Quarkonium transport in weakly and strongly coupled plasmas

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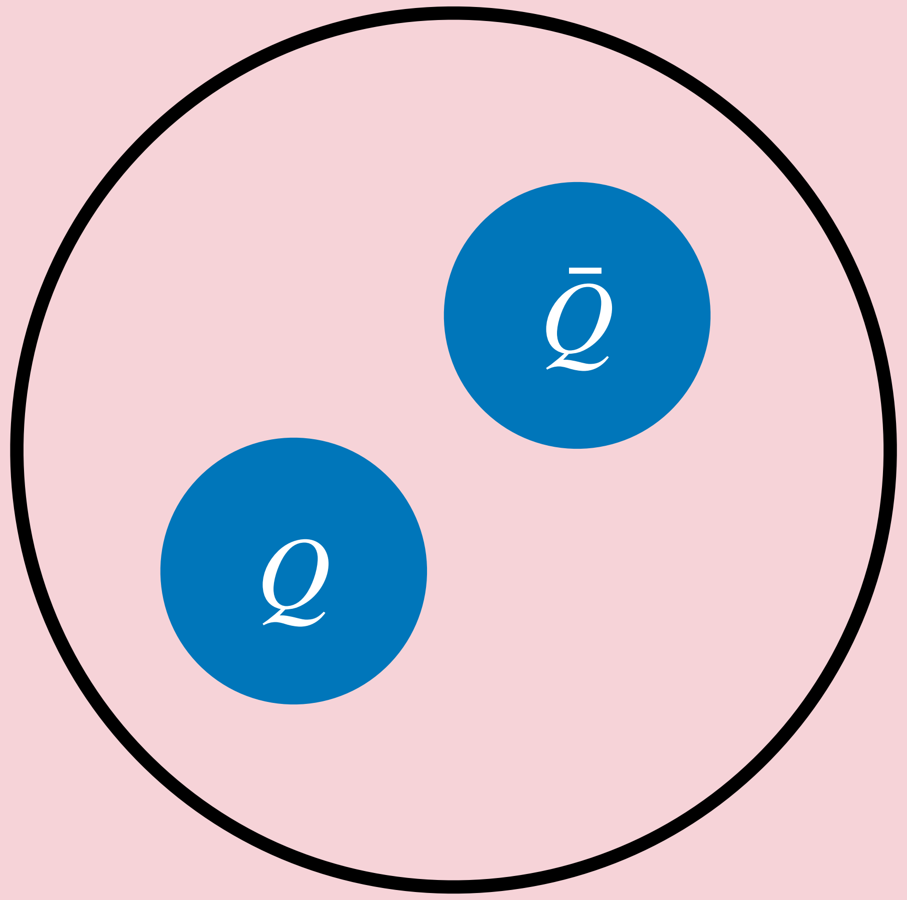
in collaboration with Xiaojun Yao (MIT) and Govert Nijs (MIT)
based on 2107.03945, 2205.04477, 22XX.XXXXX

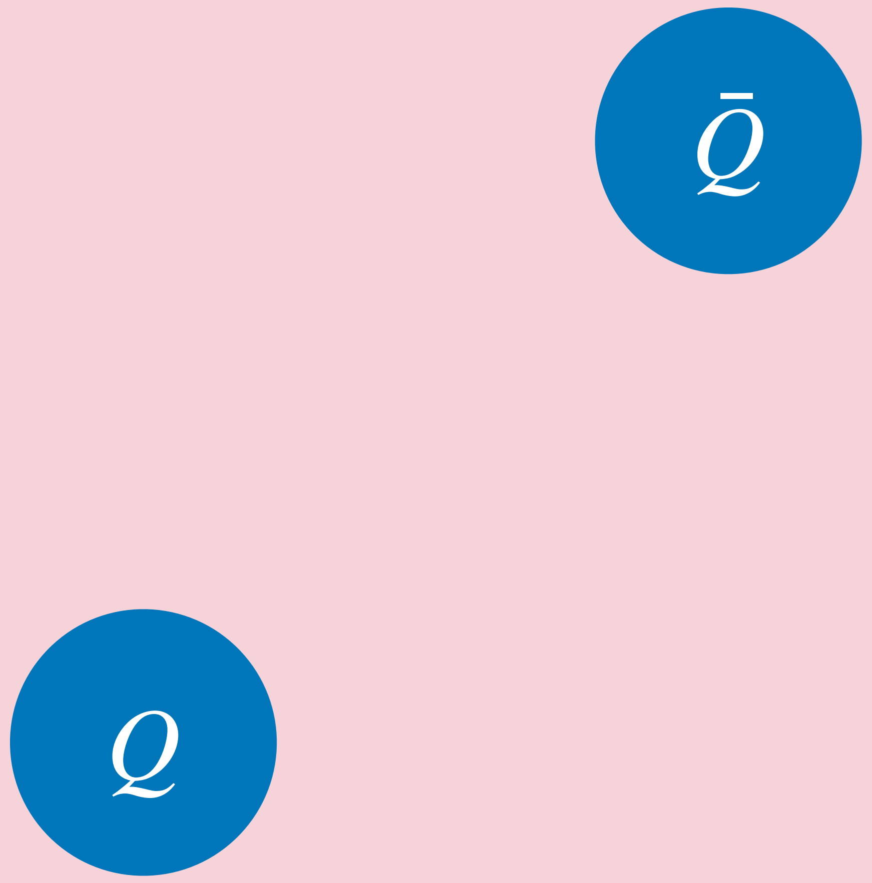
$M \gg Mv \gg Mv^2$

M: heavy quark mass
v: typical relative speed

Quarkonium in medium



color singlet;
bound state



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$T > 0$

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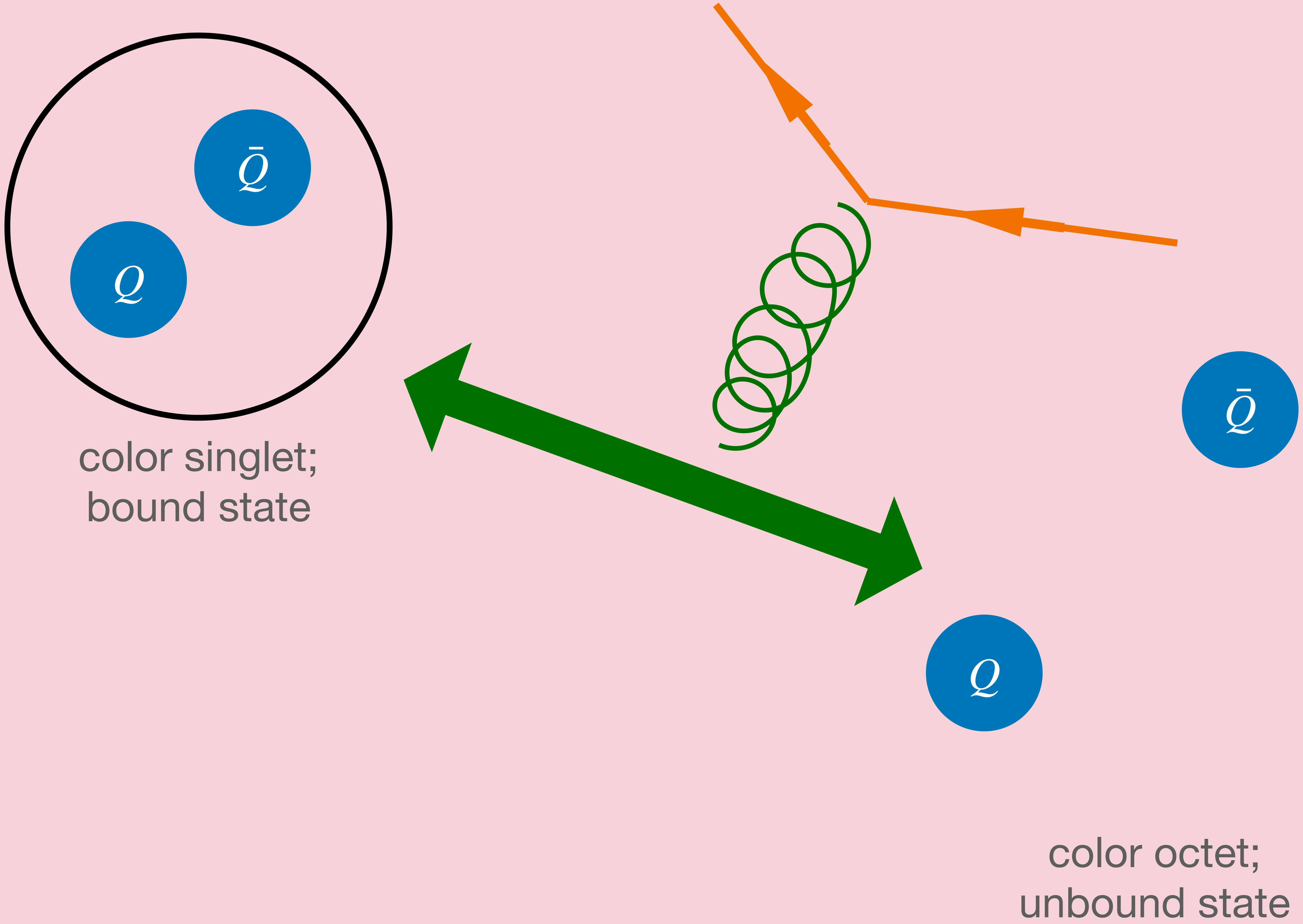

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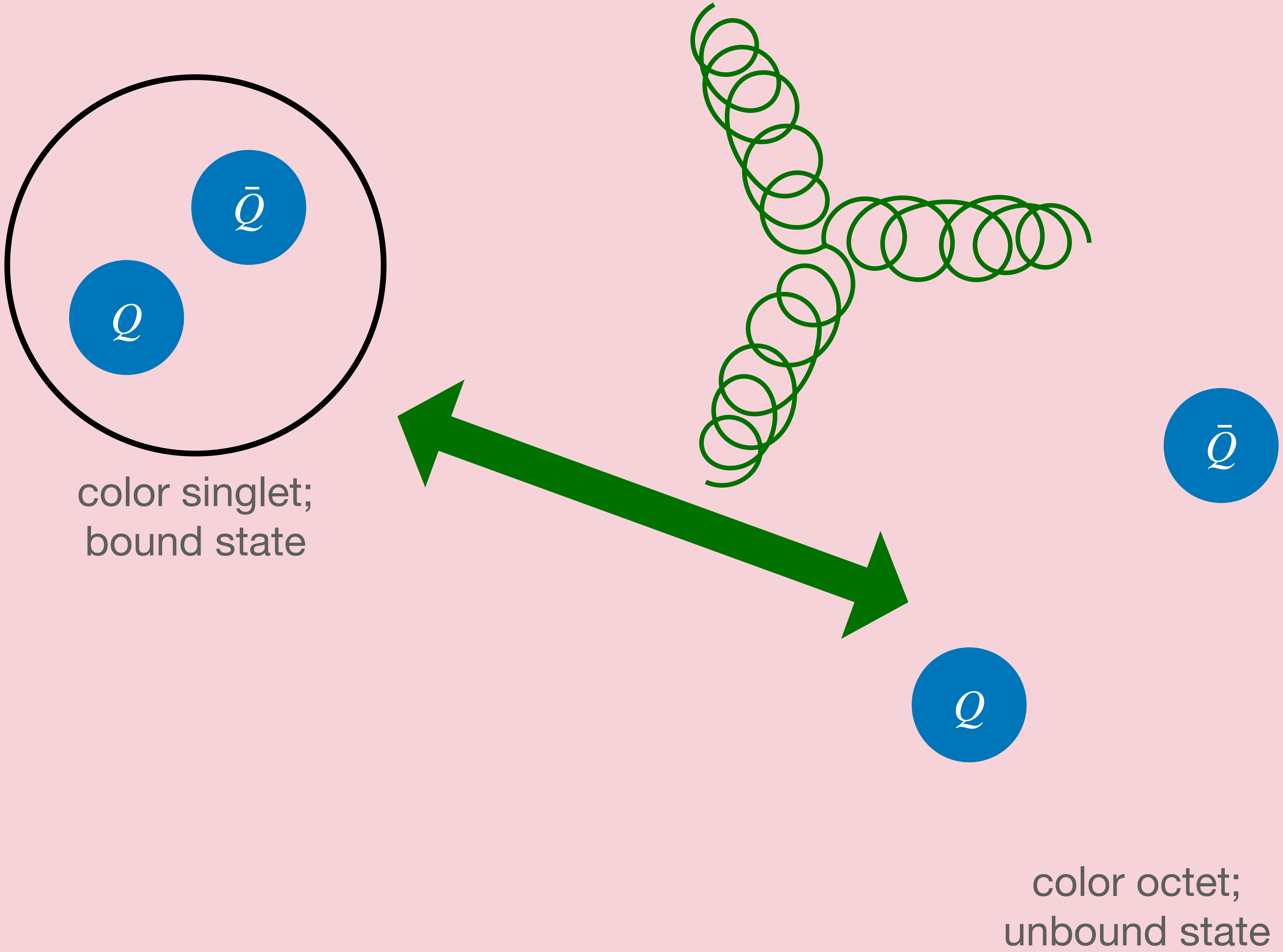

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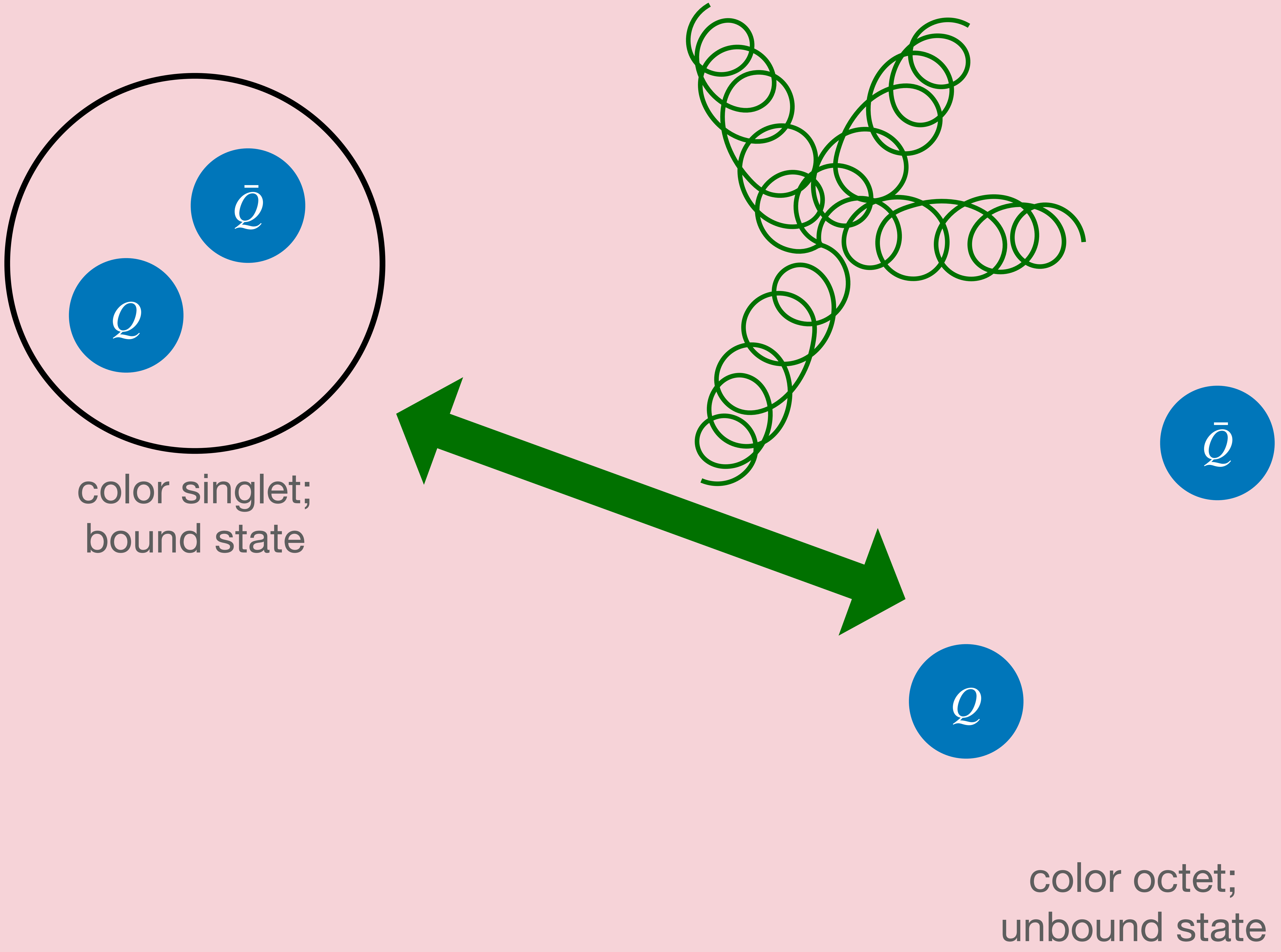

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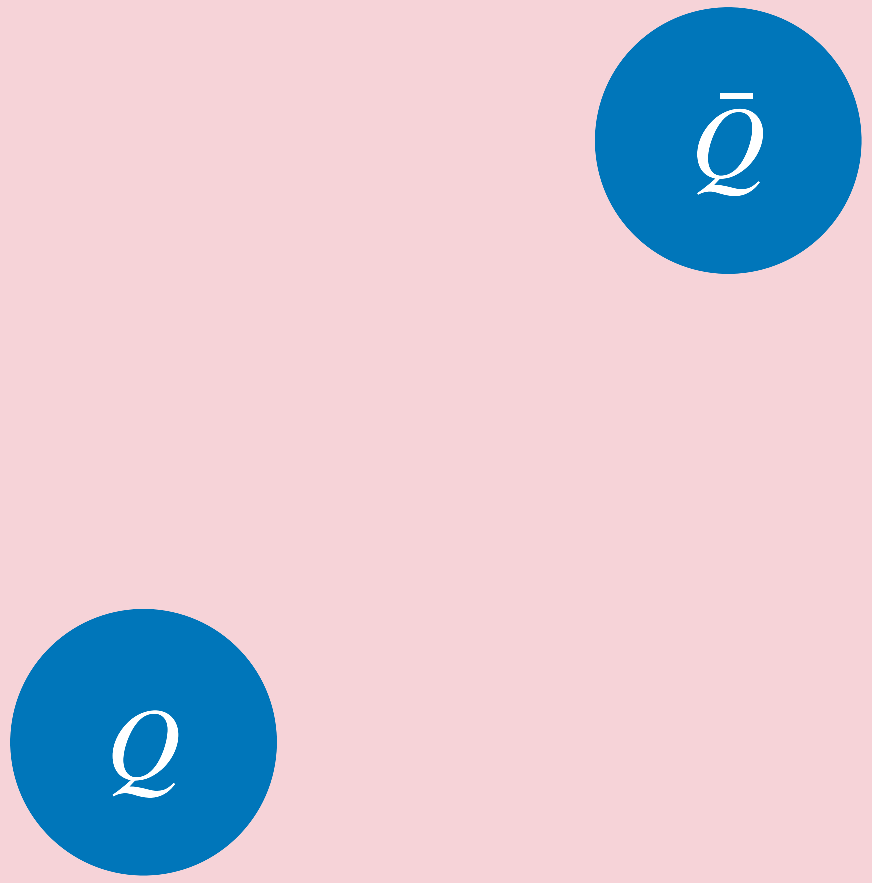
Quarkonium in medium

At high T, quarkonium "melts" because the medium screens the interactions between heavy quarks (Matsui & Satz 1986)

$$Q\bar{Q} \text{ melts if } r \sim \frac{1}{Mv} \gg \frac{1}{T}$$



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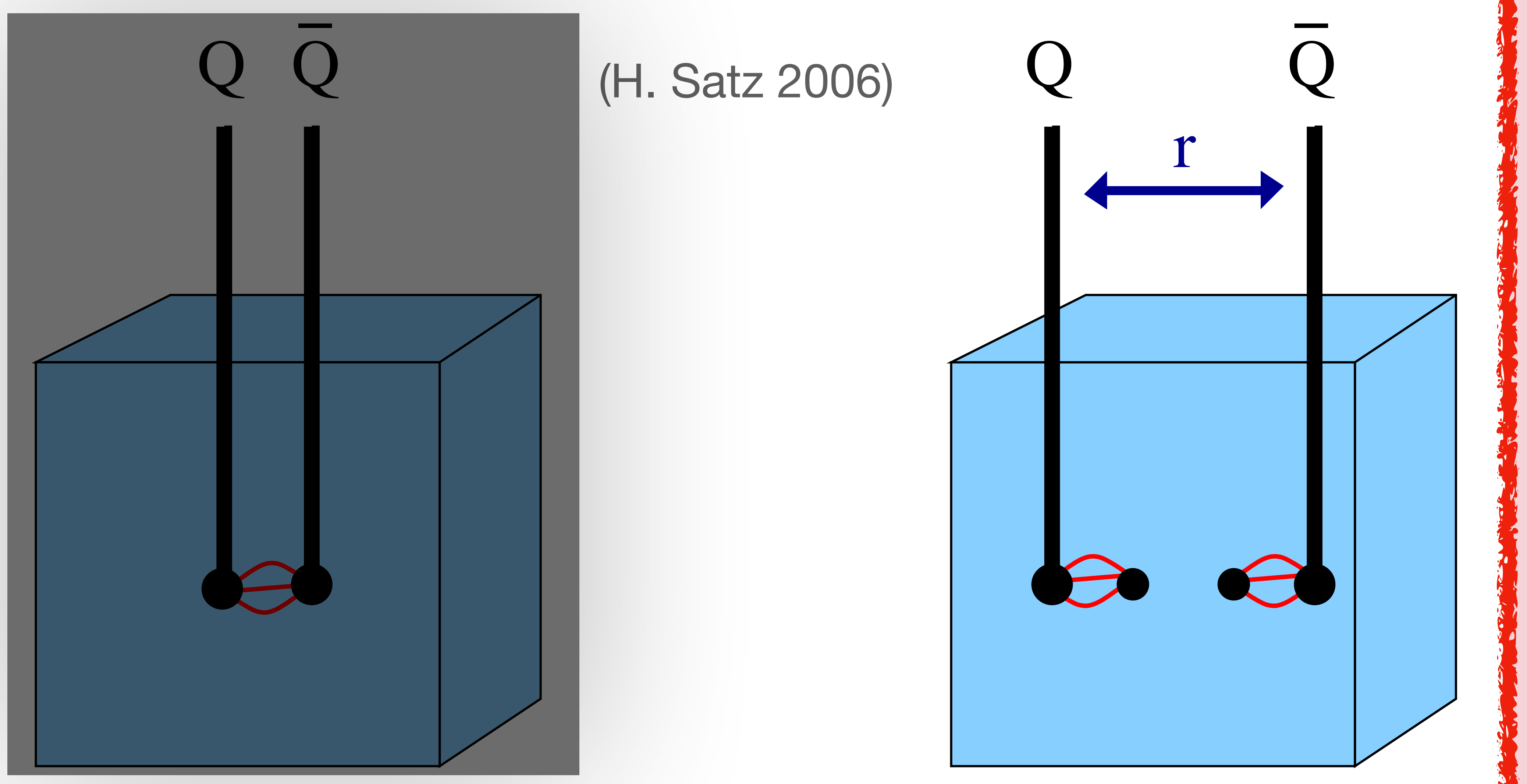

(H. Satz 2006)

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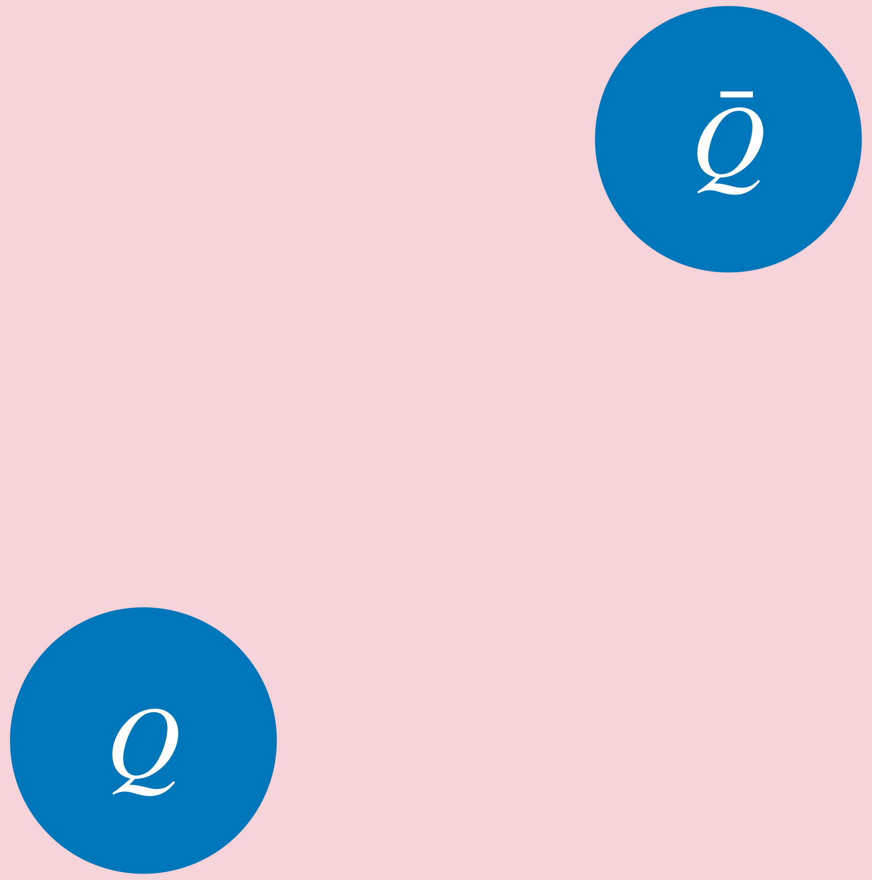

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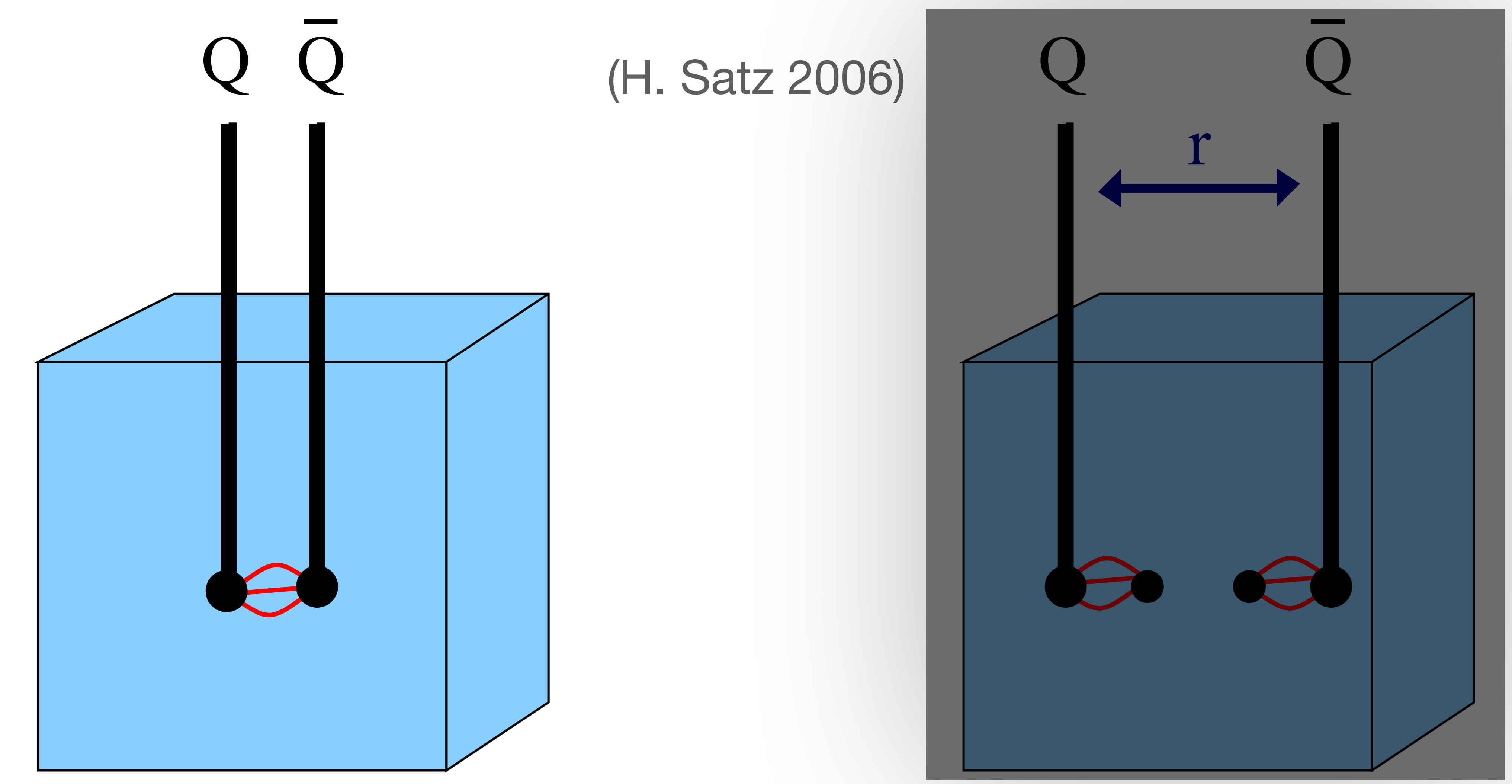

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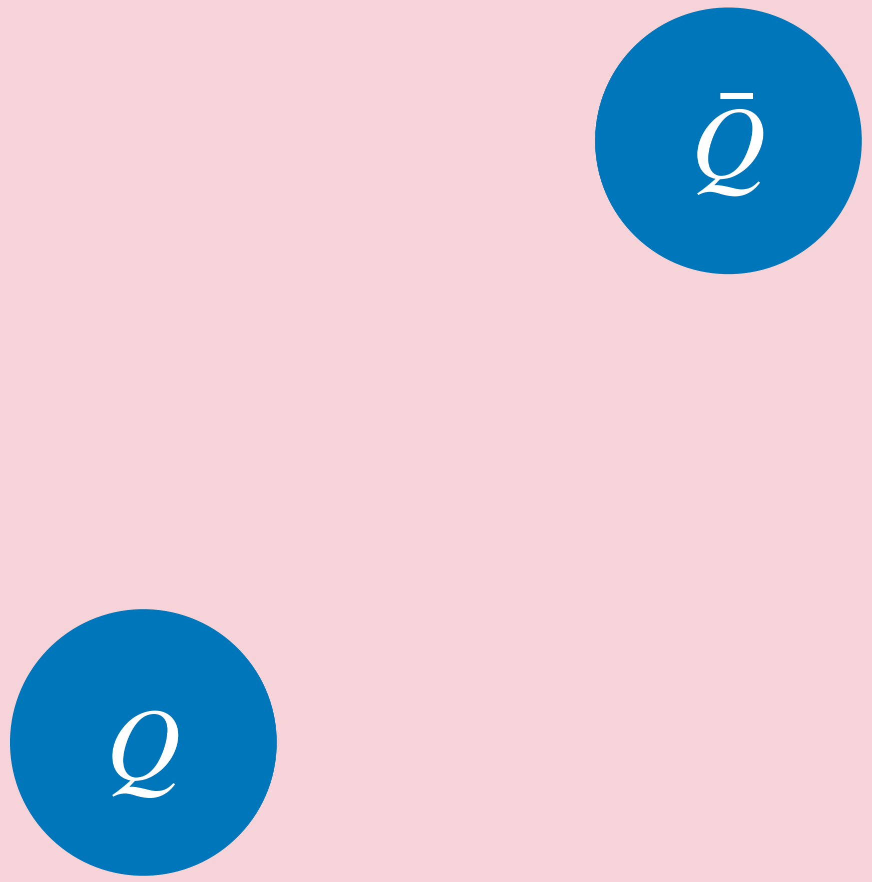

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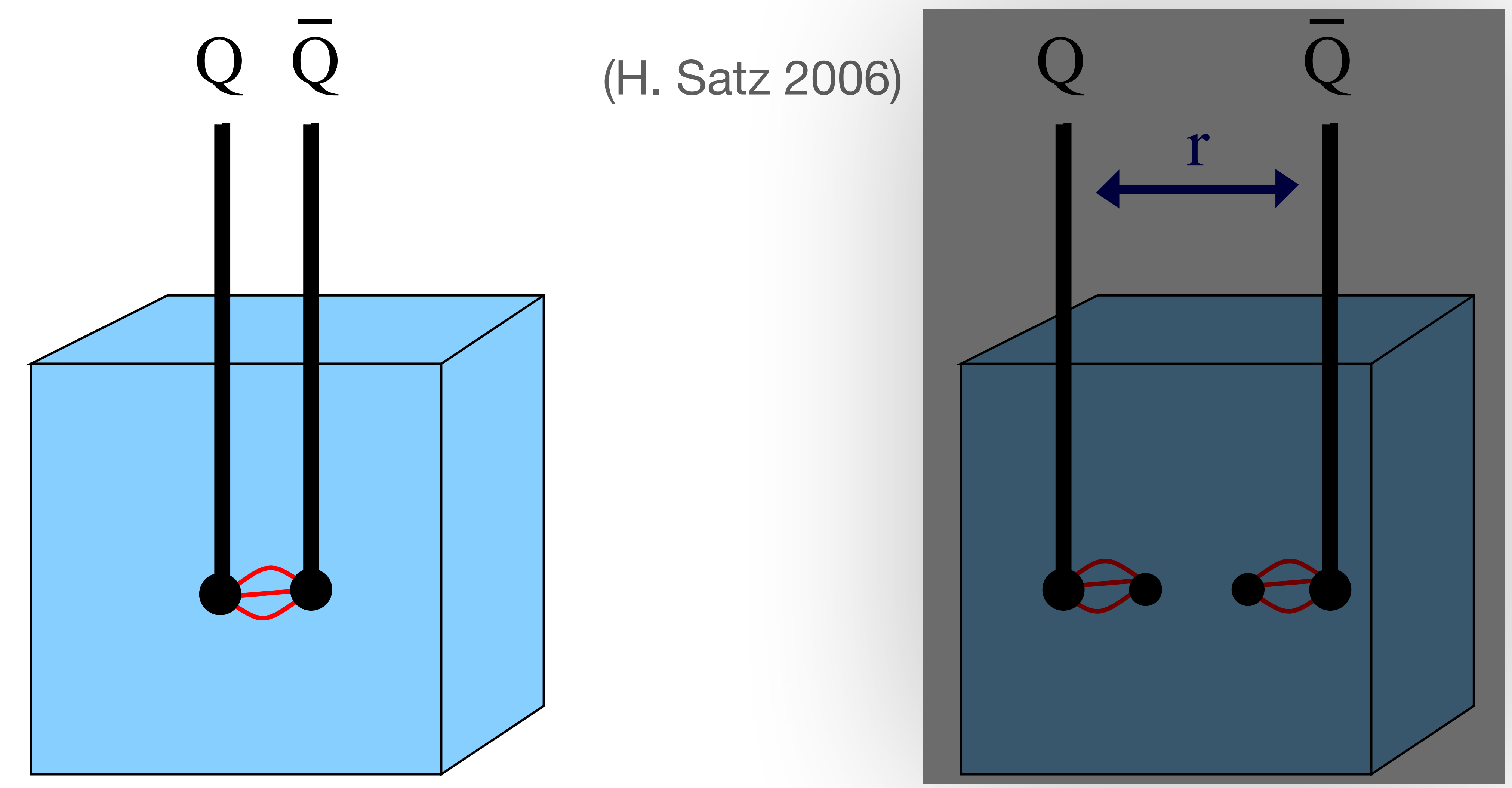
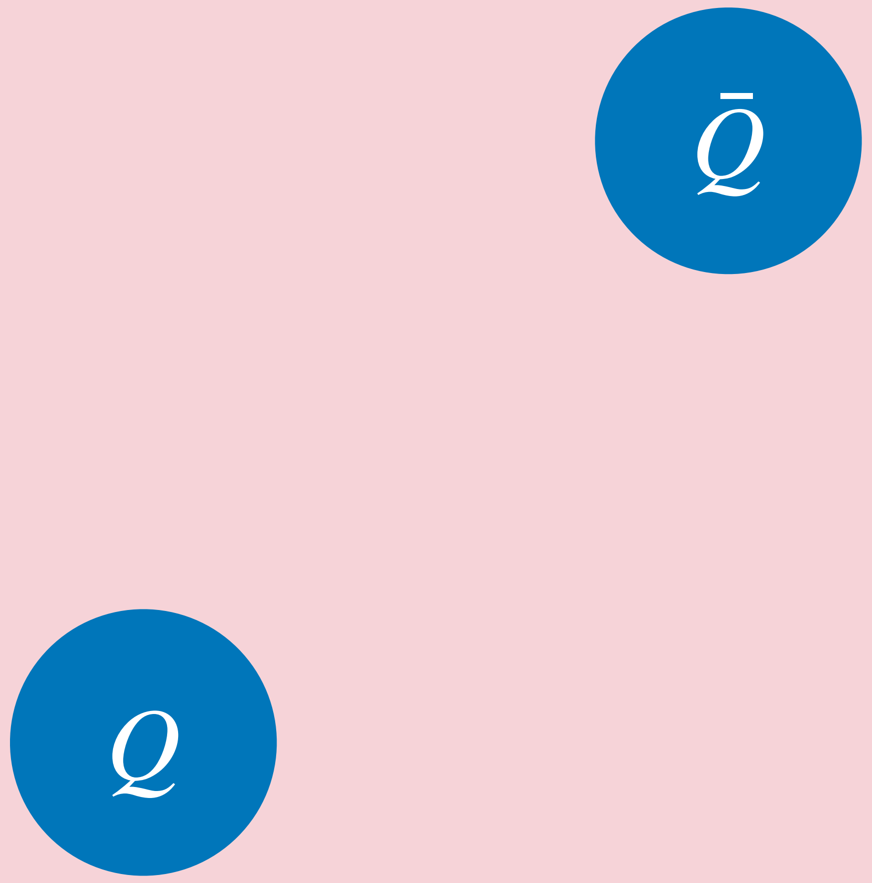


$Q\bar{Q}$ can bind if $r \sim \dfrac{1}{Mv} < \dfrac{1}{T}$

$\Longrightarrow$ most of quarkonium starts to form when $Mv \gtrsim T$



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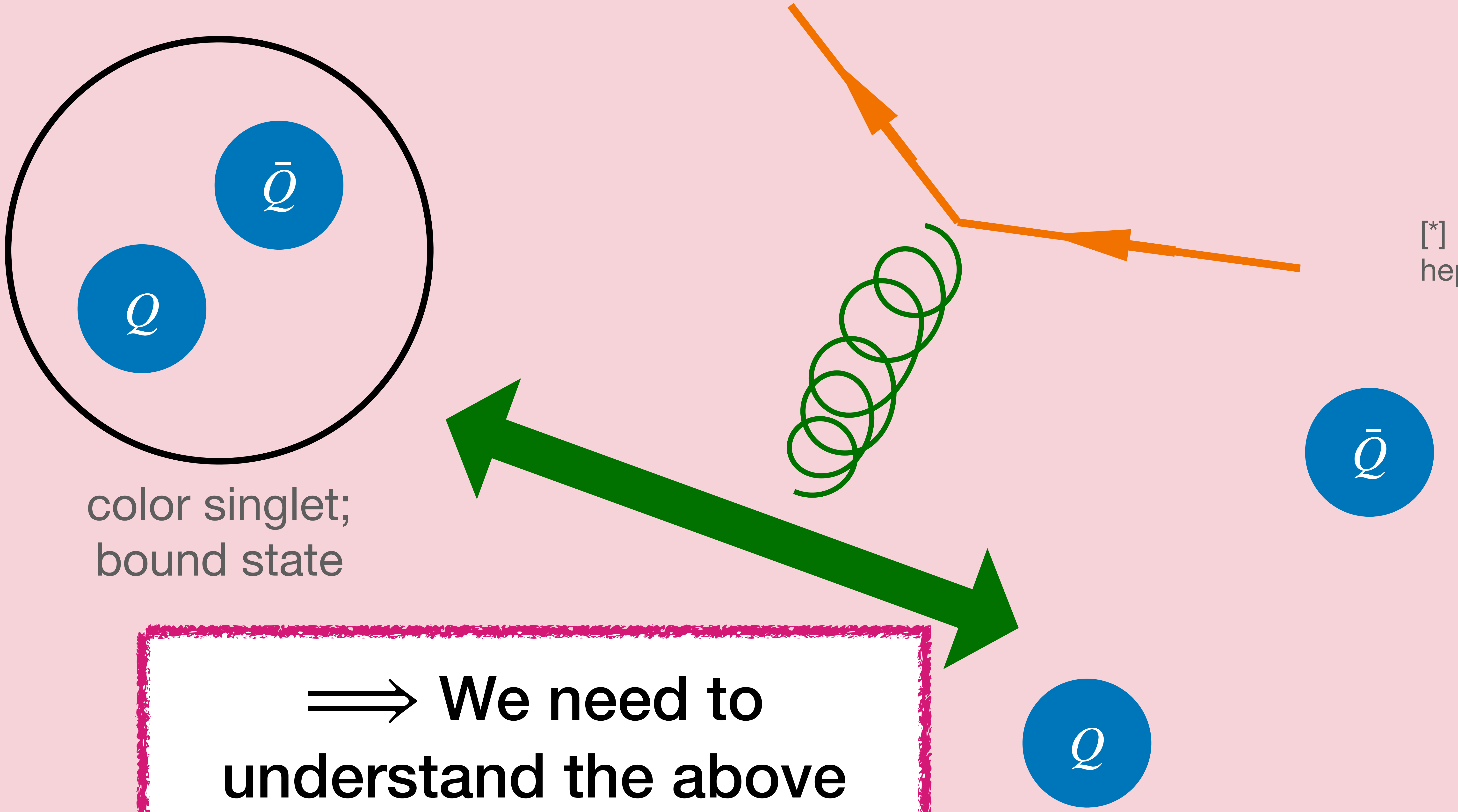

[*] N. Brambilla, A. Pineda, J. Soto. A. Vairo
hep-ph/9907240, hep-ph/0410047

color octet;
unbound state

$\Longrightarrow$ **We need to understand the above dynamics in the hierarchy**

$$Mv \gg T$$

$\Longrightarrow$ **pNRQCD [*]**

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Open quantum systems

"tracing/integrating out" the QGP

- Given an initial density matrix $\rho_{\text{tot}}(t=0)$, quarkonium coupled with the QGP evolves as

$$\rho_{\text{tot}}(t) = U(t)\rho_{\text{tot}}(t=0)U^{\dagger}(t).$$

- We will only be interested in describing the evolution of quarkonium and its final state abundances

$$\Longrightarrow \rho_S(t) = \text{Tr}_{\text{QGP}}\left[U(t)\rho_{\text{tot}}(t=0)U^{\dagger}(t)\right].$$

- Then, one derives an evolution equation for $\rho_S(t)$, assuming that at the initial time we have $\rho_{\text{tot}}(t=0) = \rho_S(t=0) \otimes e^{-H_{\text{QGP}}/T} / \mathcal{Z}_{\text{QGP}}$.

X. Yao, hep-ph/2102.01736

Time scales of quarkonia in the QGP
energy levels, interactions, and thermal environment

- The evolution of (heavy) quarkonia in a medium has three characteristic time scales:
  - The typical energy gaps $\tau_S \sim \frac{1}{\Delta E_n} \sim \frac{1}{Mv^2}$ of quarkonium energy levels
  - The interaction rate of quarkonia with the medium $\tau_R \sim \frac{T}{H_{\text{int}}^2} \sim \frac{(Mv)^2}{T^3}$
  - The medium correlation time $\tau_E \sim \frac{1}{T}$

$$\mathscr{L}_{\text{pNRQCD}} = \mathscr{L}_{\text{light quarks}} + \mathscr{L}_{\text{gluon}} + \int d^3r \text{Tr}_{\text{color}} \left[S^\dagger (i\partial_0 - H_s) S + O^\dagger (iD_0 - H_o) O + V_A \left(O^\dagger \mathbf{r} \cdot g\mathbf{E} S + \text{h.c.} \right) + \frac{V_B}{2} O^\dagger \left\{ \mathbf{r} \cdot g\mathbf{E}, O \right\} + \cdots \right]$$

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Lindblad equations for quarkonia at low T

quantum optical limit & quantum Brownian motion limit

- After tracing out the QGP degrees of freedom, one gets a Lindblad-type equation:

$$\frac{\partial\rho}{\partial t} = -i[H_{\text{eff}}, \rho] + \sum_j \gamma_j \left(L_j \rho L_j^\dagger - \frac{1}{2}\left\{ L_j^\dagger L_j, \rho \right\} \right)$$

- This can be done in two different limits within pNRQCD:

Quantum Brownian Motion:

$$\tau_R \gg \tau_E$$
$$\tau_S \gg \tau_E$$

relevant for $Mv \gg T \gg Mv^2$

Quantum Optical:

$$\tau_R \gg \tau_E$$
$$\tau_R \gg \tau_S$$

relevant for $Mv \gg Mv^2, T \gtrsim m_D$

Semiclassical transport of quarkonia

a Wigner transform of the Lindblad equation

We can study semiclassical transport by identifying

$$f_{\mathscr{B}}(\mathbf{x}, \mathbf{k}, t) \equiv \int_{k'} e^{i\mathbf{k}'\cdot\mathbf{x}} \left\langle \mathbf{k} + \frac{\mathbf{k}'}{2}, \mathscr{B} \middle| \rho_S(t) \middle| \mathbf{k} - \frac{\mathbf{k}'}{2}, \mathscr{B} \right\rangle,$$

and from the Lindblad equation derive:

- a Boltzmann equation in the Quantum Optical limit, or
- a Fokker-Planck equation in the Quantum Brownian motion limit.

The QGP input for the collision terms are chromoelectric field correlators.

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Our object of interest in this talk

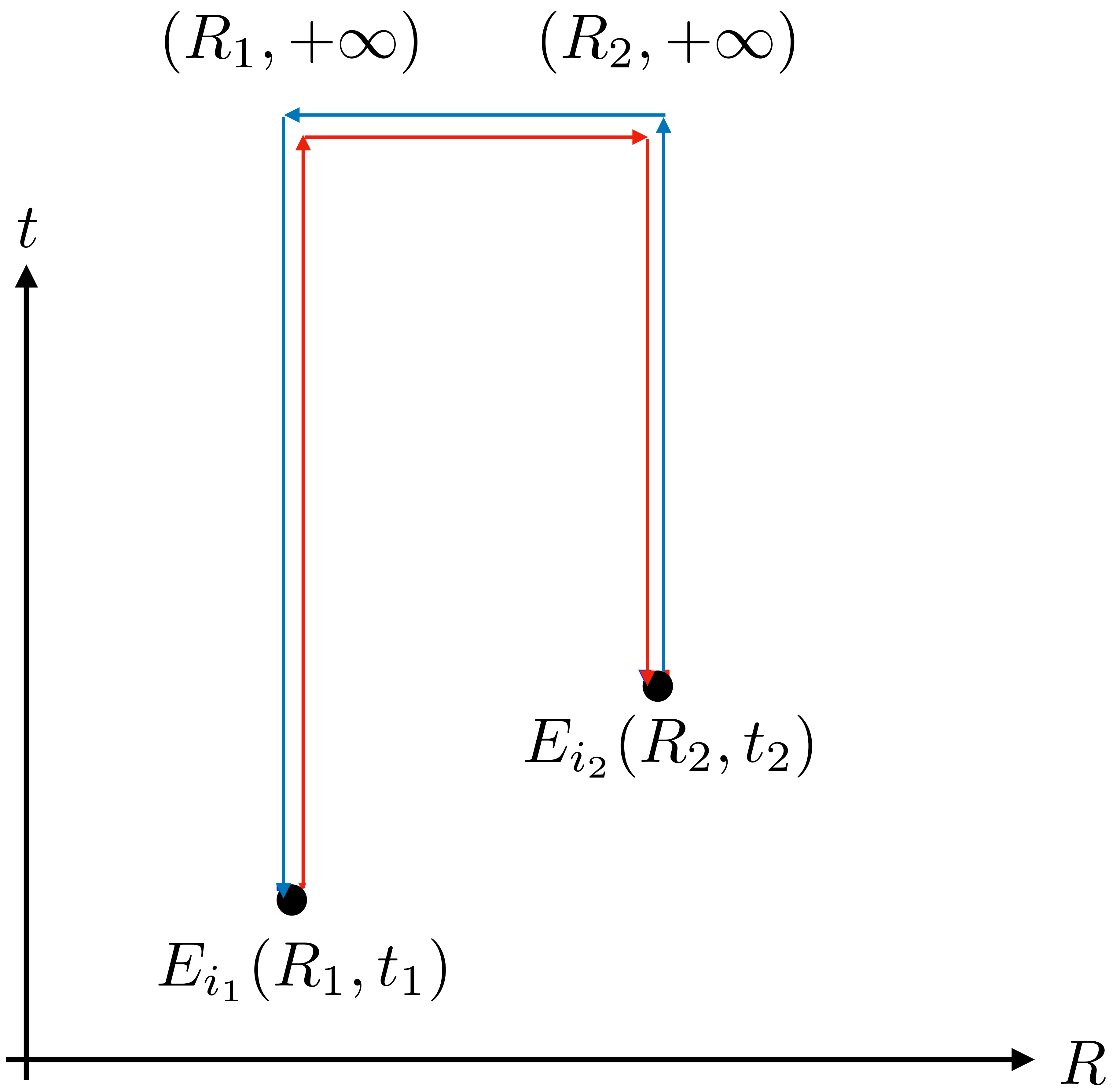
See also: N. Brambilla et al. hep-ph/1612.07248, hep-ph/1711.04515, hep-ph/2205.10289

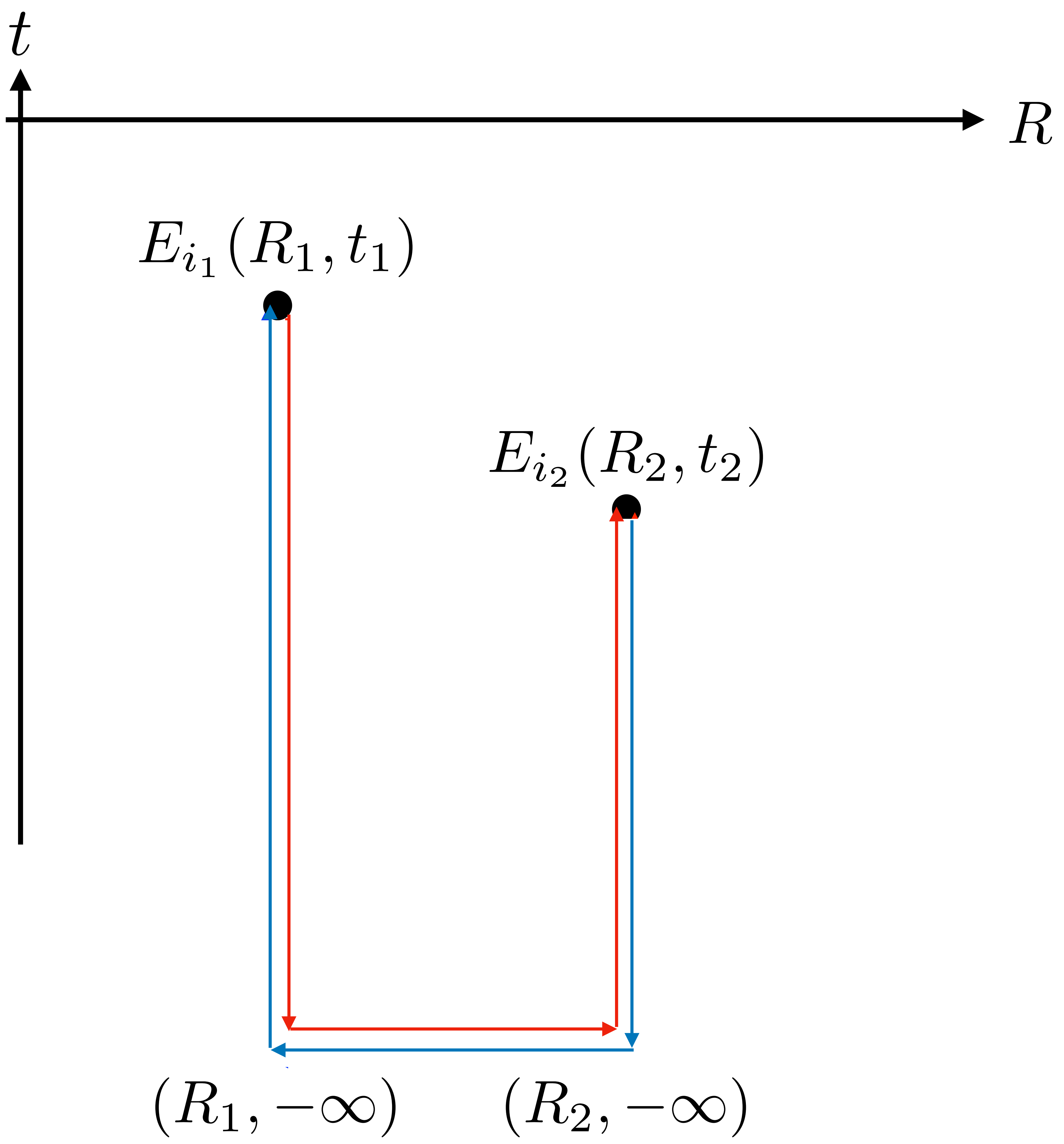
X. Yao and T. Mehen, hep-ph/2009.02408

QGP chromoelectric correlators

for quarkonia transport

$$[g_E^{--}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(\mathscr{W}_{2'} E_{i_2}(\mathbf{R}_2, t_2)\right)^a \left(E_{i_1}(\mathbf{R}_1, t_1)\mathscr{W}_{1'}\right)^a \right\rangle_T$$





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QGP chromoelectric correlators

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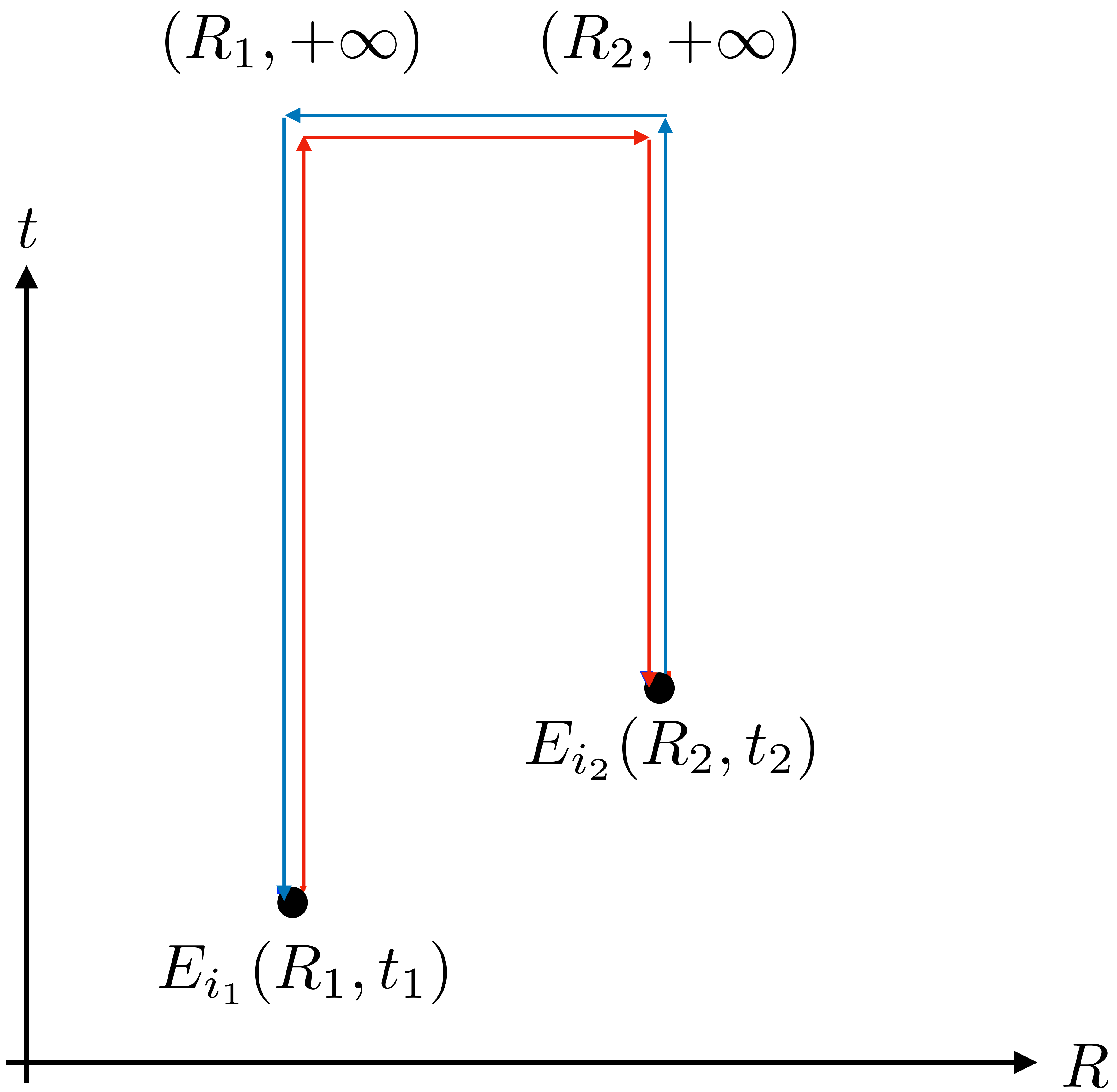

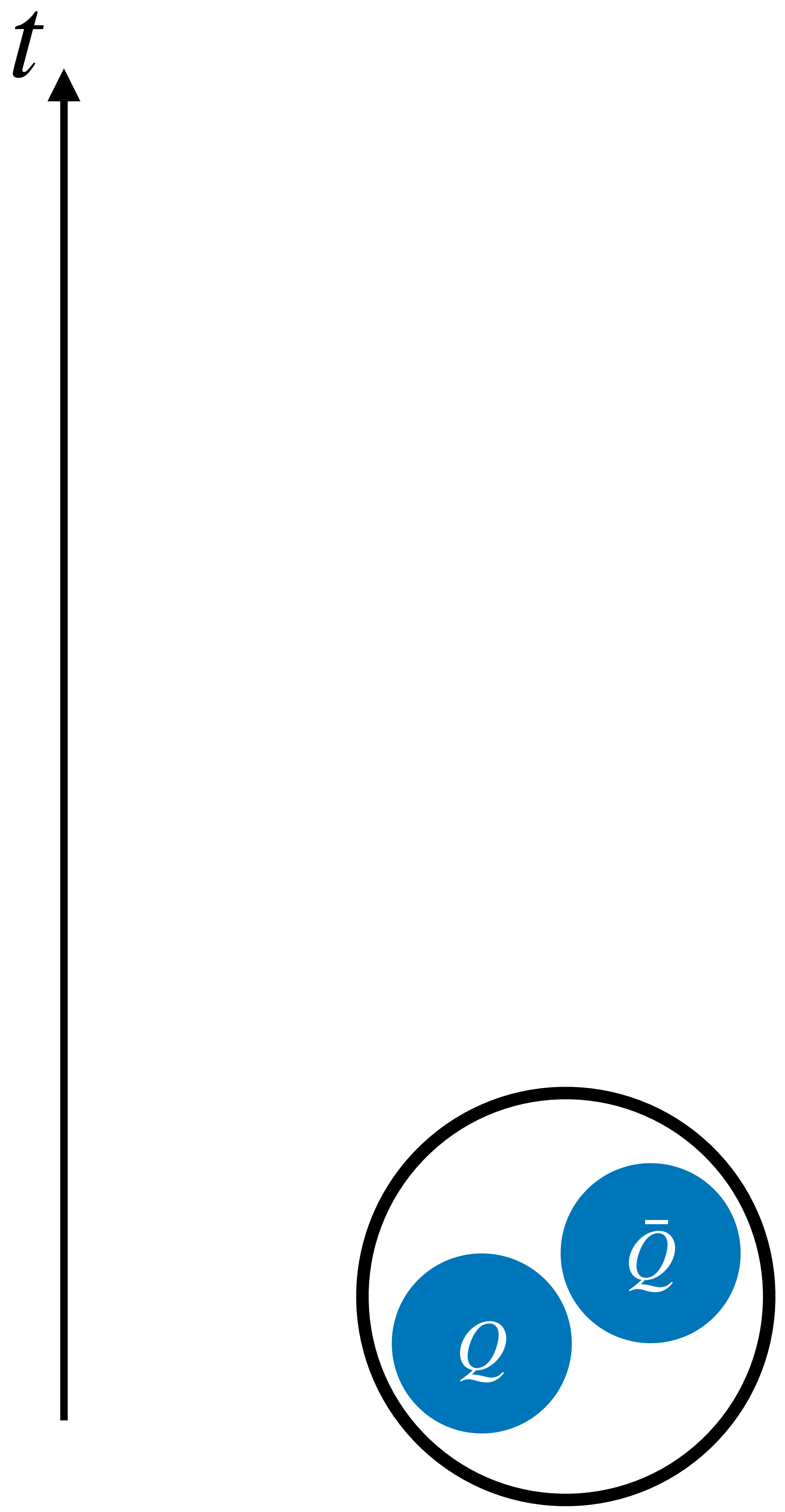

bound state:
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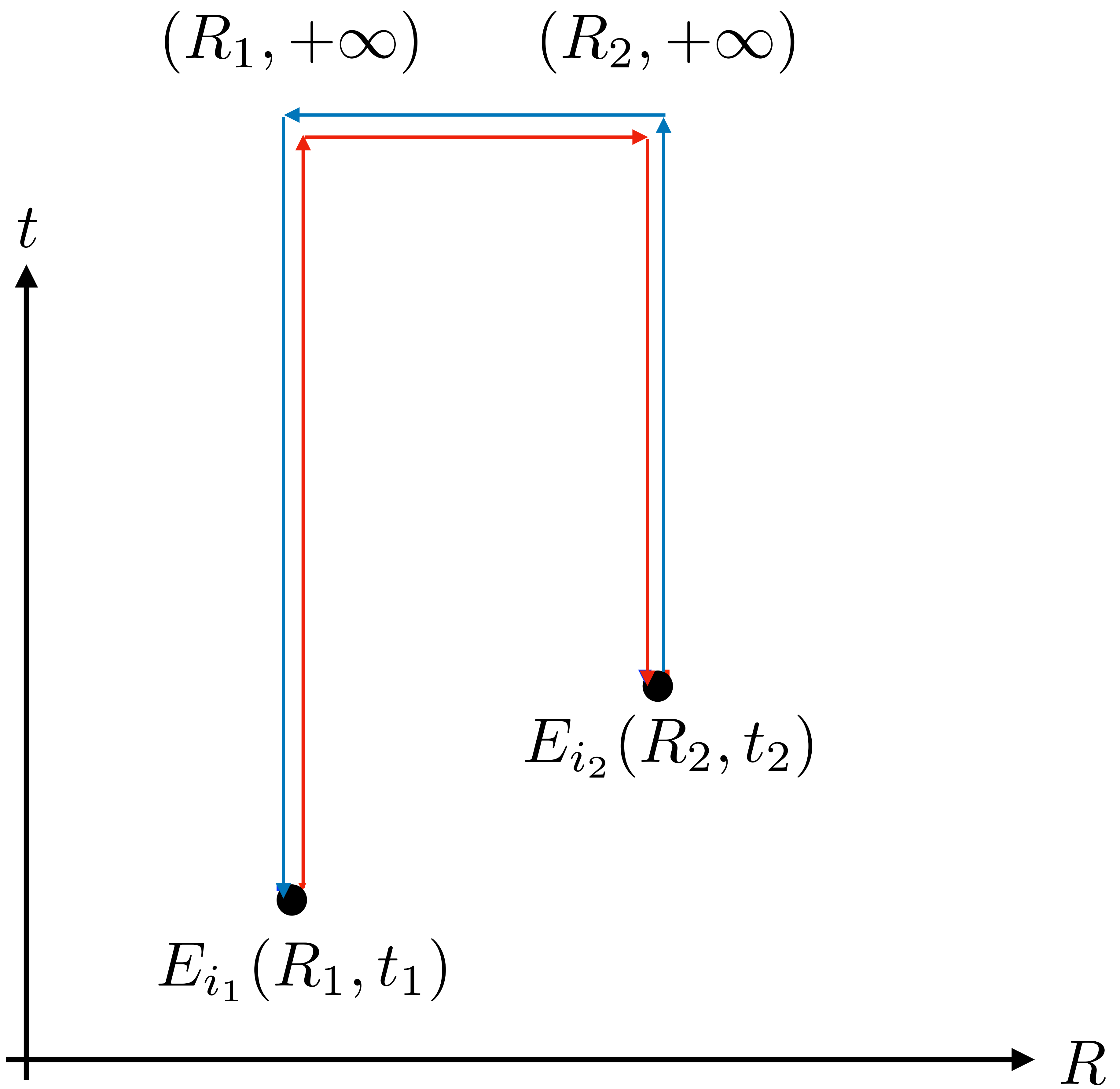

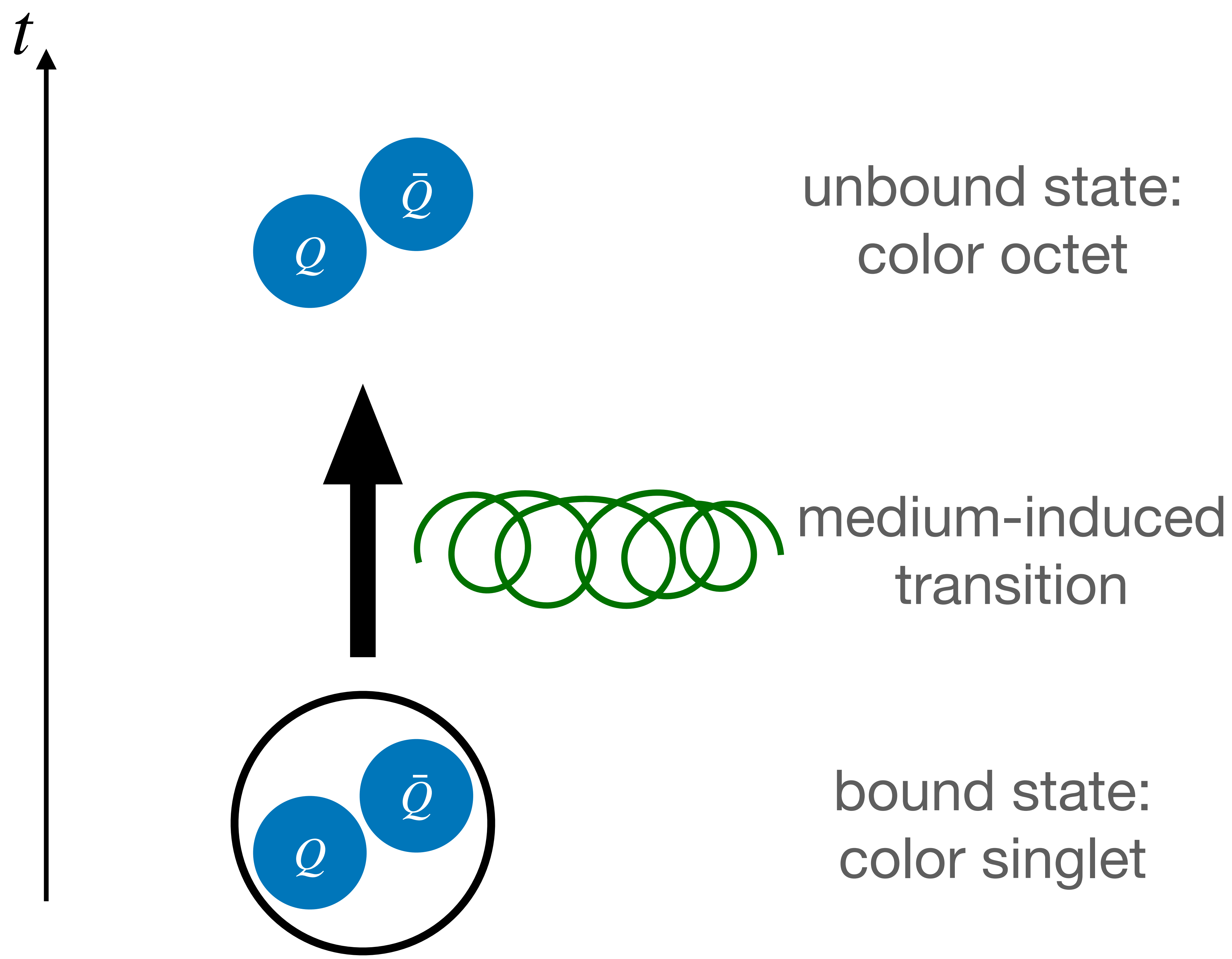

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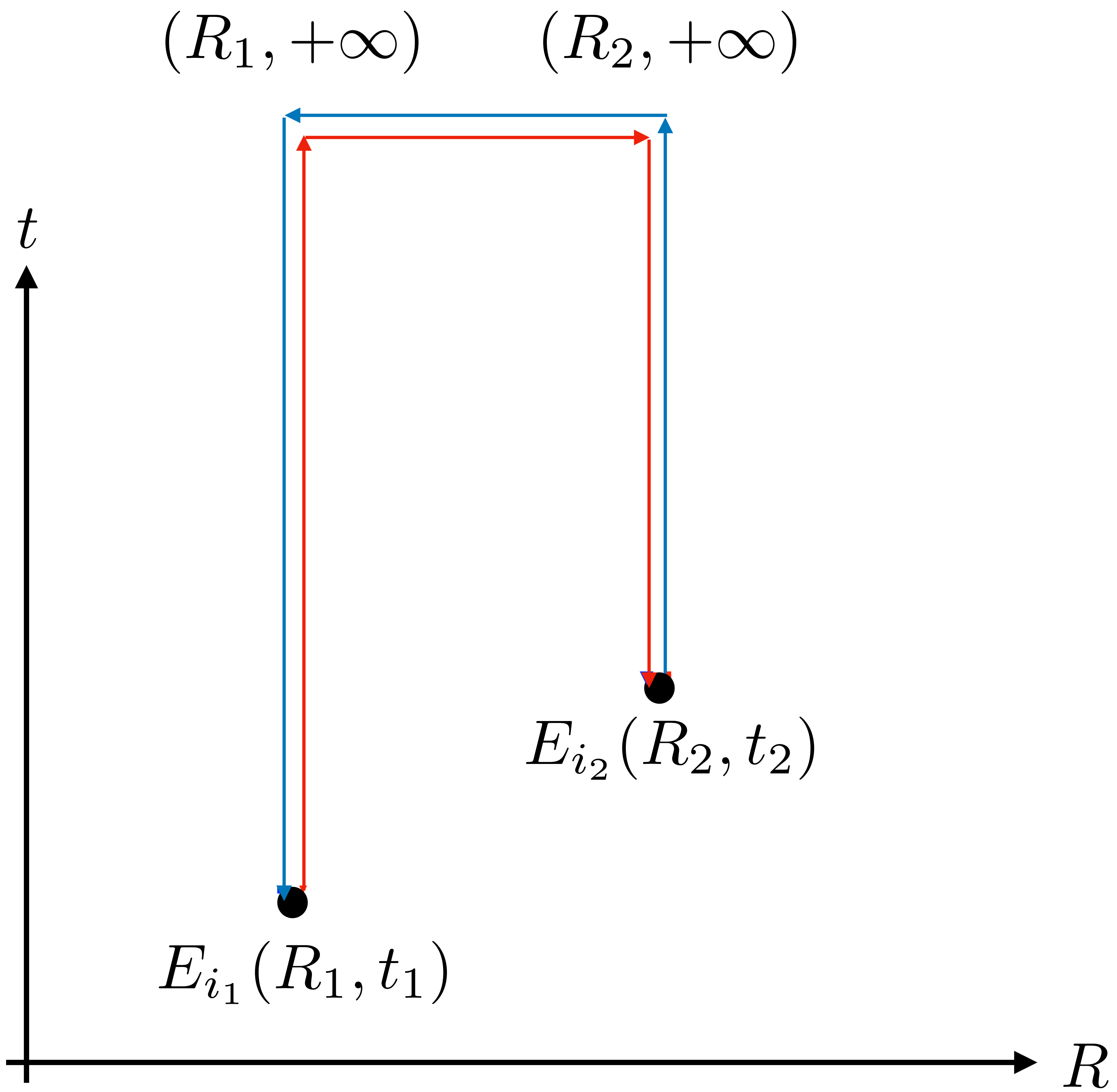

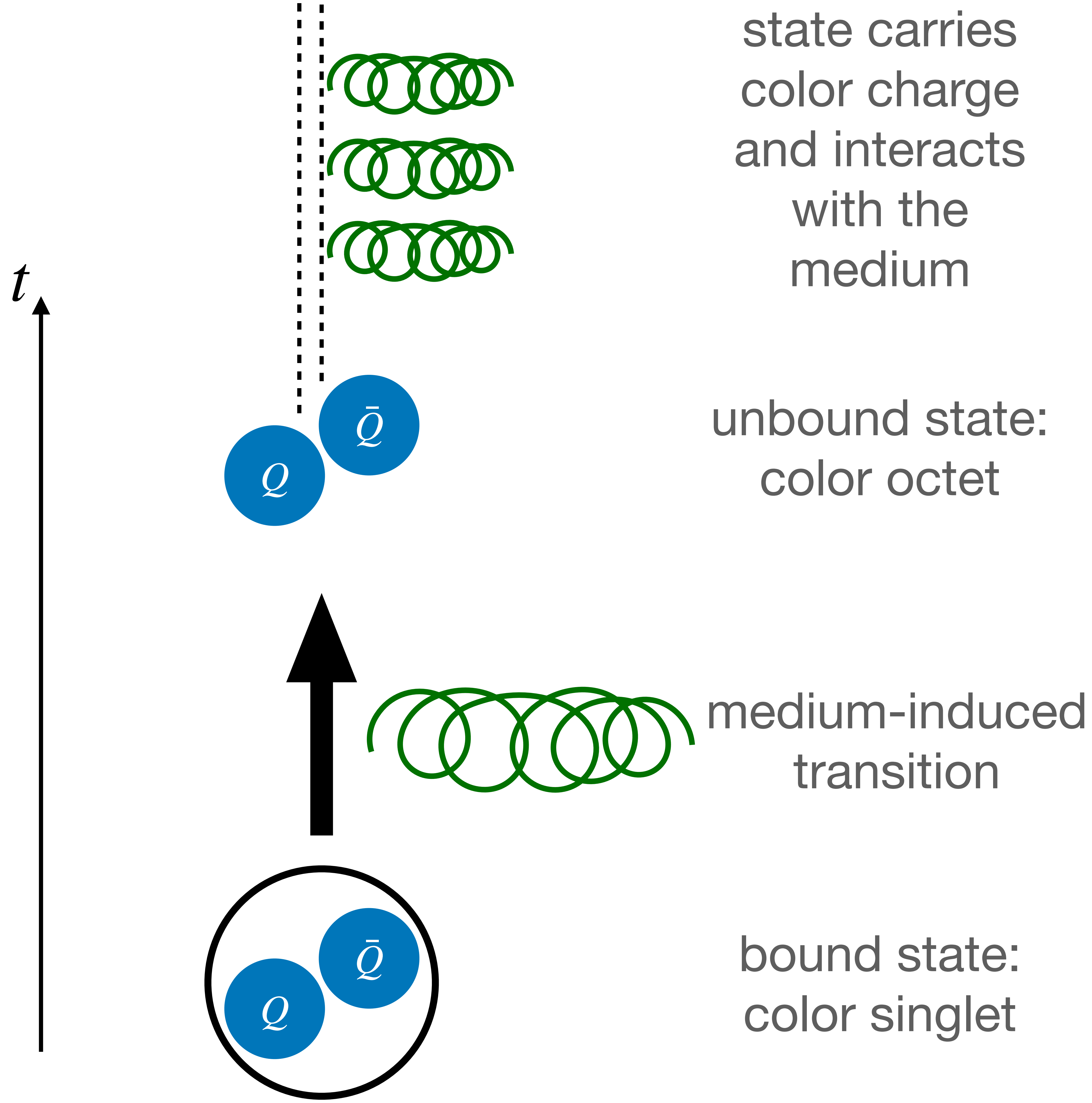

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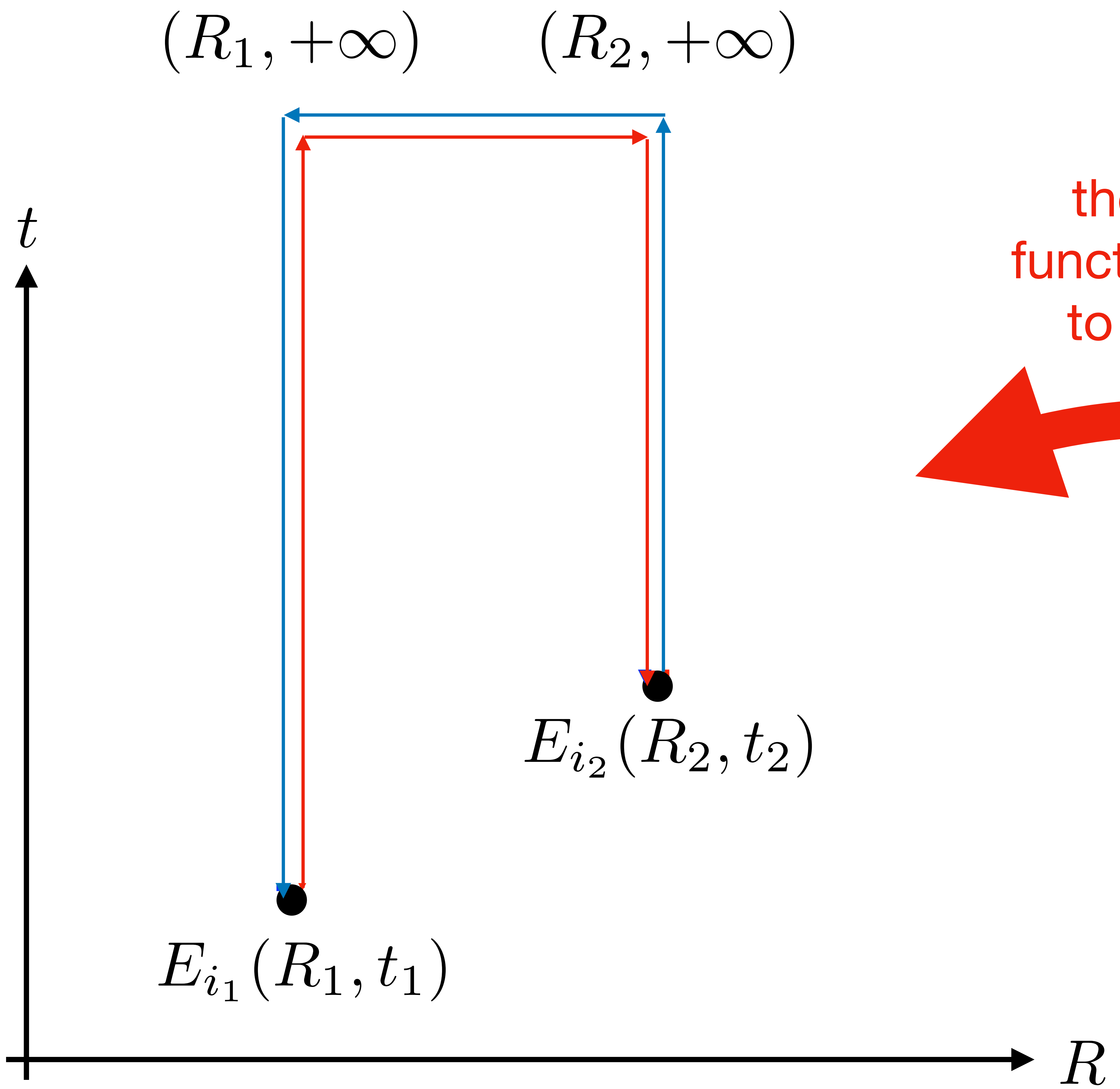
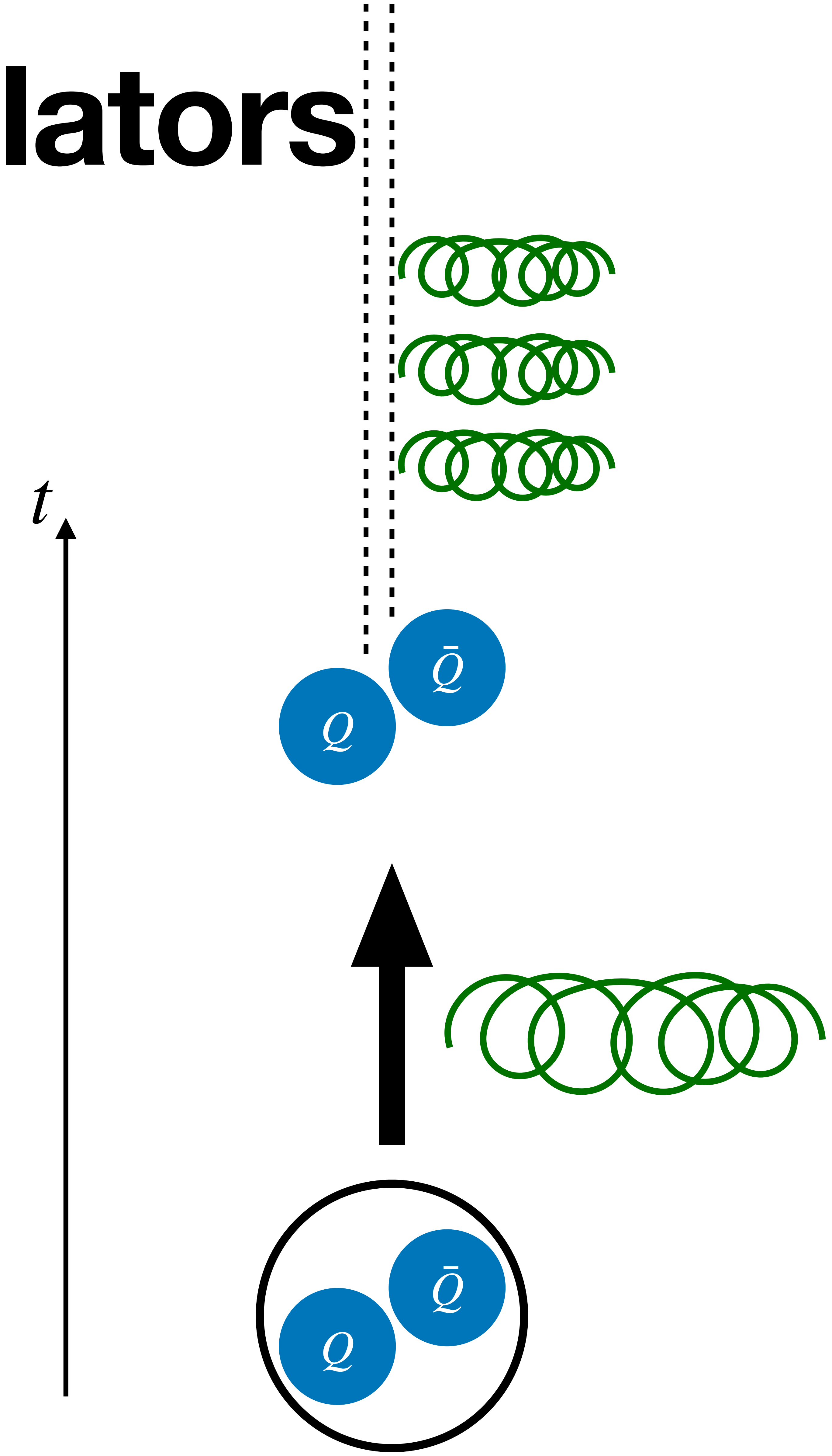


the correlation function associated to this process



the unbound state carries color charge and interacts with the medium

unbound state: color octet

medium-induced transition

bound state: color singlet

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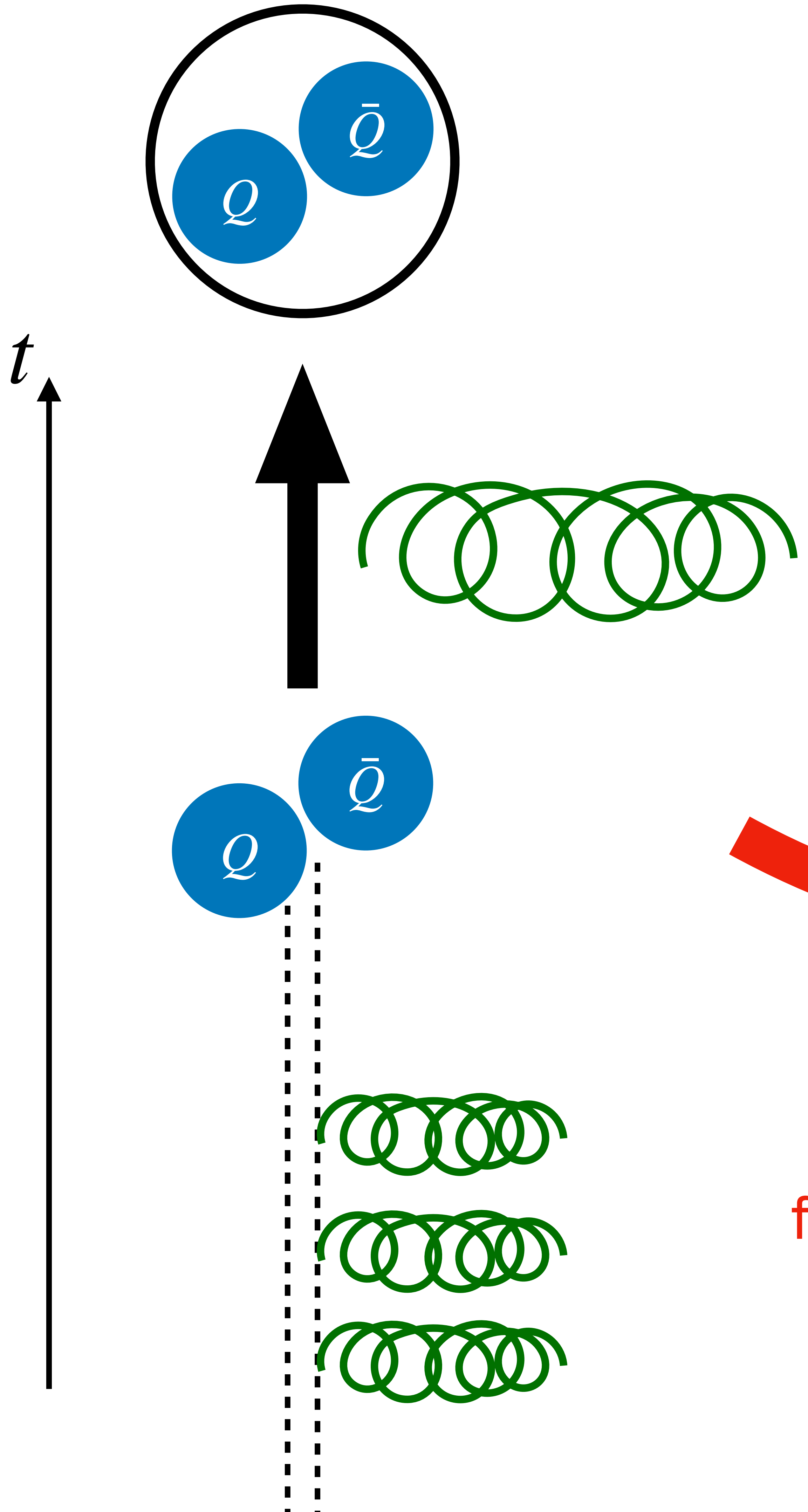

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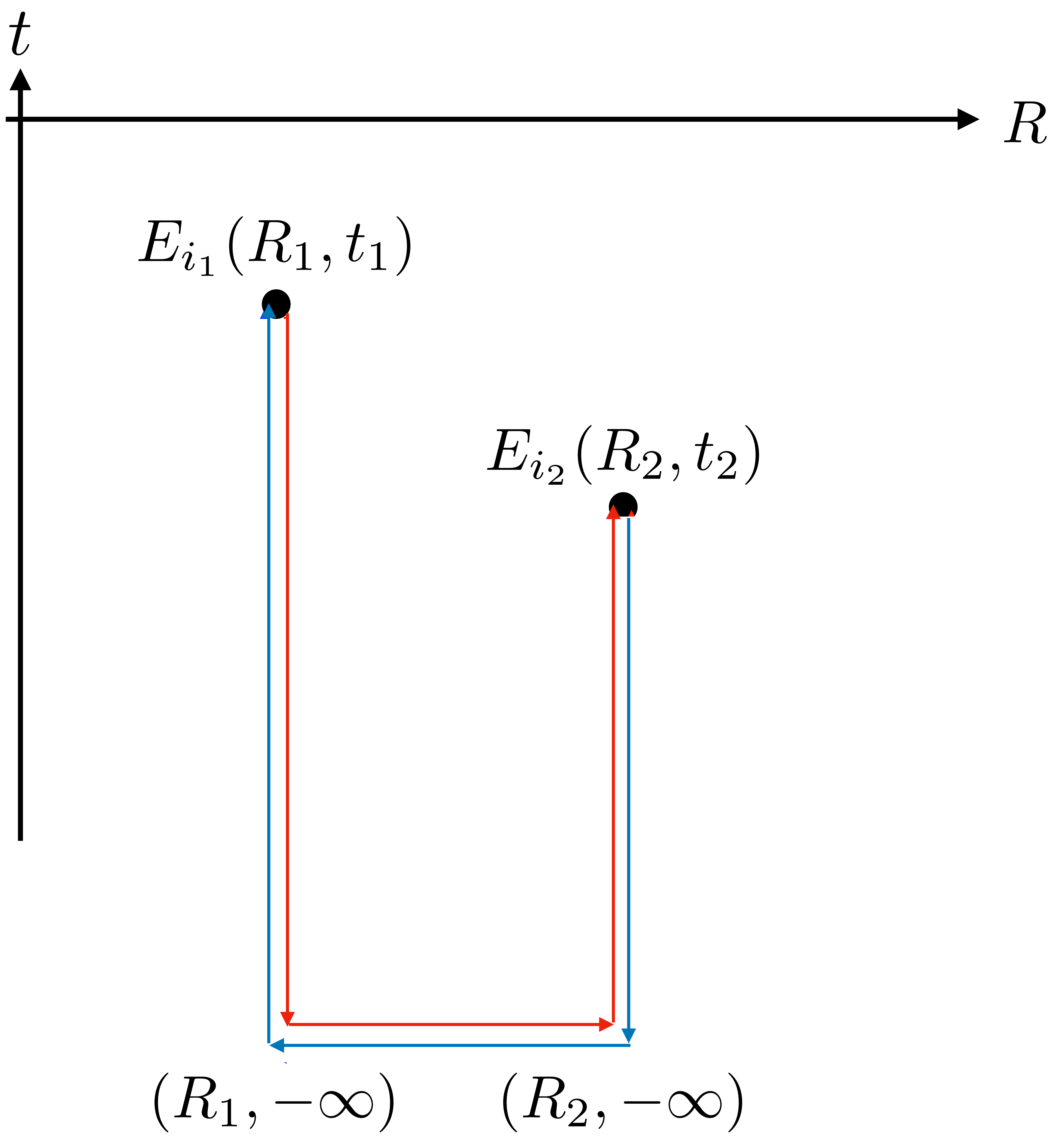

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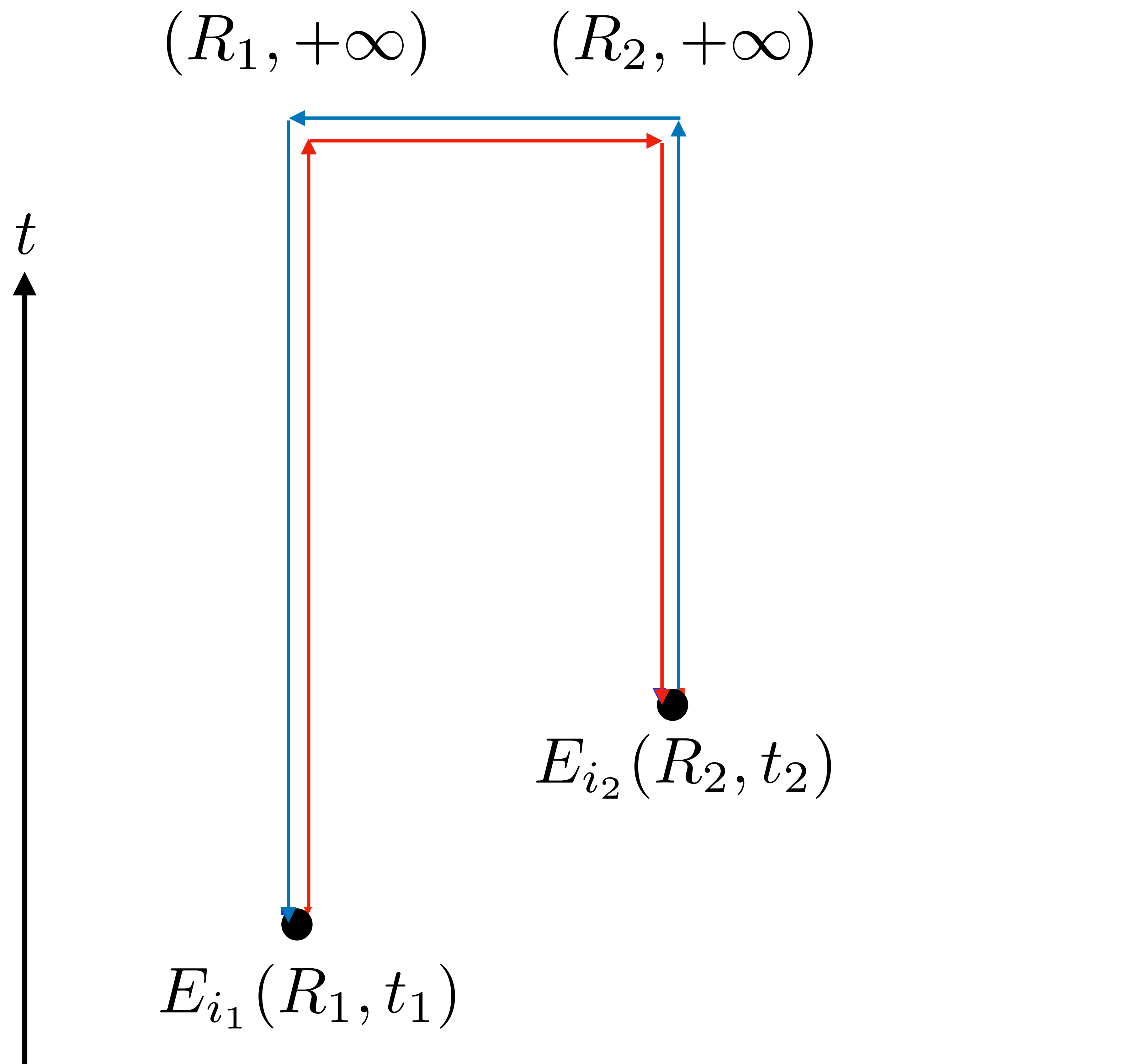

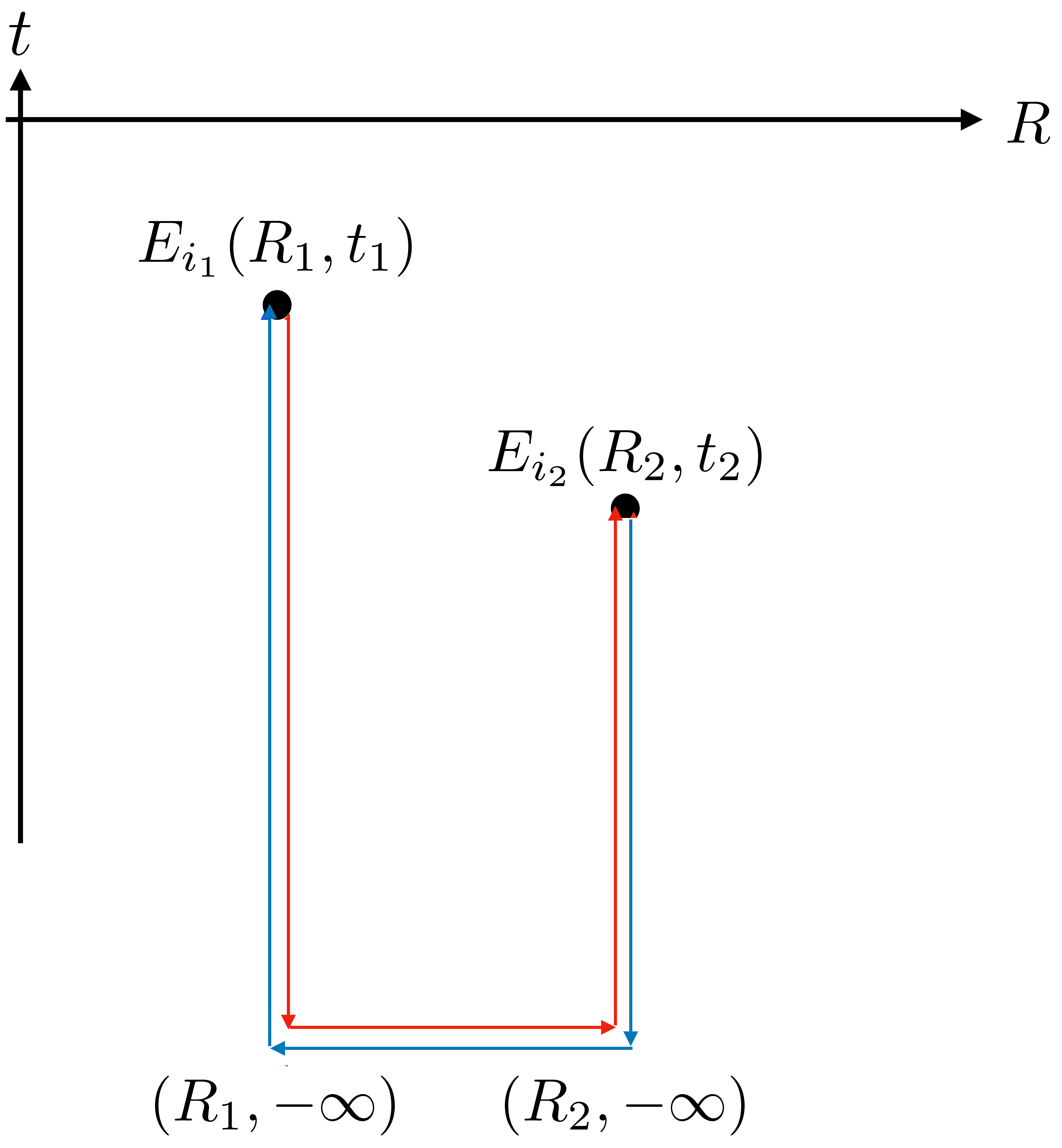

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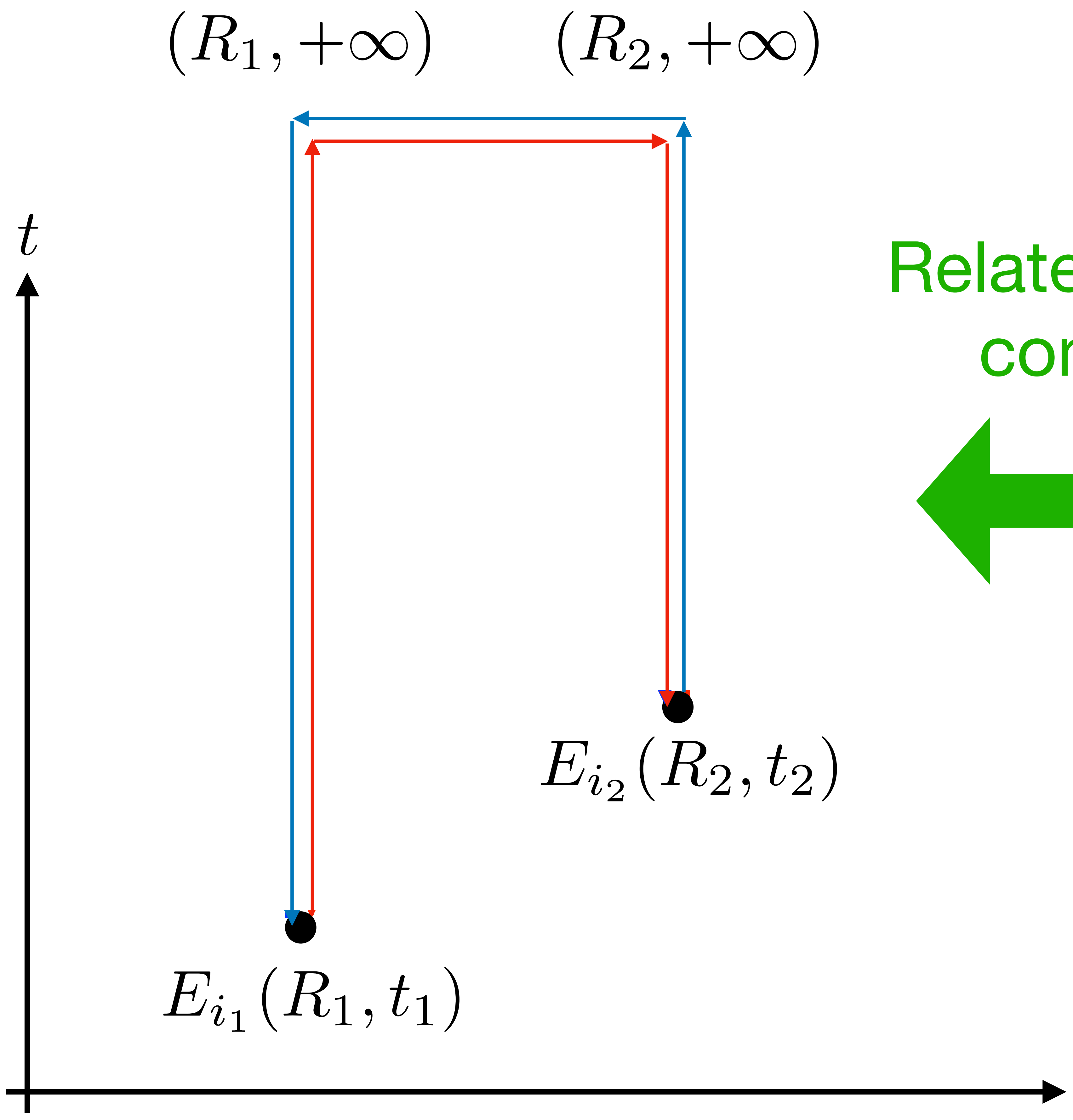
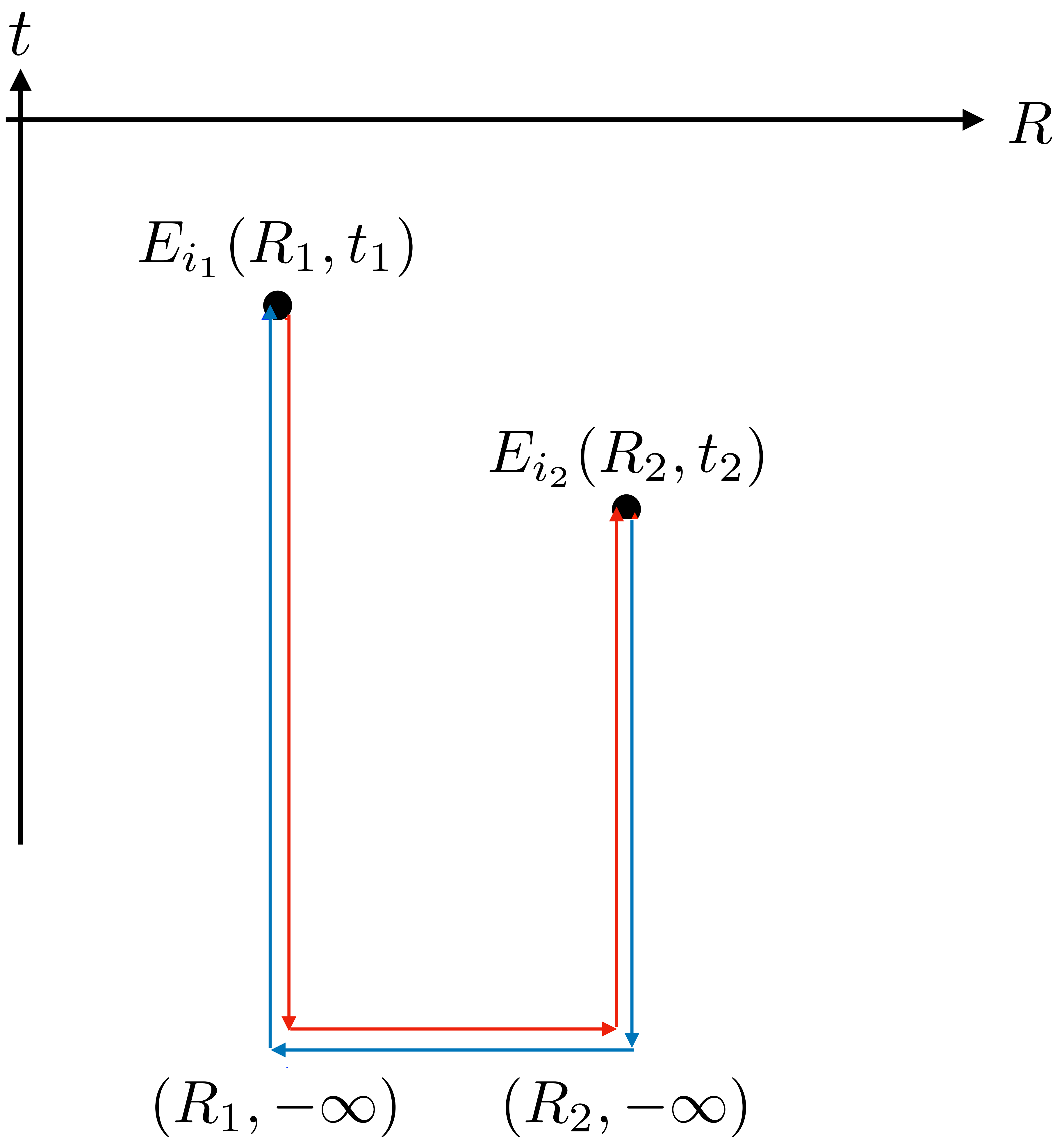


Related by KMS conditions



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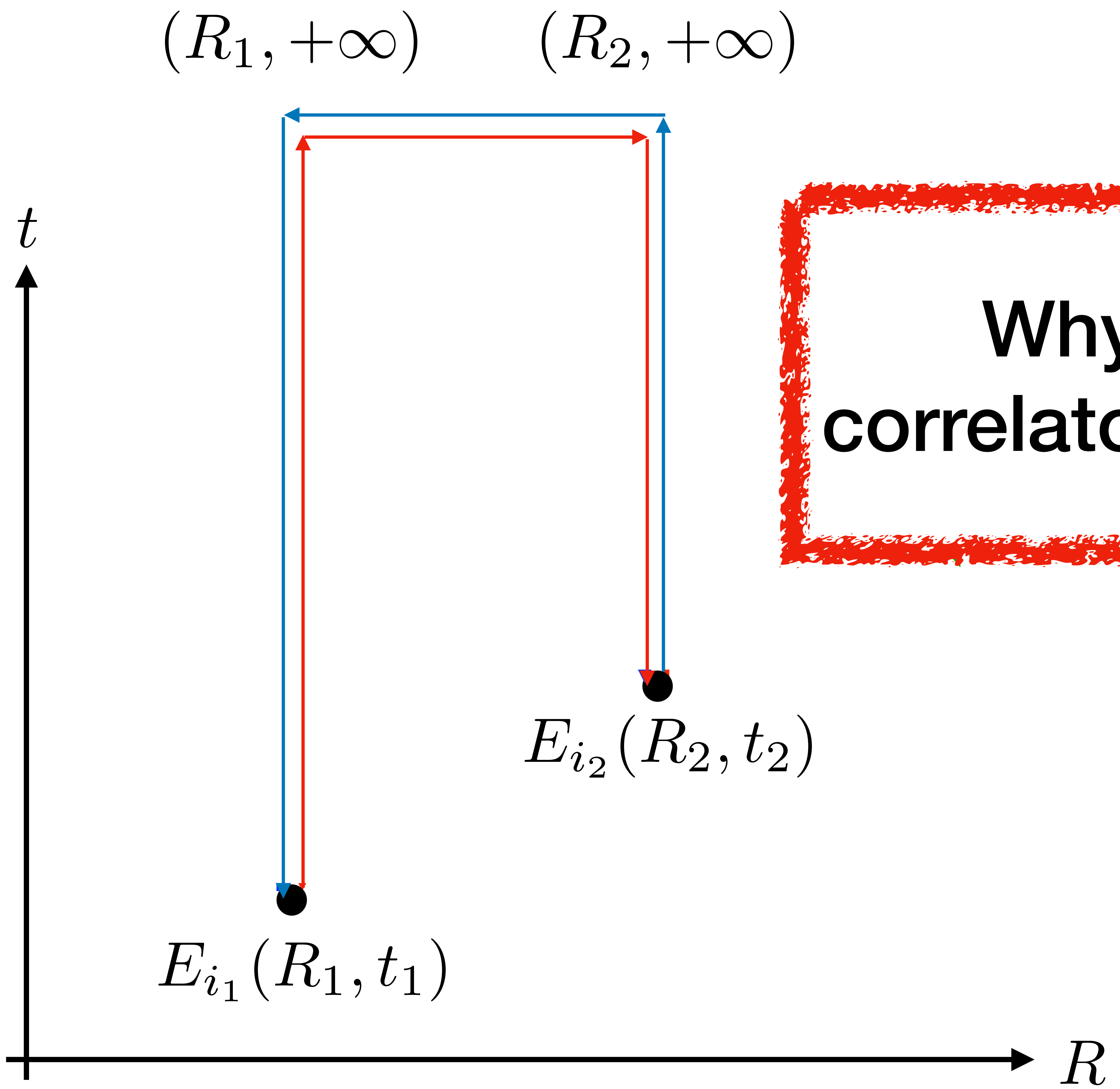
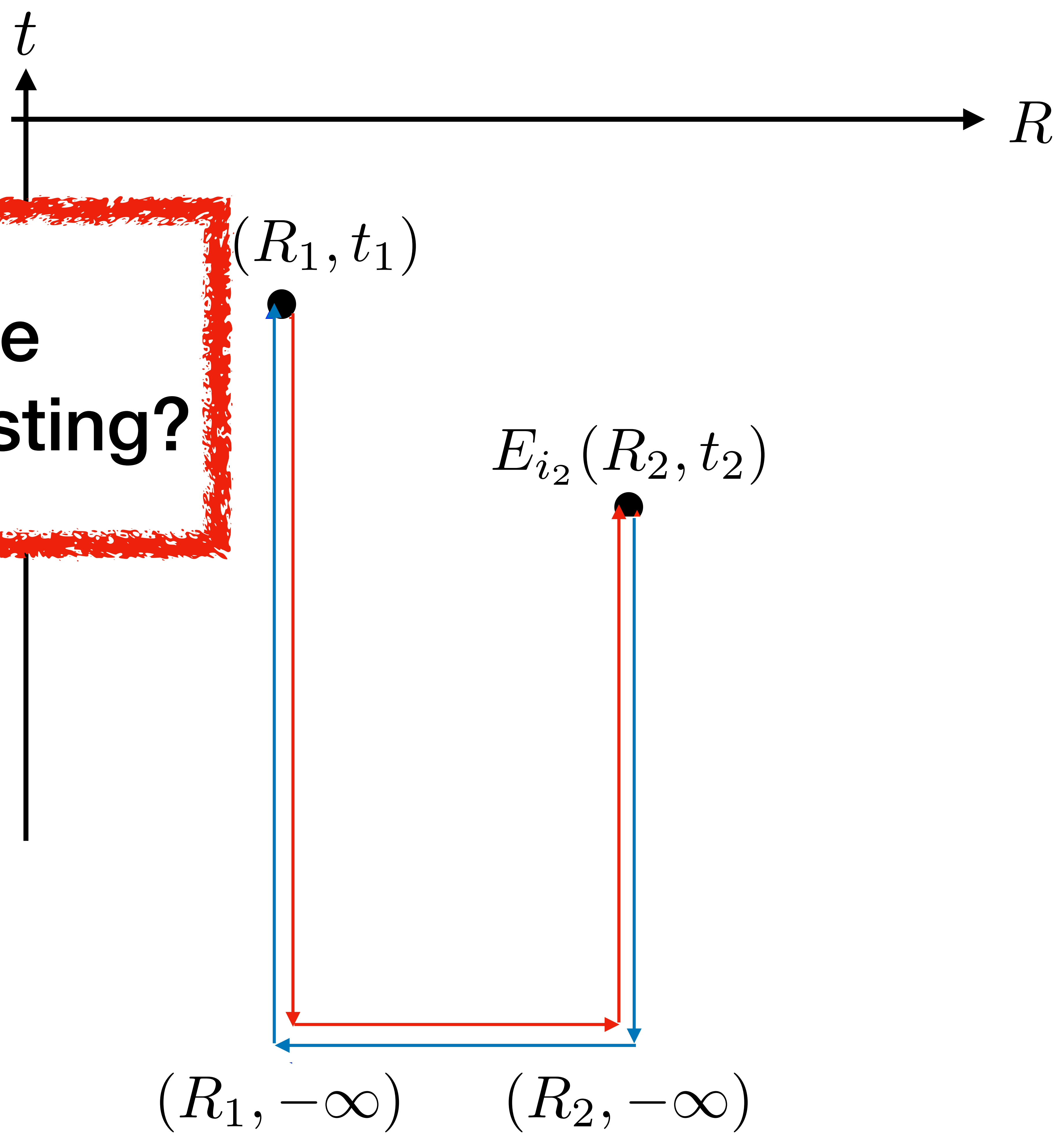


Why are these correlators interesting?



$$[g_E^{++}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(E_{i_2}(\mathbf{R}_2, t_2)\mathscr{W}_2\right)^a \left(\mathscr{W}_1 E_{i_1}(\mathbf{R}_1, t_1)\right)^a \right\rangle_T$$

electric correlators

port

Why are these correlators interesting?

$$[g_E^{--}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(\mathscr{W}_{2'} E_{i_2}(\mathbf{R}_2, t_2)\right)^a \left(E_{i_1}(\mathbf{R}_1, t_1)\mathscr{W}_{1'}\right)^a \right\rangle_T$$

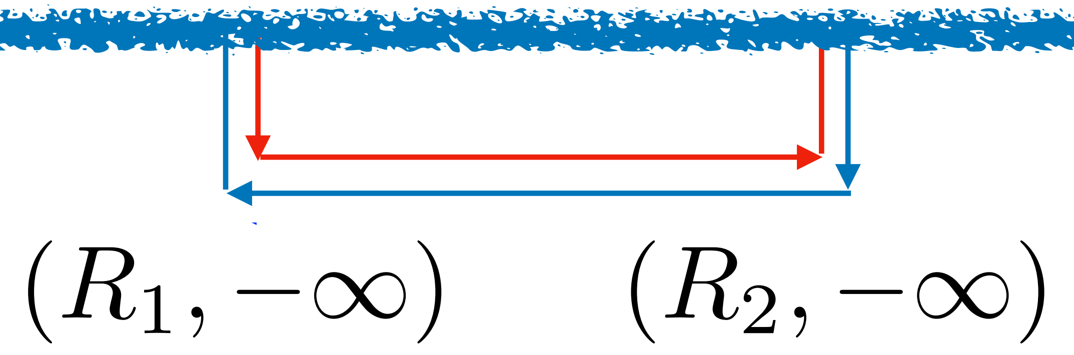
$+\infty)$

These determine the dissociation and formation rates of quarkonia:

$$\Gamma^{\mathrm{diss}} \propto \int \frac{\mathrm{d}^3\mathbf{p}_{\mathrm{rel}}}{(2\pi)^3}\frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^3} \,|\langle \psi_{\mathscr{B}}|\,\mathbf{r}\,|\Psi_{\mathbf{p}_{\mathrm{rel}}}\rangle|^2 g^{>}_{ii}\left(q^0 = E_{\mathscr{B}} - \frac{\mathbf{p}^2_{\mathrm{rel}}}{M}, \mathbf{q}\right),$$

$$\Gamma^{\mathrm{form}} \propto \int \frac{\mathrm{d}^3\mathbf{p}_{\mathrm{cm}}}{(2\pi)^3}\frac{\mathrm{d}^3\mathbf{p}_{\mathrm{rel}}}{(2\pi)^3}\frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^3} \,|\langle \psi_{\mathscr{B}}|\,\mathbf{r}\,|\Psi_{\mathbf{p}_{\mathrm{rel}}}\rangle|^2 g^{>}_{ii}\left(q^0 = \frac{\mathbf{p}^2_{\mathrm{rel}}}{M} - E_{\mathscr{B}}, \mathbf{q}\right)$$

$$\times f_{\mathscr{S}}(\mathbf{x}, \mathbf{p}_{\mathrm{cm}}, \mathbf{r} = 0, \mathbf{p}_{\mathrm{rel}}, t)\,.$$



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Strongly and weakly coupled aspects

interactions with the medium and its self-interactions

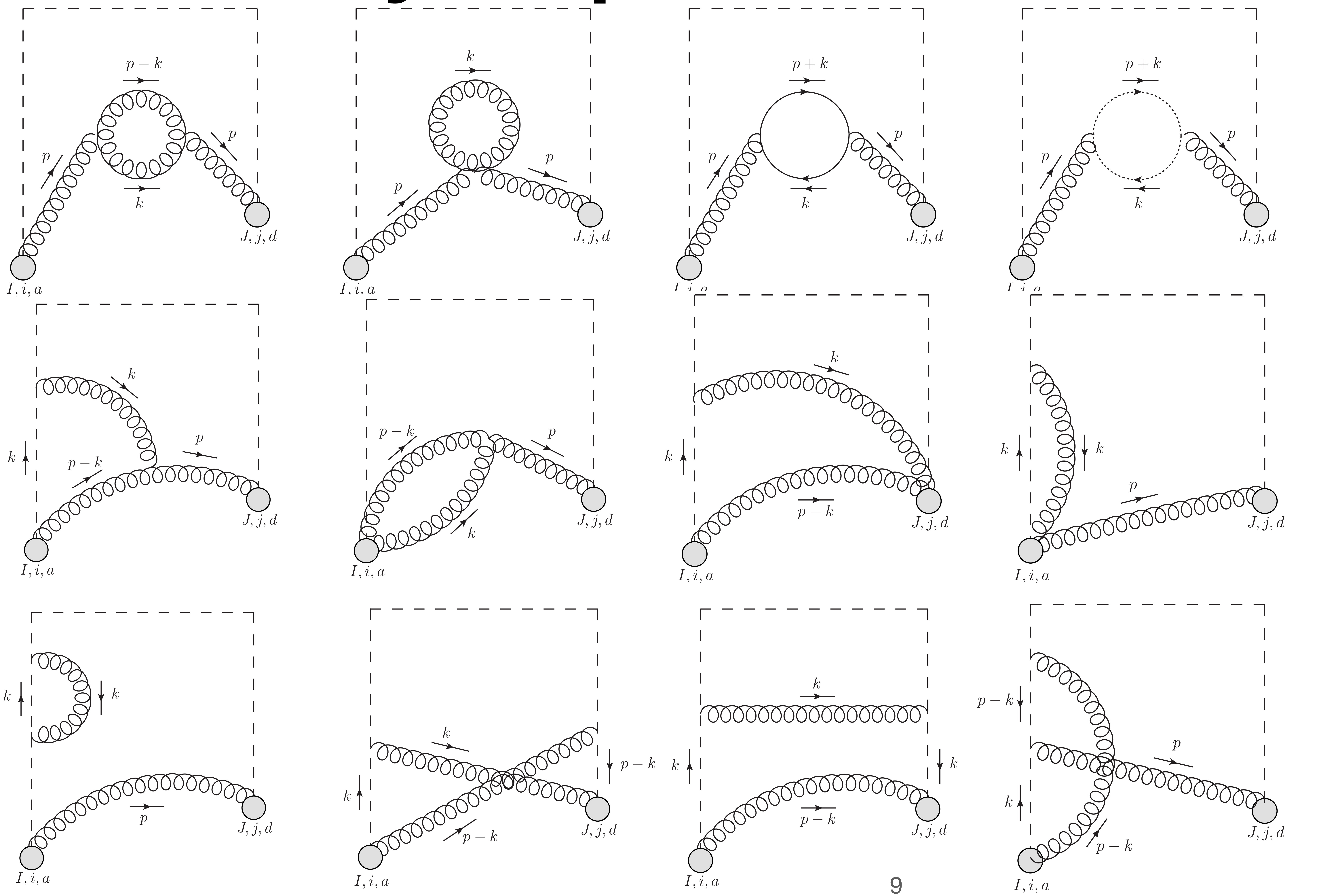
- So far, our discussion is specific to weak coupling between HQs and the medium in the sense of pNRQCD as an open quantum system ($Mv \gg T$).
- The medium can be either weakly or strongly coupled.
- In what follows:

   A. Weakly coupled calculation of the correlators in QCD at NLO relevant for quarkonium transport [2107.03945]

   B. Strongly coupled calculation of the analogous correlation functions in $\mathcal{N} = 4$ SYM [22XX.XXXXX] (outline and some preliminary results)

T. Binder, K. Mukaida, B. Scheihing-Hitschfeld and X. Yao, hep-ph/2107.03945

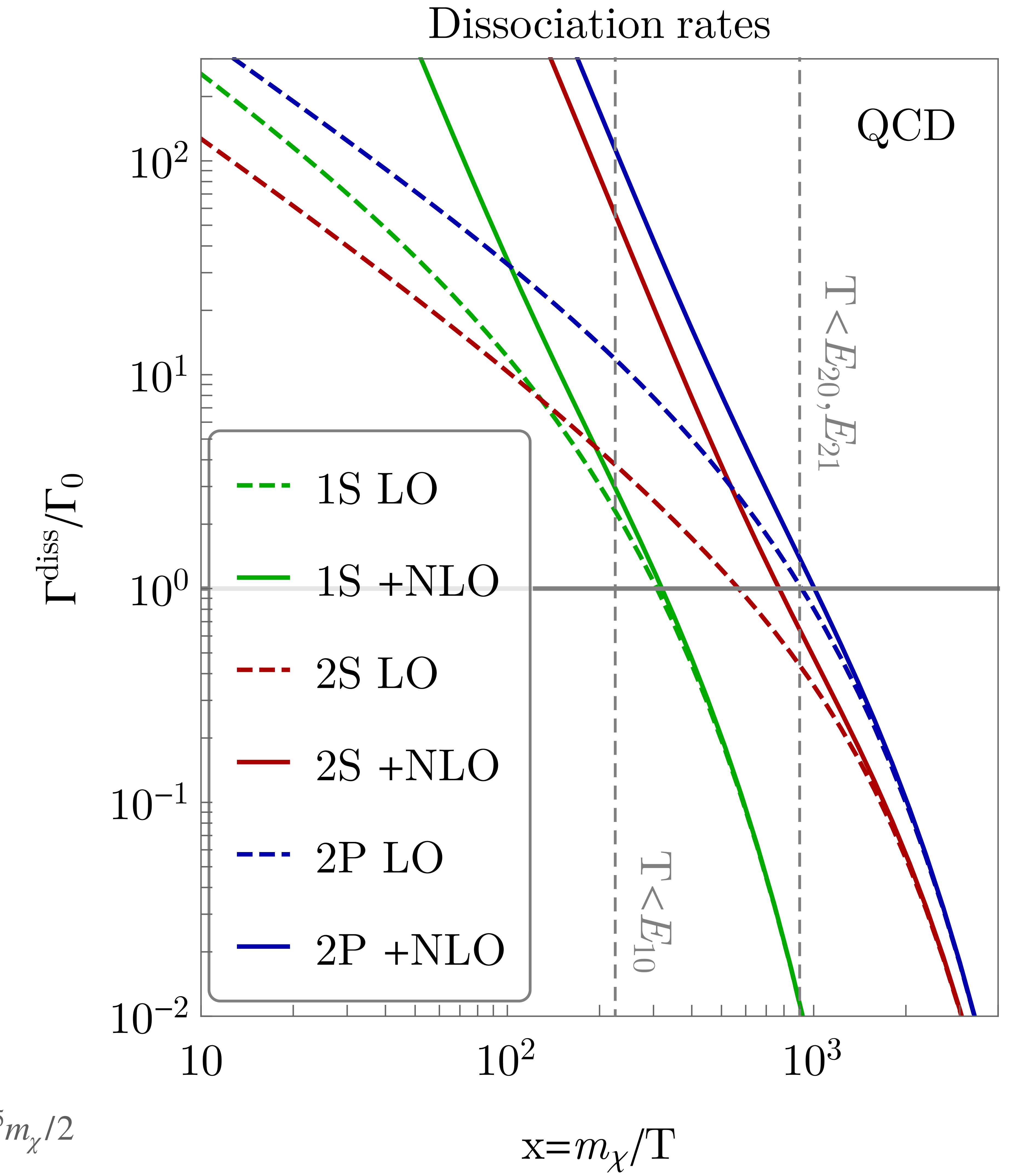
A. Weakly coupled calculation in QCD



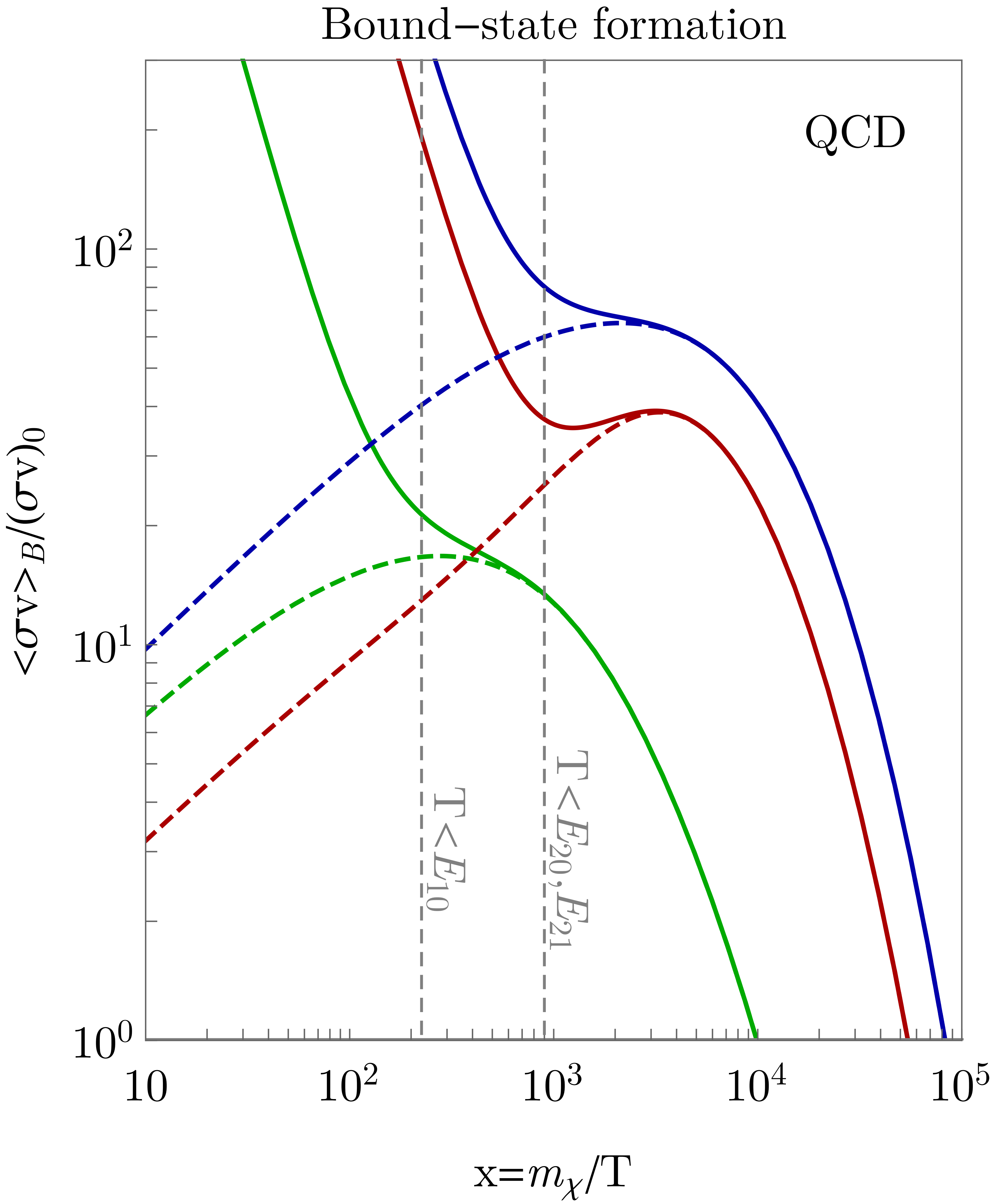
The real-time calculation proceeds by evaluating these diagrams (+ some permutations of them) on the Schwinger-Keldysh contour

A. Weakly coupled calculation in QCD

results: transition rates



$\alpha = 0.1$
$N_c = 3$



$\Gamma_0 = \alpha^5 m_\chi/2$

$(\sigma v)_0 = \pi\alpha^2/m_\chi^2$

What about explicit formulas?

The spectral function at NLO

It is simplest to write the integrated spectral function:

$$\varrho_E^{++}(p_0) = \frac{1}{2}\int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}\delta^{ad}\delta_{ij}\left[\rho_E^{++}\right]_{ji}^{da}(p_0,\mathbf{p})\,.$$

We found

$$g^2\varrho_E^{++}(p_0) = \frac{g^2(N_c^2-1)p_0^3}{(2\pi)^3}\left\{4\pi^2 + g^2\left[\left(\frac{11}{12}N_c - \frac{1}{3}N_f\right)\ln\left(\frac{\mu^2}{4p_0^2}\right) + \left(\frac{149}{36}+\frac{\pi^2}{3}\right)N_c - \frac{10}{9}N_f + F\left(\frac{p_0}{T}\right)\right]\right\}$$

T. Binder, K. Mukaida, B. Scheihing-Hitschfeld and X. Yao, hep-ph/2107.03945

The spectral function at NLO
and a comparison with its heavy quark counterpart

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and the heavy quark counterpart is, with the same T-dependent function $F(p_0/T)$,

Y. Burnier, M. Laine, J. Langelage and L. Mether, hep-ph/1006.0867

$$g^2\rho_E^{\mathrm{HQ}}(p_0) = \frac{g^2(N_c^2-1)p_0^3}{(2\pi)^3}\left\{4\pi^2 + g^2\left[\left(\frac{11}{12}N_c - \frac{1}{3}N_f\right)\ln\left(\frac{\mu^2}{4p_0^2}\right) + \left(\frac{149}{36}-\frac{2\pi^2}{3}\right)N_c - \frac{10}{9}N_f + F\left(\frac{p_0}{T}\right)\right]\right\}$$

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Heavy quark and quarkonia correlators

a small, yet consequential difference

The heavy quark diffusion coefficient can be defined from the real-time correlator A. M. Eller, J. Ghiglieri and G. D. Moore, hep-ph/1903.08064

$$\left\langle \mathrm{Tr}_{\mathrm{color}} \left[U(-\infty, t) E_i(t) U(t,0) E_i(0) U(0, -\infty) \right] \right\rangle_T,$$

whereas for quarkonia the relevant quantity is $T_F \left\langle E_i^a(t) \mathscr{W}^{ab}(t,0) E_i^b(0) \right\rangle_T$.

What we just found is simply stating that:

$$T_F \left\langle E_i^a(t) \mathscr{W}^{ab}(t,0) E_i^b(0) \right\rangle_T \neq \left\langle \mathrm{Tr}_{\mathrm{color}} \left[U(-\infty, t) E_i(t) U(t,0) E_i(0) U(0, -\infty) \right] \right\rangle_T$$

For more evidence of the difference, see also M. Eidemuller and M. Jamin, hep-ph/9709419

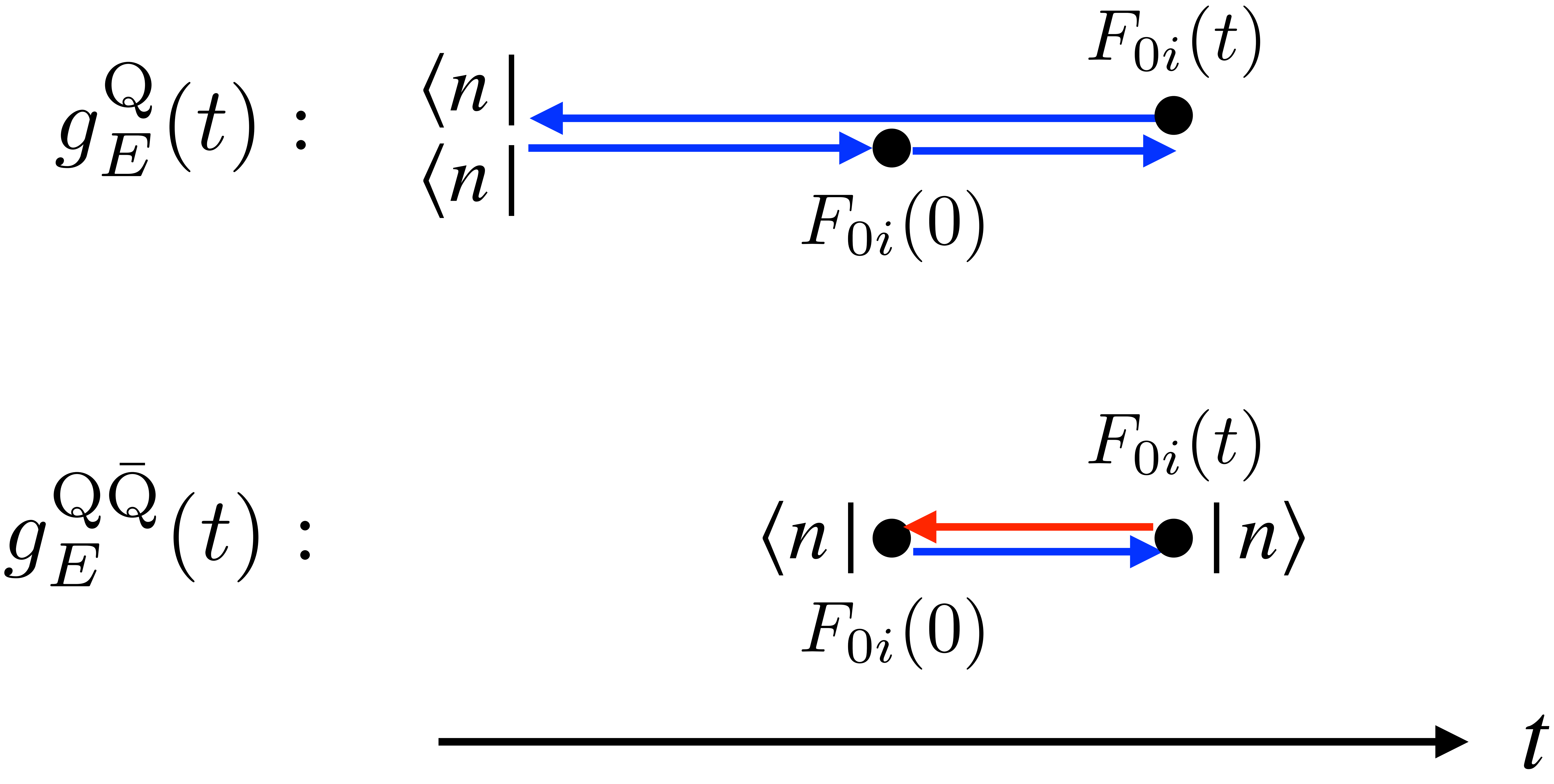
An axial gauge puzzle

an apparent (but not actual) inconsistency

- This finding presents a puzzle:
  - Let's say we were able to set set axial gauge $A_0 = 0$.
  - Then, the two correlation functions would look the same:

$$T_F \left\langle E_i^a(t) E_i^a(0) \right\rangle_T = \left\langle \mathrm{Tr}_{\mathrm{color}} \left[E_i(t) E_i(0) \right] \right\rangle_T .$$

  - If true, this would imply that one of the calculations is not gauge invariant.
- Alternatively, something must prevent us from setting $A_0 = 0$.

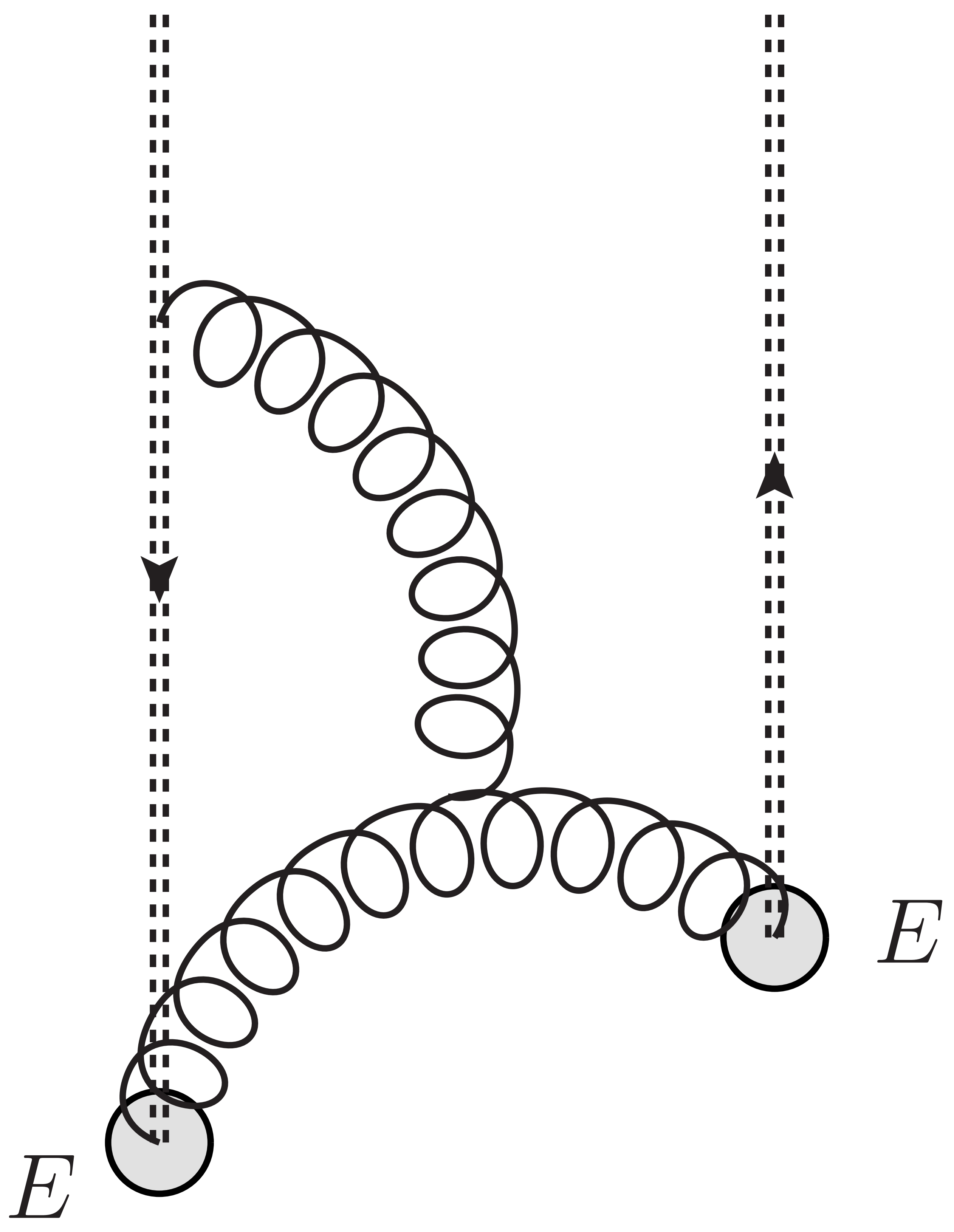


What makes them different?

The difference in terms of diagrams

operator ordering is crucial!

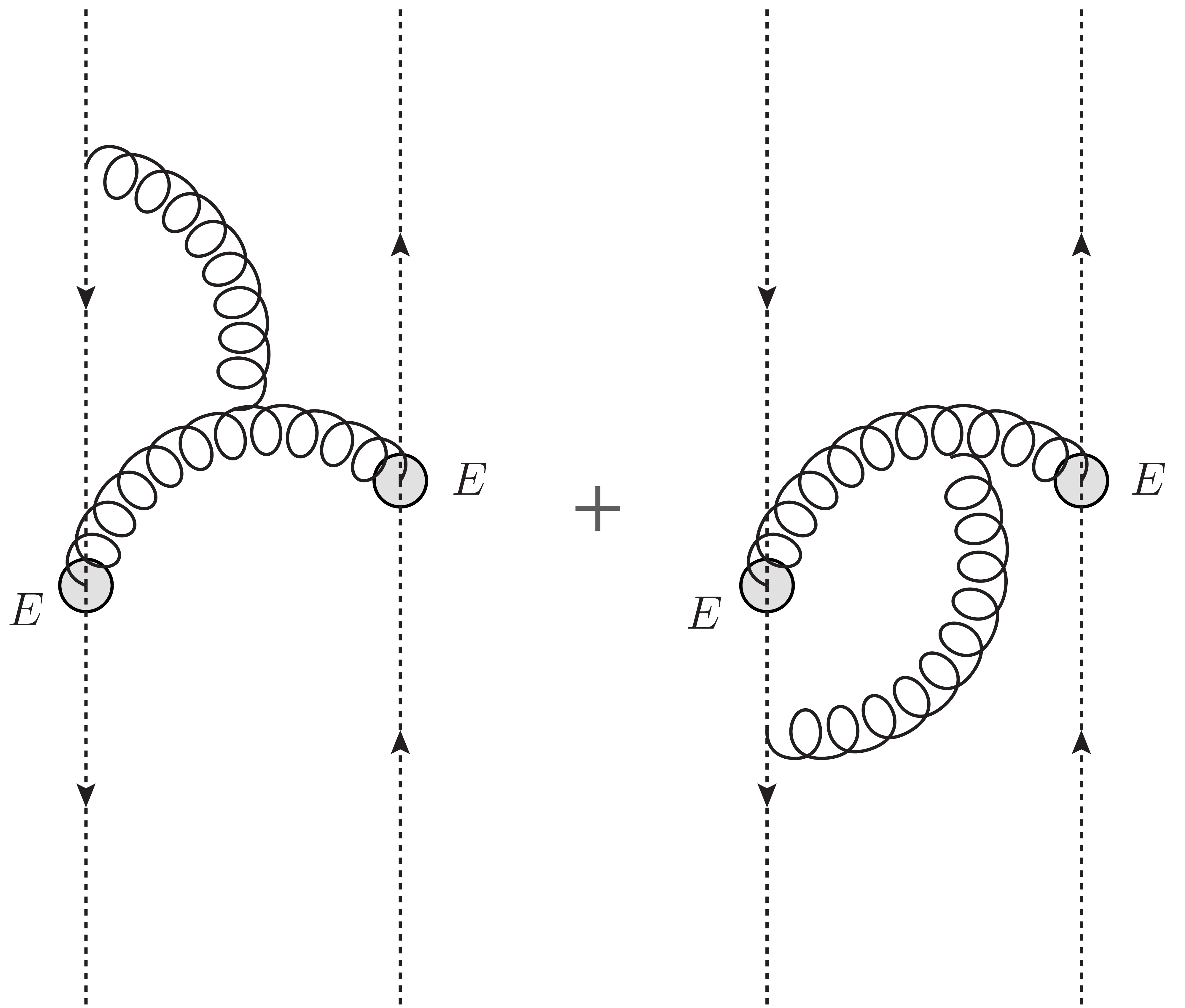
$Q\bar{Q}$



Perturbatively, one can isolate the difference between the correlators to these diagrams.

The difference is due to different operator orderings (different possible gluon insertions).

Q



Gauge independence of the difference

explicit gauge interpolation

- We performed an explicit calculation of the difference between the correlators in vacuum, with a gauge condition $G^a_M[A] = \frac{1}{\lambda} A^a_0(x) + \partial^\mu A^a_\mu(x)$.
- One finds that the difference is independent of λ, and equal to the Feynman gauge result.
- The axial gauge limit $\lambda \to 0$ is singular only if it is taken at the beginning of the calculation.

$\Longrightarrow$ The $Q\bar{Q}$ and Q correlators are different, gauge invariant quantities.

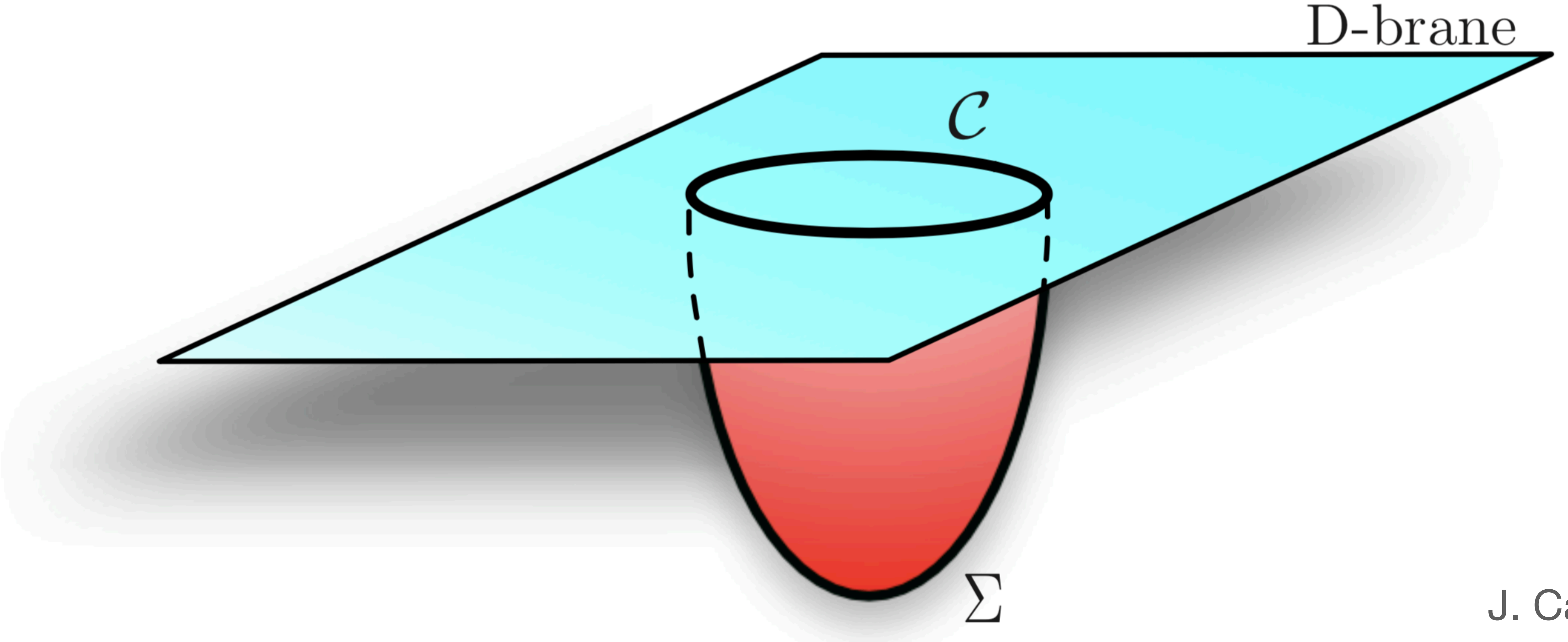
[**] J. Maldacena, hep-th/9711200

B. Strongly coupled calculation in $\mathcal{N} = 4$ SYM

setup

- The holographic duality provides a way to formulate the calculation of analogous correlators in strongly coupled theories. [**]
  - Wilson loops can be evaluated by solving classical equations of motion:

$$\langle W[\mathscr{C} = \partial\Sigma]\rangle_T = e^{iS_{\mathrm{NG}}[\Sigma]}\,.$$



J. Casalderrey-Solana, H. Liu, D. Mateos, K. Rajagopal and U. A. Wiedemann, hep-ph/1101.0618

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$$\langle W[\mathscr{C} = \partial\Sigma]\rangle_T = e^{iS_{\mathrm{NG}}[\Sigma]} .$$

    - Field strength insertions along the loop can be generated by taking variations of the path $\mathscr{C}$:

$$\frac{\delta}{\delta f^\mu(s_2)} \frac{\delta}{\delta f^\nu(s_1)} W[\mathscr{C}_f] \bigg|_{f=0} = (ig)^2 \mathrm{Tr}_{\mathrm{color}}\Big[U_{[1,s_2]} F_{\mu\rho}(\gamma(s_2))\dot{\gamma}^\rho(s_2) U_{[s_2,s_1]} F_{\nu\sigma}(\gamma(s_1))\dot{\gamma}^\sigma(s_1) U_{[s_1,0]} \Big]$$

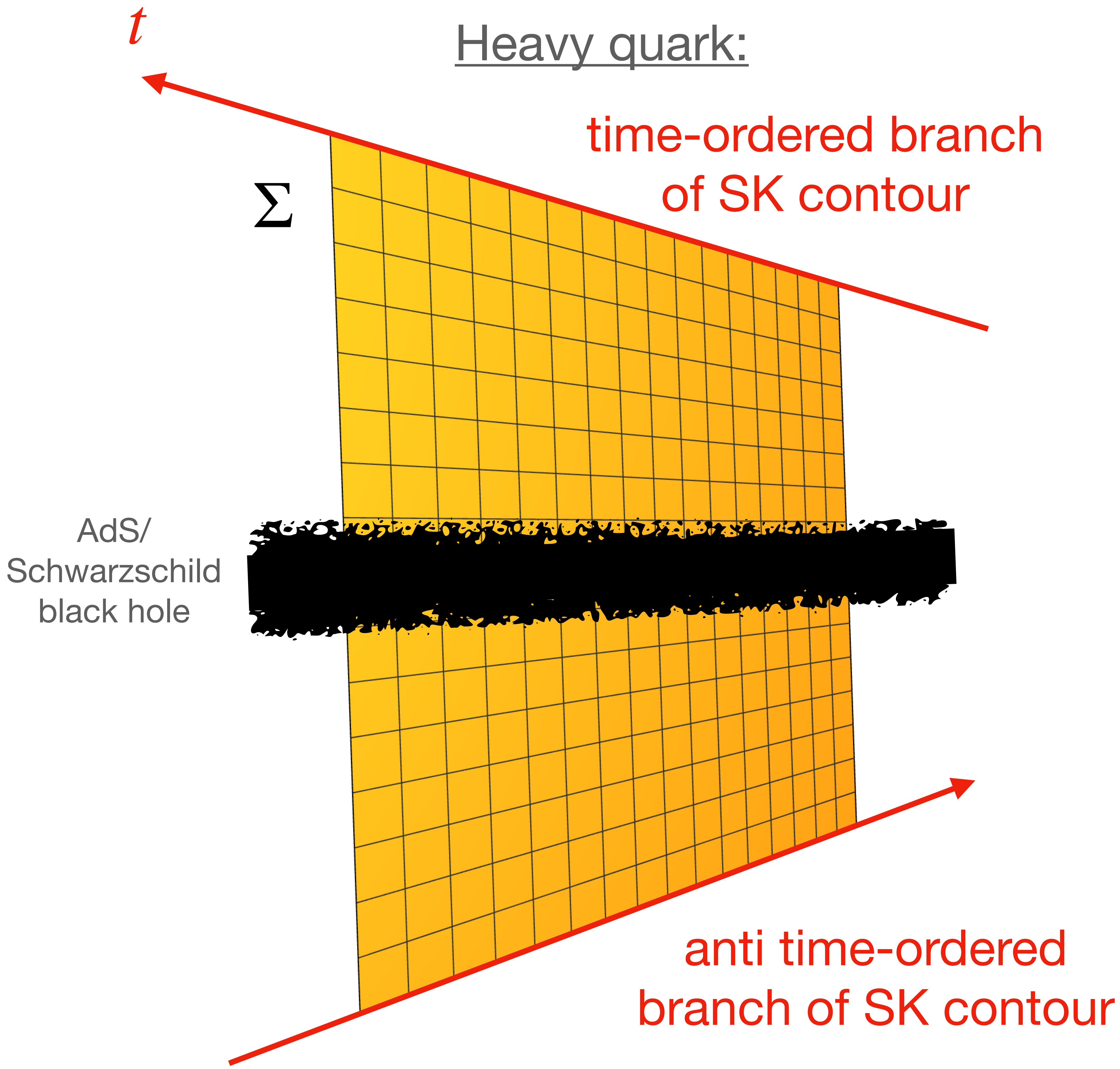
[***] J. Casalderrey-Solana and D. Teaney, hep-ph/0605199

G. Nijs, B. Scheihing-Hitschfeld and X. Yao, hep-ph/22XX.XXXXX

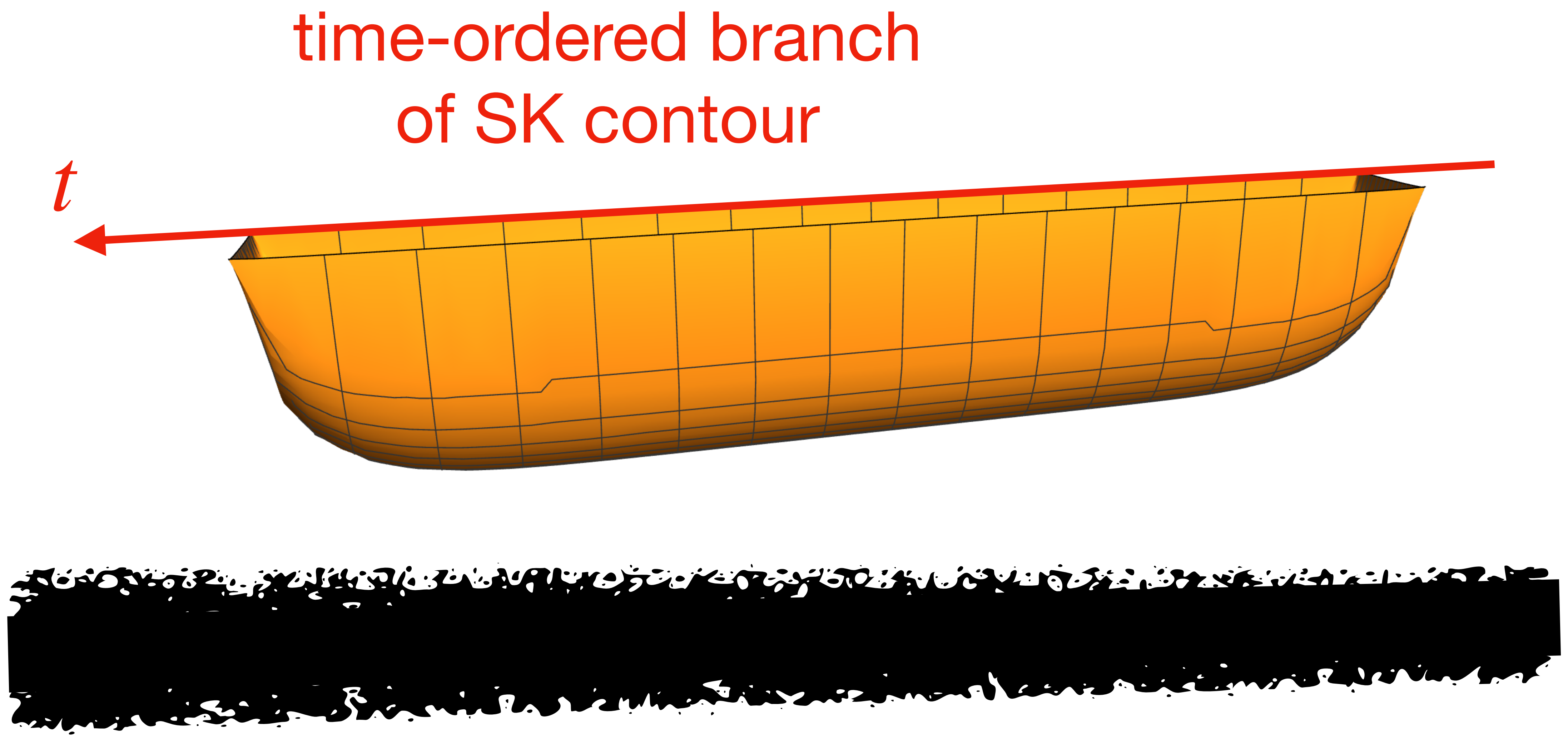
B. Strongly coupled calculation in $\mathcal{N} = 4$ SYM

the extremal worldsheets for HQ [***] and quarkonia

Heavy quark:



Quarkonia:



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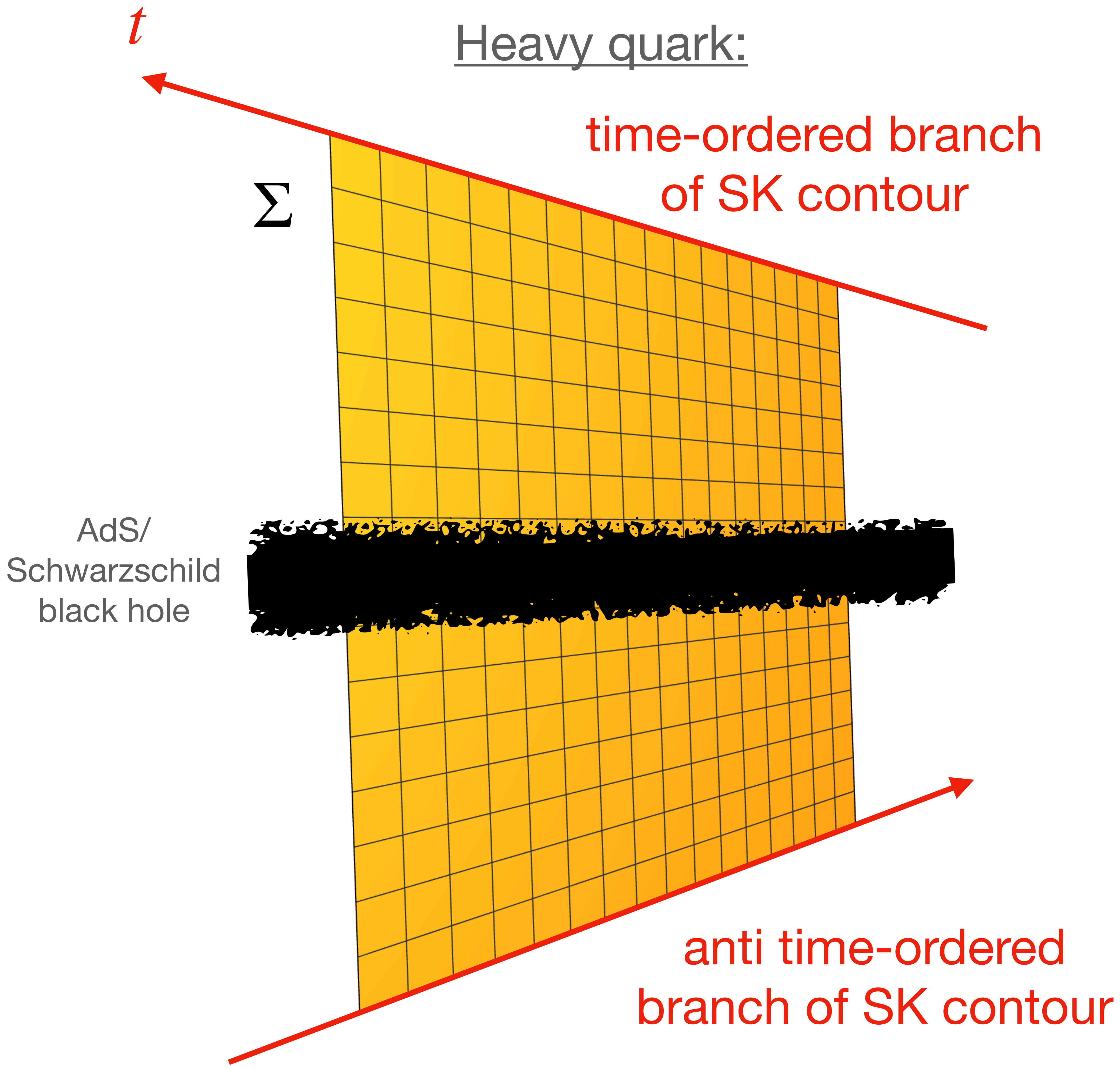
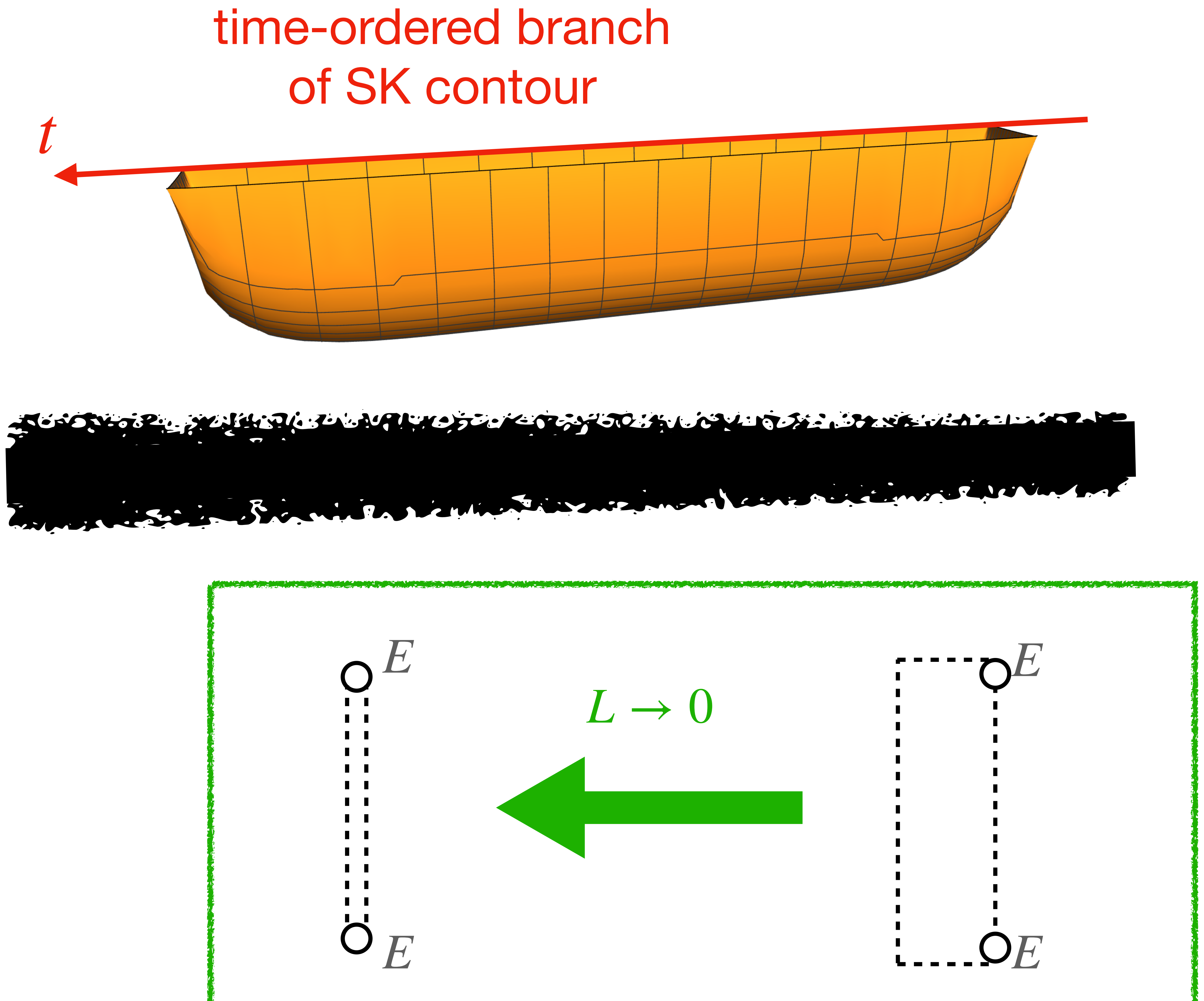


Quarkonia:



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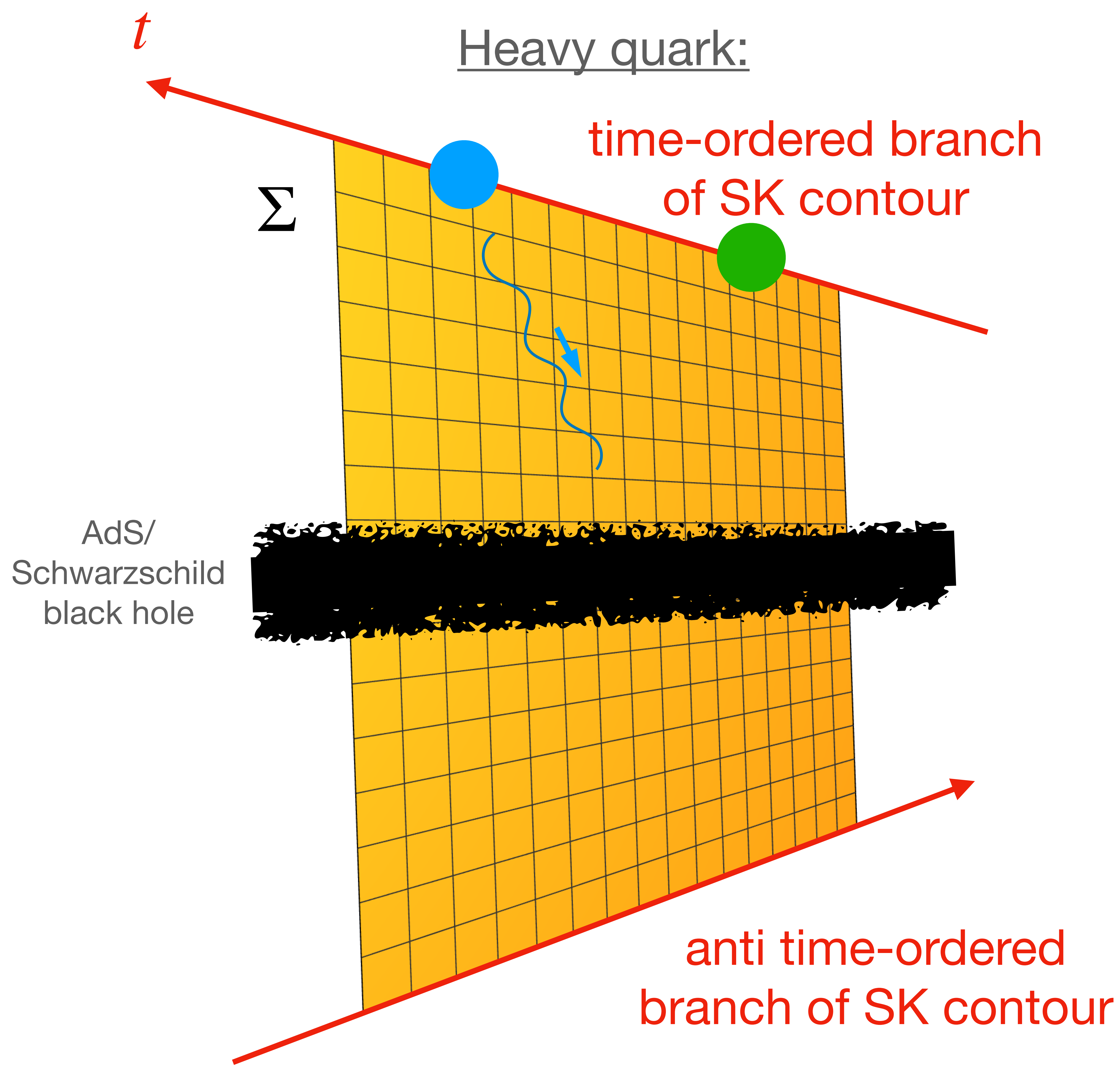
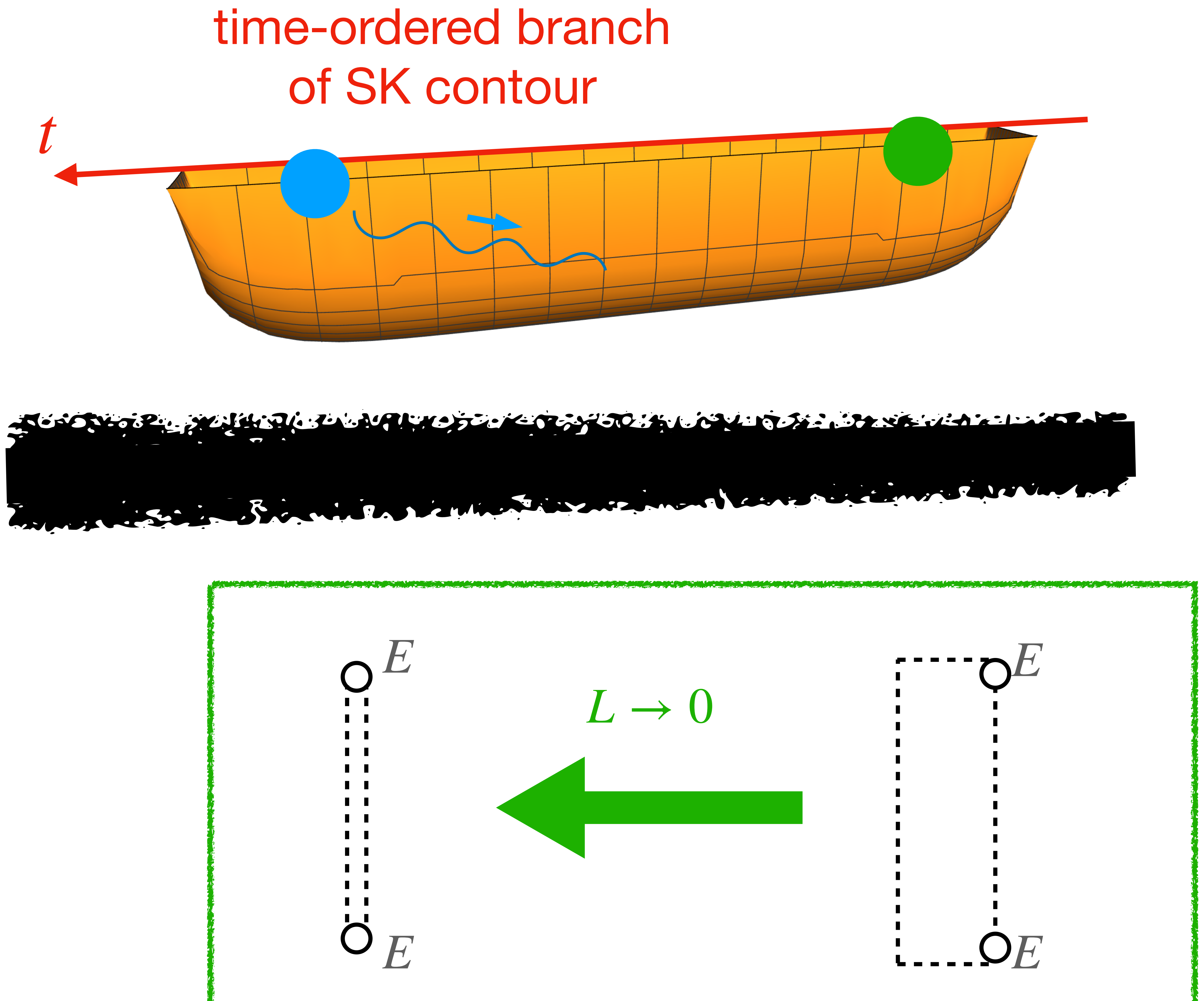


Quarkonia:



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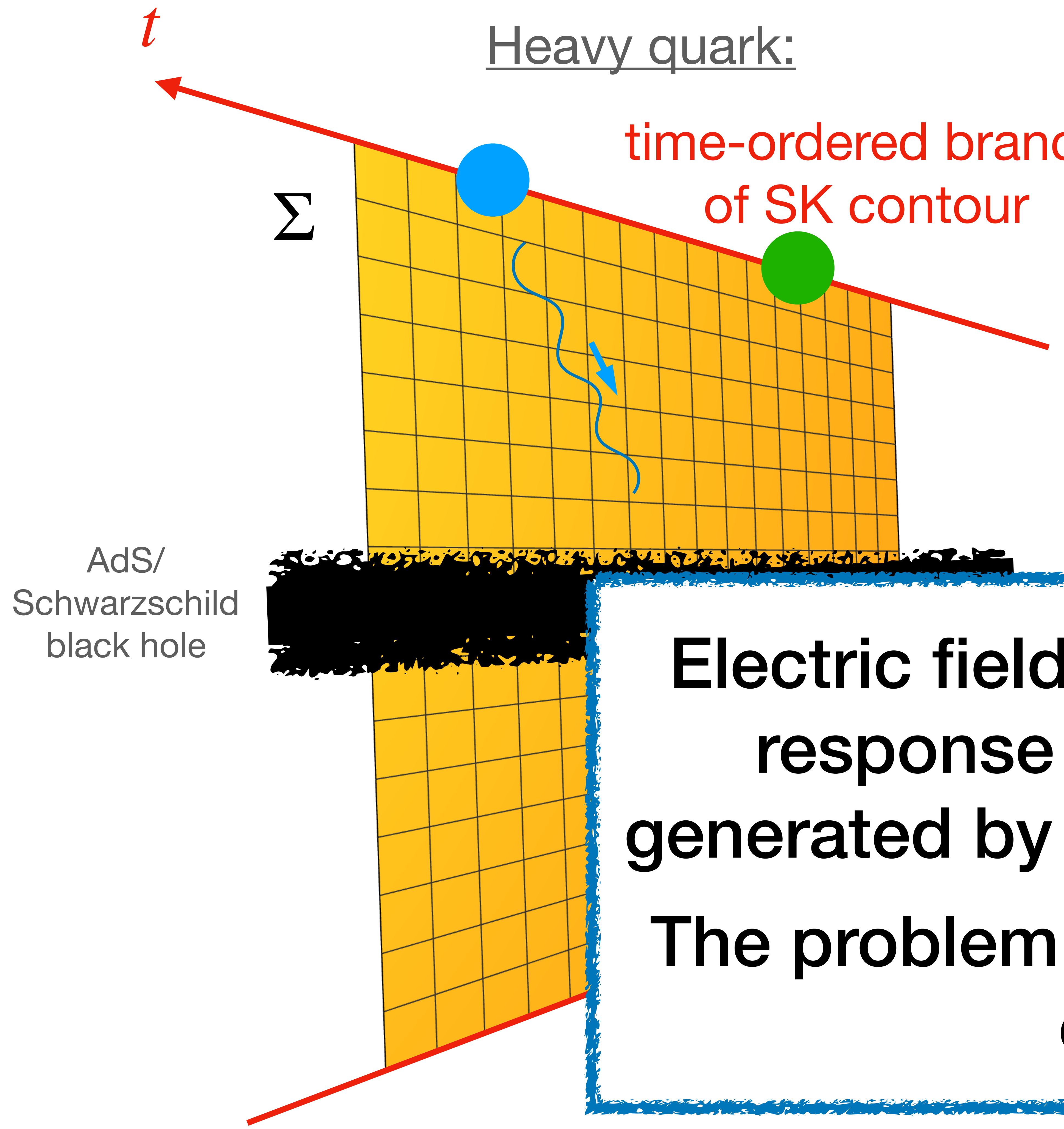
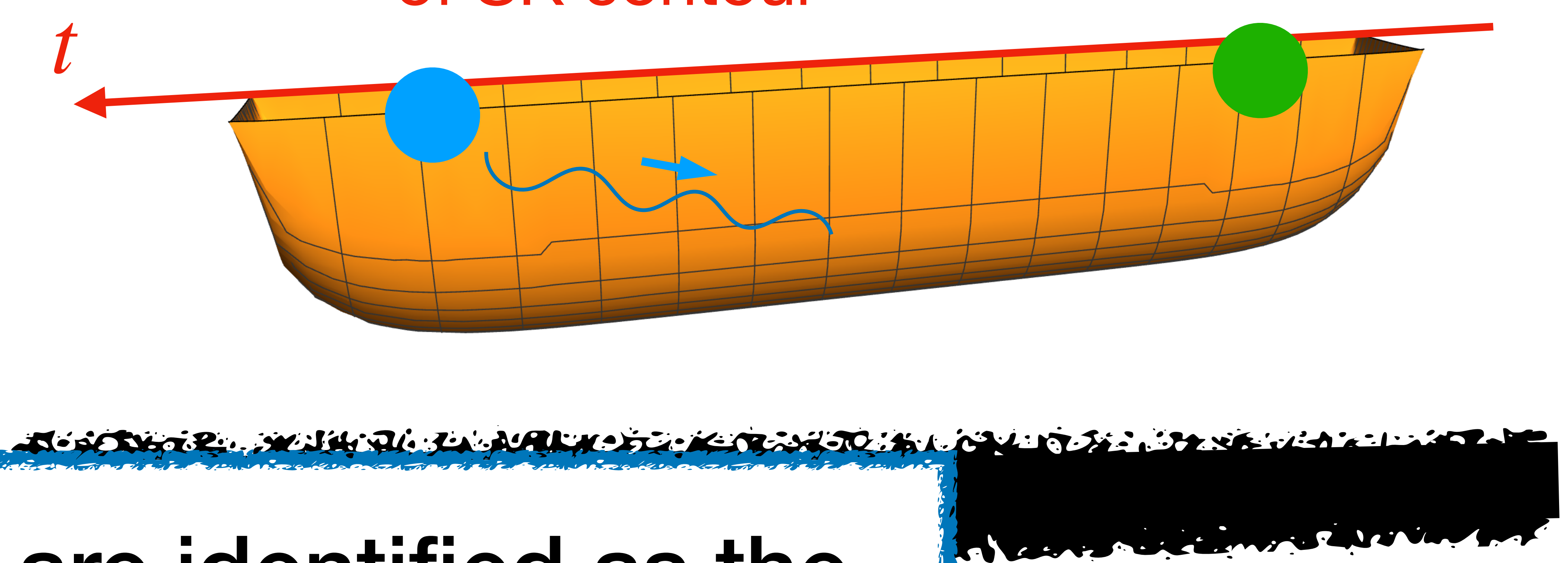
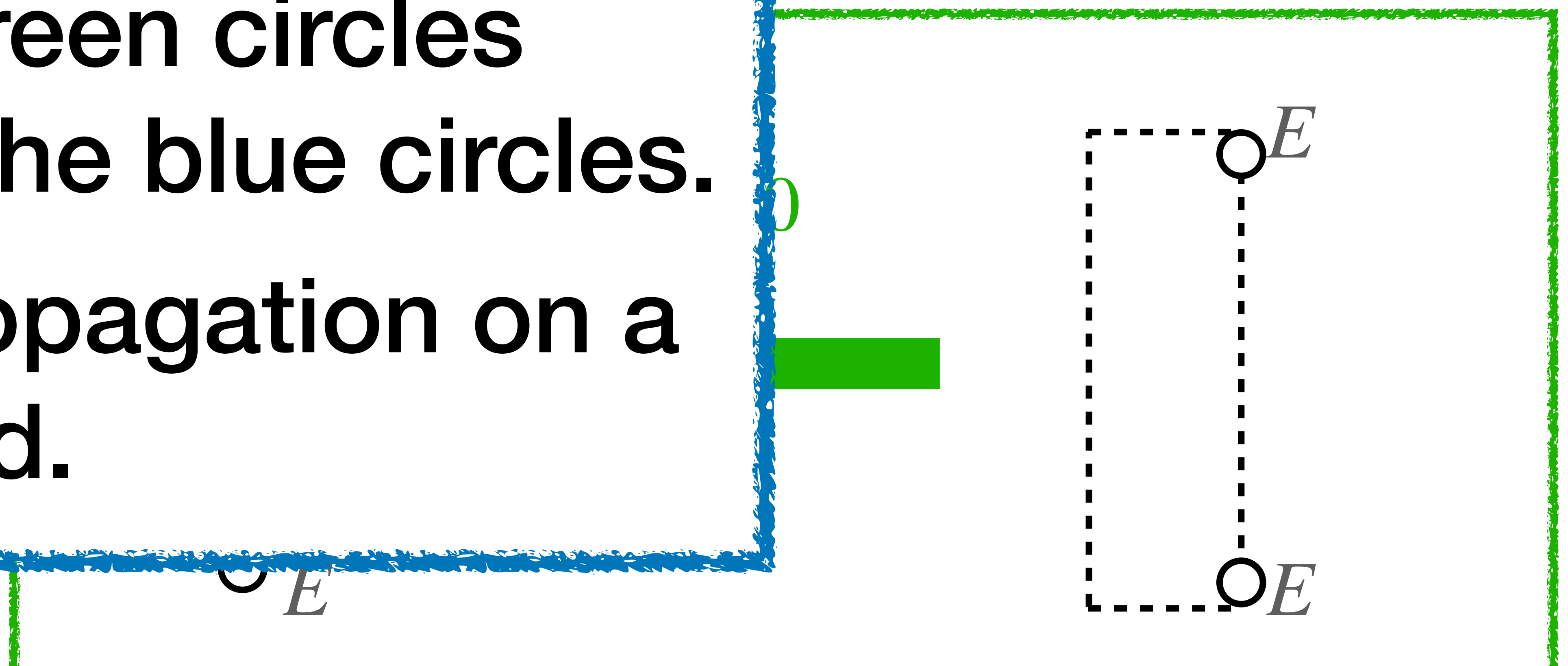


Quarkonia:





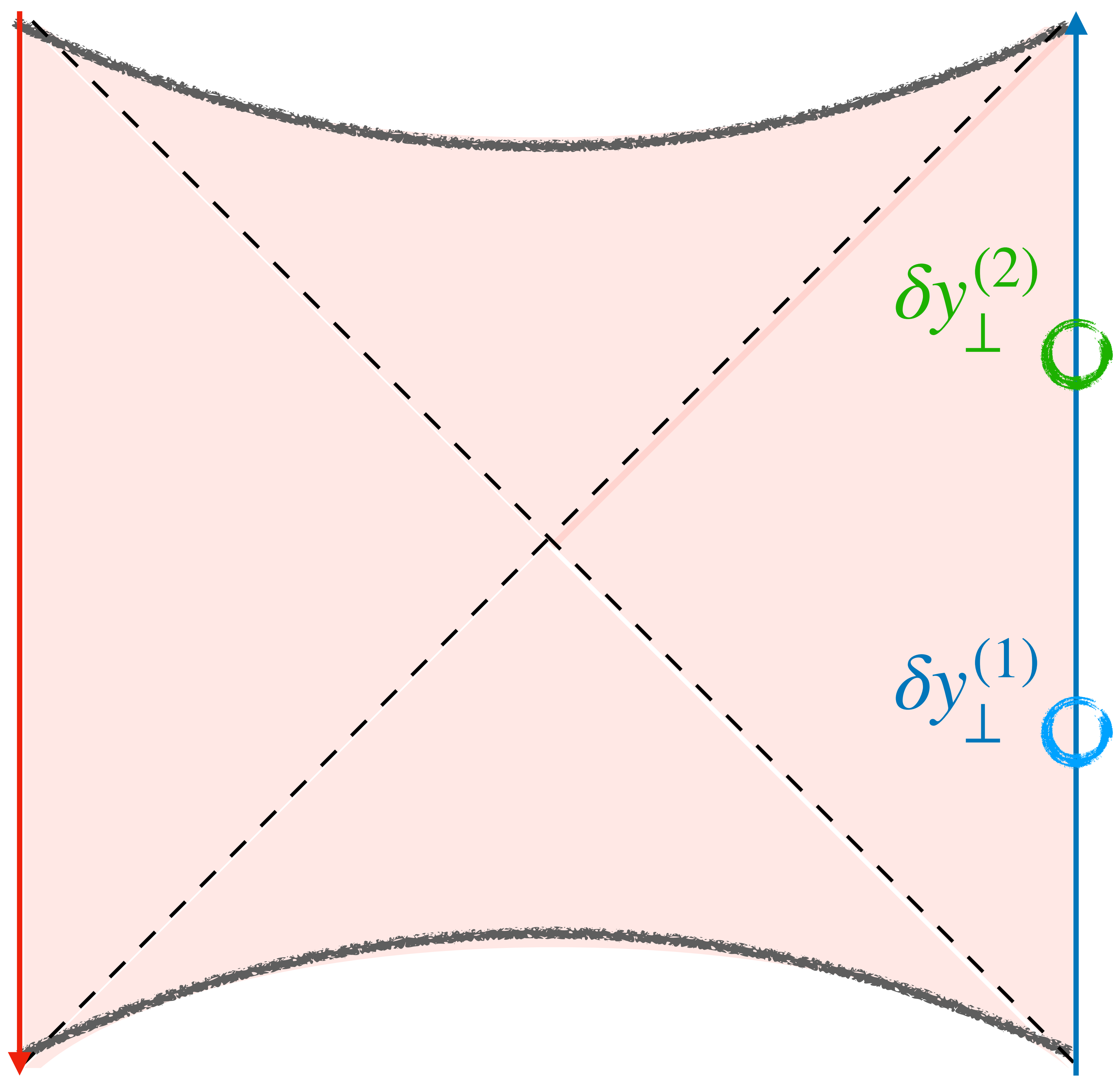
Electric field correlators are identified as the response functions at the green circles generated by a perturbation at the blue circles.

The problem is one of wave propagation on a curved background.

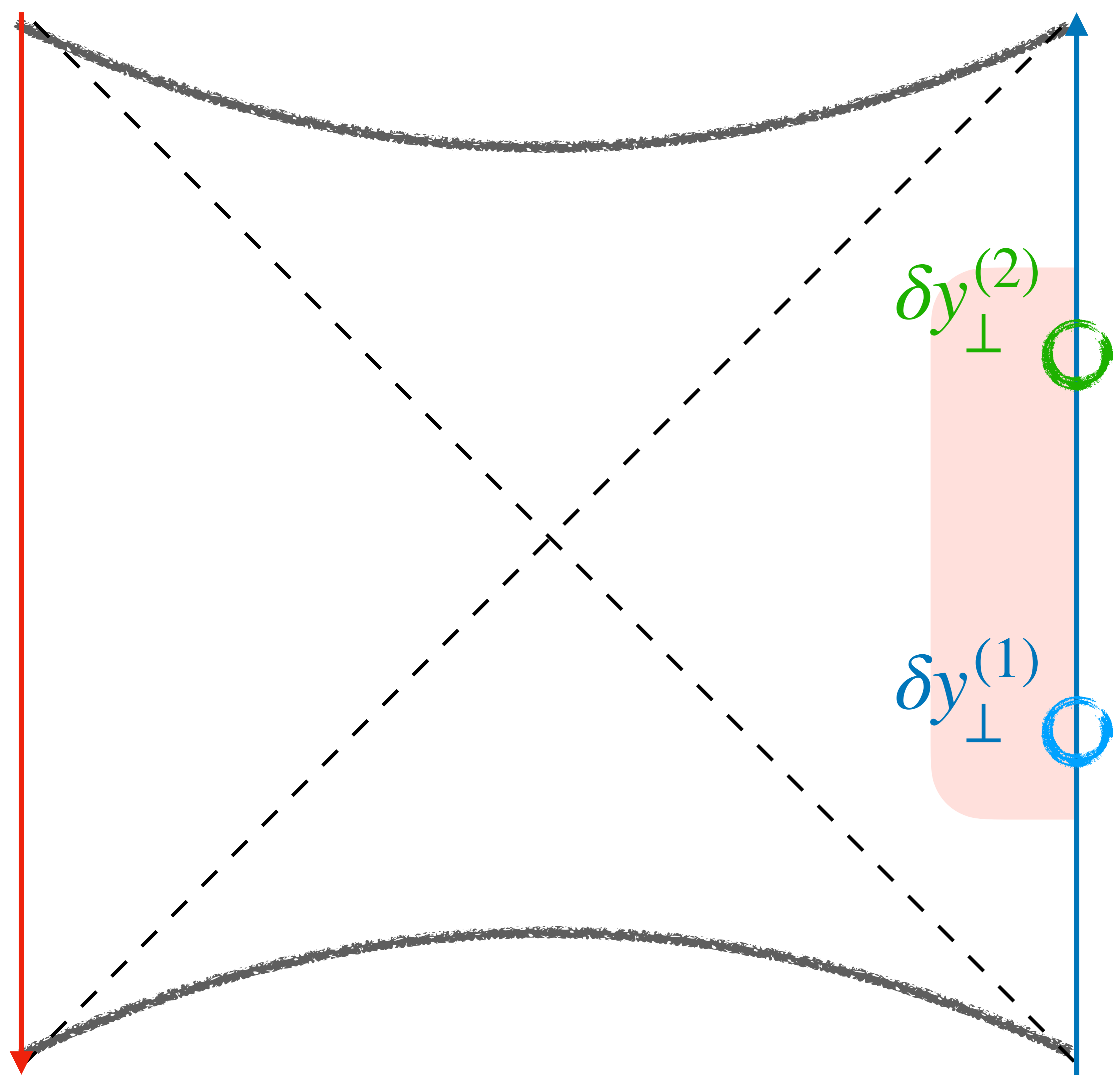
B. Strongly coupled calculation in $\mathcal{N} = 4$ SYM

Penrose diagrams

Heavy quark:



Quarkonia:



: Region covered by the world sheet

How the calculation proceeds

what equations do we need to solve?

- The classical, unperturbed equations of motion from the Nambu-Goto action to determine Σ:

$$S_{\rm NG} = -\frac{1}{2\pi\alpha'}\int d\tau d\sigma \sqrt{-\det\left(g_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu\right)}\ .$$

- The classical, linearized equation of motion with perturbations in order to be able to calculate derivatives of $\langle W[\mathscr{C}_f]\rangle_T = e^{iS_{\rm NG}[\Sigma_f]}$:

$$S_{\rm NG}[\Sigma_f] = S_{\rm NG}[\Sigma] + \int dt_1 dt_2 \left.\frac{\delta^2 S_{\rm NG}[\Sigma_f]}{\delta f(t_1)\delta f(t_2)}\right|_{f=0} f(t_1)f(t_2) + O(f^3)\ .$$

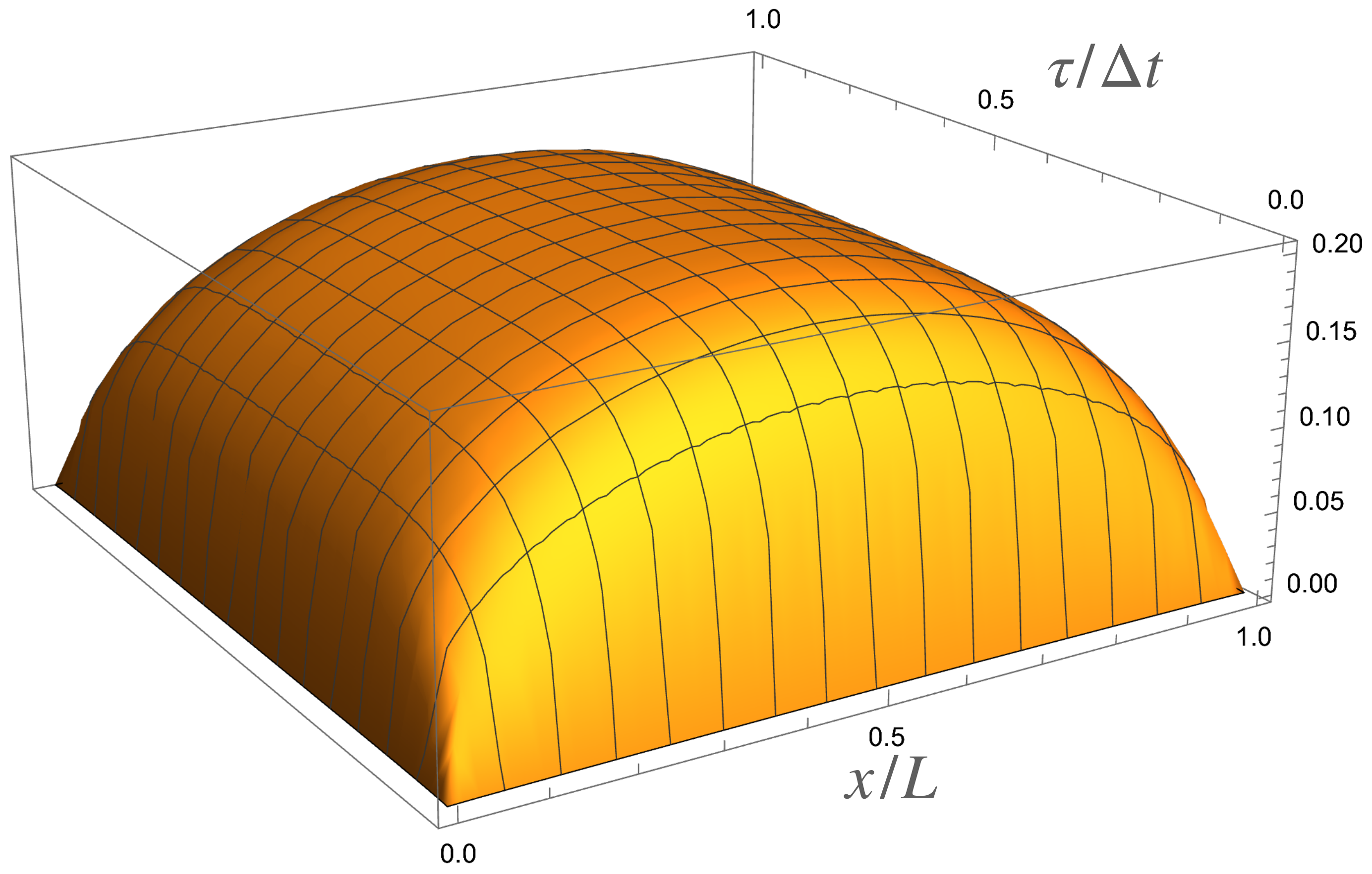
- In practice, the equations are only numerically stable in Euclidean signature, so we have to solve them and analytically continue back.

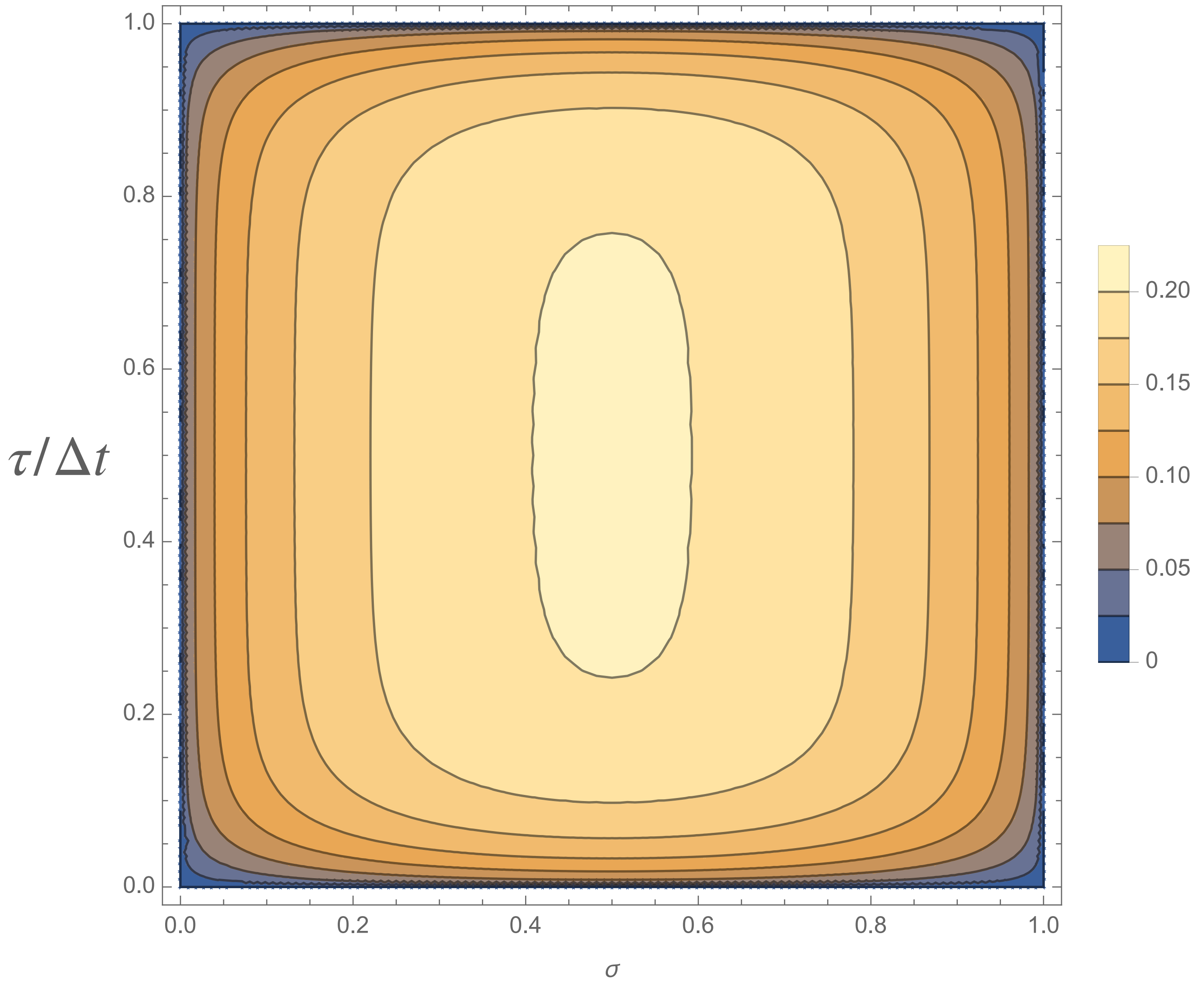
Extracting the EE correlator for quarkonia

the pipeline

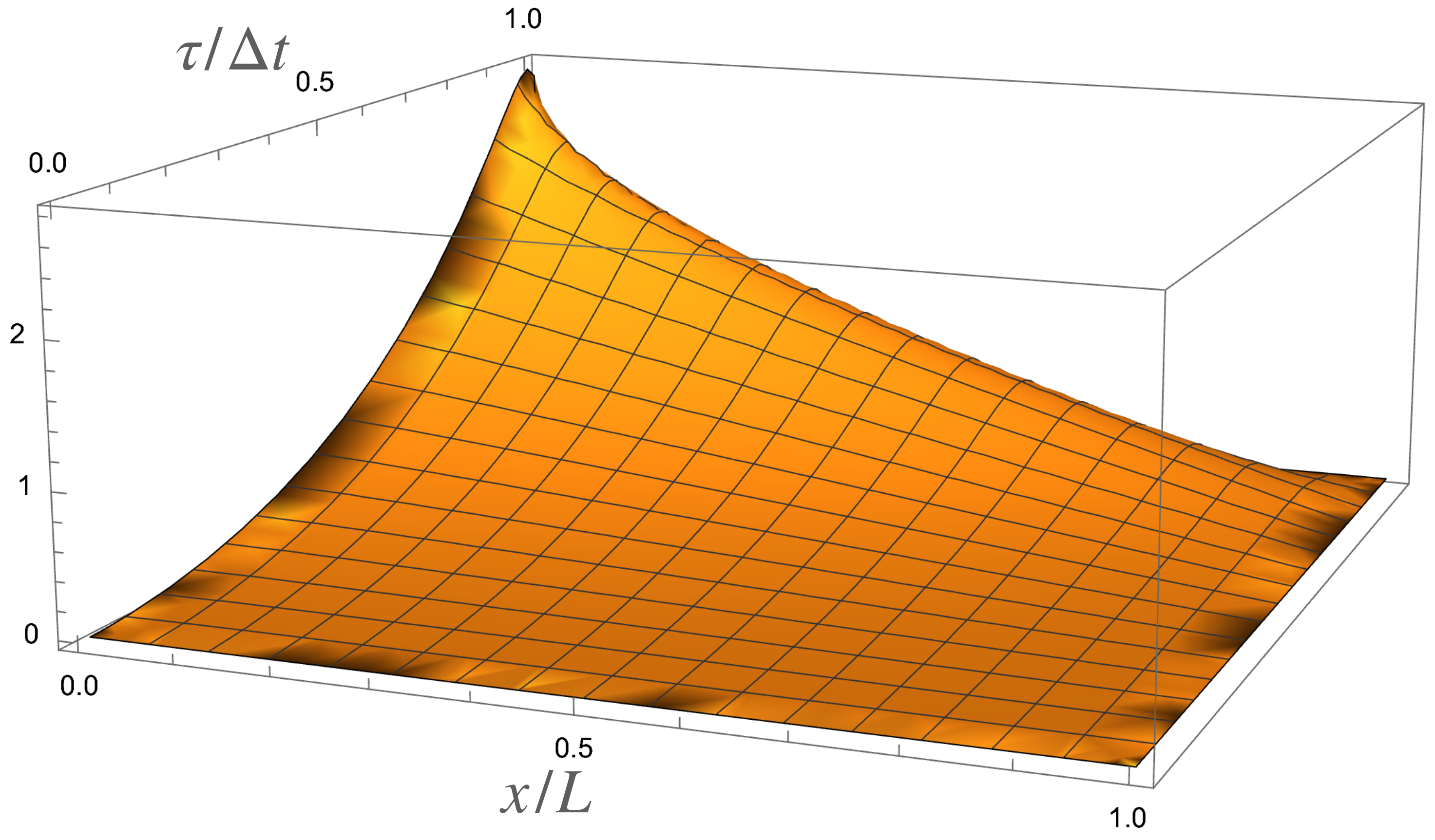
J.P. Boyd, “Chebyshev and Fourier Spectral Methods,” Dover books on Mathematics (2001)

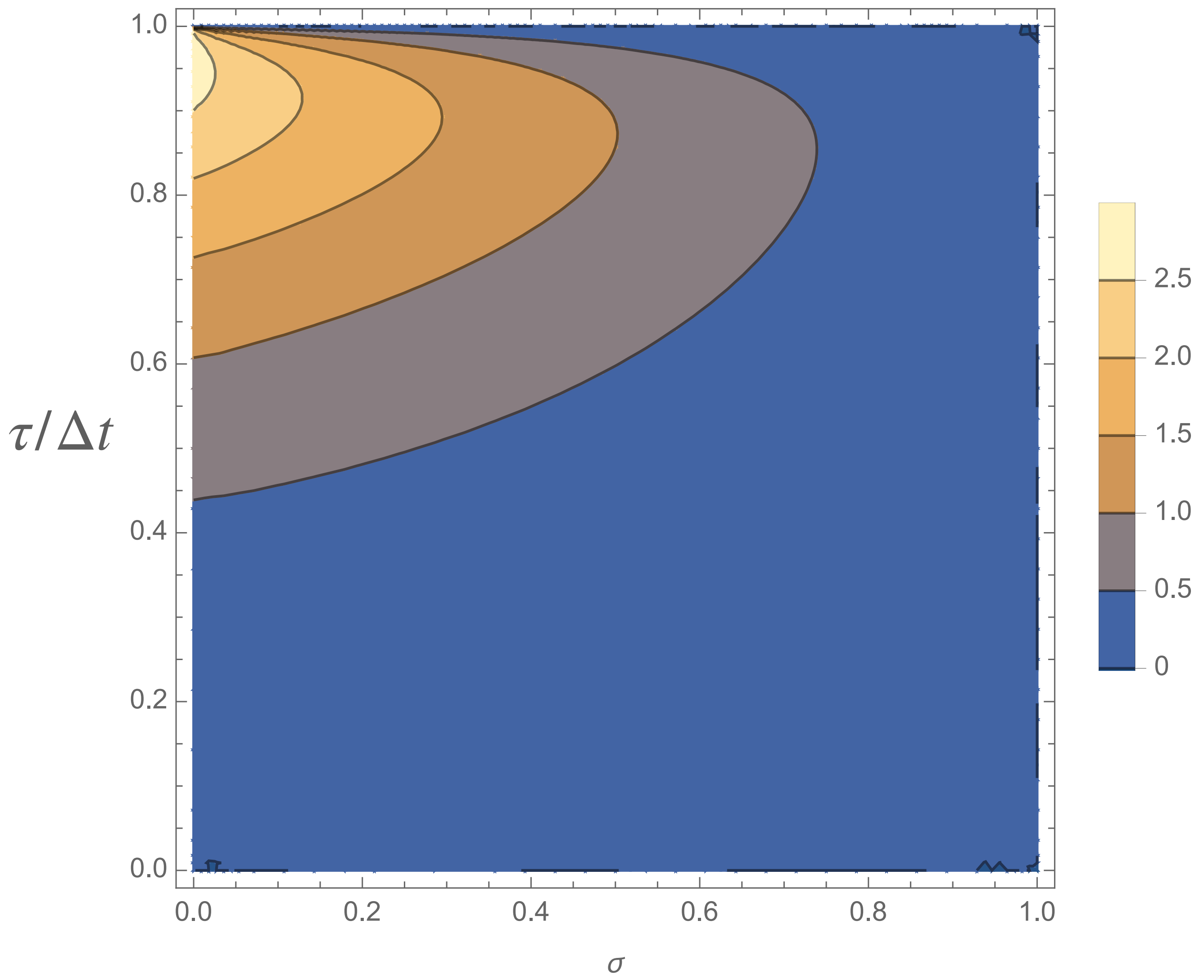
1) Solve for the background worldsheet solution:





2) Solve for the fluctuations with a source as a boundary condition:





3) Extrapolate in the limit $L \to 0$:

In progress. Stay tuned!

Summary and conclusions

- We have discussed how to calculate the chromoelectric correlators of the QGP that govern quarkonium transport

  A. at weak coupling in QCD

  B. at strong coupling in $\mathcal{N} = 4$ SYM

- We also discussed important aspects of operator ordering in QGP correlators.
- Heavy quarks, both single or in pairs, are rich probes of the QGP and give independent constraints on the underlying physics.
  - More specifically, they probe different correlation functions of the QGP
- Stay tuned for new results!

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Thank you!

Extra slides

The spectral function of quarkonia

symmetries and KMS relations

The KMS conjugates of the previous correlators are such that

$$[g_E^{++}]_{ji}^{>}(q) = e^{q^0/T}[g_E^{++}]_{ji}^{<}(q)\,, \quad [g_E^{--}]_{ji}^{>}(q) = e^{q^0/T}[g_E^{--}]_{ji}^{<}(q)\,,$$

and one can show that they are related by

$$[g_E^{++}]_{ji}^{>}(q) = [g_E^{--}]_{ji}^{<}(-q)\,, \quad [g_E^{--}]_{ji}^{>}(q) = [g_E^{++}]_{ji}^{<}(-q)\,.$$

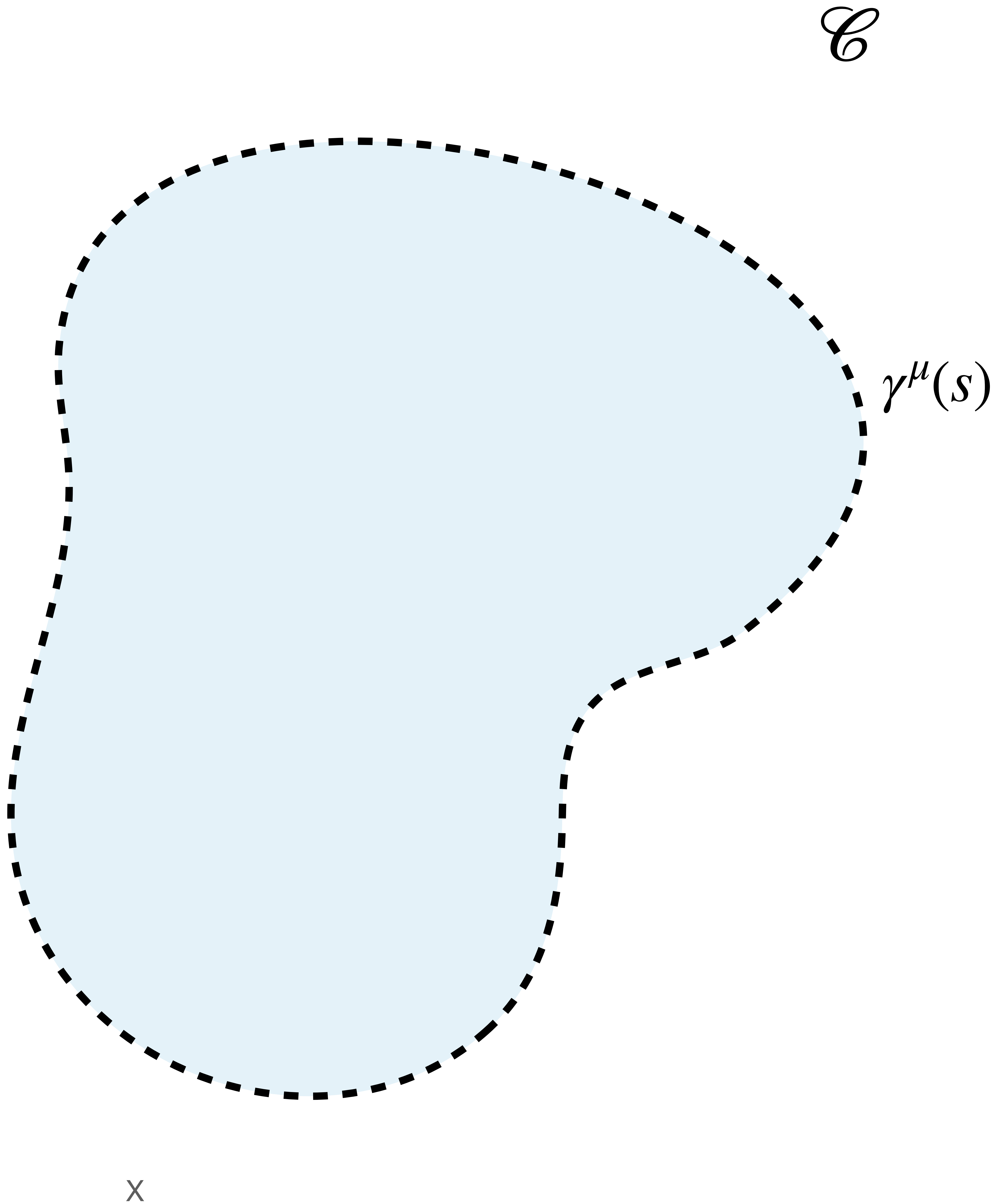
The spectral functions $[\rho_E^{++/--}]_{ji}(q) = [g_E^{++/--}]_{ji}^{>}(q) - [g_E^{++/--}]_{ji}^{<}(q)$ are not necessarily odd under $q \leftrightarrow -q$. However, they do satisfy:

$$[\rho_E^{++}]_{ji}(q) = -\,[\rho_E^{--}]_{ji}(-q)\,.$$

Appendix: derivation of field strength correlators from Wilson loop variations

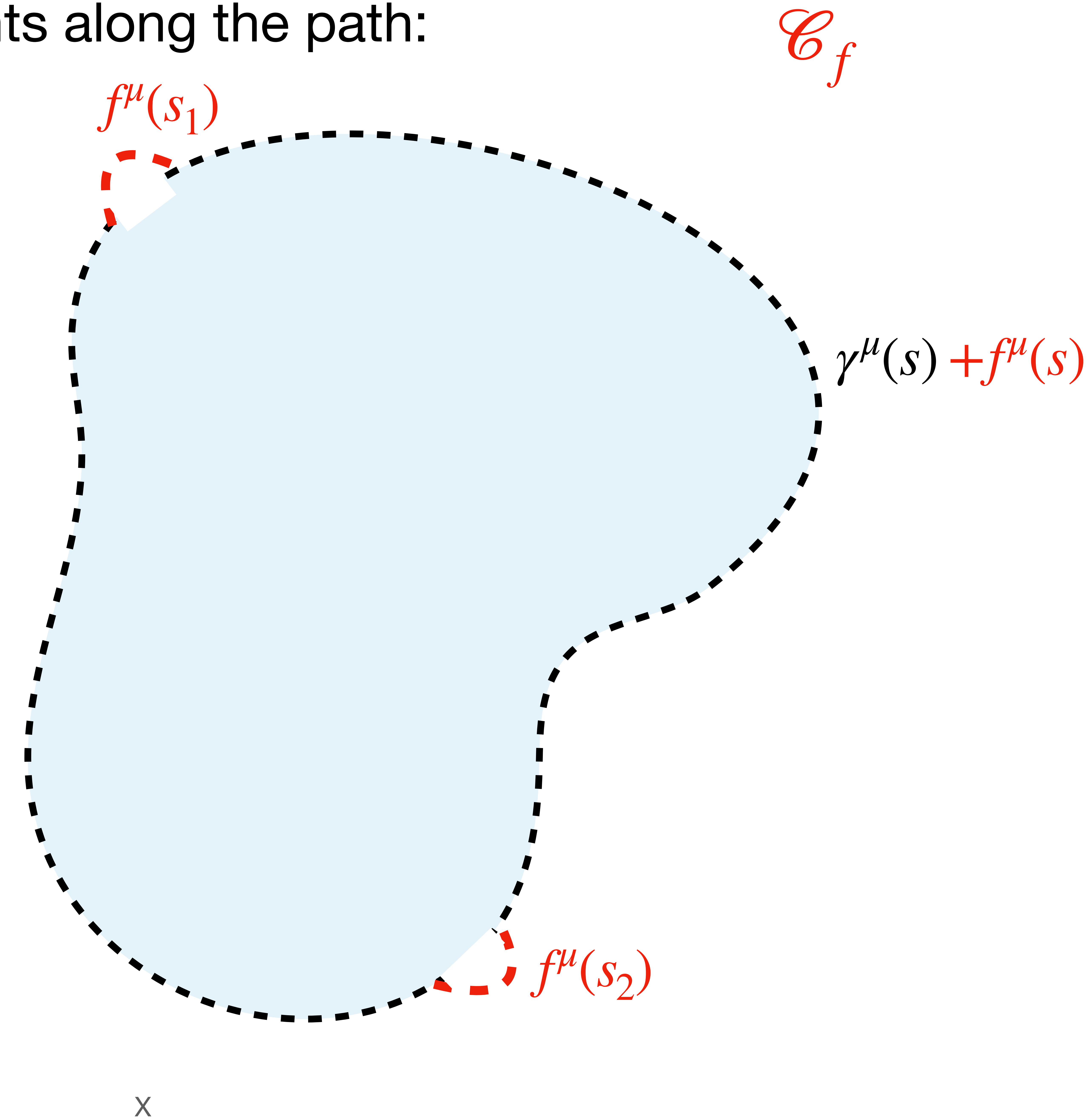
Consider a Wilson loop:

$W[\mathscr{C}] =$



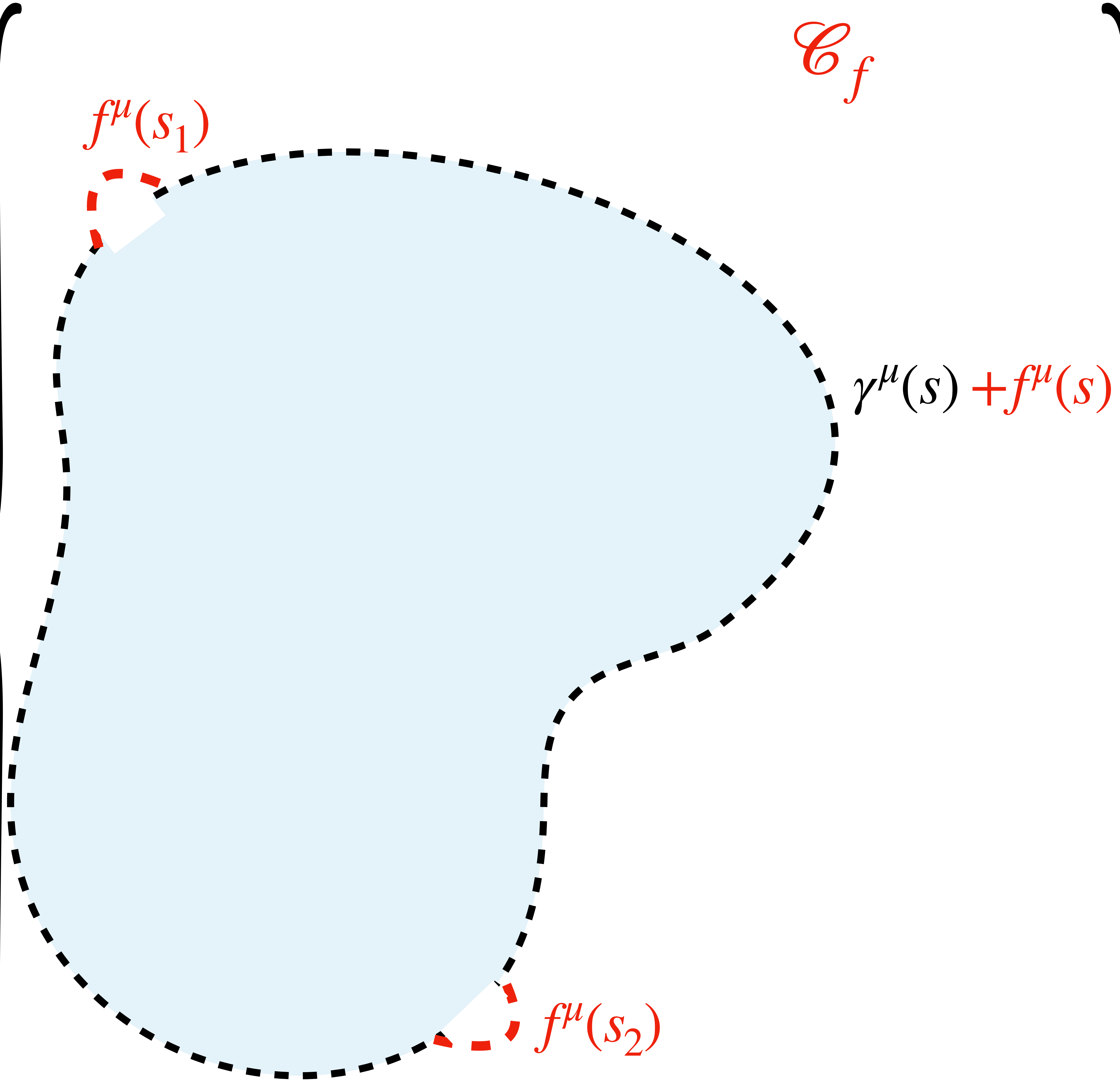
Deform it slightly around two points along the path:

$W[\mathscr{C}_f] =$



Take derivatives:

$$\frac{\delta}{\delta f^{\mu}(s_2)}\frac{\delta}{\delta f^{\nu}(s_1)}W[\mathscr{C}_f]\Big|_{f=0} = \frac{\delta}{\delta f^{\mu}(s_2)}\frac{\delta}{\delta f^{\nu}(s_1)}\Bigg\{$$



$$\Bigg\}_{f=0}$$

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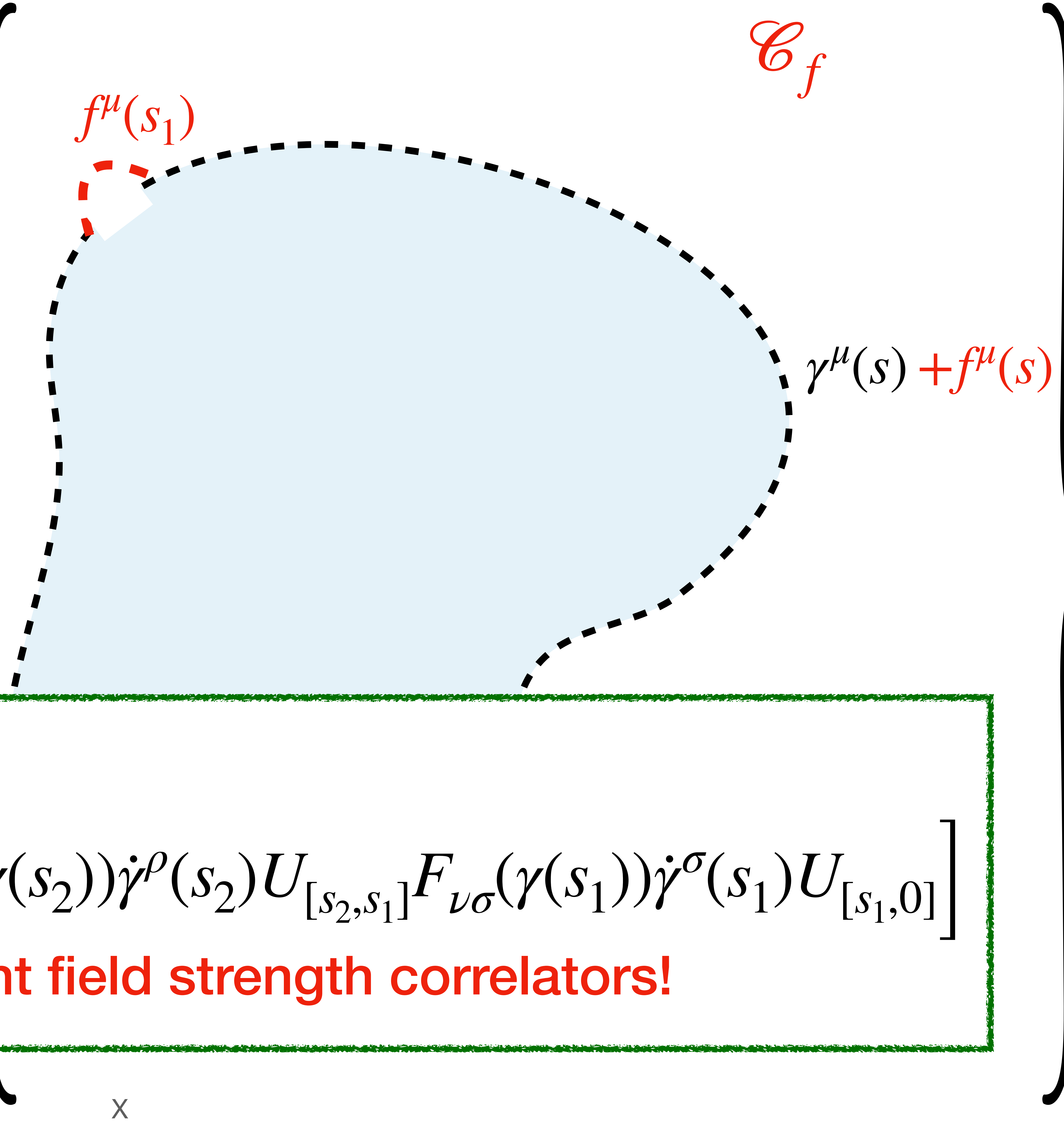
$$\frac{\delta}{\delta f^\mu(s_2)}\frac{\delta}{\delta f^\nu(s_1)}W[\mathscr{C}_f]\Big|_{f=0} = \frac{\delta}{\delta f^\mu(s_2)}\frac{\delta}{\delta f^\nu(s_1)}\left\{ \quad \right\}_{f=0}$$



$\Longrightarrow$ **The result is:**

$$= (ig)^2 \mathrm{Tr}_{\mathrm{color}}\left[U_{[1,s_2]}F_{\mu\rho}(\gamma(s_2))\dot{\gamma}^\rho(s_2)U_{[s_2,s_1]}F_{\nu\sigma}(\gamma(s_1))\dot{\gamma}^\sigma(s_1)U_{[s_1,0]}\right]$$

Gauge invariant field strength correlators!