: 'neutron skin' near surface
- proton to nucleon ratio in interior of neutron stars $\frac{Z}{A} \approx 0.025$

Isospin asymmetry: cosmology

- early Universe characterized by charge neutrality $n_Q = 0$,
  (almost perfect) baryon symmetry $n_B = 0$
  but lepton number n_L only weakly constrained by observations
  Oldengott, Schwarz '17



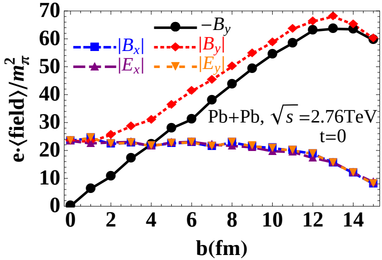
- weak equilibrium

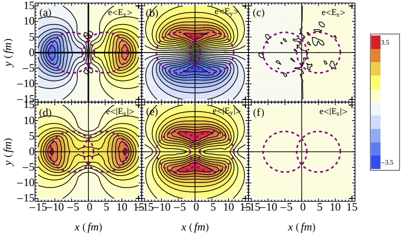
$$d \leftrightarrow u\, e^- \,\bar{\nu}_e$$

large $n_L \leftrightarrow$ large isospin asymmetry Abuki, Brauner, Warringa '09

Electromagnetic fields: heavy ion collisions

- electromagnetic fields in the early stage of heavy-ion collisions reaching m_π^2 and well beyond
  Deng et al. '12





- most probably short-lived fields Huang '15
- impact of electric field enhanced for asymmetric systems (for example Cu+Au at RHIC) Voronyuk et al. '14

Lattice simulations

Monte Carlo simulations

- Euclidean QCD path integral over gauge field $\mathcal{A}$

$$\mathcal{Z} = \int \mathcal{D}\mathcal{A}\, e^{-S_g[\mathcal{A}]} \det[\not{D}[\mathcal{A}] + m]$$

- Monte-Carlo simulations need: $\det[\not{D} + m] \in \mathbb{R}^+$
  for that one needs Γ so that

$$\Gamma \not{D}\, \Gamma^\dagger = \not{D}^\dagger, \quad \Gamma^\dagger \Gamma = 1$$

$$\det[\not{D}+m] = \det[\Gamma^\dagger \Gamma(\not{D}+m)] = \det[\Gamma(\not{D}+m)\Gamma^\dagger] = \det[\not{D}^\dagger+m] = \det[\not{D}+m]^*$$

- usually positivity can also be shown
- such a Γ exists: no complex action problem

Complex actions vs. real actions

- two-flavor Dirac operator ($\mathcal{A}_\mu$: $\mathrm{SU}(3)$ field, A_μ: $\mathrm{U}(1)$ field) (remember Wick rotation $A_4 = -iA_0$)

$$\not{D}[\mathcal{A}, A] = \gamma_\mu(\partial_\mu + i\mathcal{A}_\mu)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + i\gamma_\mu\begin{pmatrix}A^u_\mu & 0\\ 0 & A^d_\mu\end{pmatrix}$$

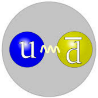
- pure QCD: $A_\mu = 0$ — $\Gamma = \gamma_5$ ✓
- magnetic field: $A^f_2 = q_f B x_1$ — $\Gamma = \gamma_5$ ✓
- imaginary baryon chem. pot.: $A^u_4 = A^d_4 = \mu$ — $\Gamma = \gamma_5$ ✓
- imaginary electric field: $A^f_4 = q_f E x_1$ — $\Gamma = \gamma_5$ ✓
- real baryon chem. pot.: $iA^u_4 = iA^d_4 = \mu$ — ↯
- real electric field: $iA^f_4 = q_f E x_1$ — ↯
- real isospin chem. pot.: $iA^u_4 = -iA^d_4 = \mu_I$ — $\Gamma = \gamma_5\tau_1$ ✓

Son, Stephanov '01

Pion condensation

Pion condensation

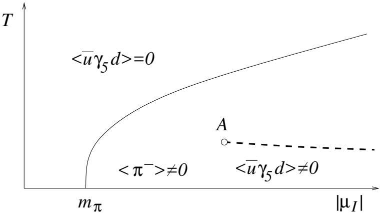
- isospin chemical potential: $\mu_u = \mu_I$, $\mu_d = -\mu_I$, $\mu_s = 0$
- QCD at low energies $\approx$ pions
  chiral perturbation theory



- charged pion chemical potential: $\mu_\pi = 2\mu_I$

at zero temperature $\mu_\pi < m_\pi$ vacuum state
$\mu_\pi \geq m_\pi$ Bose-Einstein condensation

Son, Stephanov '00



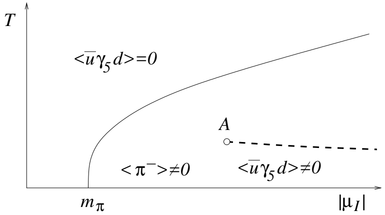
Pion condensation

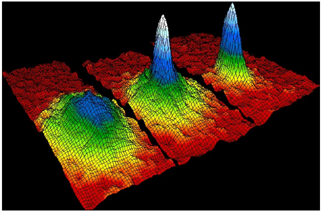
- isospin chemical potential: $\mu_u = \mu_I$, $\mu_d = -\mu_I$, $\mu_s = 0$
- QCD at low energies $\approx$ pions
  chiral perturbation theory
- charged pion chemical potential: $\mu_\pi = 2\mu_I$

at zero temperature $\mu_\pi < m_\pi$ vacuum state

$\mu_\pi \geq m_\pi$ Bose-Einstein condensation

Son, Stephanov '00





trapped Rb atoms at low temperature

Anderson et al '95 JILA-NIST/University of Colorado

Symmetry breaking

- QCD with light quark matrix

$$M = \not{D} + m_{ud}\mathbb{1}$$

- chiral symmetry (flavor-nontrivial)

$$\mathrm{SU}(2)_V$$

Symmetry breaking

- QCD with light quark matrix

$$M = \not{D} + m_{ud}\mathbb{1} + \mu_I \gamma_0 \tau_3$$

- chiral symmetry (flavor-nontrivial)

$$\mathrm{SU}(2)_V \to \mathrm{U}(1)_{\tau_3}$$

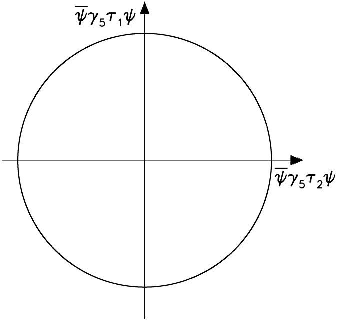
Symmetry breaking

- QCD with light quark matrix

$$M = \not{D} + m_{ud}\mathbb{1} + \mu_I\gamma_0\tau_3$$

- chiral symmetry (flavor-nontrivial)

$$\mathrm{SU}(2)_V \to \mathrm{U}(1)_{\tau_3}$$



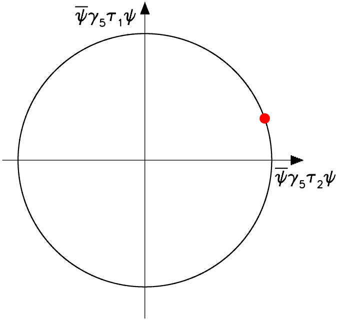
Symmetry breaking

- QCD with light quark matrix

$$M = \not{D} + m_{ud}\mathbb{1} + \mu_I \gamma_0 \tau_3$$

- chiral symmetry (flavor-nontrivial)

$$\mathrm{SU}(2)_V \to \mathrm{U}(1)_{\tau_3}$$



- spontaneously broken by a pion condensate

$$\left\langle \bar\psi\gamma_5\tau_{1,2}\psi \right\rangle = \left\langle \bar u\gamma_5 d \pm \bar d\gamma_5 u \right\rangle$$

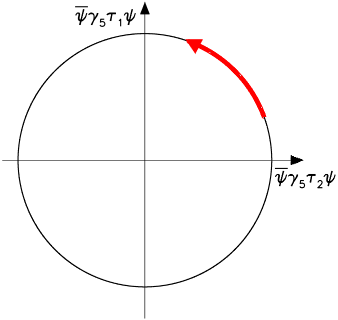
Symmetry breaking

- QCD with light quark matrix

$$M = \not{D} + m_{ud}\mathbb{1} + \mu_I \gamma_0 \tau_3$$

- chiral symmetry (flavor-nontrivial)

$$\mathrm{SU}(2)_V \to \mathrm{U}(1)_{\tau_3}$$



- spontaneously broken by a pion condensate

$$\left\langle \bar\psi\gamma_5\tau_{1,2}\psi \right\rangle = \left\langle \bar u\gamma_5 d \pm \bar d\gamma_5 u \right\rangle$$

- a Goldstone mode appears

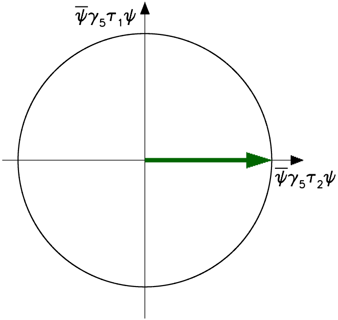
Symmetry breaking

- QCD with light quark matrix

$$M = \not{D} + m_{ud}\mathbb{1} + \mu_I \gamma_0 \tau_3 + i\lambda\gamma_5\tau_2$$

- chiral symmetry (flavor-nontrivial)

$$\mathrm{SU}(2)_V \to \mathrm{U}(1)_{\tau_3} \to \varnothing$$



- spontaneously broken by a pion condensate

$$\left\langle \bar{\psi}\gamma_5\tau_{1,2}\psi \right\rangle = \left\langle \bar{u}\gamma_5 d \pm \bar{d}\gamma_5 u \right\rangle$$

- a Goldstone mode appears
- add small explicit breaking

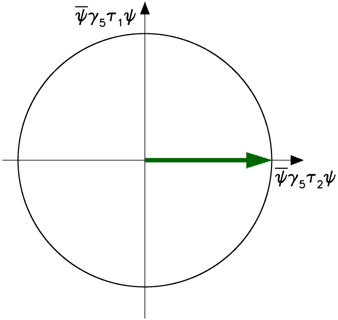
Symmetry breaking

- QCD with light quark matrix

$$M = \not{D} + m_{ud}\mathbb{1} + \mu_I \gamma_0 \tau_3 + i\lambda\gamma_5\tau_2$$

- chiral symmetry (flavor-nontrivial)

$$\mathrm{SU}(2)_V \to \mathrm{U}(1)_{\tau_3} \to \varnothing$$



- spontaneously broken by a pion condensate

$$\left\langle \bar{\psi}\gamma_5\tau_{1,2}\psi \right\rangle = \left\langle \bar{u}\gamma_5 d \pm \bar{d}\gamma_5 u \right\rangle$$

- a Goldstone mode appears
- add small explicit breaking

- extrapolate results $\lambda \to 0$

Dictionary

	pion condensation	vacuum chiral symmetry breaking
pattern	$\mathrm{U}(1)_{\tau_3} \to \varnothing$	$\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \to \mathrm{SU}(2)_V$
coset	$\mathrm{U}(1)$	$\mathrm{SU}(2)_A$
Goldstones	1	3
spontaneous	$\left\langle \bar{\psi}\gamma_5\tau_2\psi \right\rangle$	$\left\langle \bar{\psi}\psi \right\rangle$
explicit	$= \partial \log \mathcal{Z} / \partial\lambda$	$= \partial \log \mathcal{Z} / \partial m$
limit	$\lambda \to 0$	$m \to 0$

Dictionary

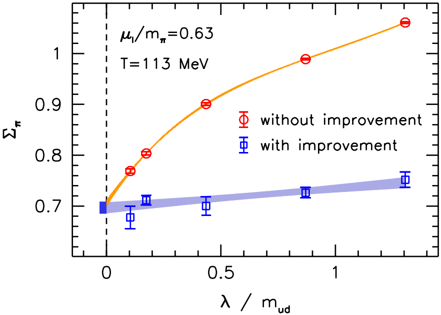
	pion condensation	vacuum chiral symmetry breaking
pattern	$\mathrm{U}(1)_{\tau_3} \to \varnothing$	$\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \to \mathrm{SU}(2)_V$
coset	$\mathrm{U}(1)$	$\mathrm{SU}(2)_A$
Goldstones	1	3
spontaneous	$\left\langle \bar{\psi}\gamma_5\tau_2\psi \right\rangle$	$\left\langle \bar{\psi}\psi \right\rangle$
explicit	$= \partial \log \mathcal{Z} / \partial\lambda$	$= \partial \log \mathcal{Z} / \partial m$
limit	$\lambda \to 0$	$m \to 0$

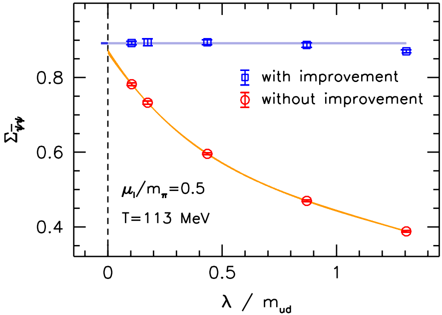
- long story short: pion condensation $1/3$ as challenging as the chiral limit of the QCD vacuum

Phase diagram: nonzero isospin

Extrapolation to $\lambda = 0$

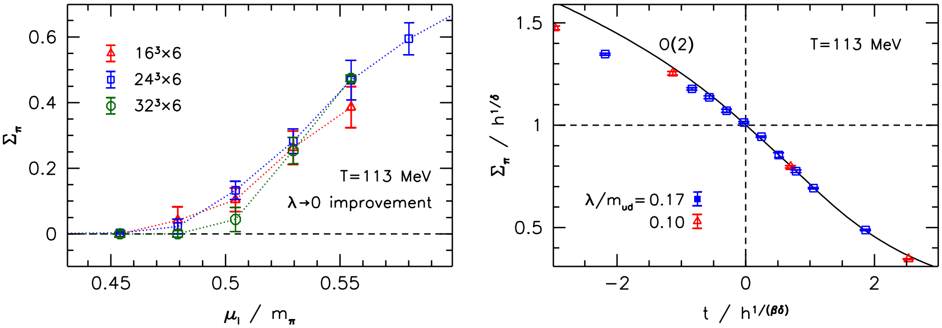
- improvement is crucial for reliable $\lambda \to 0$ extrapolation
  Brandt, Endrődi, Schmalzbauer '17 Brandt, Endrődi '19





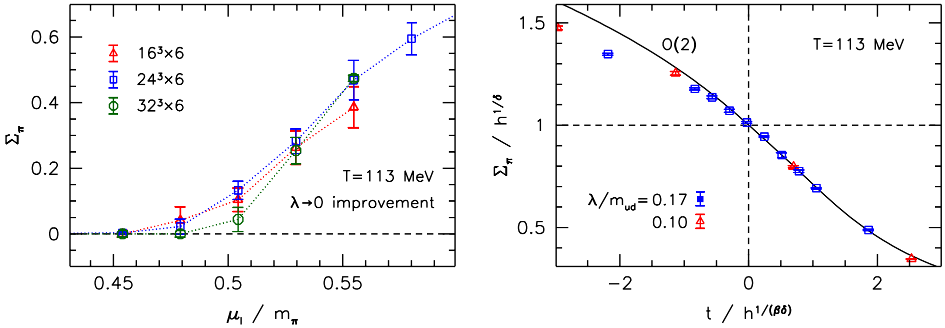
$$\Sigma_\pi = \frac{T}{V}\frac{\partial \log \mathcal{Z}}{\partial \lambda} \qquad \Sigma_{\bar{\psi}\psi} = \frac{T}{V}\frac{\partial \log \mathcal{Z}}{\partial m_{ud}}$$

Order of the transition



- volume scaling of order parameter shows typical sharpening
- collapse according to $\mathrm{O}(2)$ critical exponents Ejiri et al '09

Order of the transition

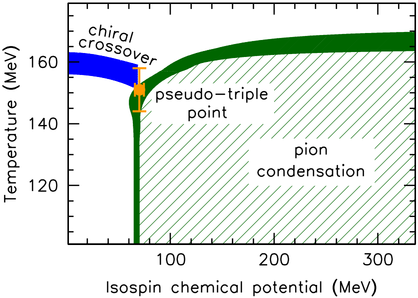


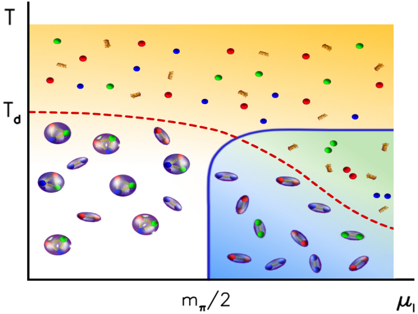
- volume scaling of order parameter shows typical sharpening
- collapse according to $\mathrm{O}(2)$ critical exponents Ejiri et al '09
- indications for a second order phase transition at $\mu_I = m_\pi/2$, in the $\mathrm{O}(2)$ universality class

Phase diagram

- phases in the $T - \mu_I$ phase diagram: hadronic (confined), quark-gluon plasma (deconfined), pion condensation (confined), BCS (deconfined)

  Brandt, Endrődi, Schmalzbauer '17 Brandt, Endrődi '19





- comparison to effective models, χPT, Q2CD, ...

  Adhikari et al. '18 Zhokhov et al. '19 Adhikari et al. '20

  Boz et al. '20 Astrakhantsev et al. '20

Equation of state: nonzero isospin

Equation of state

- equilibrium description of matter

$$\epsilon(p)$$

  relevant for:
  - neutron star physics (TOV equations)
  - cosmology, evolution of early Universe (Friedmann equation)
  - heavy-ion collision phenomenology (charge fluctuations)
- thermodynamic relations

$$p = \frac{T}{V}\log\mathcal{Z}, \qquad s = \frac{\partial p}{\partial T}, \qquad n_I = \frac{\partial p}{\partial \mu_I}, \qquad \epsilon = -p + Ts + \mu_I n_I$$

$$I = \epsilon - 3p, \qquad c_s^2 = \left.\frac{\partial p}{\partial \epsilon}\right|_{s/n_I}$$

Equation of state on the lattice

- integral method to calculate differences

$$n_I = \frac{T}{V}\frac{\partial \log \mathcal{Z}}{\partial \mu_I}, \qquad p(T,\mu_I) - p(T,0) = \int_0^{\mu_I} \mathrm{d}\mu_I'\, n_I(\mu_I')$$

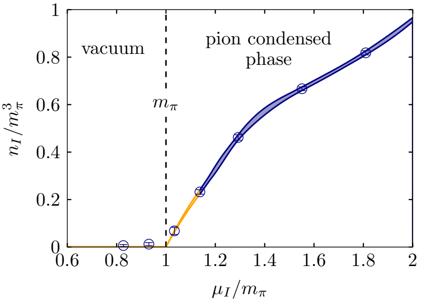
Equation of state on the lattice

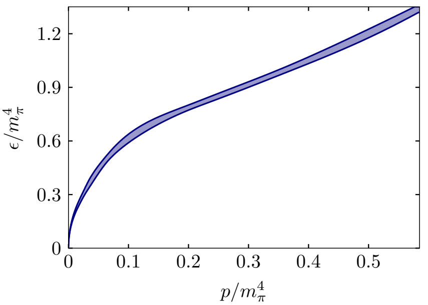
- integral method to calculate differences

$$n_I = \frac{T}{V}\frac{\partial \log \mathcal{Z}}{\partial \mu_I}, \qquad p(T,\mu_I) - p(T,0) = \int_0^{\mu_I} \mathrm{d}\mu_I' \, n_I(\mu_I')$$

- results at $T \approx 0$

  Brandt, Endrődi, Fraga, Hippert, Schaffner-Bielich, Schmalzbauer '18



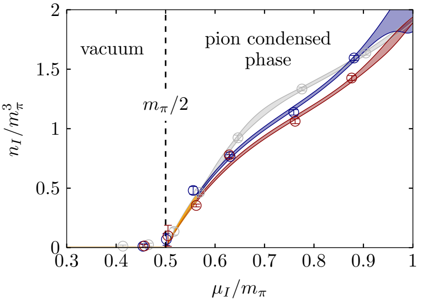


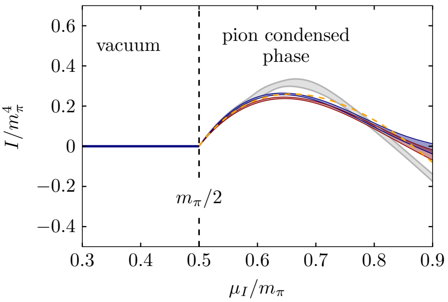
Equation of state on the lattice

- integral method to calculate differences

$$n_I = \frac{T}{V}\frac{\partial \log \mathcal{Z}}{\partial \mu_I}, \qquad p(T,\mu_I) - p(T,0) = \int_0^{\mu_I} \mathrm{d}\mu_I'\, n_I(\mu_I')$$

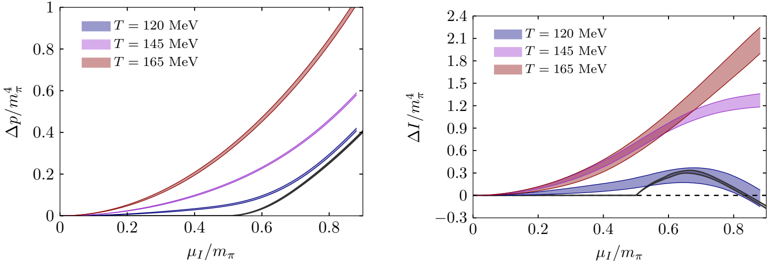
- results at $T \approx 0$
  - Brandt, Endrődi, Fraga, Hippert, Schaffner-Bielich, Schmalzbauer '18
  - Brandt, Cuteri, Endrődi, upcoming





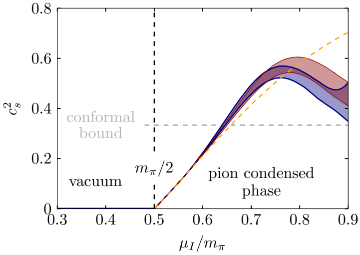
Equation of state on the lattice

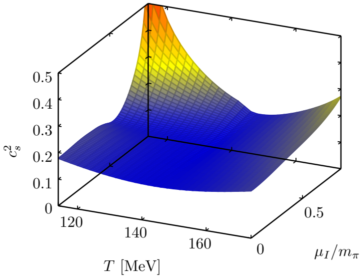
- results at $T \neq 0$ Brandt, Cuteri, Endrődi, upcoming
  Vovchenko, Brandt, Cuteri, Endrődi, Hajkarim, Schaffner-Bielich '20
- interaction measure negative at high μ_I, low T



Equation of state on the lattice

- results at $T \neq 0$ 🔗 Brandt, Cuteri, Endrődi, upcoming
  🔗 Vovchenko, Brandt, Cuteri, Endrődi, Hajkarim, Schaffner-Bielich '20
- interaction measure negative at high μ_I, low T
- speed of sound above $1/\sqrt{3}$ at high μ_I and intermediate T

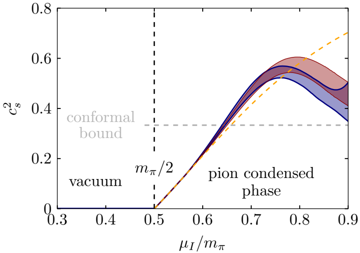


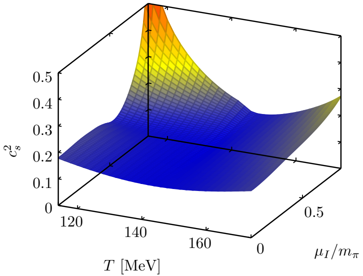


- EoS can get very stiff inside pion condensation phase

Equation of state on the lattice

- results at $T \neq 0$ 🔗 Brandt, Cuteri, Endrődi, upcoming
  🔗 Vovchenko, Brandt, Cuteri, Endrődi, Hajkarim, Schaffner-Bielich '20
- interaction measure negative at high μ_I, low T
- speed of sound above $1/\sqrt{3}$ at high μ_I and intermediate T



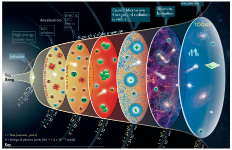


- EoS can get very stiff inside pion condensation phase
- comparison: χPT, models 🔗 Adhikari et al. '21 🔗 Avancini et al. '19

Cosmological implications

Cosmic trajectories

- early Universe



- conservation equations for isentropic expansion

$$\frac{n_B}{s} = b, \quad \frac{n_Q}{s} = 0, \quad \frac{n_{L_\alpha}}{s} = l_\alpha \quad (\alpha \in \{e, \mu, \tau\})$$

- parameters: $T, \quad \mu_B, \quad \mu_Q, \quad \mu_{L_\alpha}$
- experimental constraints Planck coll. '15 Oldengott, Schwarz '17

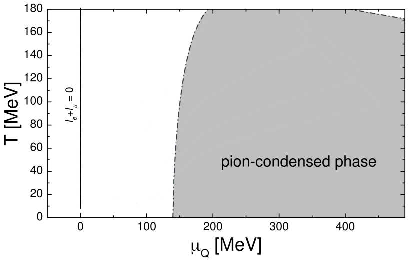
$$b = (8.60 \pm 0.06) \cdot 10^{-11}, \qquad \boxed{|l_e + l_\mu + l_\tau| < 0.012}$$

(the individual l_α may have opposite signs)

- $n_Q = 0$ with $l_e > 0$ allows equilibrium of e^-, ν_e, π^+

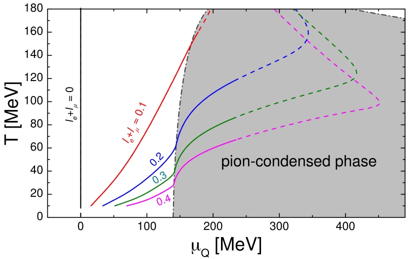
Cosmic trajectories

- cosmic trajectory $T(\mu_Q)$ is solved for
- standard scenario ($l_\alpha = 0$): $\mu_Q = 0$ for all T



Cosmic trajectories

- cosmic trajectory $T(\mu_Q)$ is solved for
- standard scenario ($l_\alpha = 0$): $\mu_Q = 0$ for all T



- cosmic trajectory enters BEC phase for lepton asymmetries allowed by observations

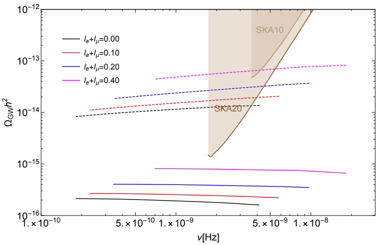
  🔗 Vovchenko, Brandt, Cuteri, Endrődi, Hajkarim, Schaffner-Bielich '20

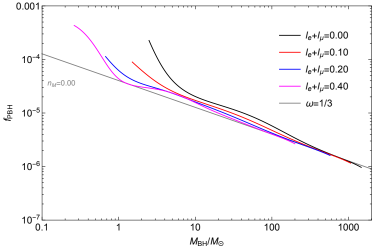
- condition for pion condensation to occur:

$$|l_e + l_\mu + l_\tau| < 0.012 \qquad |l_e + l_\mu| \gtrsim 0.1$$

Signatures of the condensed phase

- relic density of primordial gravitational waves is enhanced with respect to amplitude at $l_e + l_\mu = 0$
- fraction of primordial black holes with mass below one solar mass is enhanced





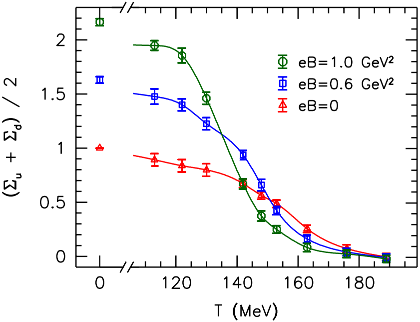
Vovchenko, Brandt, Cuteri, Endrődi, Hajkarim, Schaffner-Bielich '20

- to be detected experimentally (SKA)

Phase diagram: electromagnetic fields

Inverse catalysis and phase diagram

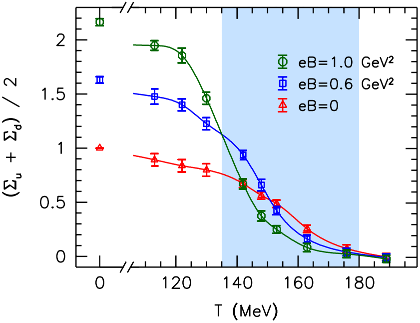
- physical m_π, staggered quarks, continuum limit
  Bali, Bruckmann, Endrődi, Fodor, Katz et al. '11 '12



- magnetic catalysis at low T (also at high T)

Inverse catalysis and phase diagram

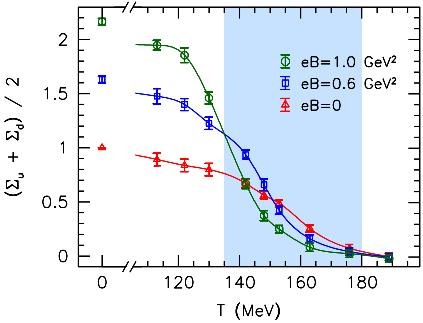
- physical m_π, staggered quarks, continuum limit
  Bali, Bruckmann, Endrődi, Fodor, Katz et al. '11 '12

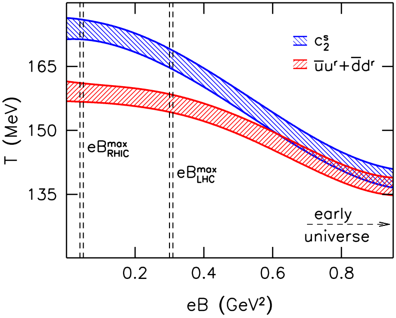


- magnetic catalysis at low T (also at high T)
- inverse magnetic catalysis (IMC) in transition region

Inverse catalysis and phase diagram

- physical m_π, staggered quarks, continuum limit
  Bali, Bruckmann, Endrődi, Fodor, Katz et al. '11 '12

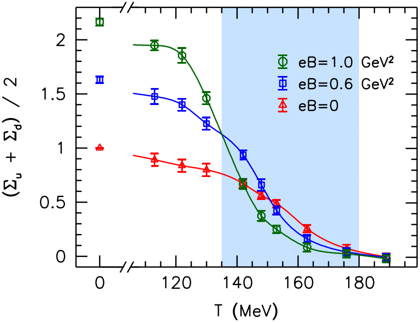


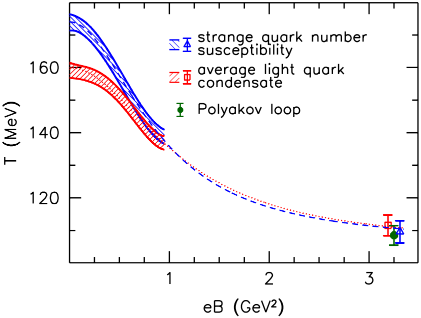


- magnetic catalysis at low T (also at high T)
- inverse magnetic catalysis (IMC) in transition region
- T_c is reduced by B

Inverse catalysis and phase diagram

- physical m_π, staggered quarks, continuum limit
  Bali, Bruckmann, Endrődi, Fodor, Katz et al. '11 '12
  Endrődi '15

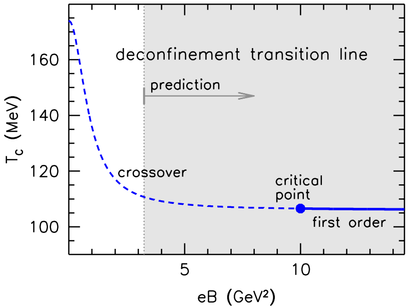




- magnetic catalysis at low T (also at high T)
- inverse magnetic catalysis (IMC) in transition region
- T_c is reduced by B

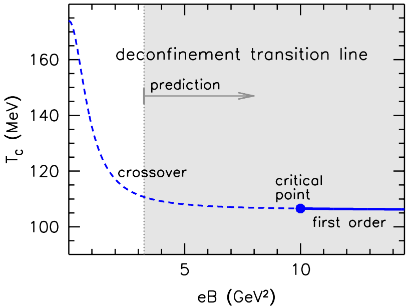
Phase diagram and critical point

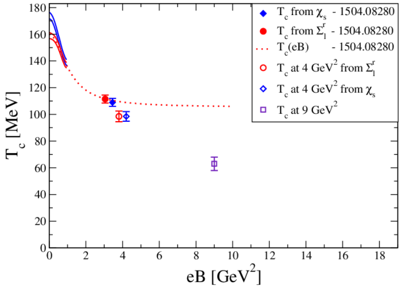
- effective theory of QCD at $B \to \infty$: first-order deconfinement transition $\Rightarrow$ critical point! Miransky, Shovkovy '02
- location of critical point based on extrapolation from $0 < eB \lesssim 3$ GeV^2 $\Rightarrow$ $\boxed{eB_c \approx 10(2)\ \text{GeV}^2}$ Endrődi '15



Phase diagram and critical point

- effective theory of QCD at $B \to \infty$: first-order deconfinement transition $\Rightarrow$ critical point! Miransky, Shovkovy '02
- location of critical point based on extrapolation from $0 < eB \lesssim 3$ GeV^2 $\Rightarrow$ $\boxed{eB_c \approx 10(2)\ \text{GeV}^2}$ Endrődi '15
- simulating up to $eB \approx 9$ GeV^2 $\Rightarrow$ $\boxed{4\ \text{GeV}^2 < eB_c < 9\ \text{GeV}^2}$
  D'Elia, Maio, Sanfilippo, Stanzione '21

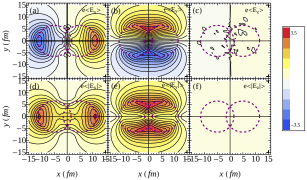




Beyond constant magnetic fields

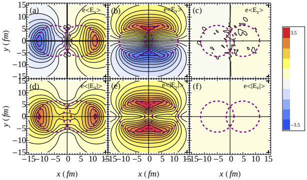
Inhomogeneous magnetic fields

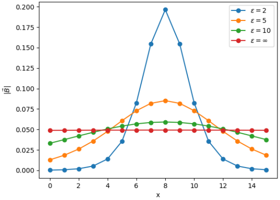
- remember HIC: inhomogeneous magnetic and electric fields



Inhomogeneous magnetic fields

- remember HIC: inhomogeneous magnetic and electric fields

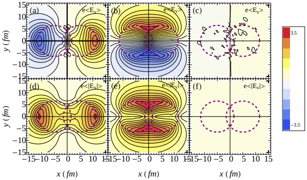


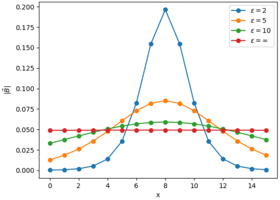


- consider profile $B(x) = B\cosh^{-2}(x/\epsilon)$ Dunne '04
  can be treated analytically (in absence of color interactions)

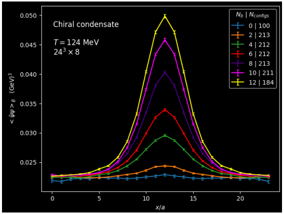
Inhomogeneous magnetic fields

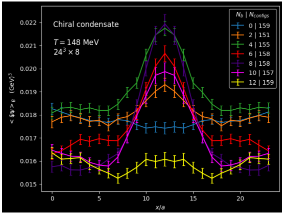
- remember HIC: inhomogeneous magnetic and electric fields





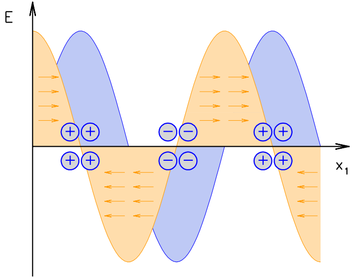
- consider profile $B(x) = B\cosh^{-2}(x/\epsilon)$ Dunne '04
  can be treated analytically (in absence of color interactions)
- impact on quark condensate D. Valois Wed 14:40





Electric fields

- static homogeneous electric field E: charges accelerated to ∞
- equilibrium requires infrared regularization
  $\rightsquigarrow$ finite wavelength $1/k_1$



- charge distribution where electric forces and pressure gradients cancel

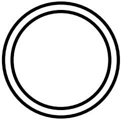
Electric susceptibility

- leading impact of E on free energy f
- here: perturbative QED at nonzero T

Electric susceptibility

- leading impact of E on free energy f
- here: perturbative QED at nonzero T
- Schwinger's approach Schwinger '51

  Loewe, Rojas '92 Elmfors, Skagerstam '95 Gies '98

$f(E) =$ 

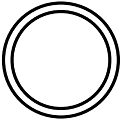
$$f = -\xi \cdot \frac{E^2}{2} + \dots$$

Electric susceptibility

- leading impact of E on free energy f
- here: perturbative QED at nonzero T
- Schwinger's approach Schwinger '51

  Loewe, Rojas '92 Elmfors, Skagerstam '95 Gies '98

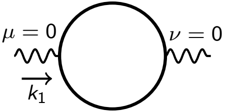
$$f(E) =$$



$$f = -\xi \cdot \frac{E^2}{2} + \dots$$

- Weldon's approach Weldon '82

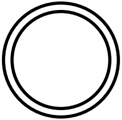
$$\xi = \lim_{k_1 \to 0} \frac{-1}{k_1^2}$$



Electric susceptibility

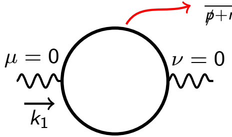
- leading impact of E on free energy f
- here: perturbative QED at nonzero T
- Schwinger's approach Schwinger '51

  Loewe, Rojas '92 Elmfors, Skagerstam '95 Gies '98

$$f(E) =$$  $$f = -\xi \cdot \frac{E^2}{2} + \dots$$

- Weldon's approach Weldon '82

$$\frac{1}{\not p + m + i\epsilon} + (\not p + m)\frac{2\pi i \delta(p^2 - m^2)}{e^{|p_0|/T} + 1}$$

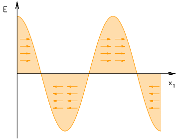
$$\xi = \lim_{k_1 \to 0} \frac{-1}{k_1^2}$$ 

- generalize calculation to $m > 0$ Endrődi, Markó in prep.

Equilibrium and mismatch

- evaluated in absence of charge distribution ($\mu = 0$)

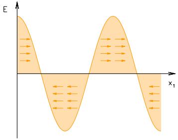
$$\xi_{\text{Weldon}} \approx \frac{T^2}{k_1^2} + \text{IR finite}$$



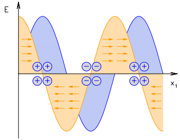
Equilibrium and mismatch

- evaluated in absence of charge distribution ($\mu = 0$)

$$\xi_{\mathrm{Weldon}} \approx \frac{T^2}{k_1^2} + \text{IR finite}$$



- evaluated "along local equilibria"
  ($N(x)$ such that $\partial\mu/\partial x = -E$)

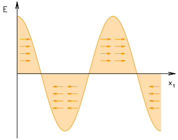


$$\xi_{\mathrm{Weldon}}^{\mathrm{equi}} = -\frac{1}{12\pi^2}\left[\log\frac{T^2}{m^2} + 1\right] + \mathcal{O}(m^2/T^2)$$

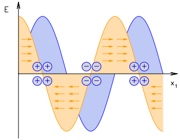
Equilibrium and mismatch

- evaluated in absence of charge distribution ($\mu = 0$)

$$\xi_{\text{Weldon}} \approx \frac{T^2}{k_1^2} + \text{IR finite}$$



- evaluated "along local equilibria" ($N(x)$ such that $\partial\mu/\partial x = -E$)



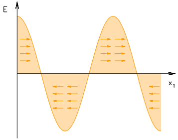
$$\xi^{\text{equi}}_{\text{Weldon}} = -\frac{1}{12\pi^2}\left[\log\frac{T^2}{m^2} + 1\right] + \mathcal{O}(m^2/T^2)$$

$$\xi_{\text{Schwinger}} = -\frac{1}{12\pi^2}\left[\log\frac{T^2}{m^2} - 1\right] + \mathcal{O}(m^2/T^2)$$

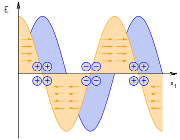
Equilibrium and mismatch

- evaluated in absence of charge distribution ($\mu = 0$)

$$\xi_{\text{Weldon}} \approx \frac{T^2}{k_1^2} + \text{IR finite}$$



- evaluated "along local equilibria" ($N(x)$ such that $\partial\mu/\partial x = -E$)



$$\xi^{\text{equi}}_{\text{Weldon}} = -\frac{1}{12\pi^2}\left[\log\frac{T^2}{m^2} + 1\right] + \mathcal{O}(m^2/T^2)$$

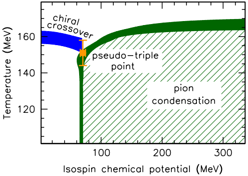
$$\xi_{\text{Schwinger}} = -\frac{1}{12\pi^2}\left[\log\frac{T^2}{m^2} - 1\right] + \mathcal{O}(m^2/T^2)$$

- Schwinger's method fulfills equilibrium construction inherently but regularizations of infrared divergence differ by finite term!

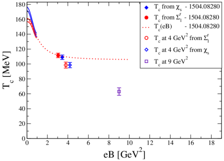
Summary

Summary

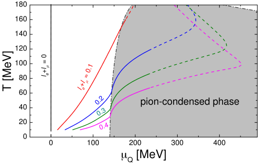
- $T - \mu_I$ phase diagram
  and pion condensation



- $T - B$ phase diagram
  and the critical point



- cosmic trajectory may enter pion
  condensed phase



- background electric fields
  and local charge distributions