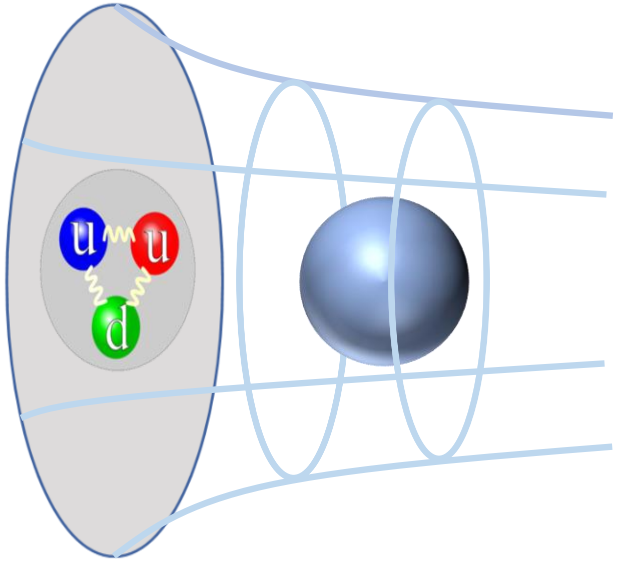
Holographic Baryons



Edwan PREAU

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[To appear soon on the ArxiV]

Outline

1) Motivation

2) Introduction 1 : Baryons in the chiral Effective Field Theory

3) Introduction 2 : The holographic correspondence

4) The V-QCD framework and its baryon

5) Summary and outlook

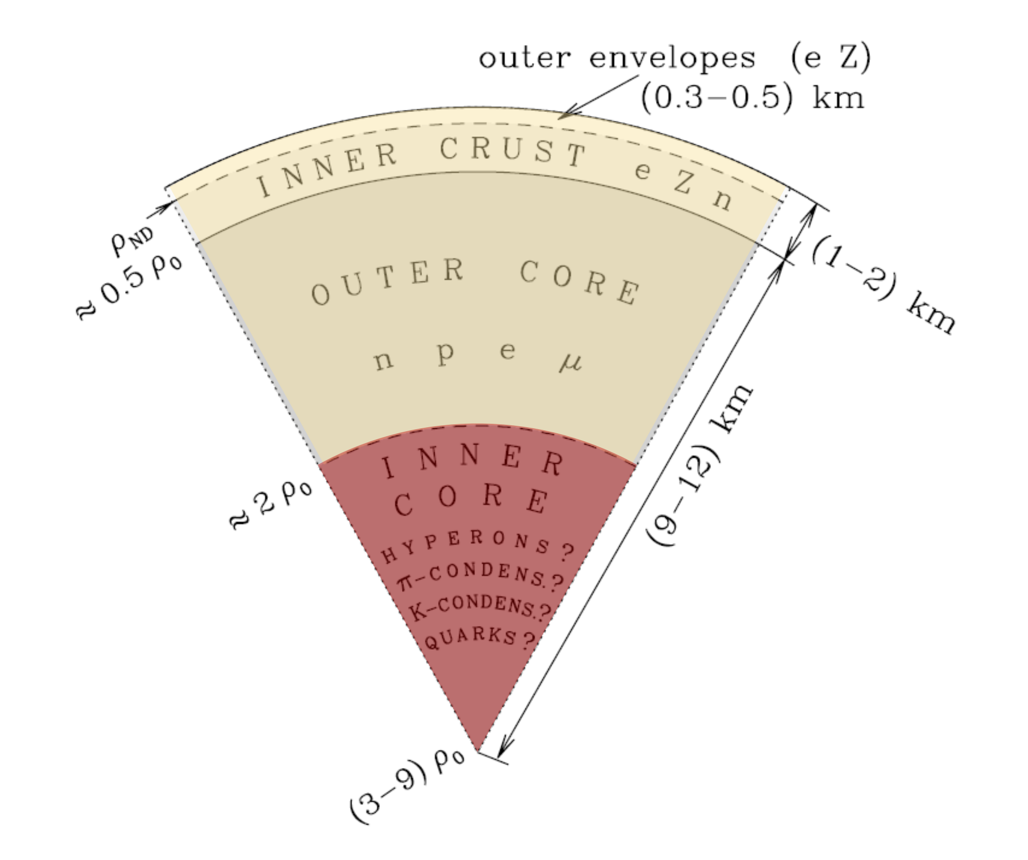
Motivation: The Neutron Star core enigma

The state of matter is known in the outer layers of Neutron Stars (NS), up to about 2 times the nuclear density $\rho_0 = 0.16\ \text{fm}^{-3}$

It is still a mystery in the inner core ($\rho \gtrsim 2\rho_0$)

There, we reach a phase where the matter is both very dense and strongly coupled

(low energy QCD)



The neutron star structure

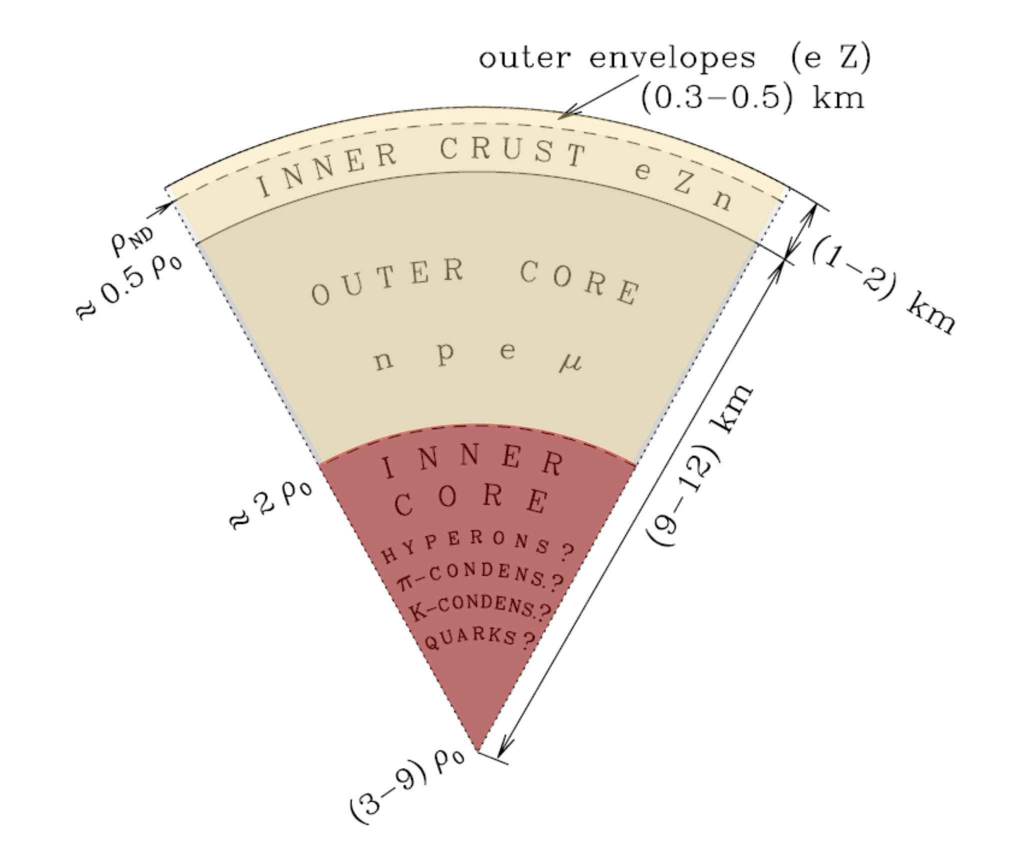
Motivation: The Neutron Star core enigma

Numerical methods (lattice QCD) are unable to reach very high densities because of the sign problem

We use the holographic method : a way of getting analytic insight into strongly coupled problems

Problem : understand baryonic physics at high density in holographic QCD

I will discuss the most elementary aspect of this problem : construct a 1-baryon state in holography



The neutron star structure

Baryon in Chiral Effective Field Theory

The low-energy Lagrangian is written as an expansion in a UV cut-off Λ

$$\mathcal{L} = \mathcal{L}^{(0)} + \Lambda^{-1}\mathcal{L}^{(1)} + \Lambda^{-2}\mathcal{L}^{(2)} + \cdots,$$

which is built out of chirally symmetric local functions of the pion matrix

$$U(x) \equiv \mathrm{e}^{2\pi i\,\Pi^{a}(x)T^{a}} \in SU(N_f)_A$$

IR QCD possesses stable baryonic bound states → Can the baryons fit in this picture?

The scaling of M_B in the large N_c limit suggests that baryons are solitons of the weakly coupled theory of mesons :

$$M_B \sim N_c \sim \frac{1}{g_{meson}}$$

[Witten, 1979]

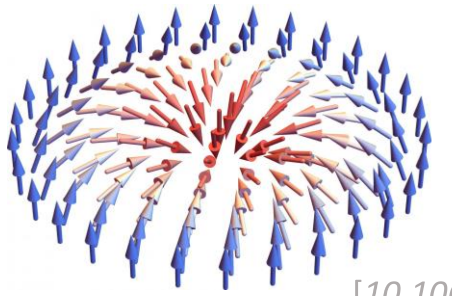
Baryon in Chiral Effective Field Theory

The leading term is the non-linear sigma model Lagrangian

$$\mathcal{L}^{(2)} = \frac{1}{4} f_\pi^2 \,\mathrm{Tr}\, \partial U^\dagger \partial U \,, \quad U \in SU\left(N_f\right)_A$$

Because $\Pi_3\left(SU\left(N_f\right)\right) = \mathbb{Z}$, this theory admits topological soliton solutions

$$N_{Skyrmion} = \frac{1}{24\pi^2} \mathrm{Tr} \int \left(U^\dagger \wedge \mathrm{d}U\right)^3 .$$



[10.1063/1.4983805]

But the skyrmion has to be of 0 size

→The Skyrme model introduces a quartic term that stabilizes the skyrmion size [Skyrme, 1961]

$$\mathcal{L}_{Skyrme} = \frac{1}{4} f_\pi^2 \,\mathrm{Tr}\, \partial U^\dagger \partial U + \frac{1}{32e^2} \mathrm{Tr}\{[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U][U^\dagger \partial^\mu U, U^\dagger \partial^\nu U]\} ,$$

Baryon in Holographic theories

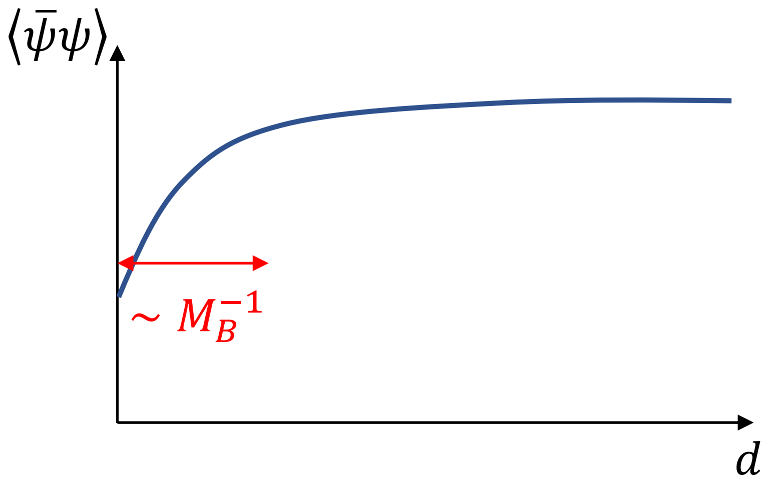
The Skyrme model gives a low-energy effective description of baryon states (↔ skyrmions)

Holographic theories reproduce a similar picture starting from a theory in 5D

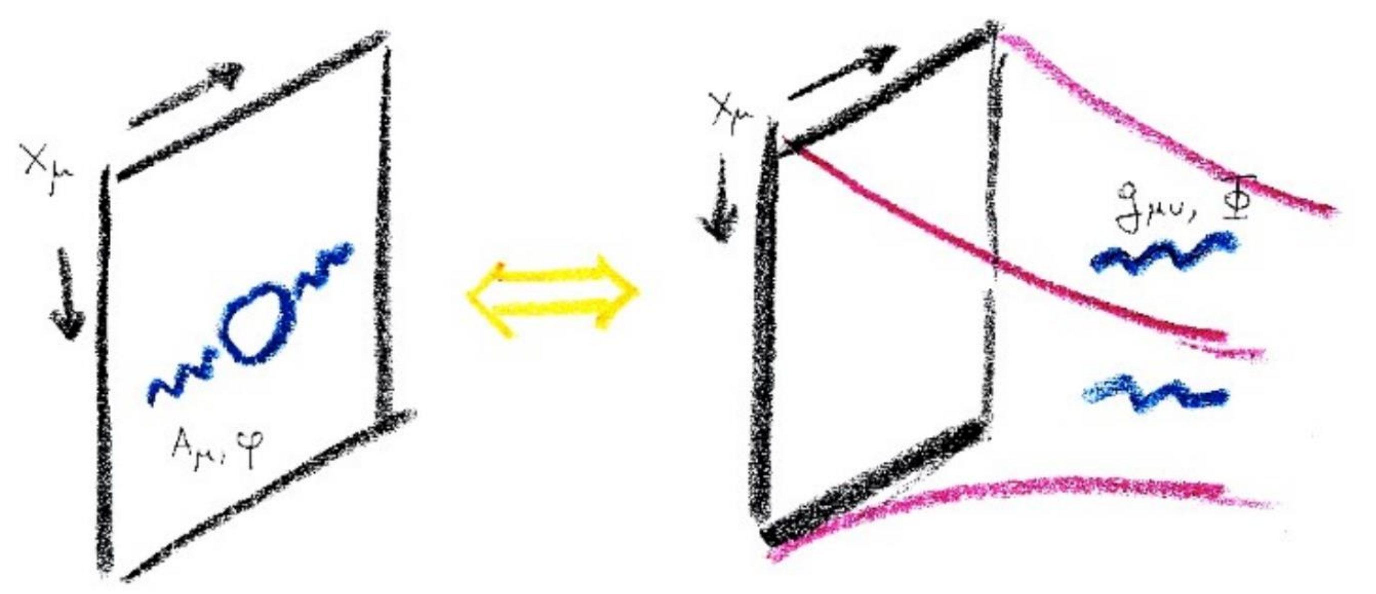
$$N_{Baryon} = N_{instanton} = N_{Skyrmion} \,.$$

V-QCD is a much more complete theory of low T QCD:

- Incorporates coupling to all higher mesonic excitations $(\sigma, \rho, \omega, \cdots)$
  → Contribute at high density
- Chiral condensate profile around the baryon
  → (partial) chiral symmetry restoration at the center
- Back-reaction of flavor onto glue → crucial at high density



The Holographic Correspondence



Duality bewteen a QFT in 4D and a semi-classical gravitational theory in 5D.

If the QFT is strongly coupled, then the dual theory is weakly curved.

The 4D space-time on which the QFT is defined is the boundary of the dual 5D space-time (bulk)

The additional dimension *z* is called the holographic coordinate and identified with the energy scale such that:

UV ⟷ boundary

IR ⟷ center

The Holographic Dictionary

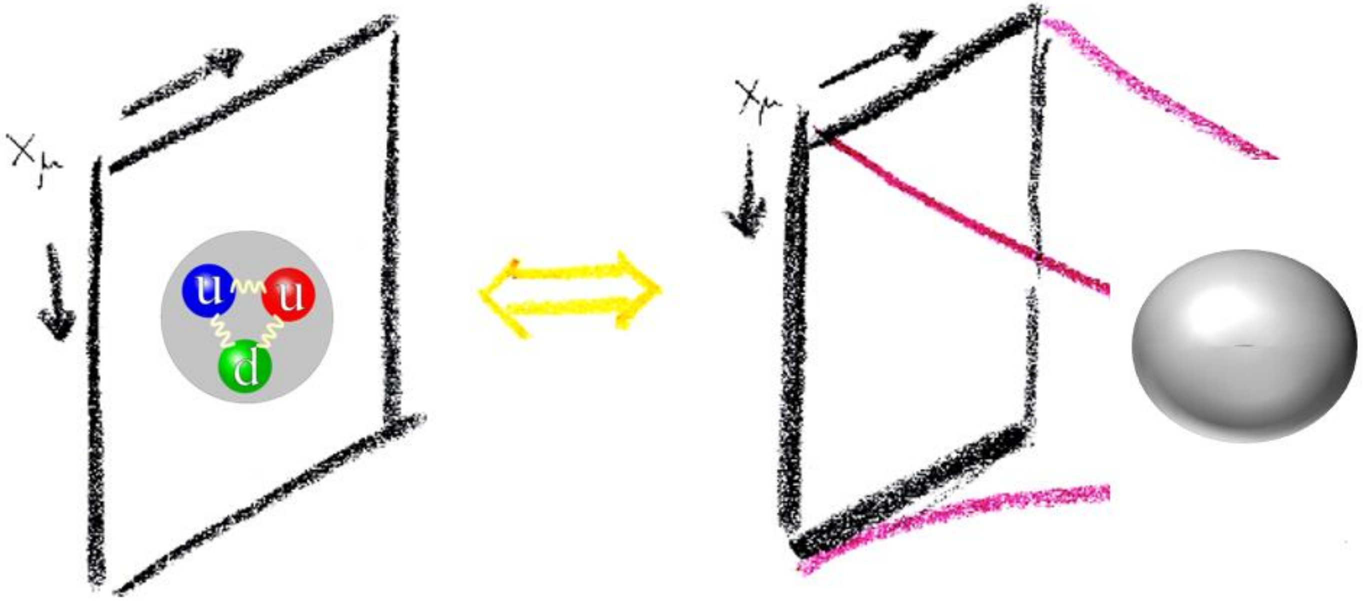
Every QFT operator has a dual field in the bulk of same spin

$T_{\mu\nu}$	$\leftrightarrow$	g_{MN}
O	$\leftrightarrow$	φ
G : $\partial_\mu J^\mu = 0$	$\leftrightarrow$	G : A^M

Holographic Baryon

[hep-th/9805112]

A baryon state in the boundary theory is dual to a soliton in the bulk



The bottom-up V-QCD framework

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{2g_{YM}^2} \text{Tr}\, F_{\mu\nu}F^{\mu\nu} + \sum_{i=1}^{N_f} \bar{\psi}^i (i\,\gamma^\mu D_\mu - m_i)\psi^i$$

- Truncate to the lowest-dimensional gauge-invariant operators
- EFT for the dual fields in a 5D bulk (x^μ, z)

$T_{\mu\nu}$	$\leftrightarrow$	g_{MN}	Color
$\text{Tr}\, F_{\mu\nu}F^{\mu\nu}$	$\leftrightarrow$	φ	
$\bar{\psi}^i\, \psi^j$	$\leftrightarrow$	$\mathcal{T}^{ij}$	Flavor
$U(N_f)_L \times U(N_f)_R : \partial_\mu J^\mu_{L/R} = 0$	$\leftrightarrow$	$U(N_f)_L \times U(N_f)_R : A^M_{L/R}$	
$\text{Tr}\, F \wedge F$	$\leftrightarrow$	a	CP-odd, θ_{QCD}

The bottom-up V-QCD framework

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{2g_{YM}^2}\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu} + \sum_{i=1}^{N_f} \bar{\psi}^i (i\,\gamma^\mu D_\mu - m_i)\psi^i$$

- Truncate to the lowest-dimensional gauge-invariant operators
- EFT for the dual fields in a 5D bulk (x^μ, z)

$T_{\mu\nu}$	$\leftrightarrow$	g_{MN}	Color
$\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu}$	$\leftrightarrow$	φ	$\frac{N_f}{N_c}$ finite
$\bar{\psi}^i\,\psi^j$	$\leftrightarrow$	$\mathcal{T}^{ij}$	Flavor
$U(N_f)_{\mathrm{L}} \times U(N_f)_{\mathrm{R}} : \partial_\mu J^\mu_{\mathrm{L/R}} = 0$	$\leftrightarrow$	$U(N_f)_{\mathrm{L}} \times U(N_f)_{\mathrm{R}} : A^M_{\mathrm{L/R}}$	
$\mathrm{Tr}\, F \wedge F$	$\leftrightarrow$	a	$\boldsymbol{\theta_{\mathrm{QCD}} = 0}$

The V-QCD Framework : Action

The V-QCD action is built by deforming what is known from string theory with phenomenological parameters of the bulk theory

$$S_{V-QCD} = S_c + S_f + S_{CS}$$

$$S_c = M_{Pl}^3 N_c^2 \int dx^5 \sqrt{-g}\left[R - \frac{4}{3}(\partial\varphi)^2 + V_g(\varphi)\right],$$

Parameters of the bulk theory	
M_{Pl}	$V_g(\varphi)$

The V-QCD Framework : Action

The V-QCD action is built by deforming what is known from string theory with phenomenological parameters of the bulk theory

$$S_{V-QCD} = S_c + S_f + S_{CS}$$

$$S_f = -\frac{1}{2} M_{Pl}^3 N_c \mathrm{STr} \int \mathrm{dx}^5 V_f(\varphi, \mathcal{T}) \left[\sqrt{-\det \mathbf{A}^{(L)}} + \sqrt{-\det \mathbf{A}^{(R)}} \right],$$ [1112.1261]

$$\mathbf{A}_{MN}^{(L)} \equiv g_{MN} + w(\varphi, \mathcal{T}) F_{MN}^{(L)} + \frac{\kappa(\varphi, \mathcal{T})}{2} \left[(D_M \mathcal{T})^\dagger (D_N \mathcal{T}) + h.c. \right],$$

Parameters of the bulk theory		
$V_f(\varphi, \mathcal{T})$	$w(\varphi, \mathcal{T})$	$\kappa(\varphi, \mathcal{T})$

The V-QCD Framework : Action

The V-QCD action is built by deforming what is known from string theory with phenomenological parameters of the bulk theory

$$S_{V-QCD} = S_c + S_f + S_{CS}$$

[hep-th/0012210]
[hep-th/0702155]

$$S_{CS} = \frac{iN_c}{4\pi^2}\int \Omega_5(\mathcal{T}, A_{(L/R)}) \ ,$$

When $\mathcal{T} \to 0$, it reduces to the CS 5-form for $A^{\mu}_{(L/R)}$:

$$\omega_5(A) = \mathrm{Tr}\left(A \wedge F^2 + \frac{i}{2}A^3 \wedge F - \frac{1}{10}A^5\right)$$

In general, it also contains the tachyon.

The dependence on the tachyon appears in terms of several functions $f_i(\mathcal{T})$.

Previous Results in V-QCD

At $T = 0$ and $n_b = 0$ [1112.1261]

- Bulk solution dual to the QCD vacuum
- Meson and glueball spectra

At $T \neq 0$ and $n_b = 0$ [1210.4516]

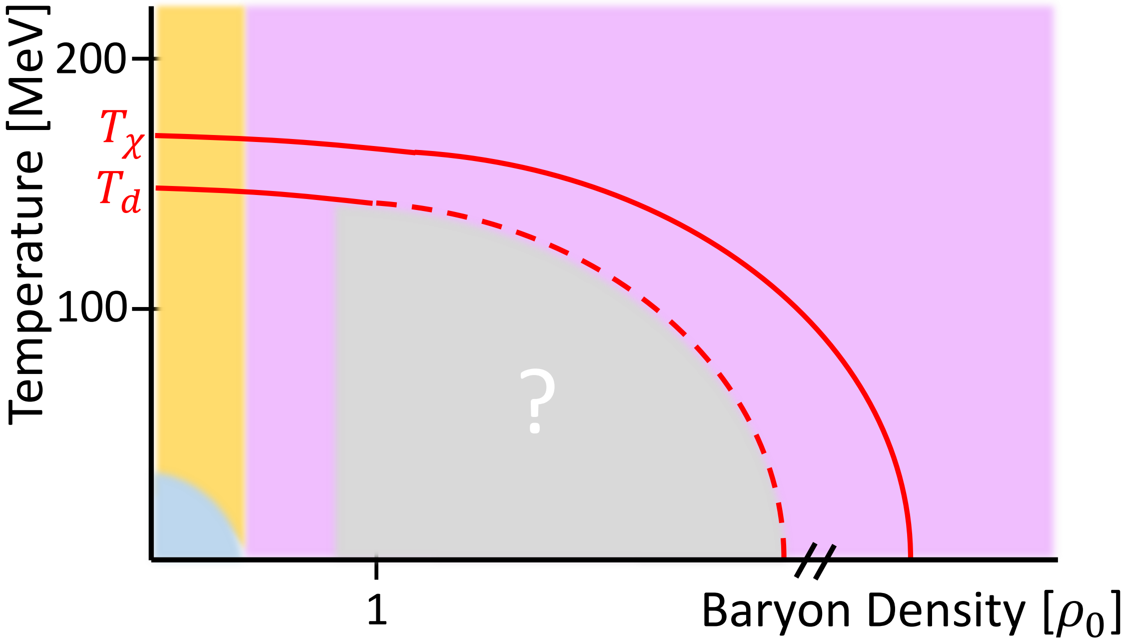
- Deconfinement & chiral phase transition
- QCD thermodynamics : $P(T)$, ...

At $T \neq 0$ and $n_b \neq 0$ [1312.5199]

Phase diagram when the baryon number is fractionalized (deconfined quarks)

We don't know how the picture is modified when we allow for baryons to appear

V-QCD phase diagram



The V-QCD Baryon

Set $m_q = 0$ and $\mathcal{T} \equiv \tau\, U$, U unitary

Consider a general solution to the bulk EoMs at $T = 0$ and baryon chemical potential $\mu_B = 0$

Under a variation $\delta A^0 = \delta\mu_B$:

$$N_B = \frac{1}{N_c}\frac{\delta S_{on-shell}}{\delta\mu_B} = \frac{1}{N_c}\frac{\delta S_{CS}}{\delta\mu_B} = \frac{1}{8\pi^2}\int_{(z,x^i)} \mathrm{Tr}\left(F_{(L)} \wedge F_{(L)} - F_{(R)} \wedge F_{(R)}\right)$$

4D Axial Instanton Density on (r, x^i)

$$\Rightarrow \frac{1}{24\pi^2}\mathrm{Tr}\int_{\partial\mathcal{M}} \left(U^\dagger \wedge \mathrm{d}U\right)^3 = N_{skyrmion}$$

$$\mathrm{A}_\mathrm{L}^\mu \to \tilde{A}_L^\mu \equiv U A_L^\mu U^\dagger - \mathrm{i}\,\partial^\mu U\, U^\dagger$$

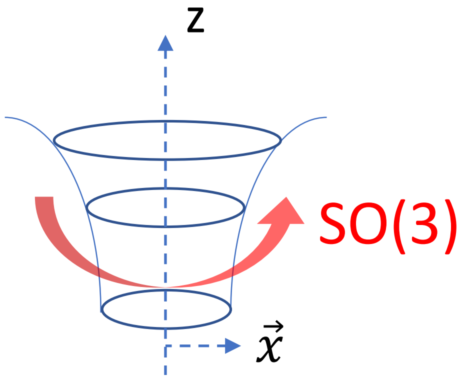
The instanton solution

We consider the approximation that the baryon is a perturbation on top of the vacuum solution :

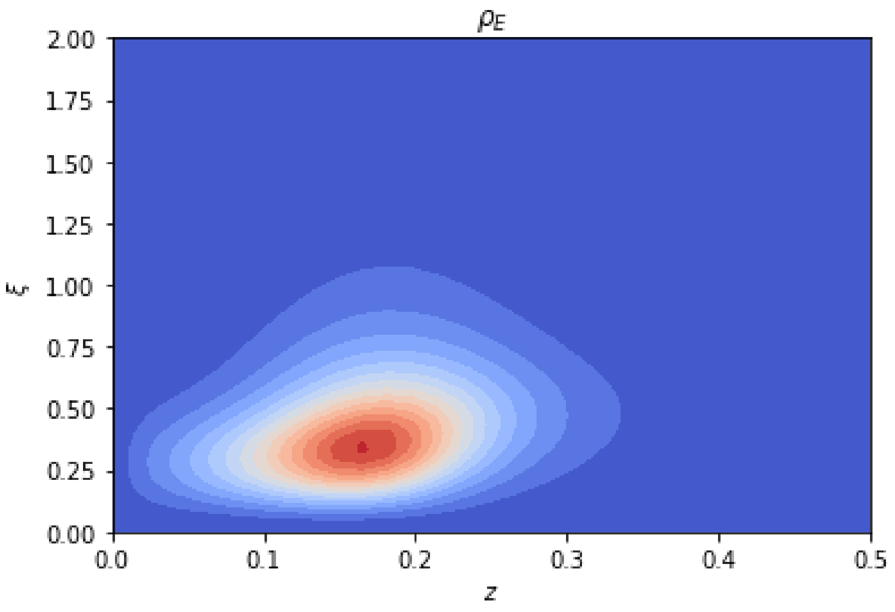
- Expand the flavor action at quadratic order in the non-abelian fields
- Neglect the back-reaction of the baryon on the color sector (g, φ)

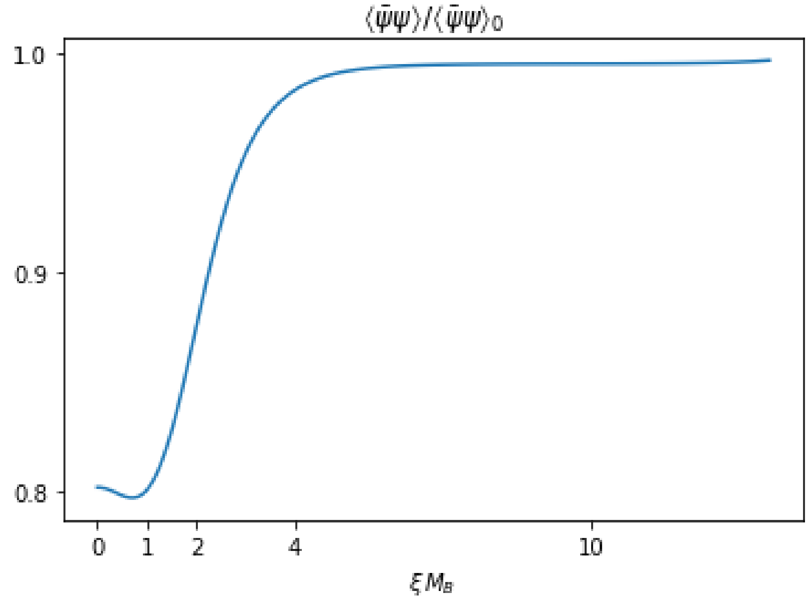
Procedure to compute the soliton solution:

- Maximally symmetric static ansatz for the fields in bulk $(SO(3), P)$
- Coupled PDE's on 2D space $(z, \xi = \sqrt{\vec{x}^2})$
- Solution obtained numerically



Numerical Results





The mass of the soliton is 1050 MeV (vs 938 MeV)

Chiral symmetry is partially restored at the center

The solution can be used as an input to compute a many baryons solution → NS matter

Summary

- Holographic models of low T QCD are more complete than χEFT
  → more realistic at high n_B

- In holography, the duals of baryons are bulk solitons

- V-QCD implements the back-reaction of the flavor sector onto the color sector
  → appropriate model to study QCD at high density (NS cores)

- Computing V-QCD solitons is challenging

- We obtained numerical solutions for parameters which give a consistent number for the baryon mass

Outlook

- Compute the rotating soliton to reproduce the baryon spectrum $(N, \Delta, \cdots)$
- Compute the soliton at finite m_{quark} → sigma term $\delta M_N = m_u \frac{\partial M_N}{\partial m_u} + m_d \frac{\partial M_N}{\partial m_d}$, both at 0 and finite density
- Bulk solution dual to a state containing many baryons → EoS for NS matter
- Generalize to finite T :
  - QCD phase diagram at strong coupling
  - Transport coefficients : flavor viscosities, flavor conductivities
    → neutrino transport in NS core

Appendix

Baryon in Chiral Effective Field Theory

One approach that has proven successful in the study of QCD at low energy is that of the chiral effective field theory

QCD possesses an approximate symmetry : the chiral symmetry

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{2g_{YM}^2}\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu} + \sum_{i=1}^{N_f} \bar{\psi}^i(i\,\gamma^\mu D_\mu - m_i)\psi^i$$

→

$U(N_f)_L \times U(N_f)_R$ symmetry

$\psi_L \to U_L\psi_L$, $\psi_R \to U_R\psi_R$

The chiral symmetry is spontaneously broken at low energies to $U(N_f)_V$: $\langle\bar{\psi}^i\psi^j\rangle \neq 0$

→ Goldstone bosons which control the low energy dynamics

$$U(x) \equiv \mathrm{e}^{2\pi i\,\Pi^a(x)T^a} \in SU(N_f)_A$$

χEFT improvements

$$\mathcal{L} = \mathcal{L}^{(0)} + \Lambda^{-1}\mathcal{L}^{(1)} + \Lambda^{-2}\mathcal{L}^{(2)} + \cdots,$$

which is built out of local functions of $U(x)$ invariant under the chiral symmetry

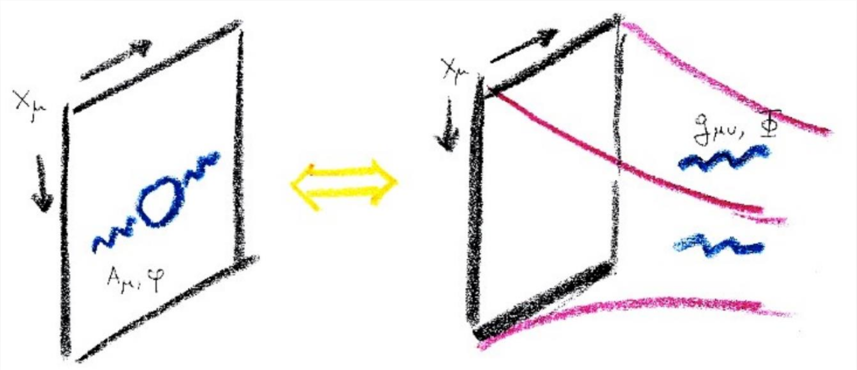
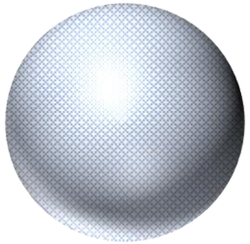
Missing ingredients :

- Coupling to the electroweak interaction and mesonic excitations in other representations of the Lorentz group introduced by coupling $U(x)$ to vector (ρ, ...), axial (ω, ...) and scalar fields (σ, ...)

- QCD anomalies
  - $U(1)_A$ anomaly : $\det(U)$
  - T'Hooft anomalies : Wess-Zumino term [Witten, 1983]

AdS^5/CFT^4

The original correspondence was formulated for an explicit 4D CFT : $\mathcal{N} = 4$ SU(N) SYM (highly supersymmetric cousin of YM) which is dual to type IIB string theory on AdS^5

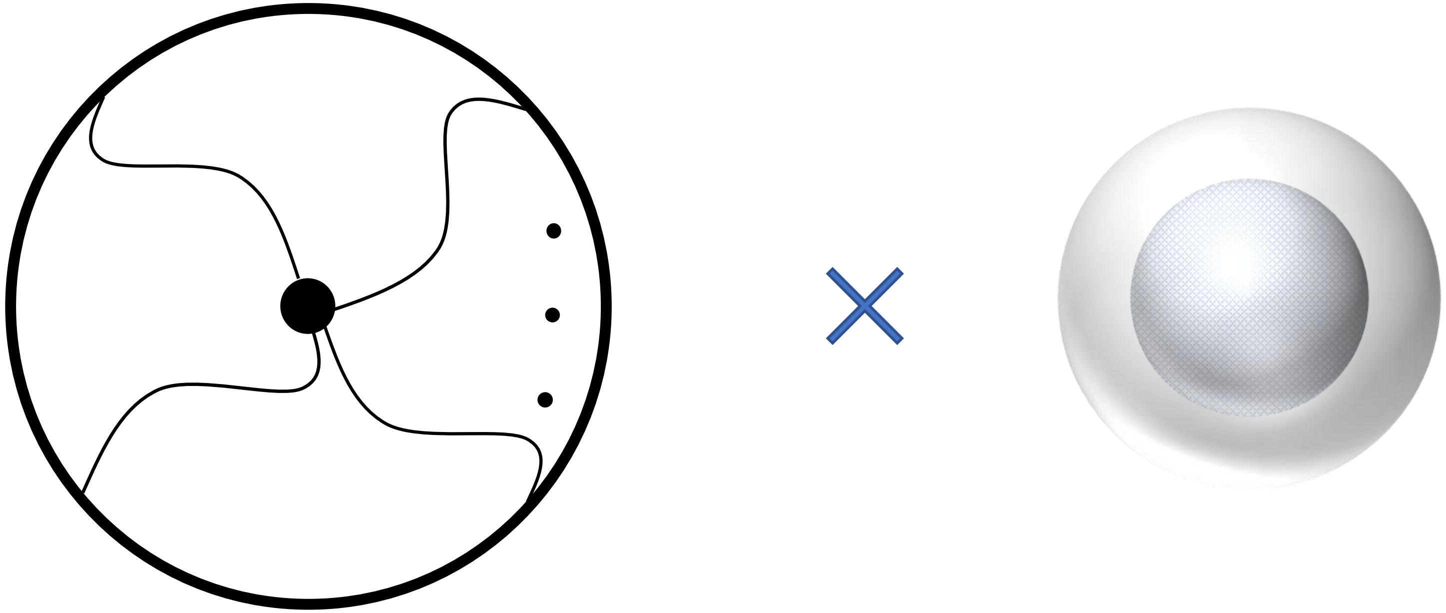
SYM  $AdS^5 \times S^5$ 

The internal space is important for the construction of the baryon in SYM

Holographic Baryons in SYM

The AdS^5/CFT^4 duality is the one that is best understood and from which others are built.

Witten argued that a baryon state of external quarks in SYM is dual to a D5-brane (5D extended object of string theory, a generalization of a membrane) wrapped around the S^5. [hep-th/9805112]



The Hard Wall Model : Geometry

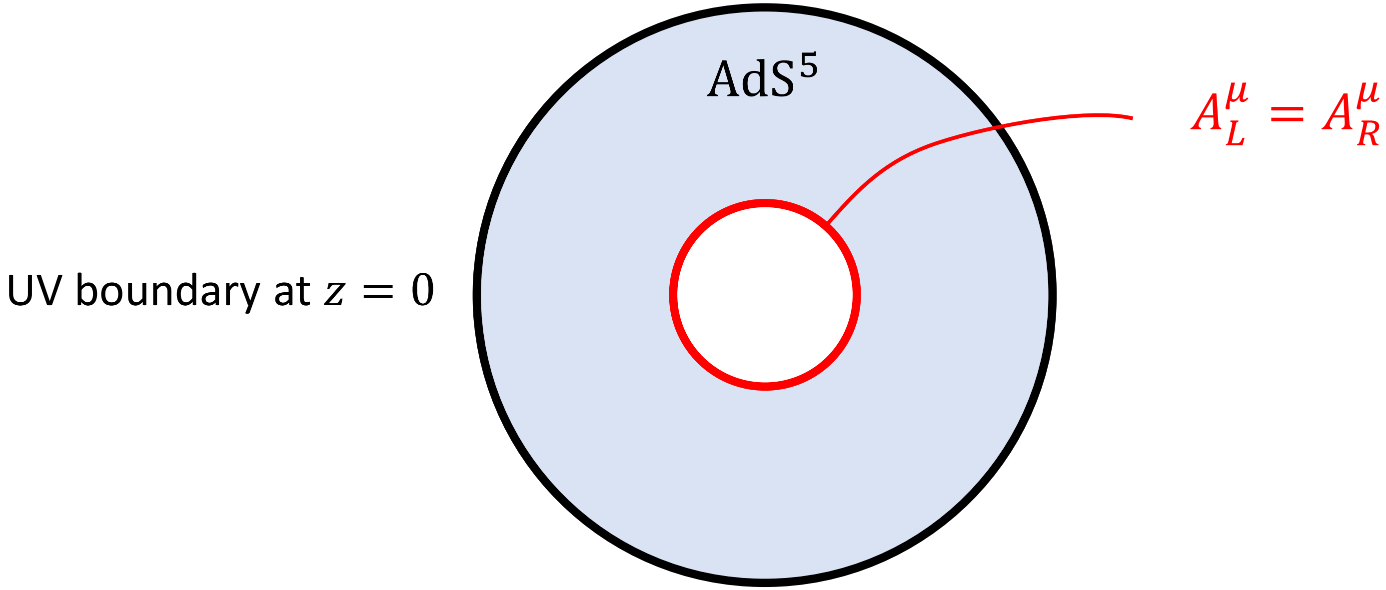
[hep-th/0003136]
[hep-ph/0501128]
[hep-ph/0501218]

The background space-time is chosen to have AdS^5 geometry

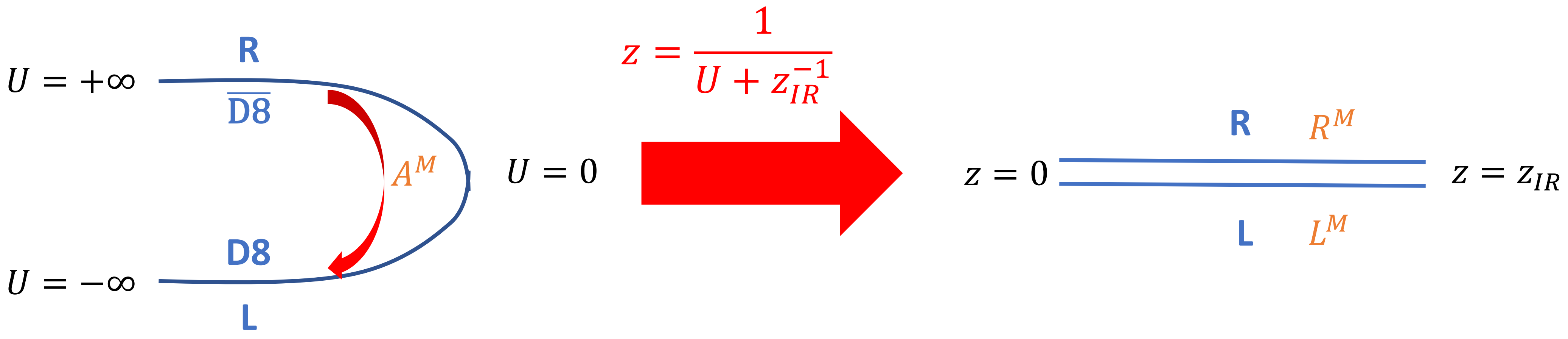
$$\mathrm{ds}^2 = a(z)^2(\mathrm{dz}^2 + \mathrm{dx}^2) \ , \qquad a(z) = \frac{\ell}{z} \ ,$$

It stops on an IR wall at $z = L$ → confinement

Chiral symmetry breaking is enforced via the boundary conditions in the IR



The Hard-Wall model from SS



Boundary conditions at $z = z_{IR}$

$$L^\mu - R^\mu = 0$$
$$L^z + R^z = 0$$
$$\partial_z(L^\mu + R^\mu) = 0$$

A. The Bulk Instanton

The Bulk Instanton

Static solution of the bulk equations of motion (EoMs) with

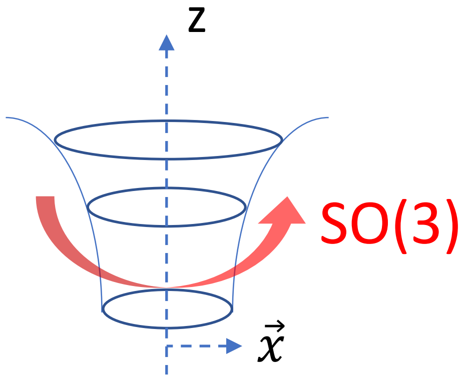
$$\frac{1}{16\pi^2}\int_{z,\vec{x}} \mathrm{Tr}\left(F_{(L)} \wedge F_{(L)} - F_{(R)} \wedge F_{(R)}\right) = 1$$

And $\pi_3\left(U(N_f)\right) = \pi_3(SU(2))$

→Focus on a subgroup $SU(2) \subset U(N_f)$

Ansatz invariant under the symmetries of the action :

- 3D rotations (up to global SU(2)) : $(t, z, \vec{x}) \to (t, z, R.\vec{x})$
- Parity : $\vec{x} \to -\vec{x}$ and $L \leftrightarrow R$



Problem : coupled non-linear PDE's defined on a 2D space $(\xi = \sqrt{\vec{x}^2}, z)$

The Bulk Instanton : ansatz

SU(2)

$$A^a_{(L),i} = -\frac{1+\phi_2(\xi,z)}{\xi}\epsilon_{iak}\hat{x}^k + \frac{\phi_1(\xi,z)}{\xi}(\delta_{ia} - \hat{x}_i\hat{x}_a) + A_\xi(\xi,z)\hat{x}_i\hat{x}_a \,,$$

$$A^a_{(L),z} = A_r(\xi,z)\,\hat{x}_a \,,$$

CS

U(1)

$$\hat{A}_{(L),0} = \Phi(\xi,z),$$

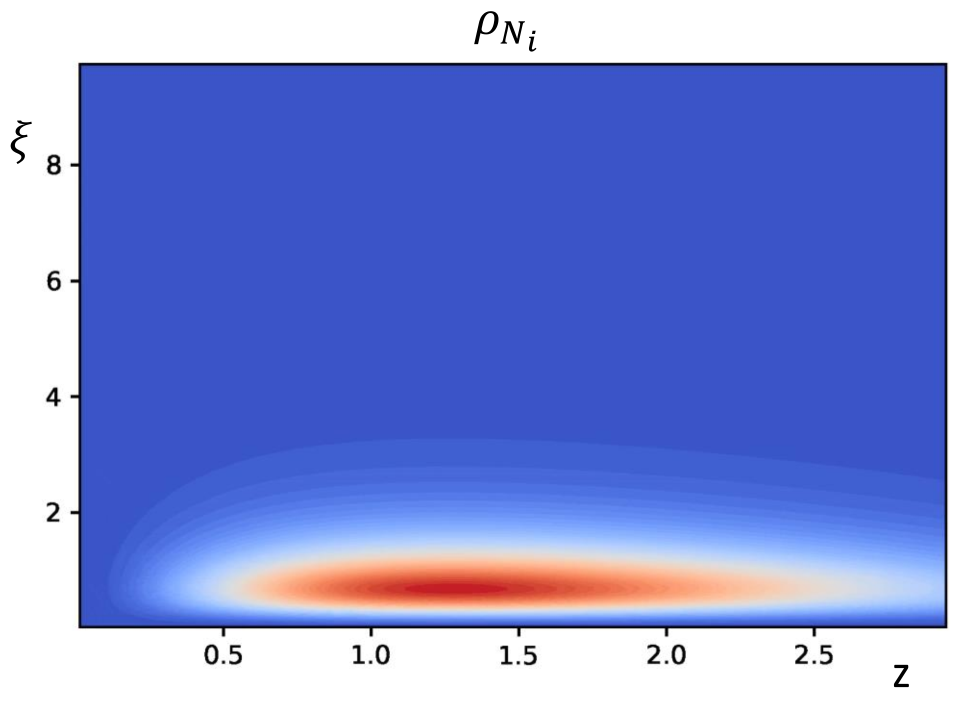
Where $\xi \equiv \sqrt{\vec{x}^2}$ and $\hat{x}^i \equiv x^i/\xi$

5 scalar fields that live on a 2D space (ξ, z)

The Bulk Instanton : Numerical Solution

- The task is to solve the 5 coupled non –linear elliptic EoMs with boundary conditions corresponding to $N_{instanton} = 1$

- The gradient descent method is adapted to the problem



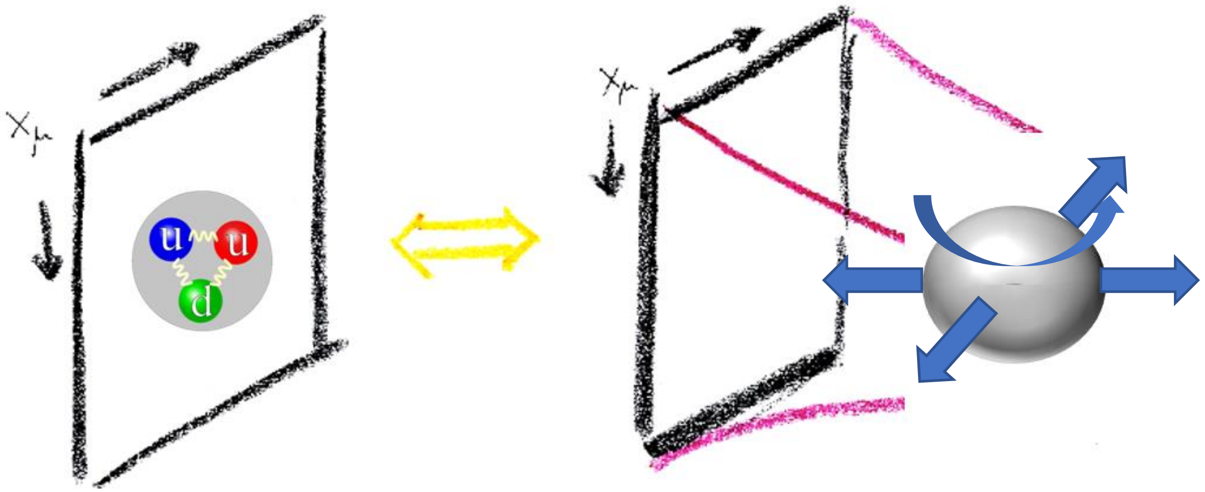
B. The Rotating soliton

Quantization of the collective modes

The baryon states are found by quantizing the collective modes of the classical instanton background

0-modes: Translation in 3D, SU(N_f) isospin rotation

Collective Modes
Translation in 3D $\longrightarrow \vec{P}$
SU(N_f) isospin rotation
Translation in z
Dilation, …

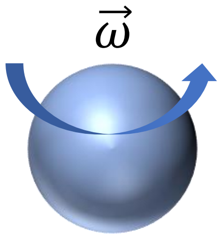


The first isospin modes give the nucleon states (I = 1/2) and isobar Δ (I = 3/2)

When $N_f = 2$, due to rotational symmetry : $SU(2)_I$ rotation $\Leftrightarrow SU(2)_S$ rotation
So the states that are computed have $S = I$

The Rotating Soliton

Solution of the classical bulk EoMs corresponding to a rotating soliton



The on-shell action at quadratic order in $\vec{\omega}$ is $L = -M_s + \frac{1}{2}\lambda_s \vec{\omega}^2$

Soliton mass

Soliton moment of inertia

The rotation is a perturbation if

$$\frac{1}{\lambda_s \ M_s} \sim \frac{1}{N_c^2} \ll 1$$

Large N_c limit : expand the fields at $\mathcal{O}(\omega)$

$$\varphi = \varphi^{(0)} + \omega \ \varphi^{(1)} ,$$

Static

Rotating

The Rotating Soliton : Quantization

$$\varphi = \varphi^{(0)} + \omega\, \varphi^{(1)}\,,$$

Substitute the ansatz into the EoMs →linear PDE's for the $\varphi^{(1)}$'s

The solution is obtained numerically

We compute the on-shell action ↔ classical Lagrangian for the rotating modes

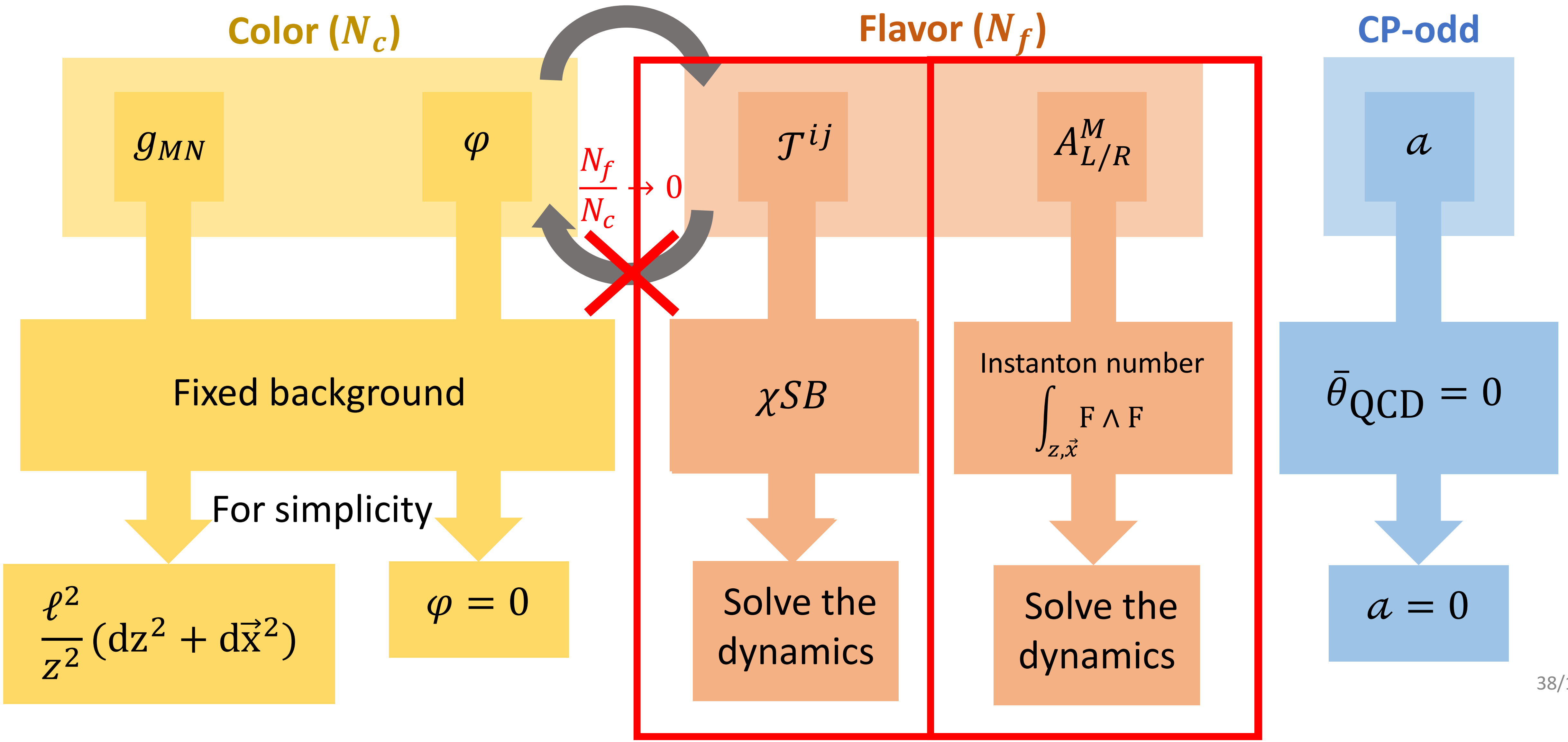
$$L = -M_s + \frac{1}{2}\, \lambda_s\, \vec{\omega}^2$$

Spin $s = \frac{1}{2}, \frac{3}{2}, \ldots$

Quantization : quantum rotor with eigenenergies $E_s = M_s + \frac{1}{2\lambda_s} s(s+1)$

C. The Hard Wall Model with χSB

The Hard Wall Model with χSB



The Hard Wall Model with χSB : Action

[hep-th/1503.04820]

The gauge invariant kinetic term of the tachyon is added to the YM action

$$S_{YM} \to S_{YM} + g_{\mathcal{T}}^2 \int dz\, dx^4\, \mathrm{Tr}\left(|D_M \mathcal{T}|\hat{}2 - m_{\mathcal{T}}^2 |\mathcal{T}|\hat{}2 \right),$$

Parameters :

- $g_{\mathcal{T}}^2$: Tachyon normalization
- $m_{\mathcal{T}}^2$: $-\frac{3}{\ell^2}$ fixed by the dimension of $< \psi^i \overline{\psi^j} >$

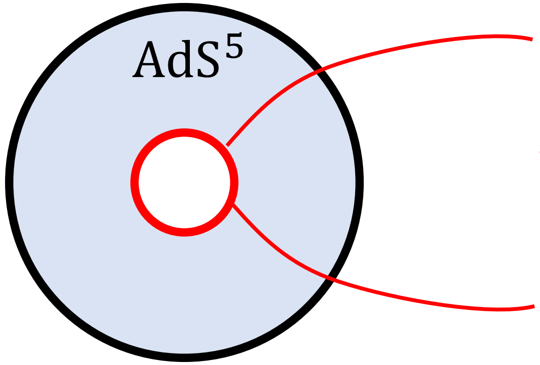
$|D_M \mathcal{T}|\hat{}2$:

$$D_M \mathcal{T} \equiv \partial_M \mathcal{T} - i\, A_{(L),M} \mathcal{T} + i\, \mathcal{T} A_{(R),M}$$

This term contains $|\mathcal{T}|^2 \left(A_{(L)} - A_{(R)}\right)^2$ which implies spontaneous χSB

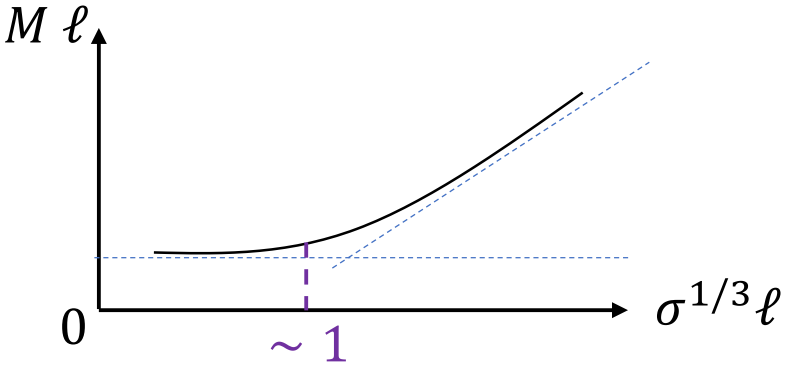
Consequences of χSB on the baryon [hep-th/1503.04820]

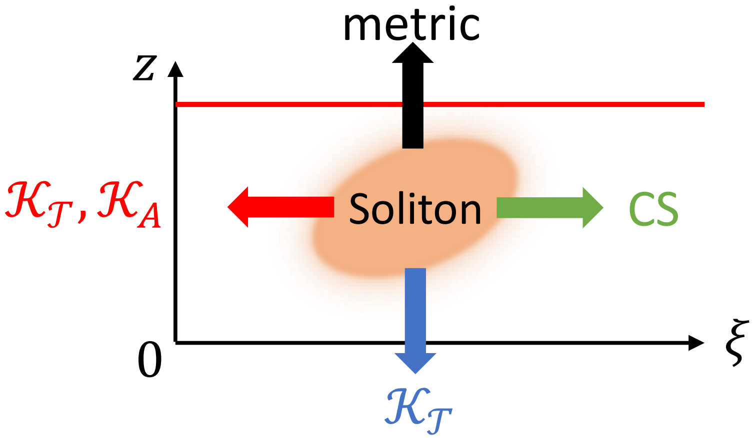
The boundary condition of the tachyon on the IR wall is taken to match the vacuum solution



$$A_L^{\mu} = A_R^{\mu}$$
$$A_L^{z} = -A_R^{z}$$
$$L_{z\mu} = -R_{z\mu}$$
$$\mathcal{T}^{ij} = (m_q\, z + \sigma\, z^3)\delta^{ij}$$

The soliton mass depends on σ and it can leave the IR wall for large σ





D. Top-down constructions

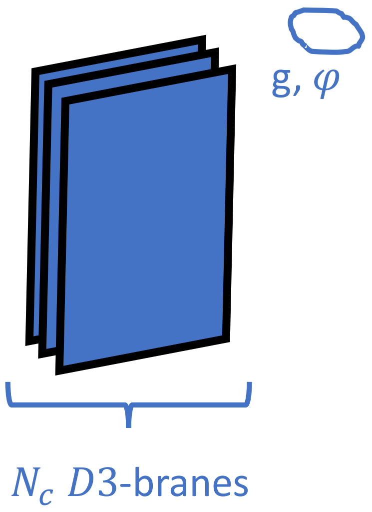
The V-QCD Model : Action

The V-QCD action is built by deforming what is known from top-down holography with phenomenological parameters of the bulk theory

$$S_{V-QCD} = S_c + S_f + S_{CS}$$

$$S_c = M_{Pl}^3 N_c^2 \int dx^5 \sqrt{-g} \left[R - \frac{4}{3} (\partial\varphi)^2 + V_g(\varphi) \right],$$

Parameters of the bulk theory	
M_{Pl}	$V_g(\varphi)$



The V-QCD Model : Action

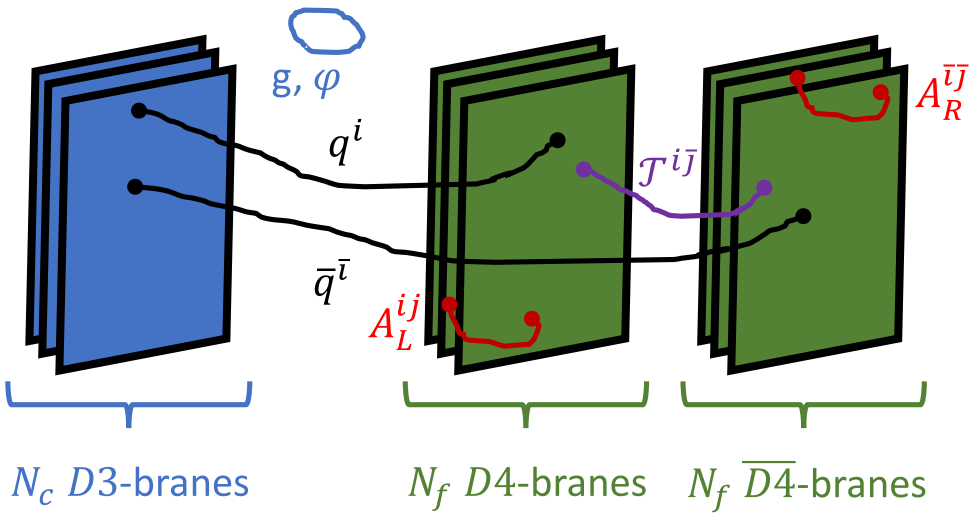
The V-QCD action is built by deforming what is known from top-down holography with phenomenological parameters of the bulk theory

$$S_{V-QCD} = S_c + S_f + S_{CS}$$

Sen :

$$\begin{cases} S_f = -\frac{1}{2} M_{Pl}^3 N_c \mathrm{STr} \int \mathrm{dx}^5 V(\mathcal{T}) \left[\sqrt{-\det \mathbf{A}^{(L)}} + \sqrt{-\det \mathbf{A}^{(R)}}\right], \\ \mathbf{A}_{MN}^{(L)} \equiv g_{MN} + F_{MN}^{(L)} + \frac{1}{2}\left[(D_M\mathcal{T})^\dagger (D_N\mathcal{T}) + h.c.\right], \end{cases}$$

[hep-th/0303057]
[hep-th/0012210]



The V-QCD Model : Action

The V-QCD action is built by deforming what is known from top-down holography with phenomenological parameters of the bulk theory

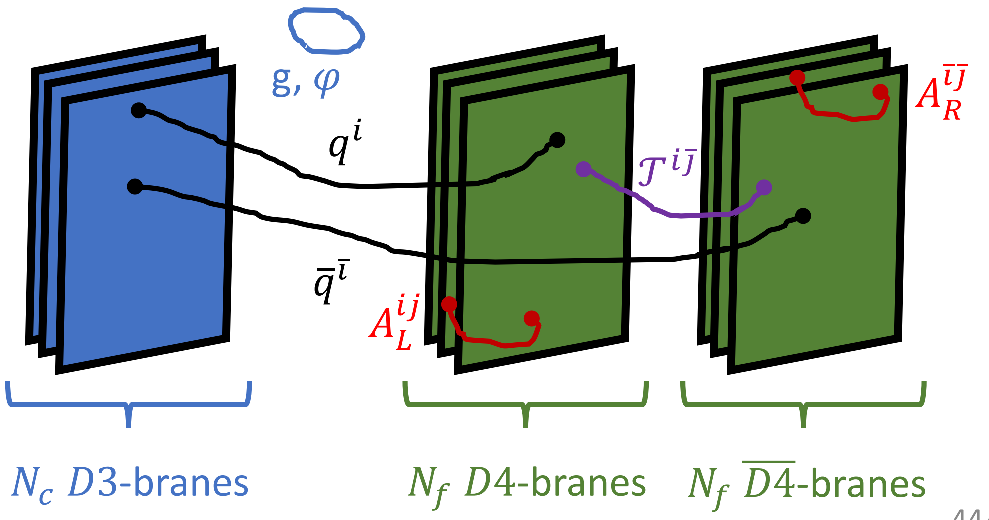
$$S_{V-QCD} = S_c + S_f + S_{CS}$$

V-QCD :

$$\begin{cases} S_f = -\frac{1}{2} M_{Pl}^3 N_c \mathrm{Tr} \int \mathrm{dx}^5 V_f(\varphi, \mathcal{T}) \left[\sqrt{-\det \mathbf{A}^{(L)}} + \sqrt{-\det \mathbf{A}^{(R)}} \right], \\ \mathbf{A}_{MN}^{(L)} \equiv g_{MN} + w(\varphi, \mathcal{T}) F_{MN}^{(L)} + \frac{\kappa(\varphi, \mathcal{T})}{2} \left[(D_M \mathcal{T})^\dagger (D_N \mathcal{T}) + h.c. \right], \end{cases}$$

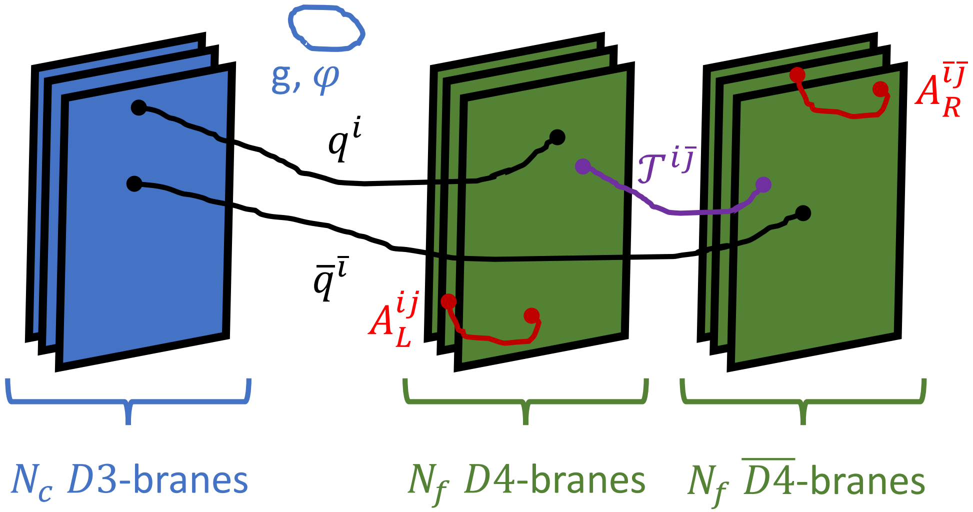
[1112.1261]

Parameters of the bulk theory		
$V_f(\varphi, \mathcal{T})$	$w(\varphi, \mathcal{T})$	$\kappa(\varphi, \mathcal{T})$



The V-QCD Model : Action

The V-QCD action is built by deforming what is known from top-down holography with phenomenological parameters of the bulk theory



$$S_{V-QCD} = S_c + S_f + S_{CS}$$

$$S_{CS} = \frac{iN_c}{4\pi^2}\int \Omega_5(\mathcal{T}, A_{(L/R)}) \ ,$$

[hep-th/0012210]
[hep-th/0702155]

Parameters of the bulk theory
$f_i(\mathcal{T})$

When $\mathcal{T} = 0$, Ω_5 is the CS 5-form

In String Theory, the tachyon dependence is known only in the maximally supersymmetric case

We generalize this result : Ω_5 is the sum of all 5-forms built from $(A, F, D\mathcal{T})$ with coefficients $f_i(\mathcal{T})$

E. DBI form

A comment on the DBI form of the action

In string theory, the flavor sector appears as the low-energy excitations of $D4 - \overline{D4}$

→ Low-energy dynamics on flat background controlled by the Sen action [1112.1261]

→ The flavor action is a deformation with potentials $V_f(\varphi, \mathcal{T}), \kappa(\varphi, \mathcal{T}), w(\varphi, \mathcal{T})$.

From a phenomenological point of view : is this necessary to use something so complicated?

→ **Ex:** expand $\sqrt{-\det \mathbf{A}}$ at quadratic order in the flavor fields.

Consider the bulk solution dual to the QCD vacuum, with $\mathcal{T} = \tau\, \mathrm{I}$

→ More general action :

$$\sqrt{-\det \mathbf{A}} = \sqrt{-g}\sqrt{\det\left(\delta^\mu_\nu + \kappa\, \partial^\mu \tau\, \partial_\nu \tau\right)} \to \sqrt{-g}\left(\det\left(\delta^\mu_\nu + \kappa\, \partial^\mu \tau\, \partial_\nu \tau\right)\right)^b,$$

Results:

- No solutions for $b < 1/2$
- For $b > 1/2$: no linear meson spectrum $m_n^2 \sim n^{4\frac{c+1}{2c+3}}$, with $c > -1/2$

F. Details about the V-QCD potentials

Details about the potentials ansätze: glue

Parameters of the bulk theory	
M_{Pl}	$V_g(\varphi)$

[1809.07770]

$$\lambda \equiv e^{\varphi}$$

$$V_g(\varphi) = 12 + V_1^{UV}\lambda + V_2^{UV}\frac{\lambda^2}{1 + \lambda/\lambda_0} + V^{IR} e^{-\frac{\lambda_0}{\lambda}}\left(\frac{\lambda}{\lambda_0}\right)^{\frac{4}{3}}\left(\log\left(1 + \frac{\lambda}{\lambda_0}\right)\right)^{\frac{1}{2}}$$

: IR asymptotics fixed by confinement and linear glueball spectrum $m_n^2 \sim n$

: UV asymptotics fixed to match the UV YM beta function

: M_{Pl}, λ_0 and V^{IR} are adjusted to fit the lattice YM thermodynamics ($N_c \to \infty$) in the deconfined phase (pressure P and interaction measure $\epsilon - 3P$)

Details about the potentials ansätze: glue

Additional QCD parameter : energy scale Λ

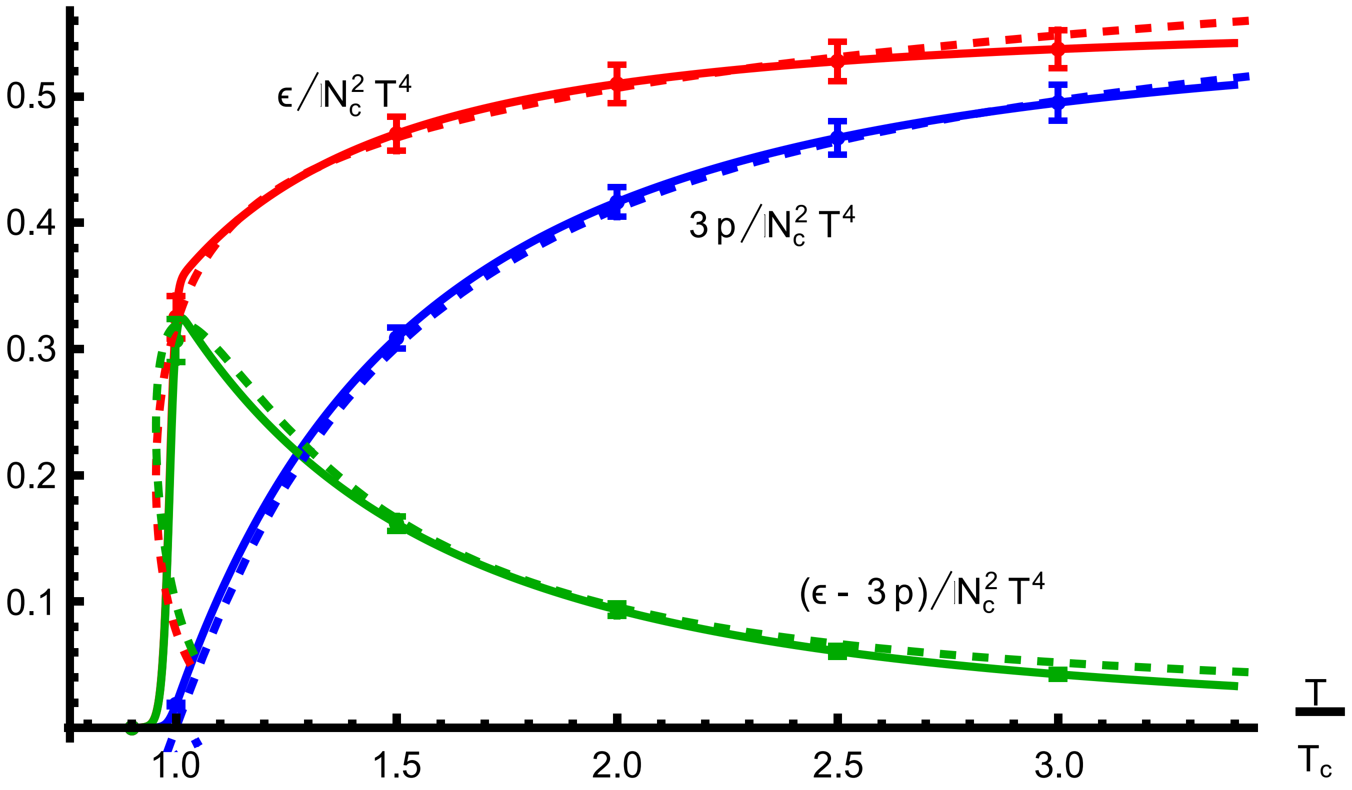
In the bulk theory, it is a source for $\varphi \sim -\frac{1}{\log r\,\Lambda}$ at $r \to 0$

$M_{Pl}, \lambda_0, V^{IR}$ and Λ are fitted to YM thermodynamics:

- M_{Pl} : overall normalization of P
- Λ : critical temperature T_c
- λ_0 and V^{IR} : shape of the curves

Fit is rigid : with fixed asymptotics, significant deviation from lattice data requires large modifications of the potentials.

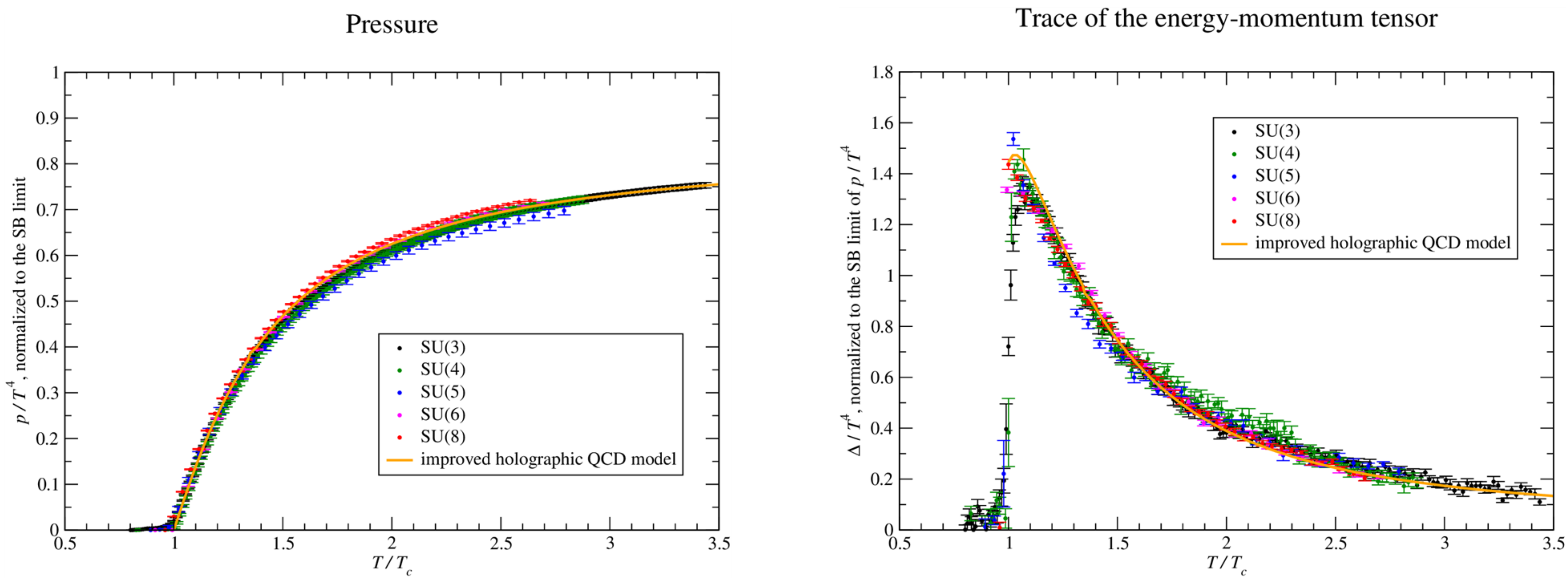
Fit to lattice YM thermodynamics



[1809.07770]

The solid lines and error bars represent the extrapolation of lattice data to Nc = ∞ and the dashed curves are the results for the holographic model.

Lattice YM thermodynamics in the large N_c limit



[0907.3719]

Thermodynamic quantities converge fast in the large N_c limit → $N_c = 3$ close to large N_c

Details about the potentials ansätze: DBI

Parameters of the bulk theory		
$V_f(\varphi,\mathcal{T})$	$w(\varphi,\mathcal{T})$	$\kappa(\varphi,\mathcal{T})$

From string theory (Sen action for $D4-\overline{D4}$ in flat space-time) : [1112.1261]

$$V_{f(\varphi,\mathcal{T})} = V_{f0}(\varphi)\mathrm{e}^{-\mathcal{T}^\dagger\mathcal{T}}, \qquad \kappa(\varphi,\mathcal{T}) = \kappa(\varphi), \qquad w(\varphi,\mathcal{T}) = w(\varphi)$$

We parametrize : $$V_{f0}(\varphi) = W_0^{UV} + W_1^{UV}\lambda + W_2^{UV}\frac{\lambda^2}{1+\lambda/\lambda_0} + W^{IR}\mathrm{e}^{-\frac{\lambda_0}{\lambda}}\left(\frac{\lambda}{\lambda_0}\right)^2$$

$$w(\varphi)^{-1} = w_0\left(1 + w_1\frac{\lambda/\lambda_0}{1+\lambda/\lambda_0} + \bar{w}_0\mathrm{e}^{-\frac{\hat{\lambda}_0}{\lambda}}\frac{(\lambda/\hat{\lambda}_0)^{\frac{4}{3}}}{\log(1+\lambda/\hat{\lambda}_0)}\right)$$

$$\kappa(\varphi)^{-1} = \kappa_0\left(1 + \bar{\kappa}_0\left(1+\frac{\bar{\kappa}_1\lambda_0}{\lambda}\right)\mathrm{e}^{-\frac{\lambda_0}{\lambda}}\frac{(\lambda/\lambda_0)^{\frac{4}{3}}}{\sqrt{\log(1+\lambda/\lambda_0)}}\right)$$

$$V_{f0}(\varphi) = W_0^{UV} + W_1^{UV}\lambda + W_2^{UV}\frac{\lambda^2}{1+\lambda/\lambda_0} + W^{IR}\mathrm{e}^{-\frac{\lambda_0}{\lambda}}\left(\frac{\lambda}{\lambda_0}\right)^{2}$$

$$w(\varphi)^{-1} = w_0\left(1 + w_1\frac{\lambda/\lambda_0}{1+\lambda/\lambda_0} + \bar{w}_0\mathrm{e}^{-\frac{\hat{\lambda}_0}{\lambda}}\frac{(\lambda/\hat{\lambda}_0)^{\frac{4}{3}}}{\log(1+\lambda/\hat{\lambda}_0)}\right)$$

$$\kappa(\varphi)^{-1} = \kappa_0\left(1 + \bar{\kappa}_0\left(1+\frac{\bar{\kappa}_1\lambda_0}{\lambda}\right)\mathrm{e}^{-\frac{\lambda_0}{\lambda}}\frac{(\lambda/\lambda_0)^{\frac{4}{3}}}{\sqrt{\log(1+\lambda/\lambda_0)}}\right)$$

: IR asymptotics fixed by :

- Annihilation of the brane action in the IR $(V_{f0}$ and $\kappa)$
- Linear meson spectrum $m_n^2 \sim n$ in all mesonic sectors $(V_{f0}, \kappa$ and $w)$
- Phase diagram at finite μ_B : presence of a hadron gas phase at low μ_B (w)
- For the V_{f0} power-law v_p : the previous constraints impose $v_p \leq 10/3$
  → Numerically, solutions are found for $v_p \sim 2$

$$V_{f0}(\varphi) = W_0^{UV} + W_1^{UV}\lambda + W_2^{UV}\frac{\lambda^2}{1+\lambda/\lambda_0} + W^{IR}e^{-\frac{\lambda_0}{\lambda}}\left(\frac{\lambda}{\lambda_0}\right)^2$$

$$w(\varphi)^{-1} = w_0\left(1 + w_1\frac{\lambda/\lambda_0}{1+\lambda/\lambda_0} + \bar{w}_0 e^{-\frac{\hat{\lambda}_0}{\lambda}}\frac{(\lambda/\hat{\lambda}_0)^{\frac{4}{3}}}{\log(1+\lambda/\hat{\lambda}_0)}\right)$$

$$\kappa(\varphi)^{-1} = \kappa_0\left(1 + \bar{\kappa}_0\left(1+\frac{\bar{\kappa}_1\lambda_0}{\lambda}\right)e^{-\frac{\lambda_0}{\lambda}}\frac{(\lambda/\lambda_0)^{\frac{4}{3}}}{\sqrt{\log(1+\lambda/\lambda_0)}}\right)$$

: The dilaton potential in the Veneziano limit is $V_{eff} = V_g - \frac{N_f}{N_c}V_{f0}$.

The UV asymptotics of V_{eff} are fixed to match the UV YM beta function in the Veneziano limit.

The UV asymptotics of κ : UV anomalous dimension of m_q .

$$V_{f0}(\varphi) = W_0 + W_1^{UV}\lambda + W_2^{UV}\frac{\lambda^2}{1+\lambda/\lambda_0} + W^{IR}e^{-\frac{\lambda_0}{\lambda}}\left(\frac{\lambda}{\lambda_0}\right)^2$$

$$w(\varphi)^{-1} = w_0\left(1 + w_1\frac{\lambda/\lambda_0}{1+\lambda/\lambda_0} + \bar{w}_0 e^{-\frac{\hat{\lambda}_0}{\lambda}}\frac{(\lambda/\hat{\lambda}_0)^{\frac{4}{3}}}{\log(1+\lambda/\hat{\lambda}_0)}\right)$$

$$\kappa(\varphi)^{-1} = \kappa_0\left(1 + +\bar{\kappa}_0\left(1+\frac{\bar{\kappa}_1\lambda_0}{\lambda}\right)e^{-\frac{\lambda_0}{\lambda}}\frac{(\lambda/\lambda_0)^{\frac{4}{3}}}{\sqrt{\log(1+\lambda/\lambda_0)}}\right)$$

: The remaining parameters are adjusted to fit the lattice YM thermodynamics in the deconfined phase (at $N_c = 3$ and $N_f = 2\ (u,d) + 1(s)$)

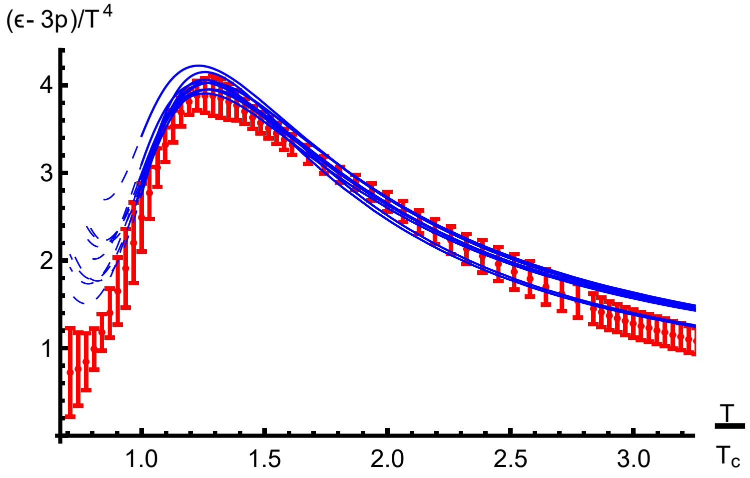
W_0 and W^{IR} are fitted to P and $\epsilon - 3P$

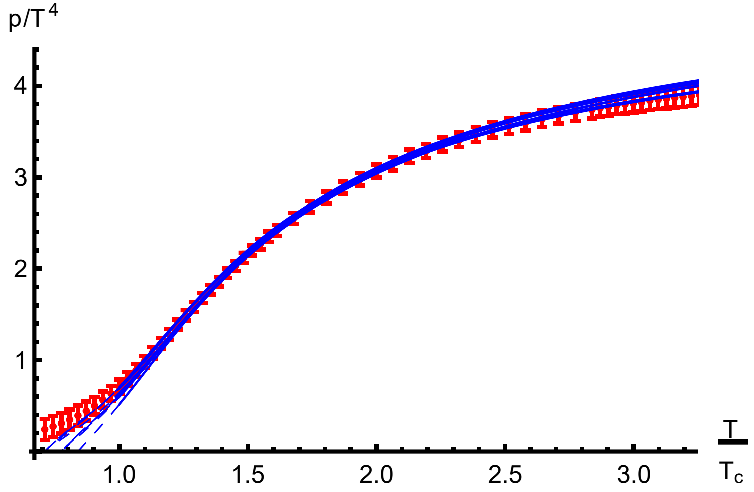
$\bar{\kappa}_0$ and $\bar{\kappa}_1$ are adjusted to the deconfinement temperature T_c

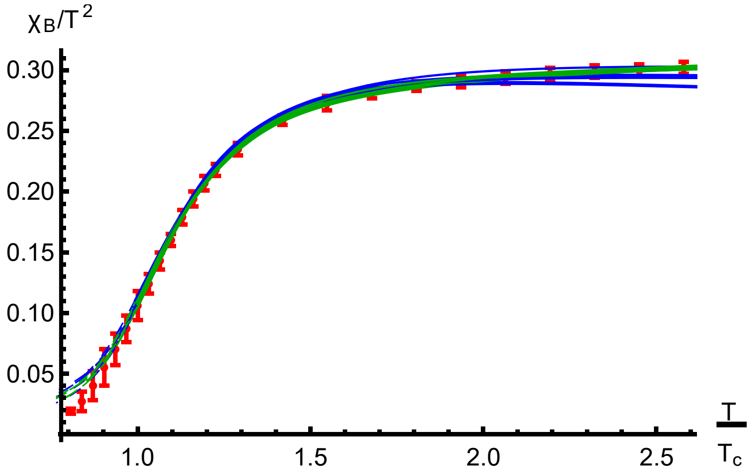
M_{Pl} and Λ are refitted to include flavor dependence

w $(w_0, w_1, \bar{w}_0$ and $\hat{\lambda}_0)$ is fitted to the baryon susceptibility $\chi_B = d^2P/d^2\mu_B$

Fit to lattice QCD thermodynamics







[1809.07770]

The error bars represent the lattice data at $N_c = 3$ and $N_f = 2\ (u, d) + 1(s)$ and the solid lines are the results for the holographic model (in the deconfined phase).

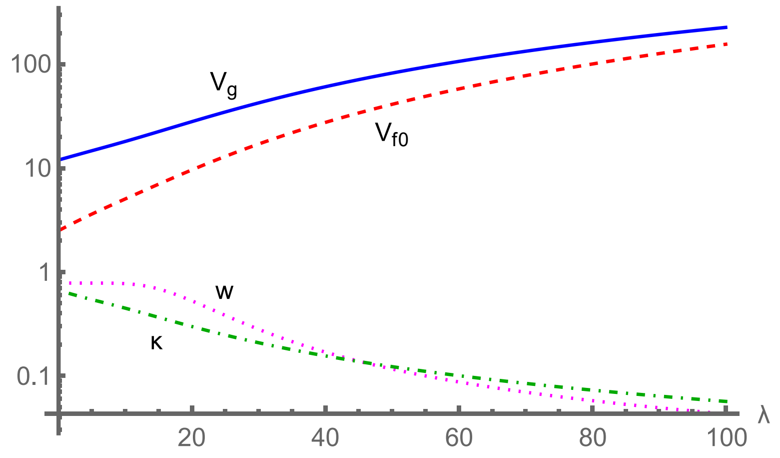
Closing remarks on the potentials

Once the UV and IR boundary conditions are fixed, 12 parameters are used to fit lattice data

The fit is rigid : dependence on all parameters is weak

The shape of the curves is a prediction of holography
→The tuning of parameters just makes the precise numerical match to QCD

The resulting potentials are actually simple monototic functions



[2110.08281]

Details about the potentials ansätze: CS

Parameters of the bulk theory
$f_i(\mathcal{T})$

$$S_{CS} = \frac{iN_c}{4\pi^2}\int \Omega_5(\mathcal{T}, A_{(L/R)}) \ ,$$

We consider the ansatz $\mathcal{T} = \tau\, U$

→ Ω_5 is the most general 5-form built from $A^{L/R}, F^{L/R}, DU$ with properties :

- Symmetry under parity P and charge conjugation C
- Invariant under bulk gauge transformations : $\mathrm{d}\delta\Omega_5 = 0$
- The boundary gauge variation $\int \delta\Omega_5$ matches the QCD flavor anomalies

$$\Omega_5 = \Omega_5^0 + \underbrace{\Omega_5^c + \mathrm{d}G_4}_{},$$

Closed part completely fixed by matching to chiral anomalies

Details about the potentials ansätze: CS

Ω_5^0 is a sum of gauge-invariant 5-forms with coefficients $f_i(\tau)$:

$$\Omega_5^0 = f_1(\tau)\Omega_5^{0,1} + f_2(\tau)\Omega_5^{0,2} + f_3(\tau)\Omega_5^{0,3} + f_4(\tau)\Omega_5^{0,4}$$

UV asymptotics : usual CS 5-form for $A^{L/R}$ recovered when $\tau \to 0$ and $U \to 1$

IR asymptotics : from string theory $f_i(\tau) \sim \mathrm{e}^{-b\,\tau^2}$

[hep-th/0303057]
[hep-th/0012210]

Ω_5 does not affect the vacuum thermodynamics nor the glueball and meson spectra

→ It affects 2-point functions in presence of external flavor fields

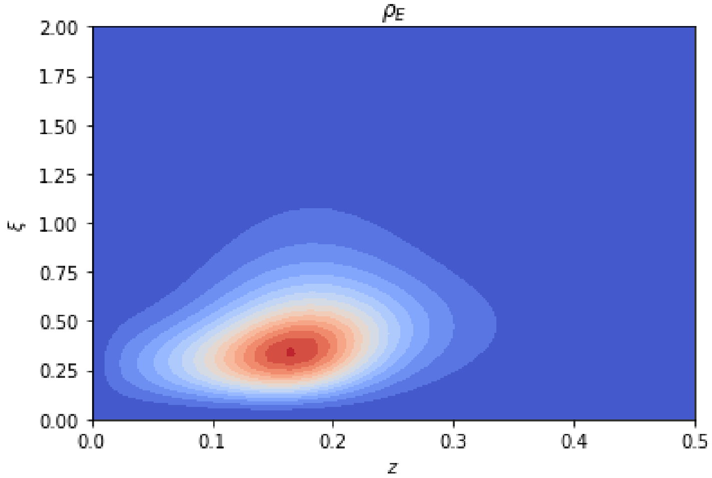
→ It generates the baryon number in the boundary theory → relevant to the baryonic phase of QCD

→ The fitting of baryonic physics in QCD will involve the $f_i(\tau)$'s

Details about the potentials ansätze: CS

The baryons appear in the middle of the bulk

UV



IR

→ Shape of the $f_i(\tau)$ in the middle may be relevant to baryonic physics

Here : choice that reproduces the flat space-time string theory result when $U = \mathrm{I}$

$$f_1(\tau) = -\frac{1}{6}\mathrm{e}^{-b\tau^2}, \qquad f_2(\tau) = \frac{i}{12}(1 + b\tau^2)\mathrm{e}^{-b\tau^2}, \qquad f_3(\tau^2) = -\frac{1}{12}\mathrm{e}^{-b\tau^2}, \qquad f_4(\tau^2) = \frac{1}{120}(2 + 2b\tau^2 + b^2\tau^4)\mathrm{e}^{-b\tau^2},$$

Numerically : • solutions found for $b > 1$

• For 3 < b < 10 the relative change of the classical M_B is $\sim 15\%$

G. Fit to f_π

Potentials updated to fit f_π

$f_\pi^2 \propto M_{Pl}^3$ is mainly increased by increasing M_{Pl}

Larger M_{Pl} → thermodynamics needs to be refitted

Only the flavor potentials are updated

$$V_{f(\varphi,\mathcal{T})} = V_{f0}(\varphi)\left(1+\left(\mathcal{T}^\dagger\mathcal{T}\right)^2\right)\mathrm{e}^{-a(\varphi)\,\mathcal{T}^\dagger\mathcal{T}}, \qquad \kappa(\varphi,\mathcal{T}) = \kappa(\varphi), \qquad w(\varphi,\mathcal{T}) = w(\varphi)$$

Step 1 : generate random potentials → select the best fit to thermodynamics
Step 2 : adjust the resulting potentials to fit the mesonic spectrum

This double fit requires more parameters (12 → 17)

Potentials updated to fit f_π

$$V_{f0}(\varphi) = W_0^{UV} + W_1^{UV}\lambda + W_2^{UV}\frac{\lambda^2}{1+\lambda/\tilde{\lambda}_0} + W^{IR} e^{-\frac{\tilde{\lambda}_0}{\lambda}}\left(\frac{\lambda}{\tilde{\lambda}_0}\right)^2\left(1 + W_1^{IR}\frac{\tilde{\lambda}_0}{\lambda}\right)$$

$$a(\varphi) = 1 + \frac{1}{2}(a_{IR} - 1)\left(1 + \tanh\left(\frac{\log(\lambda/\lambda_0) - a_h}{a_c}\right)\right)$$

$$w(\varphi)^{-1} = w_0\left(1 + w_1\frac{\lambda/\hat{\lambda}_0}{1+\lambda/\hat{\lambda}_0} + \bar{w}_0 e^{-\frac{\hat{\lambda}_0}{\lambda}}\frac{(\lambda/\hat{\lambda}_0)^{\frac{4}{3}}}{\log(1+\lambda/\hat{\lambda}_0)}\right)$$

$$\kappa(\varphi)^{-1} = \kappa_0\left(1 + +\bar{\kappa}_0\left(1 + \frac{\bar{\kappa}_1\lambda_0}{\lambda}\right)e^{-\frac{2\lambda_0}{\lambda}}\frac{(\lambda/\lambda_0)^{\frac{4}{3}}}{\sqrt{\log(1+2\lambda/\lambda_0)}}\right)$$