Hadrons at high temperature: a lattice update

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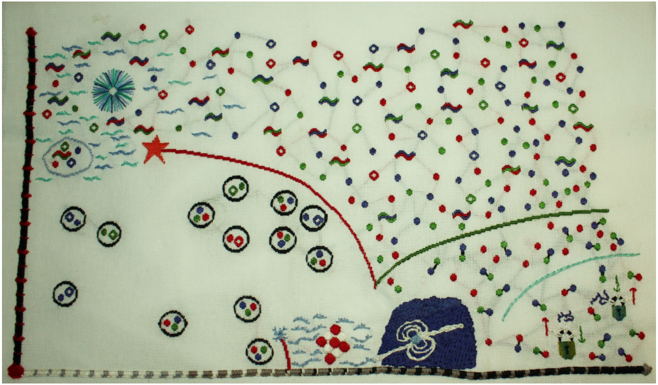
Outline

Background



Background

- Quark–gluon plasma is created in heavy-ion collisions at RHIC and LHC
- No direct observation of QGP — must infer from "fallout"
- Dynamical medium: expanding, cooling fireball
  - $\rightarrow$ transport coefficients are crucial in understanding
- In-medium mass and width modifications below T_c
  - $\rightarrow$ relevance for hadron resonance gas models?
- Sequential suppression $\longrightarrow$ quarkonia as QGP thermometers?

Lattice simulations

- QGP near crossover is strongly interacting: nonperturbative methods required
- Equilibrium thermal field theory formulated in euclidean space — suitable for Monte Carlo simulations

$$\langle \mathcal{O} \rangle = \int \mathcal{D}[\Phi]\mathcal{O}[\Phi]e^{-S[\Phi]}$$

- Temperature $T = \frac{1}{L_\tau} = (N_\tau a_\tau)^{-1}$
- Real-time quantities may be determined from spectral function

$$\rho(\omega) = \mathrm{Im}\, G_R(\omega) = \mathrm{Im} \int_0^\infty G_R(t)e^{-i\omega t}$$

$$G_E(\tau; T) = \int_0^\infty d\omega K(\omega, \tau; T)\rho(\omega; T)$$

- 2+1 active light flavours required for quantitative predictions!

Dynamical anisotropic lattices

- A large number of points in time direction required to extract spectral information
- For $T = 2T_c$, $\mathcal{O}(10)$ points $\Longrightarrow a_t \sim 0.025$ fm
- Far too expensive with isotropic lattices $a_s = a_t$!
- Fixed-scale approach
  - $\rightarrow$ vary T by varying N_τ (not a)
  - $\rightarrow$ need only 1 $T = 0$ calculation for renormalisation
  - $\rightarrow$ independent handle on temperature

- Introduces 2 additional parameters
- Non-trivial tuning problem
  [PRD **74** 014505 (2006); HadSpec Collab, PRD **79** 034502 (2009)]

Simulation parameters

[PRD **76** 194513 (2007), HadSpec Collab, PRD **79** 034502 (2009)]

Gen	N_f	ξ	a_s (fm)	a_τ^{-1} (GeV)	m_π (MeV)	N_s	L_s (fm)
1	2	6.0	0.162	7.35	490	12	1.94
2	2+1	3.45	0.123	5.63	390	24	2.95
						32	3.94
2L	2+1	3.45	0.112	6.08	240	32	3.58
2P	2+1	3.45	*0.100	*6.80	140	48	4.80
3	2+1	7.0	*0.123	*11.66	*390	32	3.94

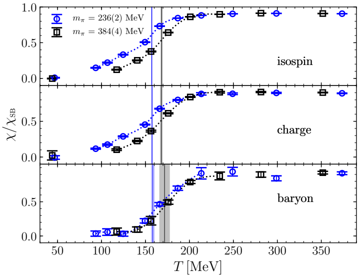
Simulation parameters: temperatures

Gen 2			Gen 2L		
N_τ	T (MeV)	T/T_c	N_τ	T (MeV)	T/T_c
128	44	0.24	128	47	0.42
			64	95	0.59
			56	109	0.67
48	117	0.63	48	127	0.78
40	141	0.76	40	152	0.94
36	156	0.84	36	169	1.04
32	176	0.95	32	190	1.17
28	201	1.09	28	217	1.34
24	235	1.27	24	253	1.56
20	281	1.52	20	304	1.87
16	352	1.90	16	380	2.34
			12	507	3.12
			8	760	4.69

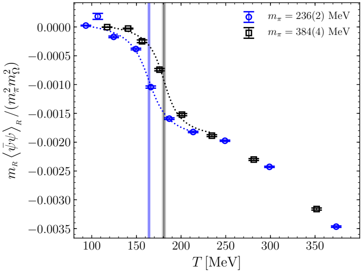
Susceptibilities and chiral transition

[PRD**105**(2022)034504]

Isospin, charge and baryon susceptibility



Renormalised chiral condensate

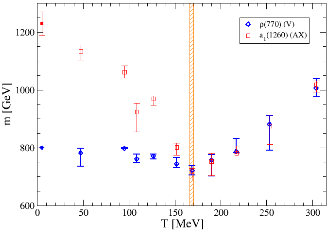


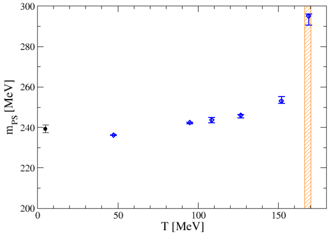
T_c is shifted by $\sim 20\,\text{MeV}$ when m_π goes from 390 to 240 MeV

Light mesons

PRELIMINARY

- Exponential fits to smeared and point correlators
- Consistent results, only showing smeared
- Chiral partners A and V become degenerate at transition
- No degeneracy seen in PS–S
- Slight increase in m_π

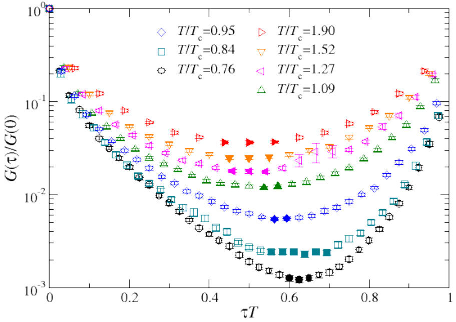




Light baryons

[PRD**92**(2015)014503; JHEP**1706**034; PRD99(2019)074503; PRD**105**(2022)034504]

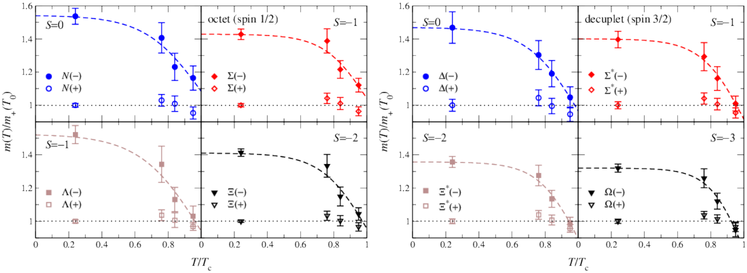
Positive and negative parity states encoded in same correlator



Forward propagating: + parity; Backward propagating: − parity

Using smeared (extended) sources to enhance ground state

Baryon mass modifications [Gen2]



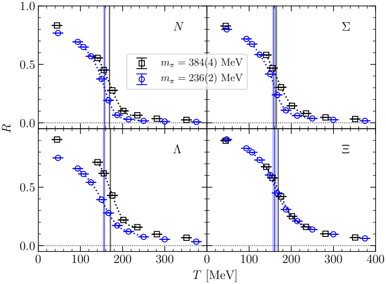
- Positive parity ground state masses unaffected by T up to T_c
- Negative parity masses decrease
- Parity restoration near T_c?

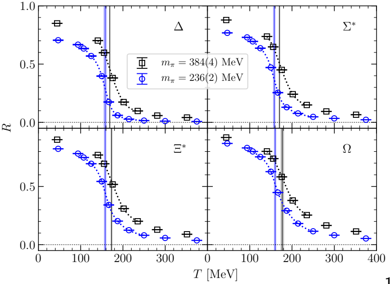
Parity restoration

Measure of parity restoration:

$$R(\tau) = \frac{G(\tau) - G\beta - \tau)}{G(\tau) + G(\beta + \tau)}$$

$$R = \frac{\sum_{n=0}^{\beta/2-1} R(\tau_n)/\sigma^2(\tau_n)}{\sum_{n=0}^{\beta/2-1} 1/\sigma^2(\tau_n)}$$





Charm

- J/ψ suppression — a probe of the quark–gluon plasma?
- Quantitative results for broadening and melting?
- To what extent do c quarks thermalise?
- How reliable are quenched lattice simulations?
- What happens to open charm and charmed baryons?

Methods

- Reconstructed correlators with model $T = 0$ spectral function
- Reconstructed correlators: direct reconstruction
- Bayesian reconstruction of spectral functions
- Model χ^2 fits, point and smeared sources

Spectral functions

- contain information about the fate of hadrons in the medium
  - $\rightarrow$ stable states $\rho(\omega) \sim \delta(\omega - m)$
  - $\rightarrow$ resonances or thermal width $\rho(\omega) \sim$ lorentzian
  - $\rightarrow$ continuum above threshold

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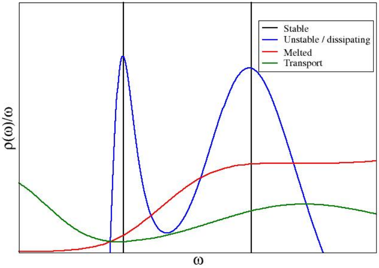

Spectral functions

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  - → stable states $\rho(\omega) \sim \delta(\omega - m)$
  - → resonances or thermal width $\rho(\omega) \sim$ lorentzian
  - → continuum above threshold
- $\rho_\Gamma(\omega, \vec{p})$ related to euclidean correlator $G_\Gamma(\tau, \vec{p})$ according to

$$G_\Gamma(\tau, \vec{p}) = \int \rho_\Gamma(\omega, \vec{p}) K(\tau, \omega) d\omega \,, \quad K(\tau, \omega) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}$$

- an ill-posed problem — requires a large number of time slices
  - → Fit to physically motivated Ansatz
  - → Use Maximum Entropy Method or other Bayesian methods
  - → Other inversion methods, eg Backus–Gilbert, machine learning

Reconstructed correlators

The systematic uncertainty of the spectral function can be avoided by studying the reconstructed correlator, defined as

$$G_r(\tau; T, T_r) = \int_0^\infty \rho(\omega; T_r) K(\tau, \omega, T) d\omega$$

where K is the kernel

$$K(\tau, \omega, T) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}$$

If $\rho(\omega; T) = \rho(\omega; T_r)$ then $G_r(\tau; T, T_r) = G(\tau; T)$

Small changes in correlators is compatible with large changes in spectral function

Direct correlator reconstruction

[Meyer (2010), Ding et al (2012)]

With

$$T = \frac{1}{a_\tau N}, \;\; T_r = \frac{1}{a_\tau N_r}, \quad \frac{N_r}{N} = m \in \mathbb{N}$$

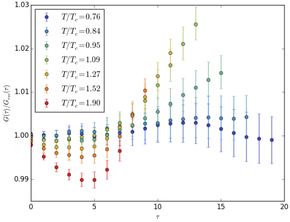
and using

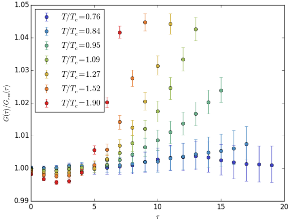
$$\frac{\cosh\left[\omega(\tau - N/2)\right]}{\sinh(\omega N/2)} = \sum_{n=0}^{m-1} \frac{\cosh\left[\omega(\tau + nN + mN/2)\right]}{\sinh(\omega m N/2)}$$

we have

$$G_r(\tau; T, T_r) = \sum_{n=0}^{m-1} G(\tau + nN, T_r)$$

Charmonium: reconstructed correlators [Gen2]

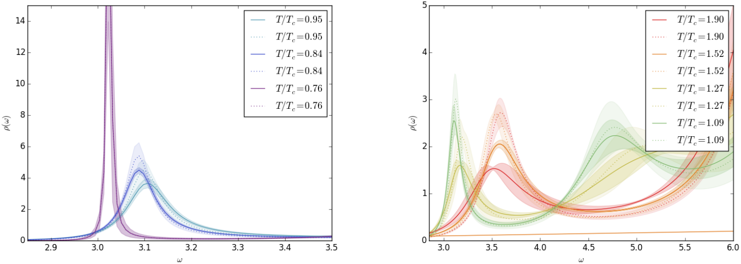




Top: pseudoscalar (η_c)
Bottom: vector (J/ψ)

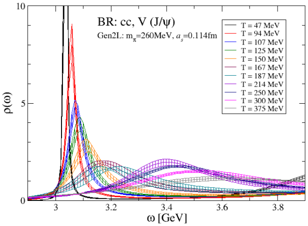
- $T \lesssim T_c$ consistent with no change
- Small but significant modifications above T_c

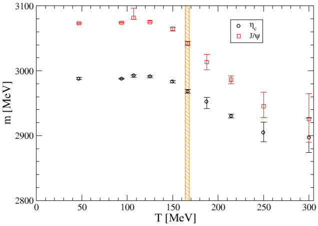
J/ψ spectral functions [Gen2]



- BR method for thermal (solid lines) and reconstructed (dotted lines) correlators
- Similar results from MEM
- Consistent with no change below T_c
- Possible weakening or melting for $T \gtrsim 1.5\,T_c$

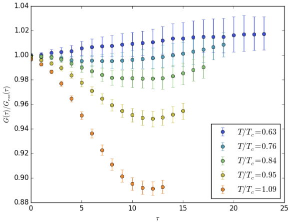
Charmonium: Gen2L

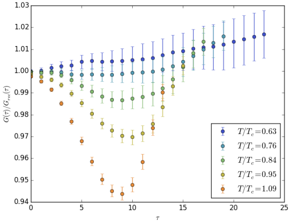




- BR on thermal correlators only
- Fits to smeared correlators suggest negative mass shift for $T > 120\text{MeV}$

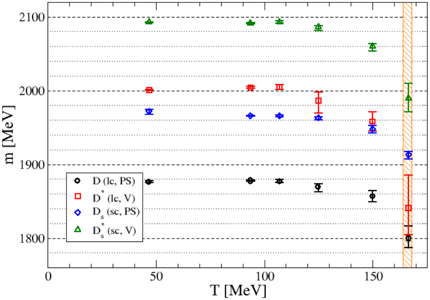
D mesons: reconstructed correlators and fits





Top: D, Bottom: D^*

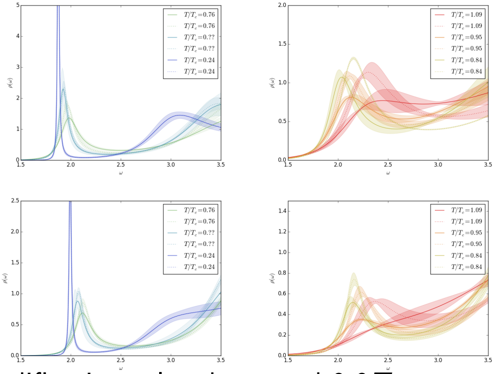
- Significant changes for $T \gtrsim T_c$
- Modifications below T_c
- Smaller for D_s



Open charm spectral functions

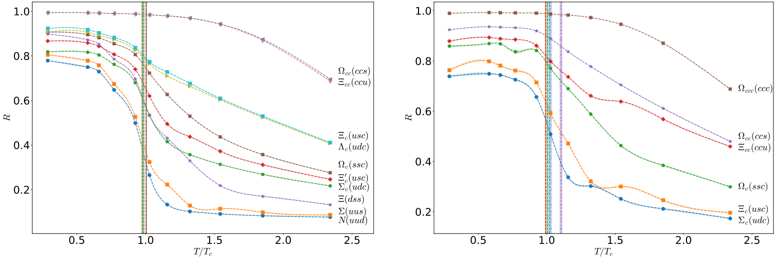
D (PS)

D^* (V)



- Significant modifications already around $0.9\,T_c$
- Clear difference at $T \approx 1.9\,T_c$

Charmed baryons



Parity doubling crossover is at same temperature for (singly) charmed as for light and strange baryons

NRQCD

Scale separation $M_Q \gg T, M_Q v$
Integrate out hard scales $\longrightarrow$ Effective theory
Expand in orders of heavy quark velocity $\mathbf{v}$; we use $\mathcal{O}(\mathbf{v}^4)$ action

Advantages

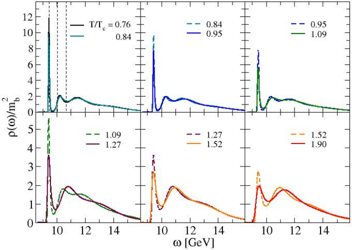
- No temperature-dependent kernel, $G(\tau) = \int \rho(\omega) e^{-\omega\tau} \frac{d\omega}{2\pi}$
- No zero-modes
- Longer euclidean time range
- Appropriate for probes not in thermal equilibrium

Disadvantages

- Not renormalisable, requires $Ma_s \gtrsim 1$
- Does not incorporate transport properties

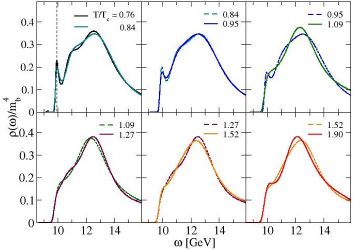
Spectral functions — MEM analysis [Gen2

S-waves



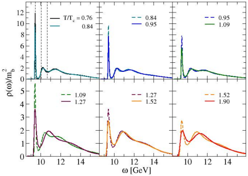
Υ (2S) melts, but ground state remains robust

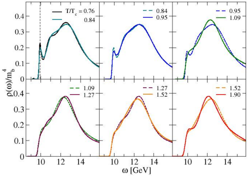
P-waves



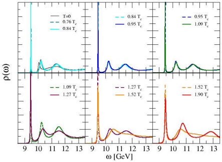
P-waves dissociate close to T_c

MEM vs BR method

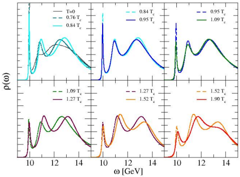




Υ

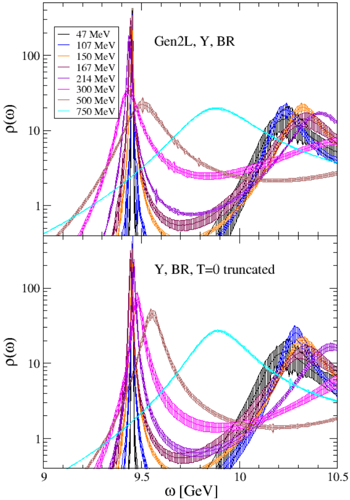


χ_{b1}



Known discrepancy: BR produces more peak structures

BR spectral functions — Gen2L



PRELIMINARY

- Negative mass shift at all T?
- No significant thermal broadening for $T \lesssim 250\,\text{MeV}$?
- Width and mass to be cross-checked with other methods — in progress
- $\Upsilon(2S)$ not resolved

Summary

- Light mesons:
  - $\rightarrow$ chiral doubling $\rho - a_1$ seen at T_c
  - $\rightarrow$ m_π increases with T
- Baryons:
  - $\rightarrow$ observed parity restoration
  - $\rightarrow$ impact on hadron resonance gas
  - $\rightarrow$ singly charmed baryons behave similarly to light
- Open charm:
  - $\rightarrow$ thermal modifications already below T_c
  - $\rightarrow$ indication of significant mass drop
  - $\rightarrow$ no bound states above T_c
- Charmonium:
  - $\rightarrow$ no significant modification in S-waves below T_c
  - $\rightarrow$ suggested survival up to $1.5T_c$
- Beautonium:
  - $\rightarrow$ S wave survival up to $T > 2T_c$, moderate mass shift
  - $\rightarrow$ Quantitative results for mass shift and width still elusive

Outlook

- Complete understanding of systematics
- Towards the physical limit with lighter quarks — underway
- Repeat with smaller a_τ — underway
- Open beauty

THANK YOU

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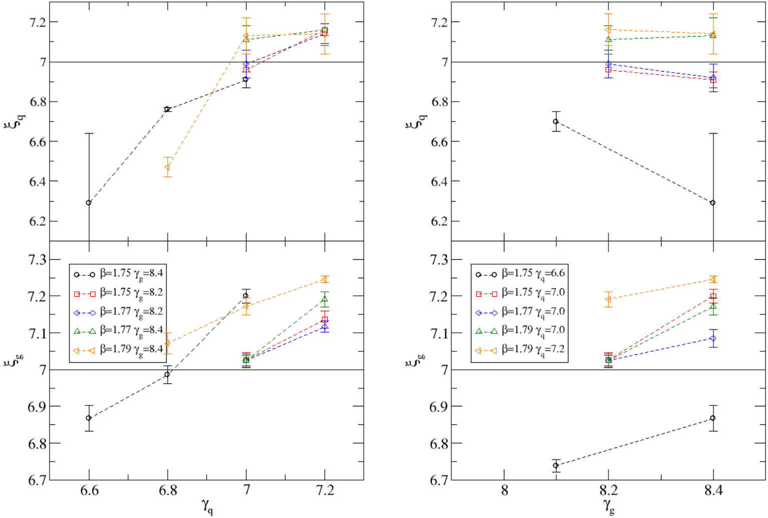
Sinéad Ryan, Maria Paola Lombardo, Seyong Kim, Ben Jäger, Simon Hands, Tim Burns, Chris Allton, Gert Aarts

Gen3 tuning


β=1.75 γ_g=8.4
β=1.75 γ_g=8.2
β=1.77 γ_g=8.2
β=1.77 γ_g=8.4
β=1.79 γ_g=8.4
ξ_q
ξ_g
γ_q
β=1.75 γ_q=6.6
β=1.75 γ_q=7.0
β=1.77 γ_q=7.0
β=1.79 γ_q=7.0
β=1.79 γ_q=7.2
γ_g