

Heavy quark diffusion from the lattice

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Based on:

Nora Brambilla¹, V.L.¹, Péter Petreczky², Antonio Vairo²: PRD102 (2020)
Nora Brambilla¹, V.L.¹, Julian Mayer-Stuedte¹, Péter Petreczky²: hep-lat/2206.02861

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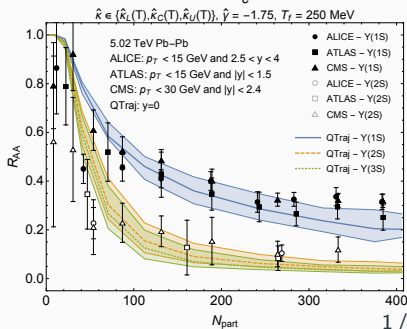
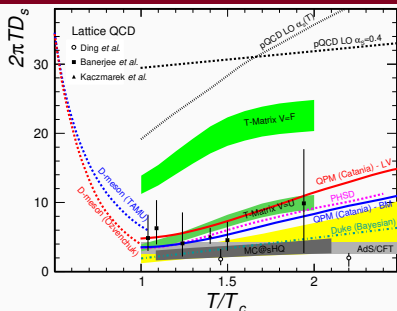
- Motivation
- Diffusion in strongly coupled plasma
- Lattice basics
- Measurements
- Results
- Conclusions

Motivation

- The strongly coupled plasma can be described in terms of transport coefficients
- R_{AA} and ν_2 described by spatial diffusion coefficient D_x
- Observed ν_2 is larger than expected from kinetic models but agrees more with hydrodynamic models
- Multiple theoretical models predicting wide range of values
- Non-perturbative lattice simulations needed
- κ dominant source of variation in R_{AA}

UP: X. Dong CIPANP (2018)

DOWN: N. Brambilla, M. Escobedo, M. Strickland, A. Vairo, P. Vander Griend and J. Weber, JHEP 05 (2021) 136



Heavy Quark diffusion

- Heavy quark energy changes only little when colliding with medium

$$E_k \sim T, \quad p \sim \sqrt{MT} \gg T$$

- HQ momentum is changed by random kicks from the medium
→ Brownian motion; Follows Langevin dynamics

$$\frac{dp_i}{dt} = -\frac{\kappa}{2MT} p_i + \xi_i(t), \quad \langle \xi(t) \xi(t') \rangle = \kappa \delta(t - t')$$

- Heavy quark momentum diffusion coefficient κ related also to:

$$\text{Spatial diffusion coefficient } D_s = 2T^2/\kappa,$$

$$\text{Drag coefficient } \eta_D = \kappa/(2MT),$$

$$\text{Heavy quark relaxation time } \tau_Q = \eta_D^{-1}$$

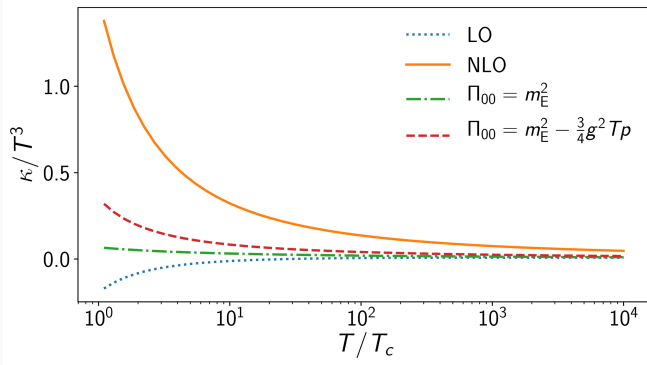
- Considering full Lorentz force:

$$F(t) = \dot{p} = q(E + v \times B)(t)$$

- $\langle v^2 \rangle \sim \mathcal{O}(\frac{T}{M})$ correction to HQ momentum diffusion

$$\kappa_{\text{tot}} \simeq \kappa_E + \frac{2}{3} \langle v^2 \rangle \kappa_B$$

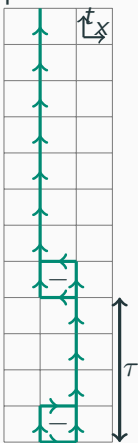
κ from perturbation theory



- Clearly $m_E \ll T$ is too strict assumption on small T
- Huge perturbative variation
 \Rightarrow needs non-perturbative measurements
- Also huge scale dependence through $m_E = g(\mu)T$
- Here we have scale from NLO EQCD $\mu \sim 2\pi T$

Heavy quark diffusion from lattice: Euclidean Correlator

periodic



periodic

- Traditional approach uses HQ current-current correlators:

Problem: Transport peak at zero

- HQEFT inspired Euclidean correlator is peak free

$$G_E(\tau) = -\frac{1}{3} \sum_{i=1}^3 \frac{\langle \text{Re Tr} [U(\beta, \tau) g E_i(\tau, 0) U(\tau, 0) g E_i(0, 0)] \rangle}{\langle \text{Re Tr} [U(\beta, 0)] \rangle}$$

$$G_B(\tau) = \sum_{i=1}^3 \frac{\langle \text{Re Tr} [U(1/T, \tau) B_i(\tau, 0) U(\tau, 0) B_i(0, 0)] \rangle}{3 \langle \text{Re Tr} U(1/T, 0) \rangle}$$

- Field strength tensor components need discretization
- Choose corner discretization for this study
- On lattice there is a self-energy contribution that generates a multiplicative renormalization
- For chromomagnetic fields there is also a finite anomalous dimension and renormalization is required

Heavy quark diffusion from lattice: Spectral function

$$G_E(\tau) = \int_0^\infty \frac{d\omega}{\pi} \rho(\omega) \frac{\cosh\left(\frac{\omega}{T} \left[\tau T - \frac{1}{2}\right]\right)}{\sinh \frac{\omega}{2T}}$$
$$\kappa = \lim_{\omega \rightarrow 0} \frac{2T}{\omega} \rho(\omega), \quad \gamma = -\frac{1}{3N_c} \int_0^\infty \frac{d\omega}{2\pi} \frac{\rho(\omega)}{\omega}$$

- Euclidean correlator related to spectral function
- Needs inversion of integral equation
- We use simple procedure of modeling $\rho(\omega)$ and comparing to lattice data
- γ not measured yet, could be measured directly from G_E or from reconstructed $\rho(\omega)$

Renormalization and spectral function: G_E

- Normalize the data with perturbative LO result (also tree-level improve)

$$G_{E,B}^{\text{norm}} = \pi^2 T^4 \left[\frac{\cos^2(\pi T)}{\sin^4(\pi T)} + \frac{1}{3 \sin^2(\pi T)} \right]$$

- On Lattice E has non-physical self-energy contribution
 $Z_E = 1 + g_0^2 \times 0.137718569 \dots + \mathcal{O}(g_0^4)$

(Christensen and Laine PLB02 (2016))

- In practice LO perturbative Z_E not enough, we normalize at a single point instead
- Model the spectral function by connecting known IR and UV behavior with ansatz:

$$\rho_{\text{IR}}(\omega) = \frac{\kappa \omega}{2T}$$
$$\rho_{\text{QCD,naive}}(\omega) = \frac{g^2 C_F \omega^3}{6\pi} \left\{ 1 + \frac{g^2}{(4\pi)^2} \left[N_c \left(\frac{11}{3} \ln \frac{\mu^2}{4\omega^2} + \frac{149}{9} - \frac{8\pi^2}{3} \right) \right] \right\}$$

- set scale such that NLO UV contribution vanishes

Renormalization and spectral function: G_B

- κ_B more complicated, requires renormalization

$$G_B^{\text{flow,UV}}(\tau, \tau_F) = (1 + \gamma_0 g^2 \ln(\mu \sqrt{8\tau_F}))^2 Z_{\text{flow}} G_B^{\overline{\text{MS}},\text{UV}}(\tau, \mu) + h_0 \cdot (\tau_F/\tau),$$

- Normalize at finite flow time with G^{flow}
- Assuming $h_0 = 0$ for simplicity. We see very little flow time dependence even with this assumption.
- We know Z_E wasn't enough, so we determine Z_{flow} similarly, i.e. just normalize at a point.
- Use same tree-level improvement as for E-correlator
- IR model the same:

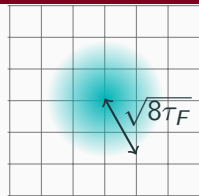
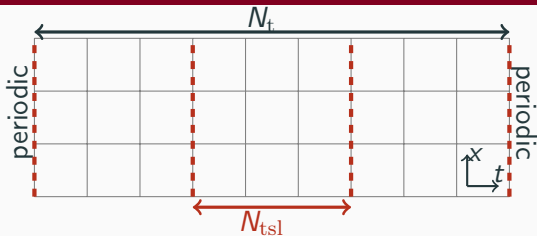
$$\rho_{\text{IR}}(\omega) = \frac{\kappa\omega}{2T}$$

- UV now depends on flow time

$$\rho_B^{\text{UV}}(\omega, \tau_F) = Z_{\text{flow}} \frac{g^2(\mu)\omega^3}{6\pi} (1 + g^2(\mu)(\beta_0 - \gamma_0) \ln(\mu^2/(A\omega^2))) + g^2(\mu)\gamma_0 \ln(8\tau_F\mu^2)$$

- Again, set the scale such that the second term vanishes

Simulation Algorithms



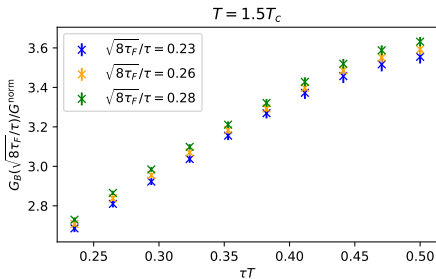
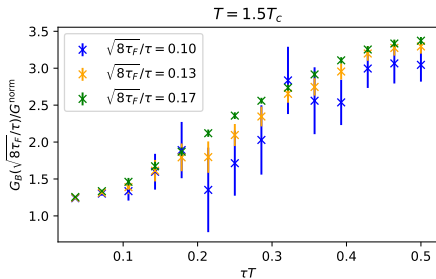
Multilevel

- Update each sub-lattice independently keeping boundaries fixed
- + Allows reaching better statistics with less configurations
- Only works in pure gauge

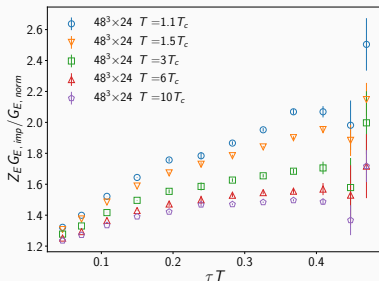
Gradient flow

- "Diffuse" with fictitious time τ_F
- Continuous smear of radius $\sqrt{8\tau_F}$
- + Could be generalized to unquenched
- + Automatically renormalizes gauge invariant observables
- Needs zero flow time limit

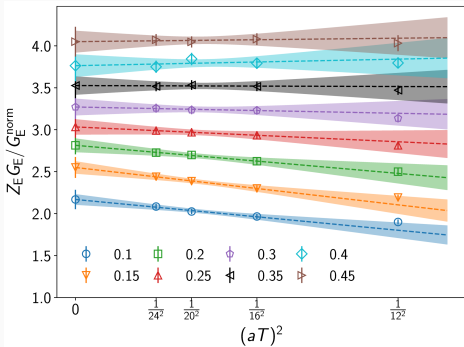
Raw (normalized*, tree-level improved) lattice data



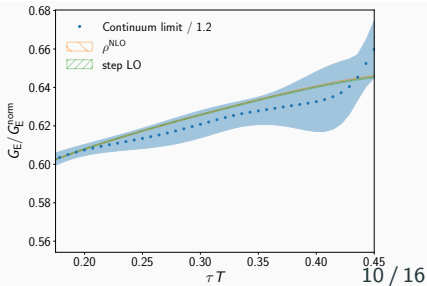
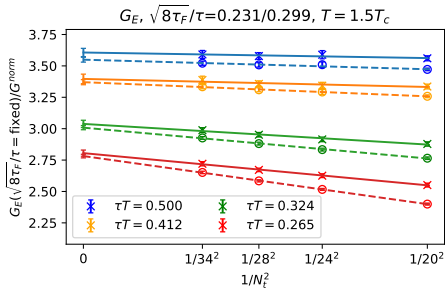
- κ dominant cause for the shape, not flow time
- Can see agreement between GF and multilevel (cross-check)
- Very similar shapes for electric and magnetic correlators



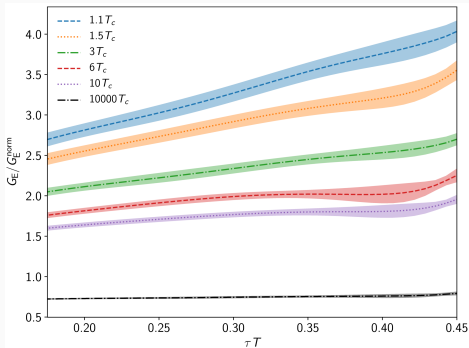
Continuum limit



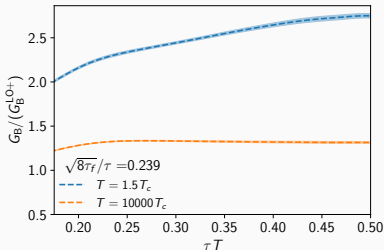
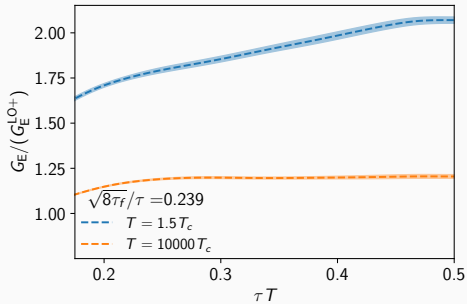
- Good limits for valid ranges of τT and τ_F
- Great agreement to perturbation theory at very high temperatures



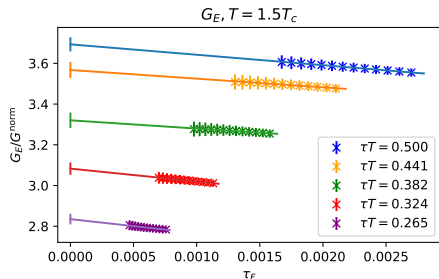
Multilevel vs GF



- Data needs additional normalization, do this at $\tau T = 0.19$

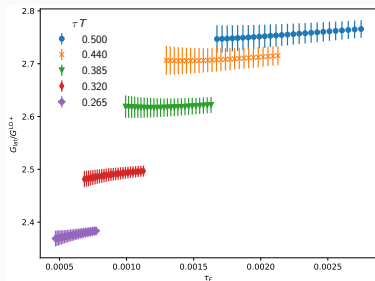


Flow time dependence of G_E and G_B

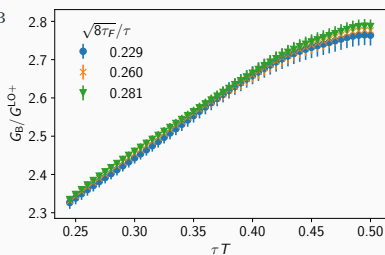


G_E

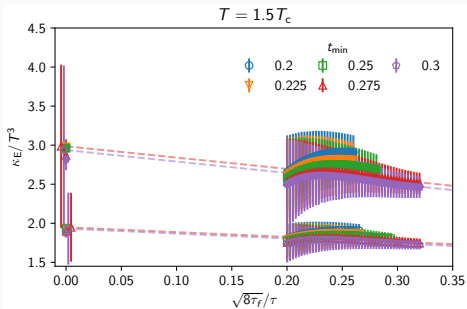
- We observe different small flow time scaling between G_E and G_B
- Flow dependent spectral function seems to remove most of flow dependence from G_B



G_B

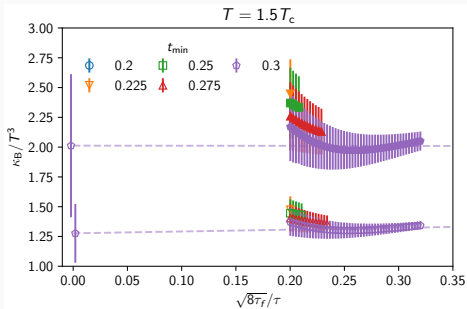


Flow time κ and order of limits

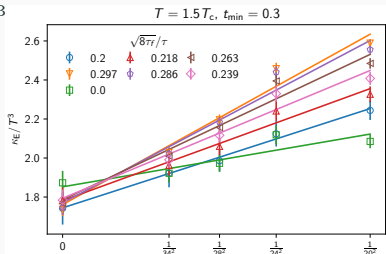


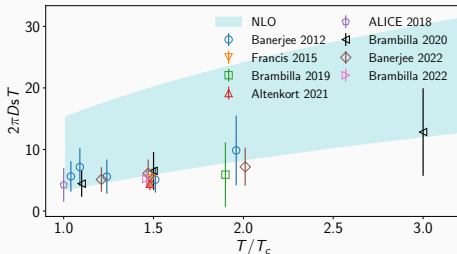
G_E

- Very little dependence on flow time
- Could even take continuum limit last



G_B





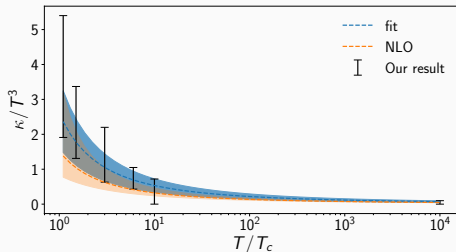
- Our results results

[Brambilla et.al. PRD102 \(2020\)](#)

[Brambilla et.al. hep-lat/2206.02861](#)

- Can fit temperature dependence:

$$\frac{\kappa^{\text{NLO}}}{T^3} = \frac{g^4 C_F N_c}{18\pi} \left[\ln \frac{2T}{m_E} + \xi + C \frac{m_E}{T} \right].$$



- Other lattice studies

[Meyer NJP13 \(2011\),](#)

[Ding et.al.JPG38 \(2011\),](#)

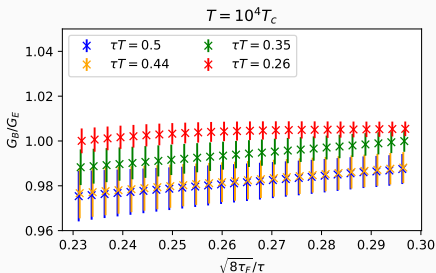
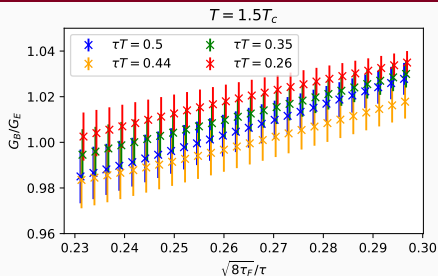
[Banerjee et.al. PRD85 \(2012\),](#)

[Francis et.al. PRD92 \(2015\)](#)

[Altenkort et.al. PRD103 \(2021\)](#)

[Banerjee et.al. hep-lat/2204.14075](#)

κ_B Results



- Our results results [Brambilla et.al. hep-lat/2206.02861](#)
- We get $1.03 \leq \kappa_B/T^3 \leq 2.61$
- In agreement with the earlier result ([Banerjee et.al. hep-lat/2204.14075](#))

$$\kappa_{\text{tot}} \simeq \kappa_E + \frac{2}{3} \langle v^2 \rangle \kappa_B$$

- Using $\langle v^2 \rangle$ from ([Petreczky et.al. Eur. Phys. J. C62 \(2009\)](#))
- $\langle v_{\text{charm}}^2 \rangle \simeq 0.51$ and $\langle v_{\text{bottom}}^2 \rangle \simeq 0.3$, we get that the mass suppressed effects on the heavy quark diffusion coefficient is 34% and 20% for the charm and bottom quarks respectively.

Conclusions and Future prospects

- Measured in wide range of temperatures
- Fit temperature dependence
- Agreement to perturbation theory at high T
- Agreement to previous results at small T
- Measured $1/M$ corrections
 - Good agreement with other recent study
 - The mass correction indicated to be 20 to 30% for bottom and charm quarks
- Future prospects:
 - Measure γ from our data
 - Go un-quenched
 - Check adjoint representation to study quarkonium

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Thank you for your attention!