Constraining the strength of $U(1)$- symmetry breaking using a non-local NJL model

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Outline

- Introduction
- Motivation
- Methodology
- Results
- Conclusion

Introduction

- With the help of the NJL model one can study the chiral properties of QCD. [Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345(1961); 124, 246(1961)]
- In NJL model, the chiral symmetry is spontaneously broken by the chiral condensate.
- We observe magnetic catalysis (MC) and inverse magnetic catalysis (IMC) in presence of a magnetic field. [G. S. Bali et al., Phys. Rev. D 86, 071502]
- One can consider non-local interaction in NJL model which is more realistic as it captures some aspects of the asymptotic freedom of QCD through the non-local form factor. [V. P. Pagura et al., Phys. Rev. D 95, 034013]

NJL model with different $U(1)_A$ breaking strength

- The NJL Lagrangian [M. Frank, M Buballa, M. Oertel, Phys. Lett. B 562 (2003) 221-226]

$$\mathcal{L}_{\mathrm{NJL}} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2$$

$$\mathcal{L}_0 = \bar{\psi}\left(i\partial\!\!\!/ - m\right)\psi$$

$$\mathcal{L}_1 = G_1\left\{(\bar{\psi}\psi)^2 + (\bar{\psi}\vec{\tau}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right\}$$

$$\mathcal{L}_2 = G_2\left\{(\bar{\psi}\psi)^2 - (\bar{\psi}\vec{\tau}\psi)^2 - (\bar{\psi}i\gamma_5\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right\}$$

- $\mathcal{L}_2$ explicitly breaks $U(1)_A$.
- Symmetry only allows the $\langle\bar{\psi}\psi\rangle$ condensate, which depends on $(G_1 + G_2)$.
- With μ_I or magnetic field as the $SU(2)$ symmetry is broken one can have $\langle\bar{\psi}\tau_3\psi\rangle$ which has a $(G_1 - G_2)$ dependence.
- G_1 and G_2 can be parametrized as $G_1 = (1-c)G_0/2$ and $G_2 = cG_0/2$
- $c = 1/2$ corresponds to the standard NJL model.

Motivation

- Standard NJL model assume the strength of the axial U(1) symmetry breaking ’t Hooft determinant term to be equal to that of the axial U(1) symmetric term, even in the non-local one.

- Our main goals:

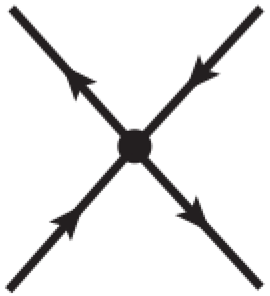
a) To constrain the strength of the $U(1)_A$ symmetry breaking interactions using lattice QCD results.

b) And once it is fixed, study the effect of T and eB on it.

c) Further test the constrained value by calculating some other known observable.

Non-local NJL model with arbitrary $U(1)$- breaking strength

- The NJL Lagrangian with non-local interaction



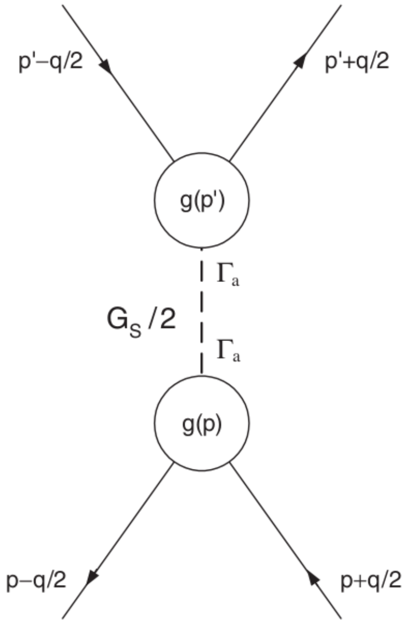
$$\mathcal{L} = \bar{\psi}\left(i\partial\!\!\!/ - m\right)\psi + \mathcal{L}_1 + \mathcal{L}_2.$$

- $\mathcal{L}_1$ respects $U(1)_A$ but $\mathcal{L}_2$ does not,

$$\mathcal{L}_1 = G_1\left\{j_a(x)j_a(x) + j_b(x)j_b(x)\right\}$$

$$\mathcal{L}_2 = G_2\left\{j_a(x)j_a(x) - j_b(x)j_b(x)\right\}$$

  with the above definition of $j_{a/b}(x)$ where $\Gamma_a = (\mathbb{I}, i\gamma_5\vec{\tau})$ and $\Gamma_b = (i\gamma_5, \vec{\tau})$



- $j_{a/b}(x)$ are the non-local currents, given by

$$j_{a/b}(x) = \int d^4z\, \mathcal{G}(z)\bar{\psi}\left(x + \tfrac{z}{2}\right)\Gamma_{a/b}\psi(x - \tfrac{z}{2}),$$

$$\Gamma_a = (\mathbb{I}, i\gamma_5\vec{\tau}) \text{ and } \Gamma_b = (i\gamma_5, \vec{\tau})$$

- $\mathcal{G}(z)$ is the non-locality form factor.
- Symmetries

$$SU(2)_V \times SU(2)_A \times U(1)_V$$

Non-local NJL model with arbitrary $U(1)_A$ breaking strength

- With non-zero mean field $\langle\bar{\psi}\psi\rangle$, the free energy becomes

$$\frac{\Omega_{\mathrm{MF}}}{V^{(4)}} = \frac{\sigma^2}{2G_0} - 2N_c \sum_f \int \frac{d^4p}{(2\pi)^4} \log\left[p^2 + M_f^2(p)\right]$$

  with $M(p) = m + g(p^2)\sigma$

- With Lorentz symmetry the Fourier transformation $\mathcal{G}(z)$ can only depends on p^2, which is written as $g(p^2)$.

- We have consider $g(p^2)$ to be Gaussian in nature

$$g(p^2) = \exp[-p^2/\Lambda^2]$$

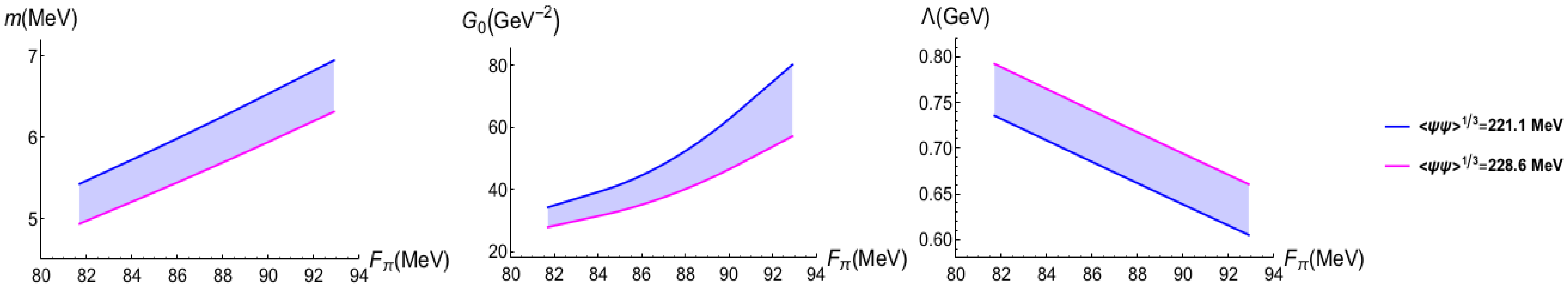
Parameter dependence

- Condensate $\langle\bar{\psi}\psi\rangle$ and F_π taken from LQCD calculation with $m_\pi = 135$ MeV. [H. Fukaya et al.(JLQCD), Phys. Rev. D 77, 074503 (2008)]

$$\langle\bar{\psi}\psi\rangle^{1/3}|_{\mu=2\ \mathrm{GeV}} = 240(4)\ \mathrm{MeV}\ \text{and}$$
$$F_\pi = 87.3(5.6)\ \mathrm{MeV}.$$

- We have used perturbative RG running to obtain the condensate at 1 GeV following the L. Giusti's work. [L.Giusti et al., Nuclear Physics B 538 (1999) 249-277]

$$\langle\bar{\psi}\psi\rangle^{1/3}|_{\mu=1\ \mathrm{GeV}} = 224.8(3.7)\ \mathrm{MeV}.$$



Parameter dependence

- Central and the four corner parameter sets:

	$\langle \bar{\psi}_f \psi_f \rangle^{1/3}$(MeV)	m_π(MeV)	F_π(MeV)	$F_{\pi,0}$(MeV)	m(MeV)	G_0(GeV^{-2})	Λ(MeV)
Parameter Set CC	224.8	135	87.3	84.25	5.87	43.34	697.22
Parameter Set HH	228.6	135	92.9	90.63	6.31	57.15	660.46
Parameter Set HL	228.6	135	81.7	77.04	4.94	27.90	792.22
Parameter Set LH	221.1	135	92.9	91.00	6.94	80.26	605.05
Parameter Set LL	221.1	135	81.7	77.61	5.42	34.32	735.38

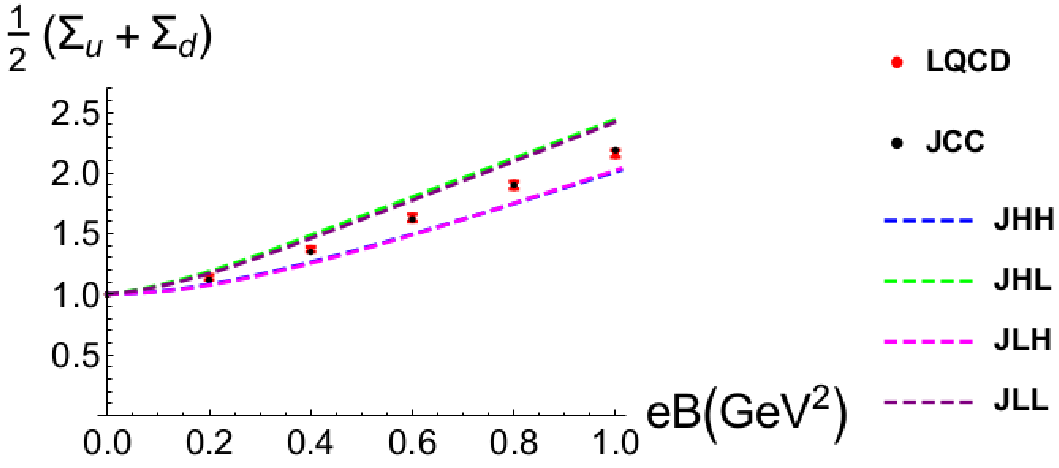
- We will see that only some of the parameter sets will eventually survive and can give desired results.

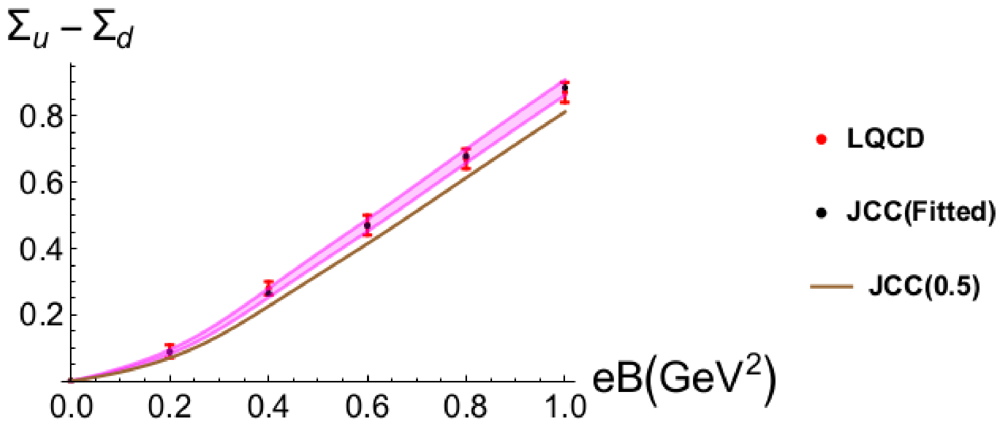
a) Can produce the IMC effect

b) And can generate phenomenologically reasonable mass for $\boldsymbol{\eta}^*$, which is an isoscalar pseudoscalar having mass roughly of the order of 400 MeV.

Fitting of c using LQCD data

- As mentioned earlier the average condensate does not depend on c.

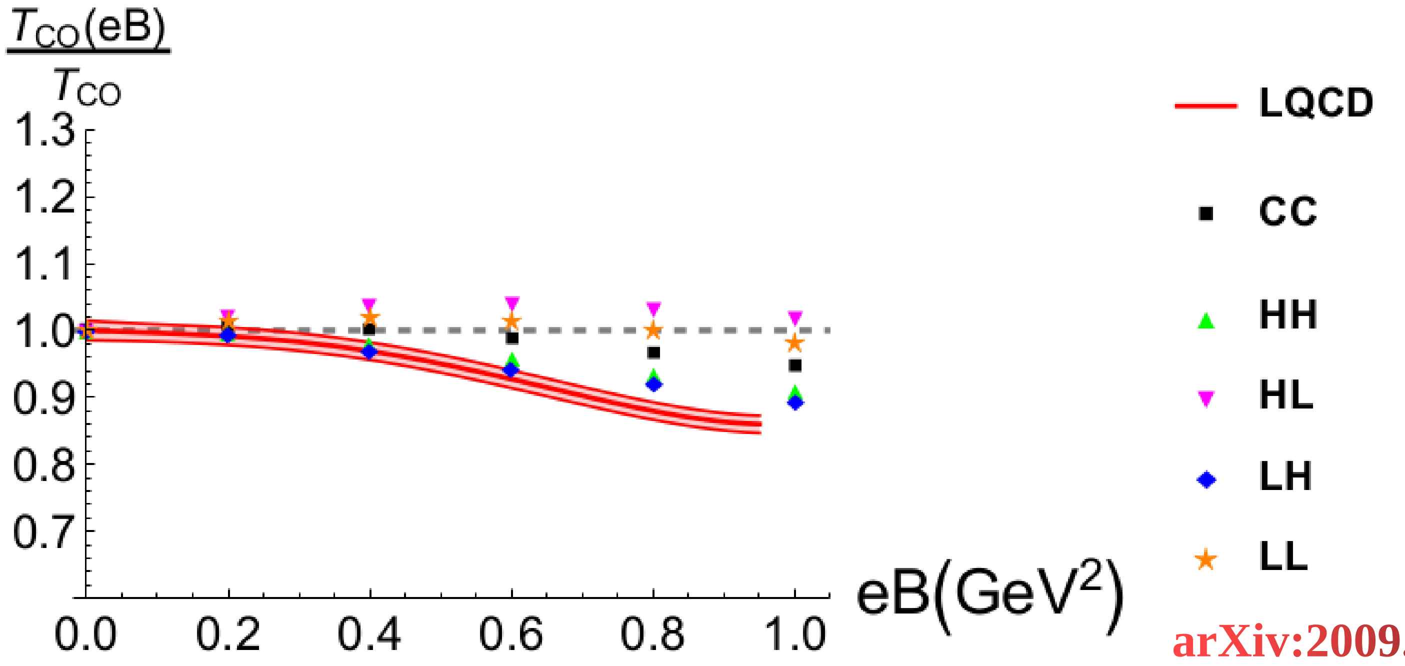




	c	χ^2 per DoF
Parameter Set CC	0.276 ± 0.068	0.211
Parameter Set HH	0.044 ± 0.079	0.149
Parameter Set HL	0.374 ± 0.051	0.290
Parameter Set LH	0.149 ± 0.103	0.634
Parameter Set LL	0.465 ± 0.062	0.551

QCD phase diagram in $T - eB$ plane

- Transition temperature does not depend on c as the condensate is almost independent of c.



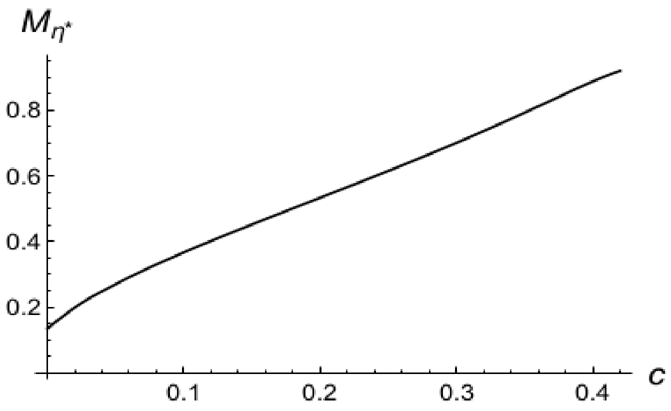
arXiv:2009.13563

- Low condensate value, $\langle\bar{\psi}\psi\rangle$ and high F_π produce a stronger IMC effect around the crossover temperature.

- HH and LH parameter sets reproduce the best IMC effect compared to LQCD.

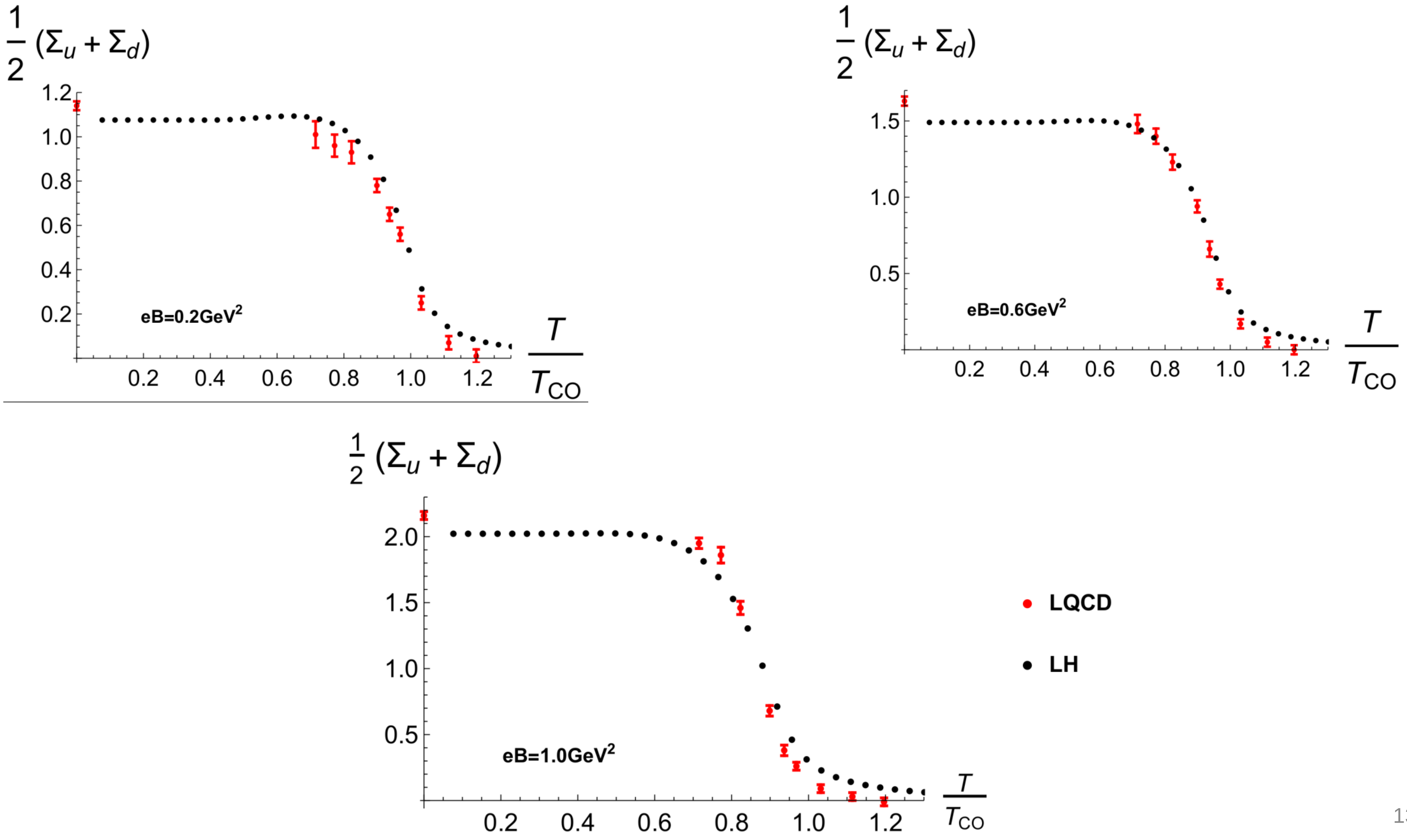
Constraint from η^*

- To choose between HH and LH we look into η^* phenomenology which is associated with the pseudoscalar-isoscalar channel.
- η^* mass is very sensitive to c. At $c = 0$, π^0 and η^* become degenerate.
- Considering η^* to be admixture of η and η' of three flavor, we can impose that $M_{\eta^*} > 400$ MeV.

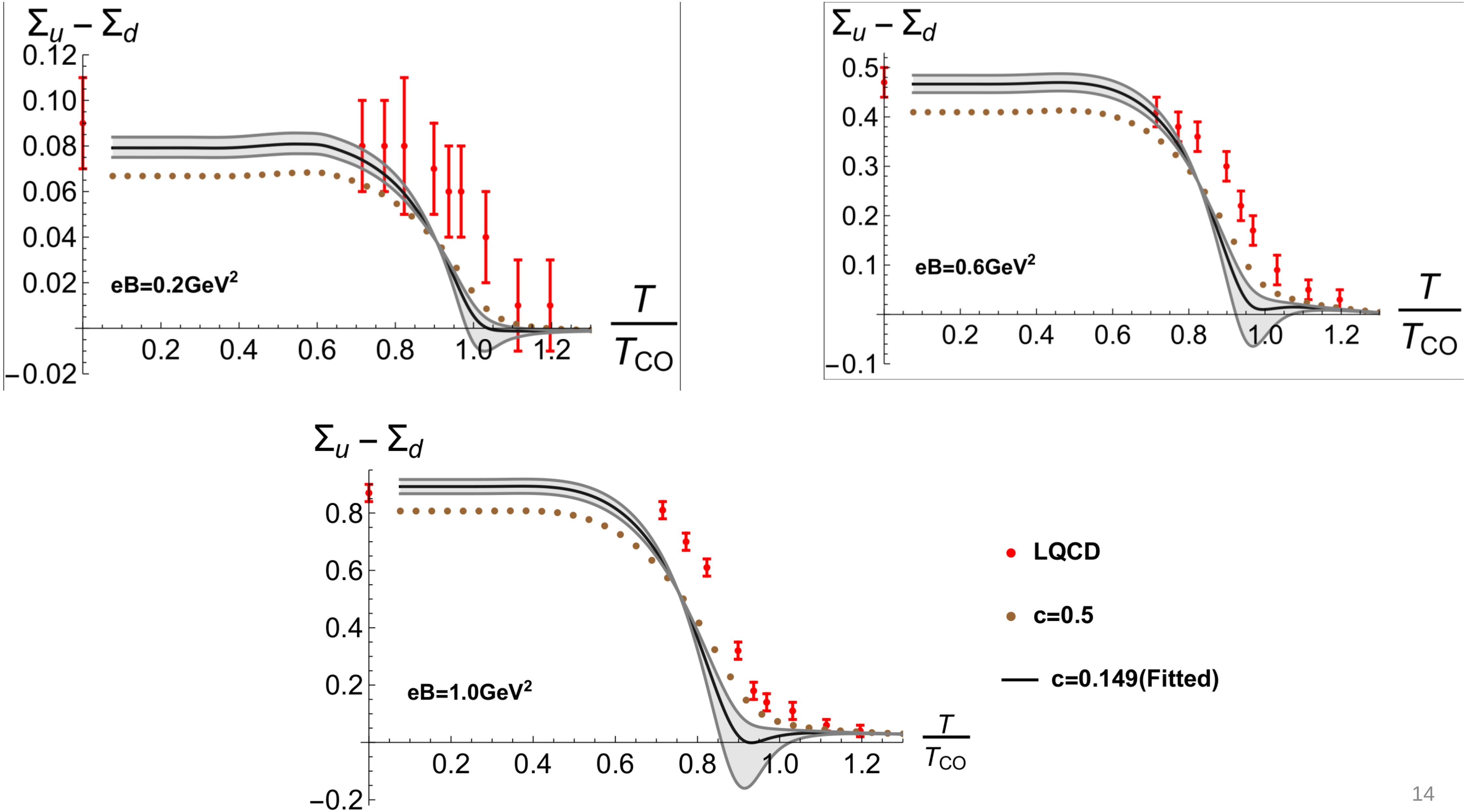


- The above constrain set the lower bound to be $c \geq 0.12$.
- We eliminate HH as the fitted c is less than the lower bound.

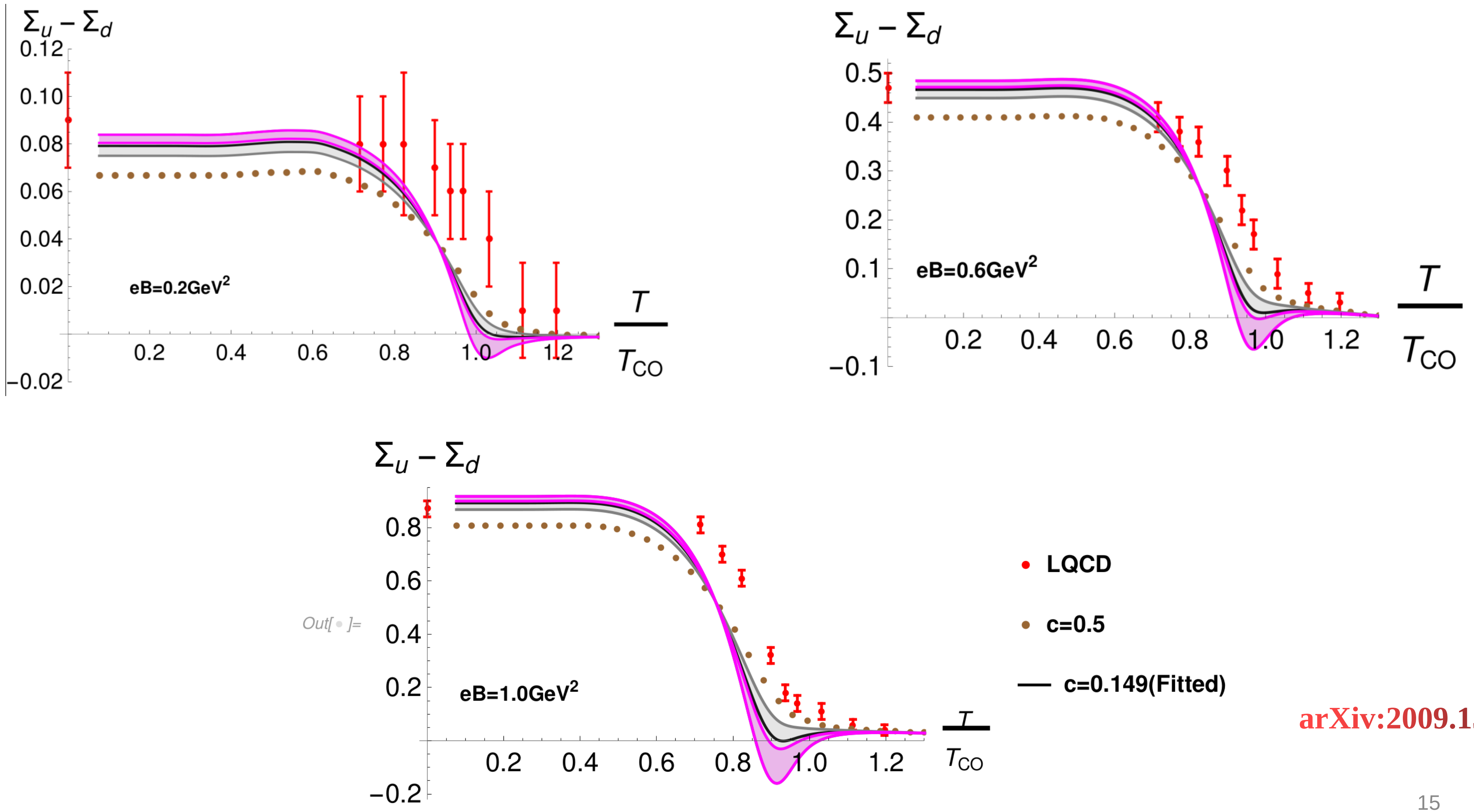
Condensate average at finite temperature



Condensate difference at finite temperature



Condensate difference at finite temperature with constraint from M_{η^*} >400 MeV



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Topological susceptibility

- Topological susceptibility(χ_t) can be formally related to the mass of the axion field.[S. Weinberg, Phys. Rev. Lett. 40, 223 (1978)]
- The topological term $\frac{\theta g^2}{32\pi^2}\mathcal{F}\tilde{\mathcal{F}}$ breaks the CP symmetry of strong interaction.
- As the dynamical axion is considered to be a possible solution to strong CP problem, θ can be related to the axion fields, $\theta = a/f_a$.

Topological susceptibility

- With a chiral rotation of the quark fields by an angle $\theta/4$ one obtain

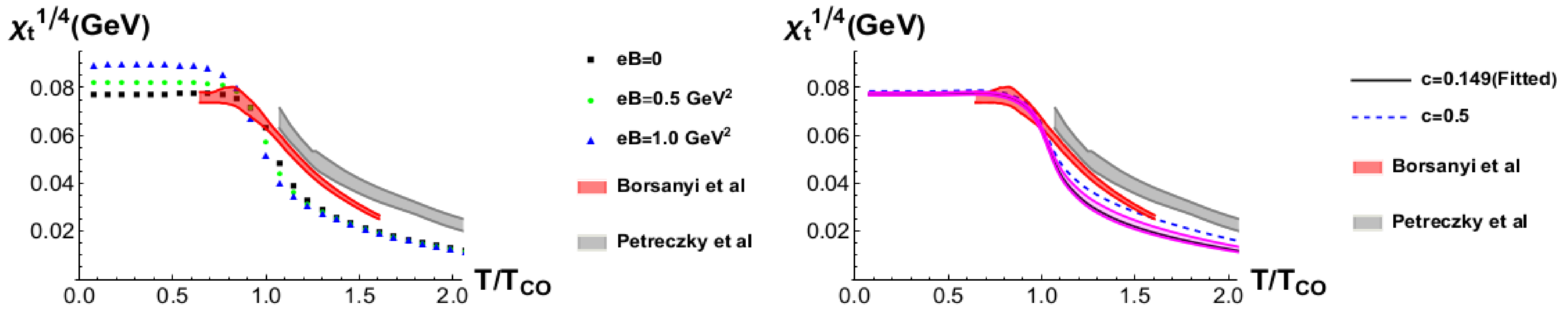
$$\begin{aligned}\mathcal{L}_2 \to \mathcal{L}_a &= G_2 \left[\cos\theta \left\{ j_a(x) j_a(x) - \tilde{j}_a(x)\tilde{j}_a(x) \right\} \right. \\ &\left. +2\sin\theta \left\{ j_0(x)\tilde{j}_0(x) - j_i(x)\tilde{j}_i(x) \right\} \right]\end{aligned}$$

- $\Omega(T, eB, \theta)$ being the free energy with these modification, the topological susceptibility is

$$\chi_t = \left. \frac{d^2\Omega(T,eB,\theta)}{d\theta^2} \right|_{\theta=0} .$$

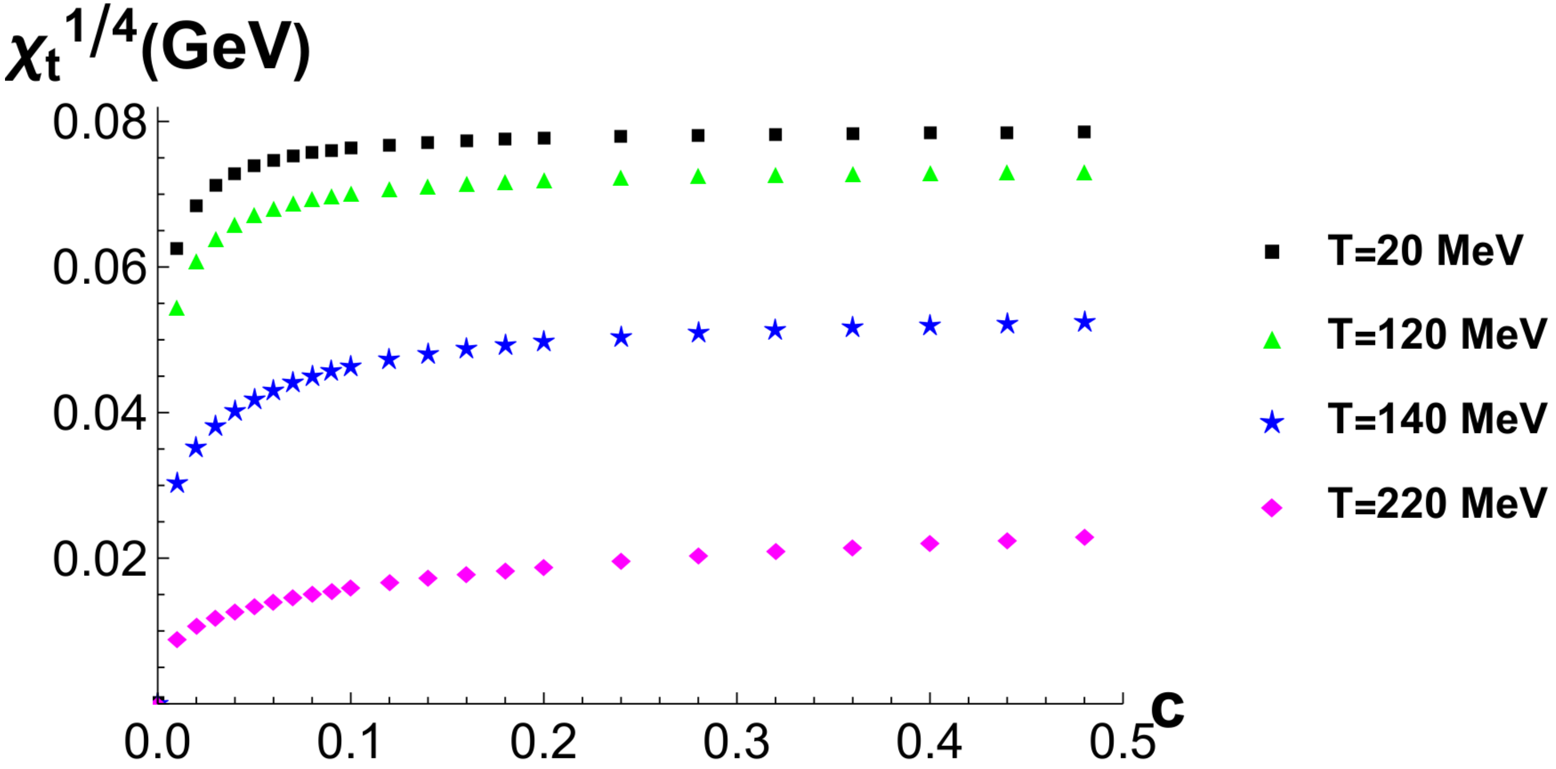
Topological susceptibility

- Topological susceptibility for the LH parameter set as a funtion of scaled T. [MSA, Chowdhury Aminul Islam, Rishi Sharma, Phys. Rev. D 104 (2021) 11, 114026, S. Borsanyi et al., Nature 539, 69 (2016), P. Petreczky, H.-P. Schadler, and S. Sharma, Phys. Lett. B 762, 498 (2016)]



- χ_t for different magnetic fields at fitted c is compared in the left figure.
- The effect of c has been explored in the right figure. The magenta band corresponds to the uncirtainty in c.

Topological susceptibility as a function of c



Upshots

- Consistence lattice data has been used to constrain the strength of the axial U(1) symmetry breaking strength (c).

- Low condensate value, $\langle\bar{\psi}\psi\rangle$ and high F_{π} produce a stronger IMC effect around the crossover temperature.

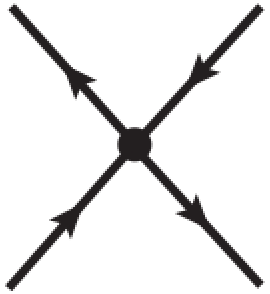
- Our estimated value for c, which is done for the first time to the best of our knowledge, is **0.149** within some error bars.

- We further test our fitted value of c by calculating topological susceptibility.

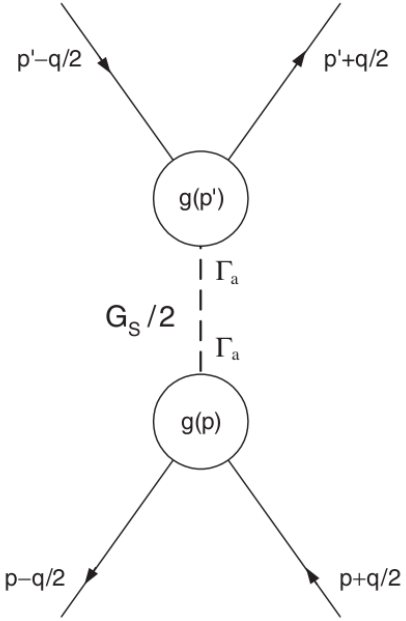
Thank You

Difference between local and non-local NJL models

- Local NJL:



- Non-local NJL:



Non-local NJL model in presence of eB

- For the non-local interaction the currents should transform as

$$j_{a/b}(x) \rightarrow$$
$$\int d^4z\ \mathcal{G}(z)\bar{\psi}\left(x+\frac{z}{2}\right) W^\dagger(x+z/2,x)\Gamma_a W(x,x-z/2)\psi(x-\frac{z}{2})$$

$$W(s,t) = P\ \exp\left[-iQ\int_s^t dr_\mu A_\mu(r)\right]$$

- The bosonized action

$$S_{\rm bos} = -\ln\det(\mathcal{D}) + \int \mathrm{d}^4\mathrm{x}\left[\frac{\sigma^2(\mathrm{x})}{2\mathrm{G}_0} + \frac{\Delta\sigma^2(\mathrm{x})}{2(1-2\mathrm{c})\mathrm{G}_0}\right]$$

- In Lorentz gauge the inverse of fermionic propagator is given by

$$\mathcal{D}_{\rm MF}(x,x') = \delta^{(4)}(x-x')(-i\partial\!\!\!/ - QBx_1\gamma_2 + m) + \mathcal{G}(x-x')$$
$$*(\sigma+\tau_3\Delta\sigma)\exp\left[\frac{i}{2}QB(x_2-x_2')(x_1+x_1')\right].$$

- Using Ritus eigenfunction one obtain the Fourier transform of the above.