Extending AMY Viscosity Calculations to Finite Baryon Chemical Potentials

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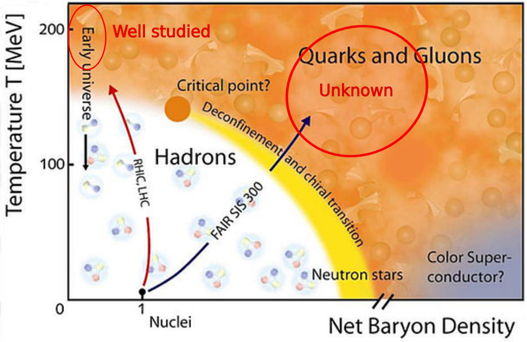
JHEP 11, 001 (2000) - Arnold, Moore and Yaffe [arXiv:hep-ph/0010177]

Outline

- Introduction
- Effective kinetic theory
- Collision integrals at high μ
- Variational solution
- Results
- Summary

Motivation

- Lower-energy collisions, and neutron star mergers, create a hot but extremely dense environment
- High density high temperature region is very unknown
- Perturbation theory should work better



[GSI Helmhotz Center for Heavy-Ion Physics]

What is η?

In a fluid system near equilibrium:

- η express the strength of the interaction in this fluid
- Conservation law (Hydrodynamics):

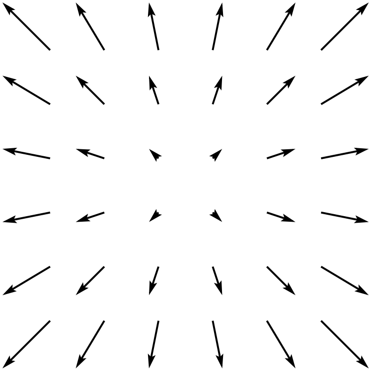
$$\partial_\mu \mathrm{T}^{\mu\nu} = 0$$

- T^{ij} corrected from its equilibrium form:

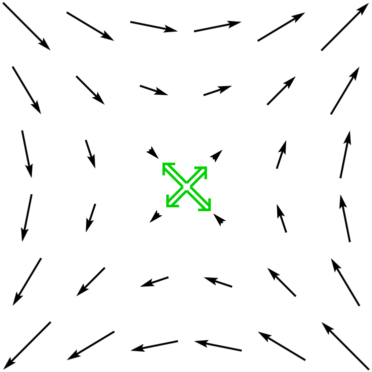
$$\mathrm{T}^{\mathrm{ij}} = \mathrm{P}\delta^{\mathrm{ij}} - \eta\left(\partial^{\mathrm{i}}\mathrm{u}^{\mathrm{j}} + \partial^{\mathrm{j}}\mathrm{u}^{\mathrm{i}} - \frac{2}{3}\delta^{\mathrm{ij}}\partial\cdot\mathrm{u}\right) - \zeta\delta^{\mathrm{ij}}\partial\cdot\mathrm{u}$$

What is η?

Bulk Viscosity



Shear Viscosity

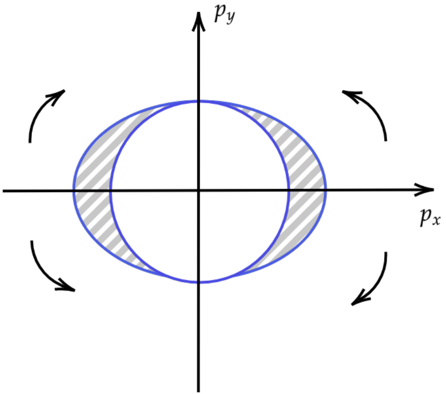


Figures from Guy Moore

Kinetic Theory

In kinetic theory, phase space density of particles, $f(x, p, t)$ describes the system, and changes in momentum occur through scattering:

$$\left[\frac{\partial}{\partial t} + \vec{v_p} \cdot \frac{\partial}{\partial \vec{x}}\right] f(\vec{p}, \vec{x}, t) = -\mathcal{C}[f]$$



Linearized Boltzmann Equation

Keeping only first order in the collision term:

$$\left[\frac{\partial}{\partial \mathrm{t}} + \mathrm{v}_{\vec{\mathrm{p}}} \cdot \frac{\partial}{\partial \vec{\mathrm{x}}}\right] \mathrm{f}_0^{\mathrm{a}}(\vec{\mathrm{p}}, \vec{\mathrm{x}}, \mathrm{t}) = -(\mathcal{C}[\mathrm{f}_1^{\mathrm{a}}])(\vec{\mathrm{p}}, \vec{\mathrm{x}}, \mathrm{t})$$

Where

$$\mathrm{f}^{\mathrm{a}} = \mathrm{f}_0^{\mathrm{a}} + \mathrm{f}_0^{\mathrm{a}}(1 - \mathrm{f}_0^{\mathrm{a}})\mathrm{f}_1^{\mathrm{a}}$$

and

$$\mathrm{f}_0^{\mathrm{q}/\bar{\mathrm{q}}} = \{\exp[\beta(\mathrm{p} \pm \mu)] + 1\}^{-1}$$

$$\mathrm{f}_0^{\mathrm{g}} = \{\exp[\beta \mathrm{p}] - 1\}^{-1}$$

Linearized Boltzmann Equation

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$$\mathrm{f}_0^\mathrm{g} = \{\mathrm{exp}[\beta \mathrm{p}] - 1\}^{-1}$$

Linearized Boltzmann Equation

The $f_1(\vec{p})$ that will solve the linearized Boltzmann equation:

$$f_1(\vec{p}) = \frac{\beta^2}{\sqrt{6}}\left(\partial_i u_j + \partial_j u_i - \frac{2\delta_{ij}\partial_k u_k}{3}\right)\chi_{ij}(\vec{p})$$

$$\chi_{ij}(\vec{p}) = \sqrt{\frac{2}{3}}\left(\hat{p}_i\hat{p}_j - \frac{1}{3}\delta_{ij}\right)\chi(p)$$

Solving the Boltzmann equation means determining $\chi(p)$
Now we define:

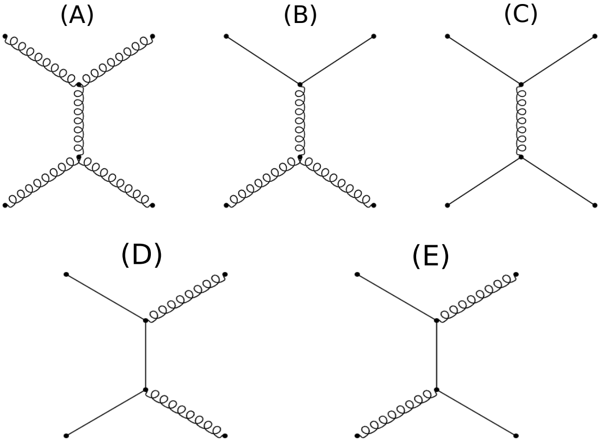
$$S_{i...j}(\vec{p}) = -\frac{1}{\beta}|\vec{p}|f_0(\vec{p})[1 \pm f_0(\vec{p})]\sqrt{\frac{2}{3}}\left(\hat{p}_i\hat{p}_j - \frac{1}{3}\delta_{ij}\right)$$

We can simplify the equation:

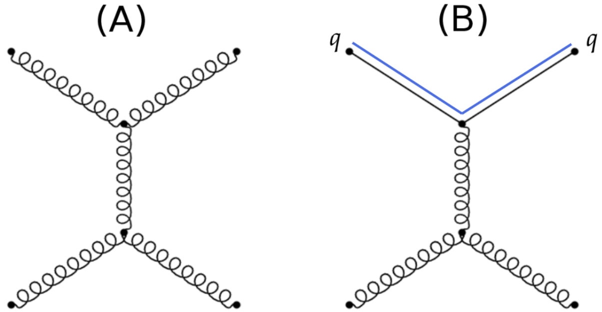
$$S_{i...j}(\vec{p}) = (\mathcal{C}\chi_{i...j})(\vec{p})$$

Feynman Diagrams

At leading-log the diagrams we need to compute are:

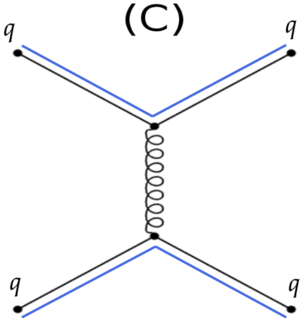


Diagrams (A) and (B) at high μ



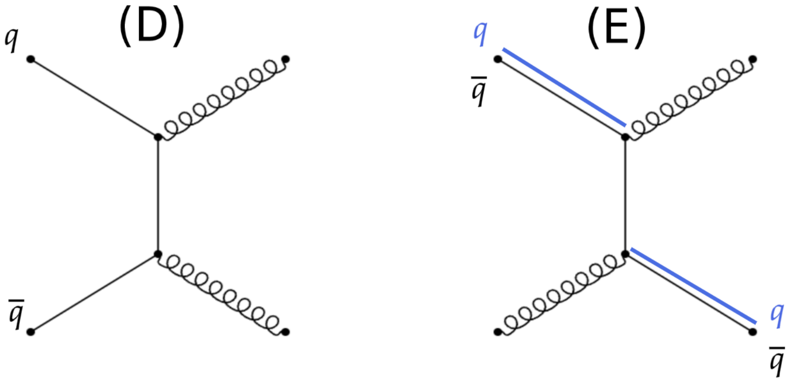
- Number of quarks increases
- Number of gluons remains the same

Diagram (C) at high μ



- Number of quarks increases
- This diagram gets more important
- screening mass increases - $m_D \sim g^2(T^2 + \mu^2) \Rightarrow$ perturbation theory should work better

Diagrams (D) and (E) at high μ



- Number of anti-quarks decreases $\Rightarrow$ diagram (D) is highly suppressed
- Diagram (E) is also highly suppressed $\Rightarrow$ difficult to transfer all momentum from quark to gluon

Variational Solution

To solve this equation we will convert this into an equivalent variational problem, using the fact that the collision operator is hermitian with respect to the inner product:

$$(\mathrm{f}, \mathrm{g}) = \beta^3 \sum_{\mathrm{a}}^{\mathrm{f\bar{f}hc}} \int_{\vec{\mathrm{p}}} \mathrm{f}^{\mathrm{a}}(\vec{\mathrm{p}}) \mathrm{g}^{\mathrm{a}}(\vec{\mathrm{p}})$$

Now, we define the functional:

$$\mathcal{Q}[\chi] = (\chi_{\mathrm{i}\ldots\mathrm{j}}, \mathrm{S}_{\mathrm{i}\ldots\mathrm{j}}) - \frac{1}{2}(\chi_{\mathrm{i}\ldots\mathrm{j}}, \mathcal{C}\chi_{\mathrm{i}\ldots\mathrm{j}})$$

Viscosity comes from maximizing $\mathcal{Q}$:

$$\eta = \frac{2}{15}\mathcal{Q}_{\mathrm{max}}$$

Variational Solution

The source term:

$$(\chi_{i...j}, S_{i...j}) = -\beta^2 \sum_{a}^{f\bar{f}hc} \int_{\vec{p}} f_0^a(p)[1 \pm f_0^a(p)]|\vec{p}|\chi^a(p)$$

The collision integrals:

$$\begin{aligned}(\chi_{i...j}, \mathcal{C}\chi_{i...j}) \quad = \quad & \beta^3 \sum_{abcd}^{f\bar{f}hc} \int_{\vec{p},\vec{k},\vec{p'},\vec{k'}} |M_{cd}^{ab}(p,k,p',k')|^2 (2\pi)^4 \\ & \delta^4(p+k-p'-k') f_0^a(\vec{p}) f_0^b(\vec{k})[1 \pm f_0^c(\vec{p'})][1 \pm f_0^d(\vec{k'})] \\ & [\chi_{i...j}^a(\vec{p}) + \chi_{i...j}^b(\vec{k}) - \chi_{i...j}^c(\vec{p'}) - \chi_{i...j}^d(\vec{k'})]^2\end{aligned}$$

Basis Set

We rewrite our χ as:

$$\begin{aligned}
\chi^{g}(p) &= \sum_{m=1}^{N} a_m \phi^{(m)}(p) \\
\chi^{q}(p) &= \sum_{m=1}^{N} a_{m+N} \phi^{(m)}(p) \\
\chi^{\bar{q}}(p) &= \sum_{m=1}^{N} a_{m+2N} \phi^{(m)}(p)
\end{aligned}$$

The choice of basis set was:

$$\phi^{(m)} = \frac{p(p/T)^{m}}{(1+p/T)^{N-1}} \quad m = 1, ..., N$$

Shear Viscosity

Putting these back into the source integrals, we get the source vector:

$$\left(\mathrm{S_{ij}}, \chi_{\mathrm{ij}}\right) = \sum_{\mathrm{m}} \mathrm{a_m} \tilde{\mathrm{S}}_{\mathrm{m}}$$

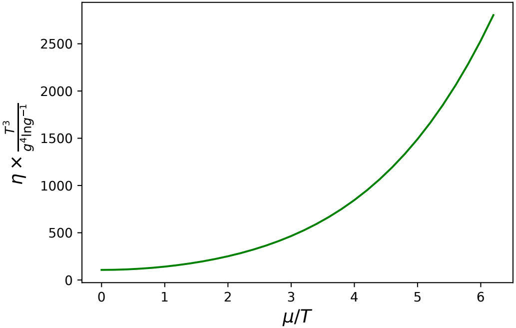
And the scattering matrix:

$$(\chi_{\mathrm{ij}}, \mathrm{C}\chi_{\mathrm{ij}}) = \sum_{\mathrm{m,n}} \mathrm{a_m} \tilde{\mathrm{C}}_{\mathrm{mn}} \mathrm{a_n}$$

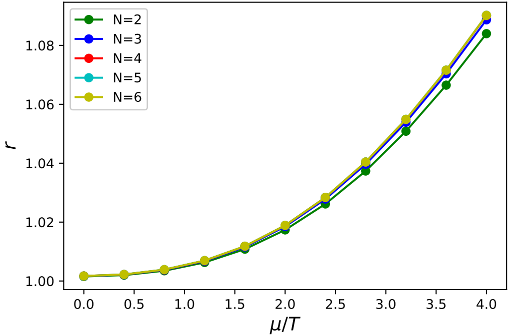
Inserting it in the functional we find that the viscosity is given by:

$$\eta = \frac{1}{15} \tilde{\mathrm{S}}^{\mathrm{T}} \tilde{\mathrm{C}}^{-1} \tilde{\mathrm{S}}$$

Shear Viscosity



Shear viscosity for $N = 6$



r = (N-parameter) / (1-parameter) for $N = 2$ to $N = 6$.

Power Estimation

The momentum-diffusion coefficient $\hat{q}$ for $\mu \gg T$ is:

$$\hat{q} \sim g^4 \mu^2 T\,.$$

For the system to equilibrate momentum has to change as:

$$(\Delta p)^2 \sim \mu^2$$

The time scale is:

$$t \sim \frac{(\Delta p)^2}{\hat{q}} \sim \frac{1}{g^4 T}$$

The shear viscosity is $\eta \sim Pt$:

$$\eta \sim \frac{\mu^4}{g^4 T}$$

Entropy Density

In weakly coupled QCD the entropy density is given by:

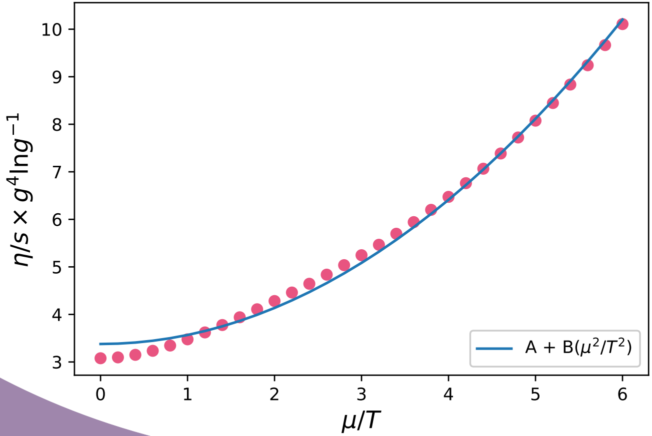
$$\mathrm{s} = 4\frac{79\pi^2}{90}\mathrm{T}^3 + 6\mu^2\mathrm{T}$$

$\Rightarrow$ the entropy density scales as $\mathrm{s} \sim \mu^2\mathrm{T}$

η/s

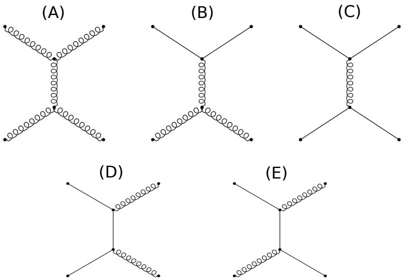
$$\Rightarrow \eta/\mathrm{s} \sim \mu^2/(\mathrm{g}^4\mathrm{T}^2)$$

$$\mathrm{g}^4 \log(\mathrm{g}^{-1})\eta/\mathrm{s} = \mathrm{A} + \mathrm{B}\mu^2/\mathrm{T}^2$$



η/s

- Small μ/T: $\eta/s \rightarrow$ constant
- Large μ/T: $\eta/s \rightarrow$ parabola
  Fermions dominate s, the scatterings and the source term
  Diagram C $\rightarrow$ most important
  Diagrams A and B $\rightarrow$ not important
  Diagrams D and E $\rightarrow$ highly suppressed
- Around $\mu/T = 2$: $\eta/s \rightarrow$ more complicated.
  Quarks and gluons play important part



Summary

For high μ:

- we can obtain η for hot dense QCD at leading log
- the relation $\eta/\mathrm{s} \sim \mu^2/(\mathrm{g}^4\mathrm{T}^2)$ is a good fit
- Fermions dominate both scatterings and the source term
- Next step: leading order calculations

Thank you!

Backup Slide

Shear viscosity to enthaply density ratio: