Quark-antiQuark potential from Wilson Line Correlators at finite temperature: A comparison between different methods

Rasmus Larsen

University of Stavanger

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Overview

- Objective
  - Find the effective potential between a quark and an anti-quark at finite temperature
- Method
  - Calculate correlator between 2 wilson lines of length τ
  - Extract energies from behavior $C \sim \exp(-E\tau)$
- Techniques
  - Zero-temperature continuum subtraction
  - Pade interpolation
  - Hard Thermal Loop inspired fit
- Results
  - Energies of the potential
  - Spectral width of the potential
- Dibyendu Bala, Olaf Kaczmarek, Rasmus Larsen, Swagato Mukherjee, Gaurang Parkar, Peter Petreczky, Alexander Rothkopf, Johannes Heinrich Weber [arxiv:2110.11659]

Approach

- Measure the energy of 2 infinitely heavy quarks, separated by distance r

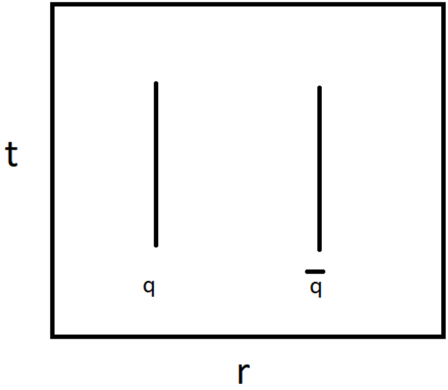


Figure: Illustration of a Wilson line correlation measurement.

- Measurement is not gauge invariant
  - Gauge fix to Coulomb gauge

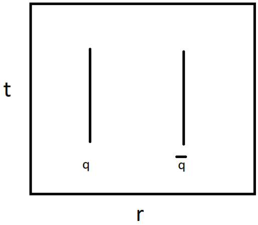
Wilson Line Correlator

- Wilson line is the product of Links

$$W(t,x) = \prod_{i}^{t} U_4(i,x) \quad (1)$$

- Infinitely heavy quarks stay fixed at same position
- Propagating from $\tau = 0$ to $\tau = t$ will be done by a wilson line of length t
- A quark and anti-quark will interfere with each other, to create different states based on the possible energies

$$C(t,x) =< Tr(W(t,0)W(t,x)^{\dagger}) > \quad (2)$$



Correlation function

- Correlation function $C(\tau, r)$ calculated on finite temperature lattice ensembles
- 2+1 flavor HotQCD configurations from $T = 151MeV$ to $T = 667MeV$
- Pion mass 160MeV, Kaon mass physical (the 3 highest temp use larger quark mass)
- $N_x = 48$, $N_\tau = 12$

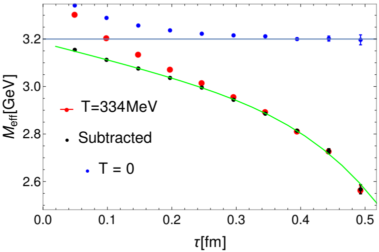
$$C(\tau, r) = \int_0^\infty \rho(\omega, r) \exp(-\omega\tau) d\omega \quad (3)$$

- Invert equation to find spectral function $\rho(\omega, r)$
  - Inversion problem very hard

Effective Mass and continuum subtraction

- Plateaus of the effective mass M_{eff} $->$ Mass state exists in $\rho(\omega)$

$$M_{eff} = \frac{1}{a}\log[C(\tau)/C(\tau+a)] = -\frac{\partial}{\partial_\tau}\log(C(\tau)) \quad (4)$$



$$\begin{aligned} C(\tau) &= Ae^{-M\tau} + C_{high}(\tau) \\ C_{sub}(\tau, T) &= C(\tau, T) - C_{high}(\tau) \end{aligned}$$

- Small τ behavior similar at $T = 0$ and $T \neq 0$
- Extract continuum $C_{high}(\tau)$ from $T = 0$ results

Extractions technique 1: Fit on continuum subtracted data

- Measurements contain contribution from continuum
- Remove the continuum

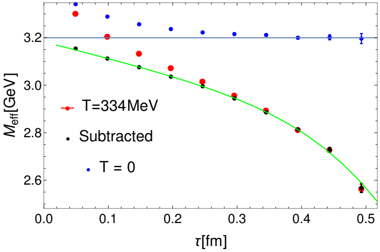


Figure: T=334MeV, r=0.44fm.

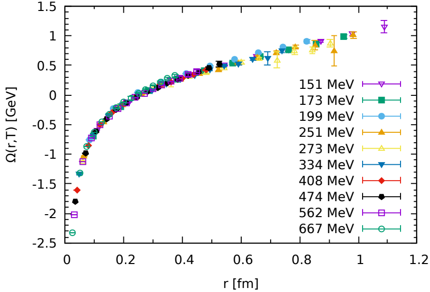
- Information in correlation function (Black points) is thus

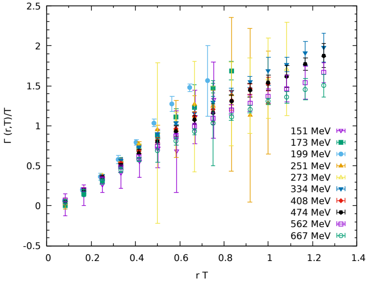
$$C_{sub}(\tau, T) \sim \exp(-\Omega\tau + \frac{1}{2}\Gamma^2\tau^2 + O(\tau^3)) \tag{5}$$

$$\rho_r(\omega, T) = A(T)\exp\left(-\frac{[\omega - \Omega(T)]^2}{2\Gamma^2(T)}\right) + A^{\text{cut}}(T)\,\delta\left(\omega - \omega^{\text{cut}}(T)\right)$$

Energy and Width from Wilson Line Correlator

- Almost no change in energy Ω (position of peak), but increasing width Γ [arxiv:2110.11659]

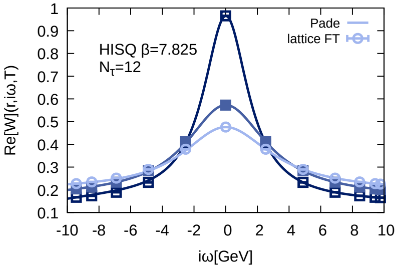


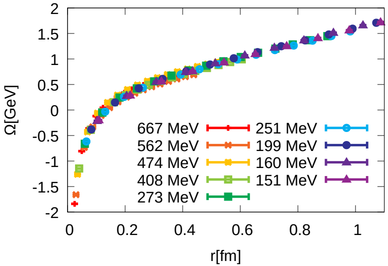


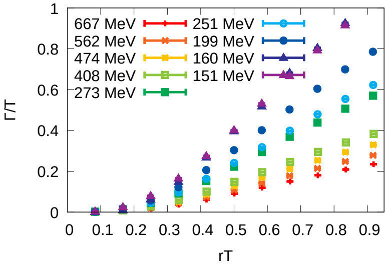
- Note difference to quenched QCD that showed screening with increased temperature

Extraction technique 2: Pade interpolation

- Fourier transform complex time correlator $W(r, i\omega, T)$
- Calculate Pade function (pol divided by pol) that goes exactly through all points
- Rotate fit from complex time to real time

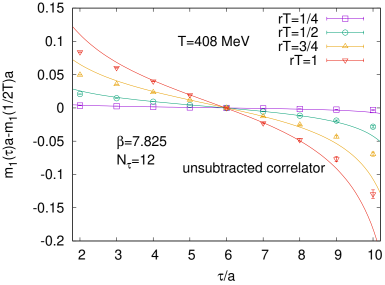


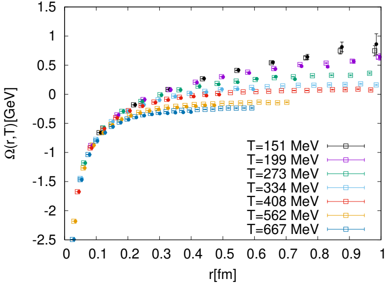


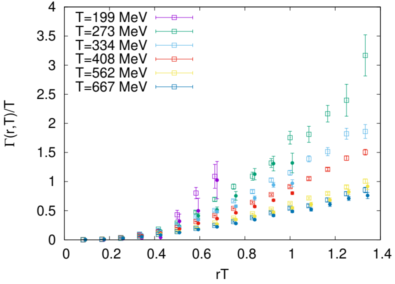


Extraction technique 3: Hard Thermal Loop inspired fit

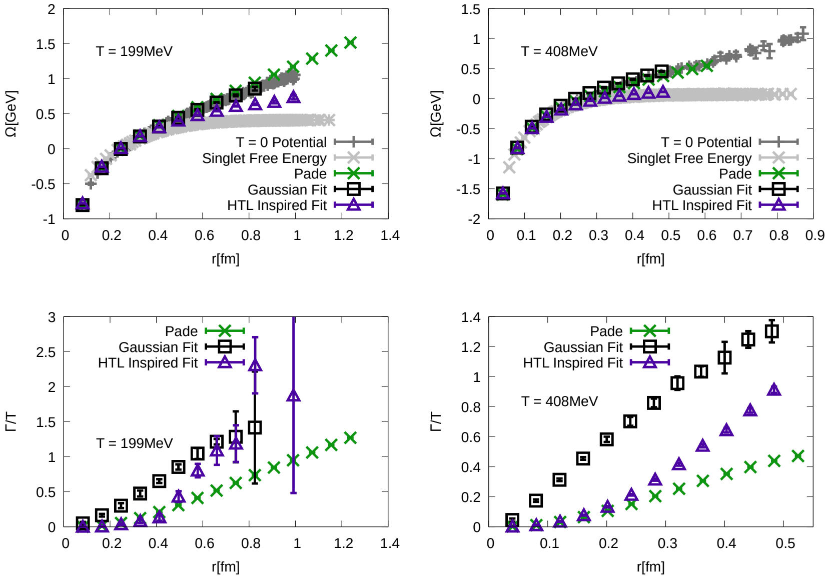
- Fit form expanded around $\tau = \beta/2$ obtained from Hard Thermal Loop expansion
- $m_1(r, n_\tau = \tau/a)a = \Omega(r,T) - \frac{\Gamma(r,T)aN_\tau}{\pi}\log(\frac{\sin(\pi n_\tau(N_\tau)}{\sin(\pi(n_\tau+1)(N_\tau)})$
- Fitted in a variable range around center of lattice





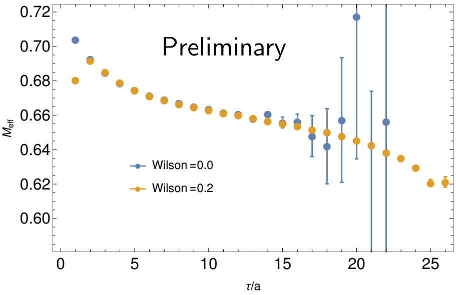


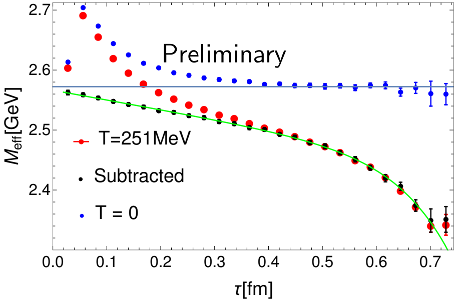
Comparison of Results



Wilson Line correlator results from 96^3 lattices

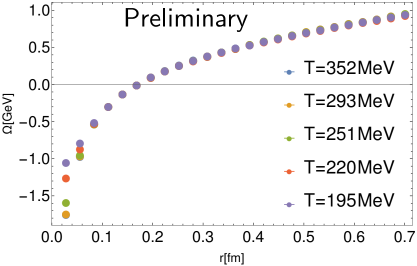
- Larger lattices generated with heavy quarks ($m_s/m_l = 5$) $96^3 * N_\tau$ using grant from PRACE
- High Energy fluctuations become dominating for large τ/a
- Wilson smearing used to remove high energy contributions
  - Affects results at small τ (both ends) and small distances

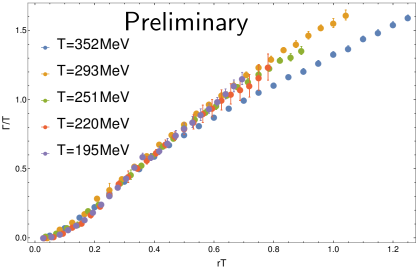




Wilson Line correlator results from 96^3 lattices

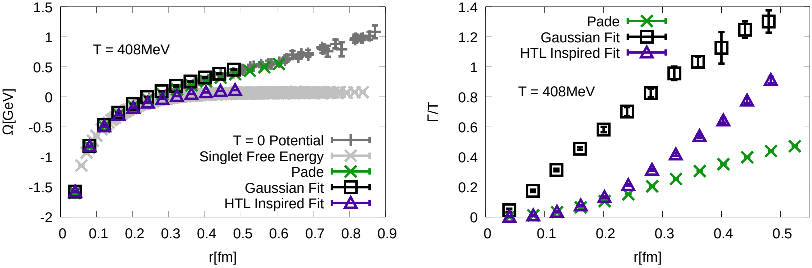
- Gaussian fits on subtracted correlator
- No Significant difference observed between 0 and finite temperature energy
- Results consistent with same methods on $N_x = 48$





- Smearing affects results at small r

Conclusion



- Most methods show no significant energy change with temperature
- Size of spectral width not consistent between different methods
- Attempts on improving fits with larger/finer lattices continue