TECHNISCHE UNIVERSITÄT WIEN

Der Wissenschaftsfonds.

Jet momentum broadening from effective kinetic theory

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With K. Boguslavski, A. Kurkela, T. Lappi, J. Peuron, in preparation

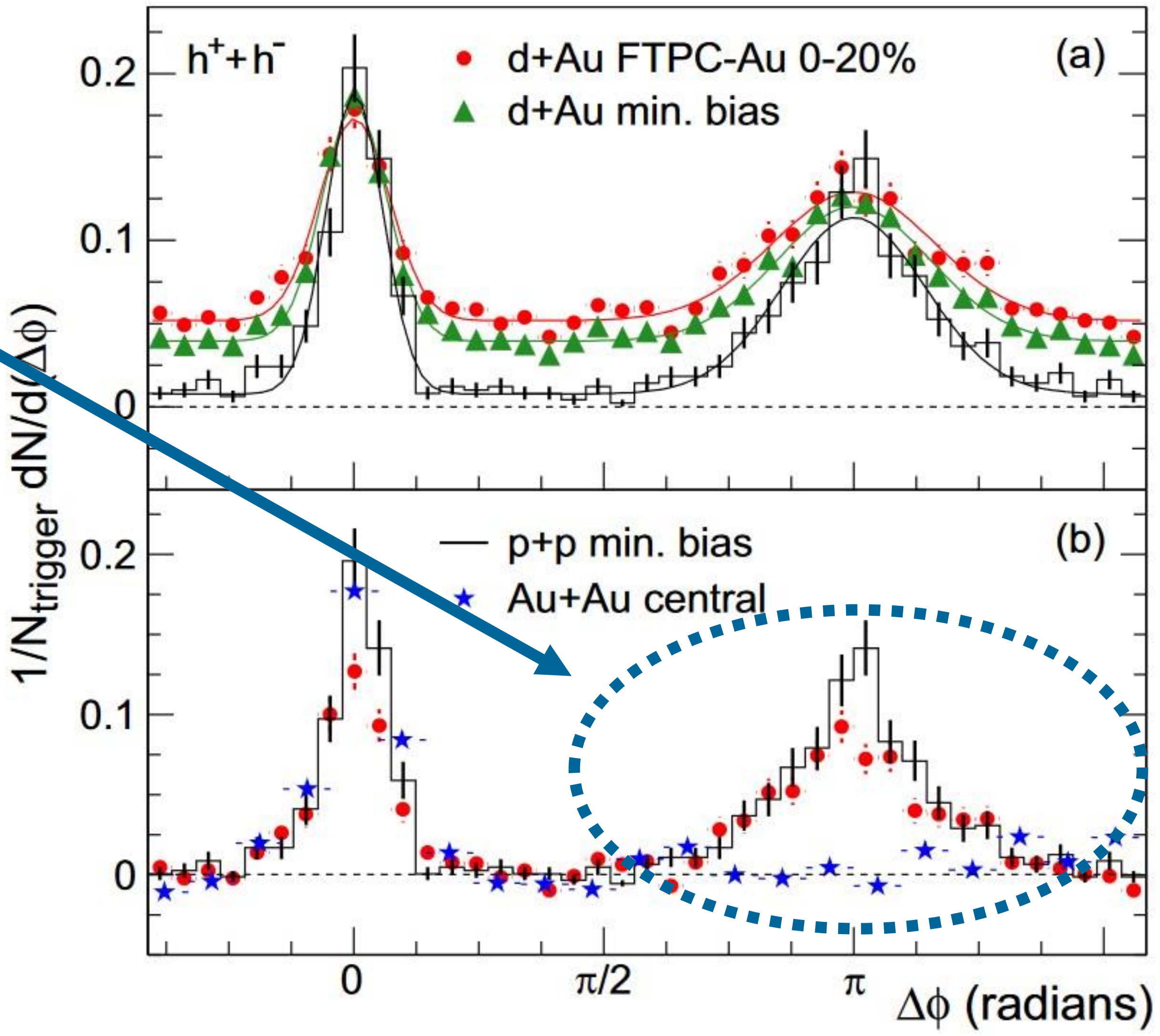
XQCD Conference, Jul 29, 2022

Contents

Jet quenching in heavy-ion collisions

- Back-to-back peak is suppressed in Au+Au
- Quark-Gluon plasma (QGP) is created
- Different description at different times:
  - Glasma – kinetic theory – hydrodynamics

Rev.Mod.Phys. 93 (2021) 3, 035003 [Berges, Heller, Mazeliauskas, Venugopalan]



Phys.Rev.Lett.91:072304,2003 (STAR Collaboration)

Jet quenching parameter

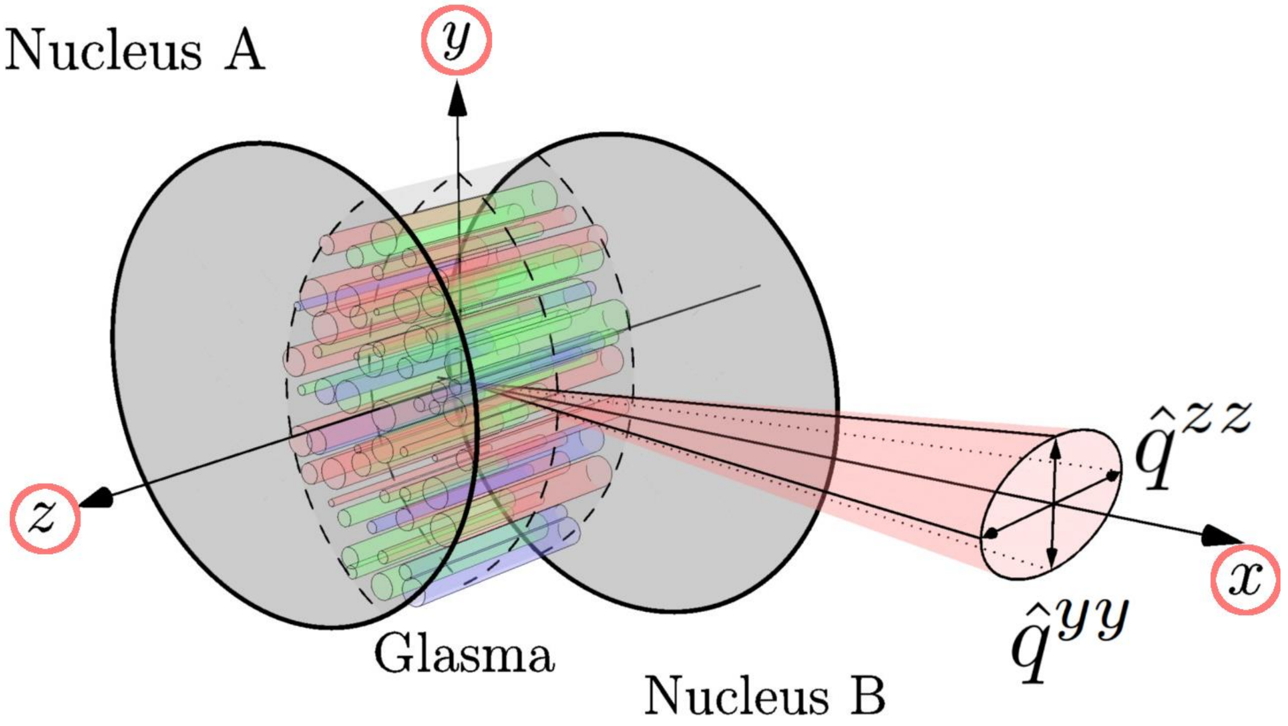
- $\hat{q}$ is defined via

$$\hat{q} = \frac{\mathrm{d}\langle p_\perp^2 \rangle}{\mathrm{d}L} = \frac{\mathrm{d}\langle p_\perp^2 \rangle}{\mathrm{d}t}.$$

- Quantifies momentum broadening
- Can be written in terms of the elastic scattering rate Γ_{el}

$$\hat{q}^{ij}(\Lambda_\perp) = \int_{q_\perp < \Lambda_\perp} \mathrm{d}^2\mathbf{q}_\perp \; q_\perp^i q_\perp^j \frac{\mathrm{d}\Gamma_{\mathrm{el}}}{\mathrm{d}^2 q_\perp}$$

$$\hat{q} = \hat{q}^{yy} + \hat{q}^{zz}$$



Phys.Lett.B 810 (2020) 135810 [Ipp, Müller, Schuh], adapted

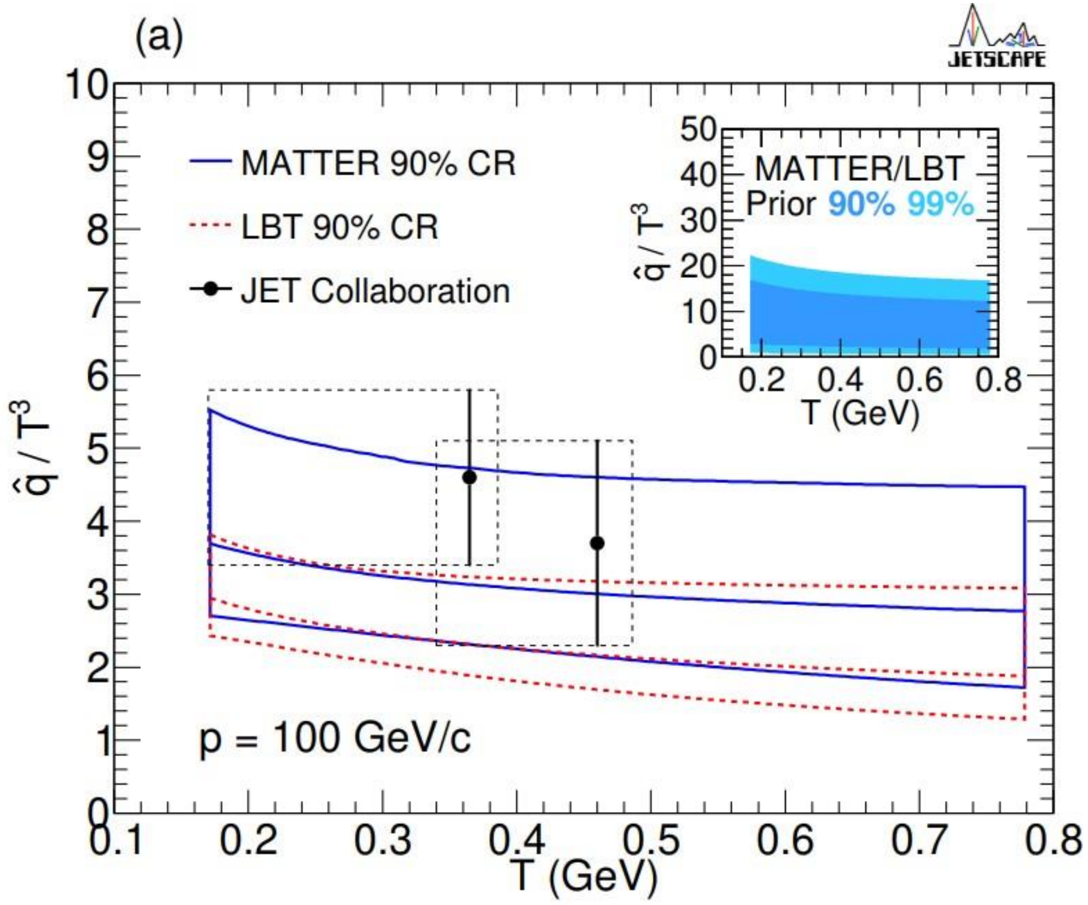
Estimates of the jet quenching parameter

- Mostly evaluated at later stages (hydrodynamics) or in thermal equilibrium

- Also discussed for anisotropic systems
  Phys.Rev.D 71 (2005) 125008 [Romatschke, Strickland]
  Phys.Rev.C 78 (2008) 024909 [Dumitru, Nara, Schenke, Strickland]

- Recently also considered in glasma
  Phys.Lett.B 810 (2020) 135810 [Ipp, Müller, Schuh]
  arXiv 2202.00357 [Carrington, Czajka, Mrowczynski]

- Want to consider $\hat{q}$ during thermalization
  → between glasma and hydro



Phys.Rev.C 104 (2021) 2, 024905 [JETSCAPE Collaboration]

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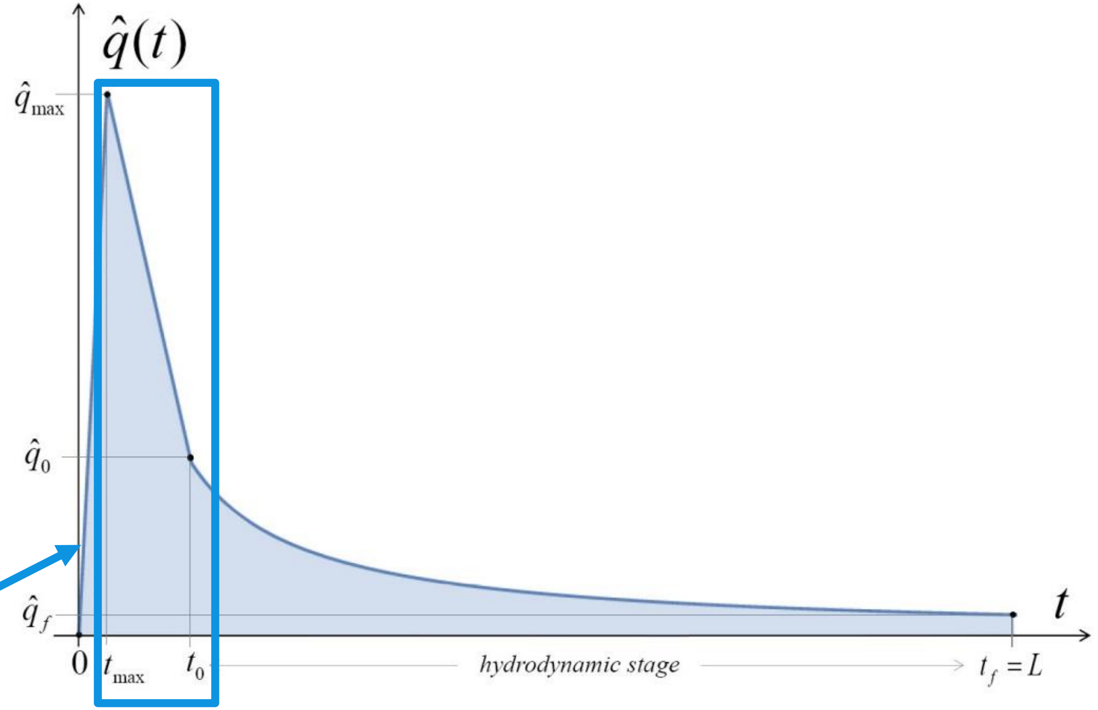
Schematic overview of $\hat{q}$ evolution



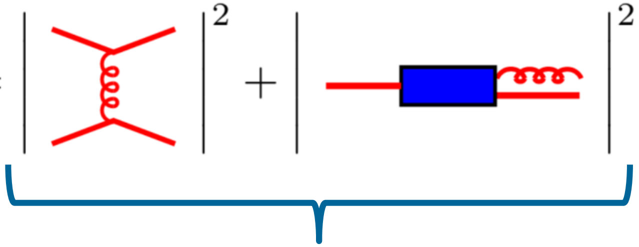
arXiv 2112.06812 [Carrington, Czajka, Mrowczynski]

Effective kinetic theory description of the QGP

- Quasi-particles with distribution function $f(t, \vec{p})$

- Time evolution described by Boltzmann equation at LO

$$(\partial_t + \boldsymbol{v} \cdot \boldsymbol{\nabla}) f =$$



Collision term

JHEP01(2003)030 [Arnold, Moore, Yaffe]
Int.J.Mod.Phys.E 16 (2007) 2555-2594 [Arnold]

- Solved numerically using Monte Carlo techniques

Phys.Rev.Lett. 115 (2015) 18, 182301 [Kurkela, Zhu]

Jet quenching parameter in kinetic theory

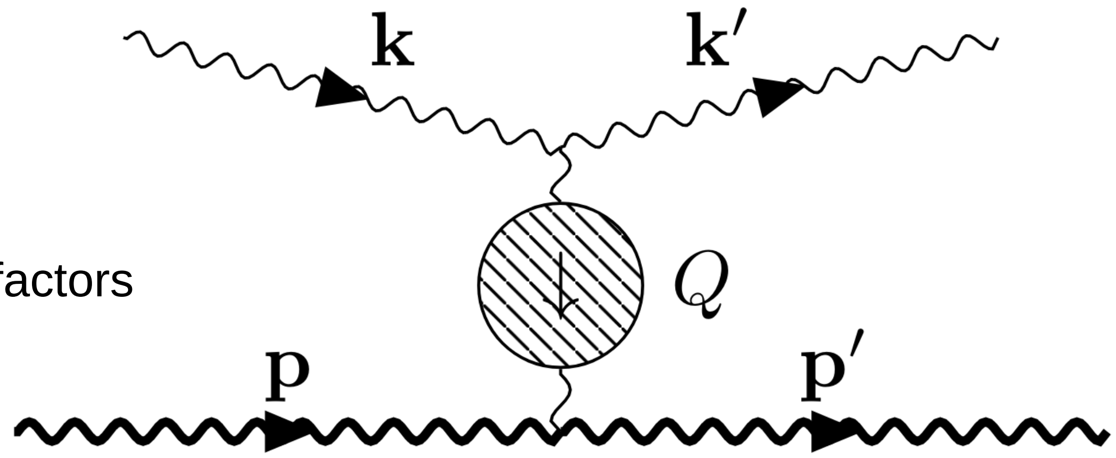
- Provided we know $f(\mathbf{k})$:

$$\hat{q}^{ij}(\Lambda_\perp) = \int_{q_\perp < \Lambda_\perp} \mathrm{d}\Gamma_{PS}\, q^i q^j\, |\mathcal{M}|^2\, f(\mathbf{k})(1 + f(\mathbf{k}'))$$

Appropriate phase-space measure

With momentum cutoff $q_\perp < \Lambda_\perp$

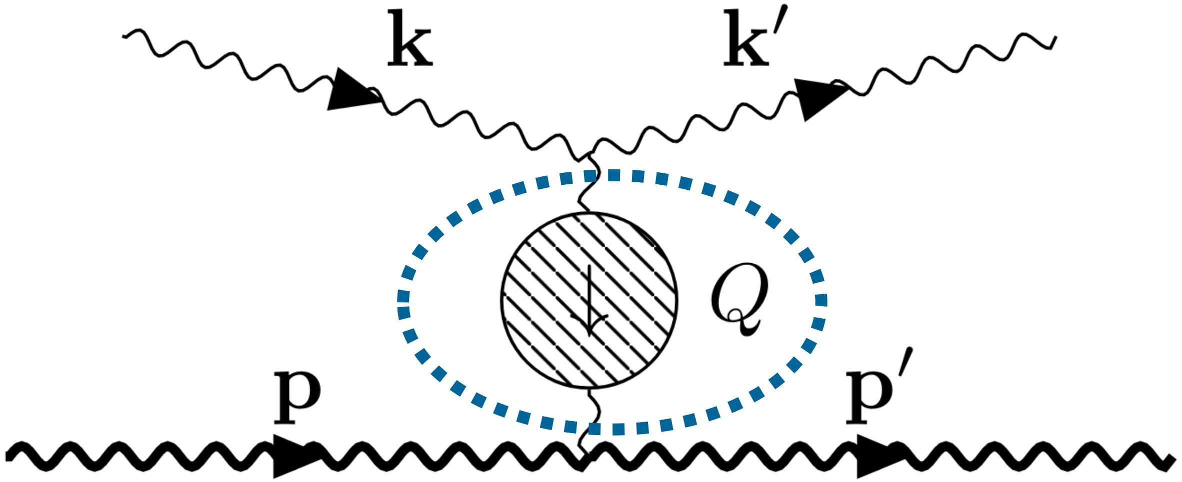
Ingoing/outgoing state factors



Matrix element with medium corrections (self-energy)

Screening in the matrix element

- Scattering matrix element includes in-medium propagator
- Receives self-energy corrections
- Anisotropic Hard thermal loop (HTL) self-energy → Unstable modes
- Approximation: Use isotropic HTL matrix element



Screening in the matrix element II

- Use isotropic HTL matrix element

- Also use approximation using single screening constant ξ

$$\frac{su}{t^2} \rightarrow \frac{su}{t^2} \frac{q^4}{(q^2 + \xi^2 m_D^2)^2}$$

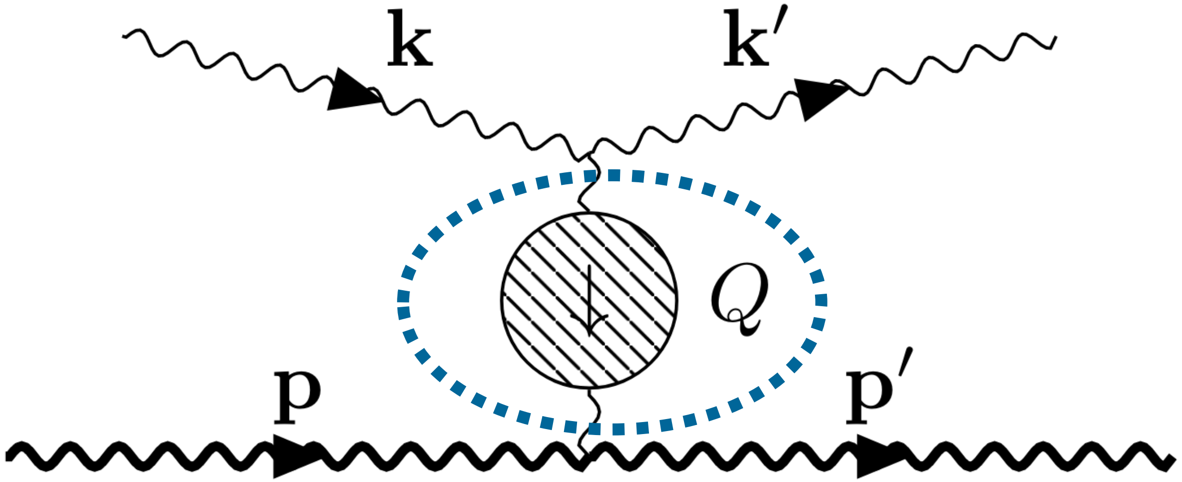
- Different for longitudinal

$$\xi_L = e^{5/6}/\sqrt{8}$$

Phys.Rev.D 89 (2014) 7, 074036 [York, Kurkela, Lu, Moore]

- and transverse momentum broadening

$$\xi_T = e^{1/3}/2$$



m_D is the Debye mass,
s, u and t are the Mandelstam variables

Over- and underoccupied systems: Scaled thermal distribution

- During thermalization: Over- and underoccupied systems
- $\hat{q}$ in thermal systems known
- Model for over- and underoccupation: scaled thermal distribution

$$f_g(k; N_0, \beta) = \frac{N_0}{\exp(\beta k) - 1}$$

- We have derived analytical expressions for $\hat{q}$ for $\Lambda_\perp \ll \mathrm{T}$, and $\Lambda_\perp \gg \mathrm{T}$

  (Formulae on backup slides)

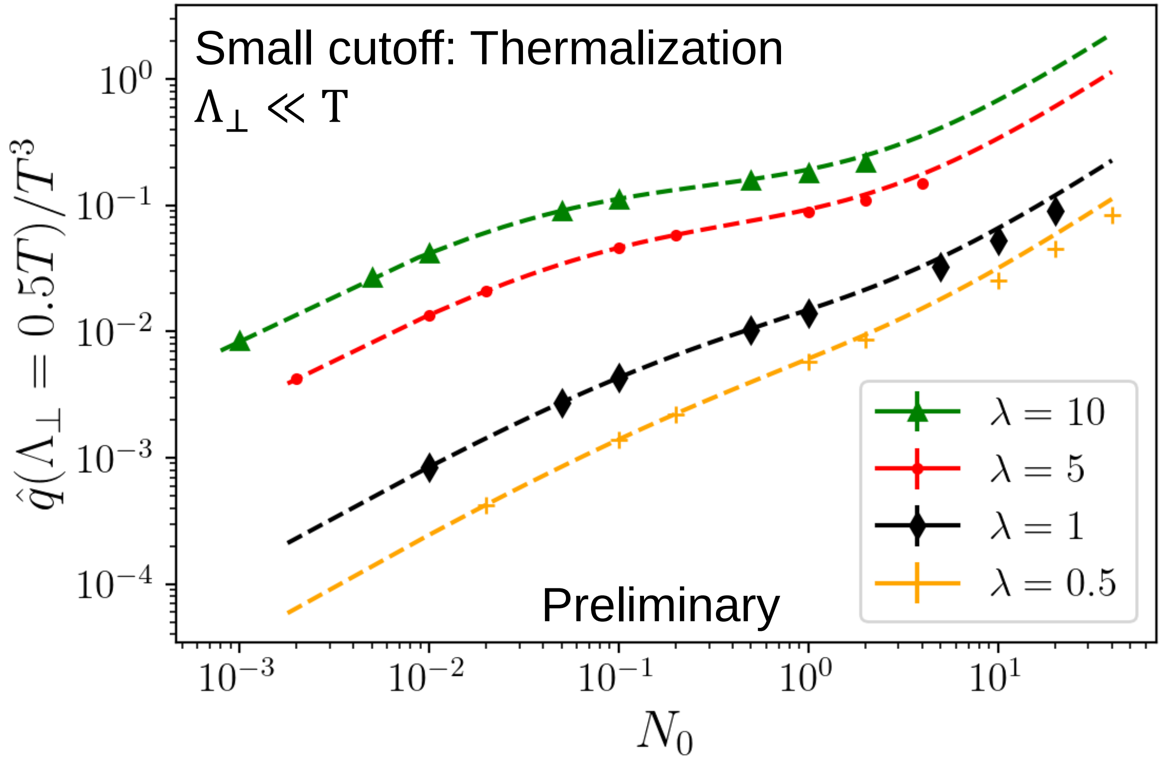
Physical meaning of the momentum cutoff

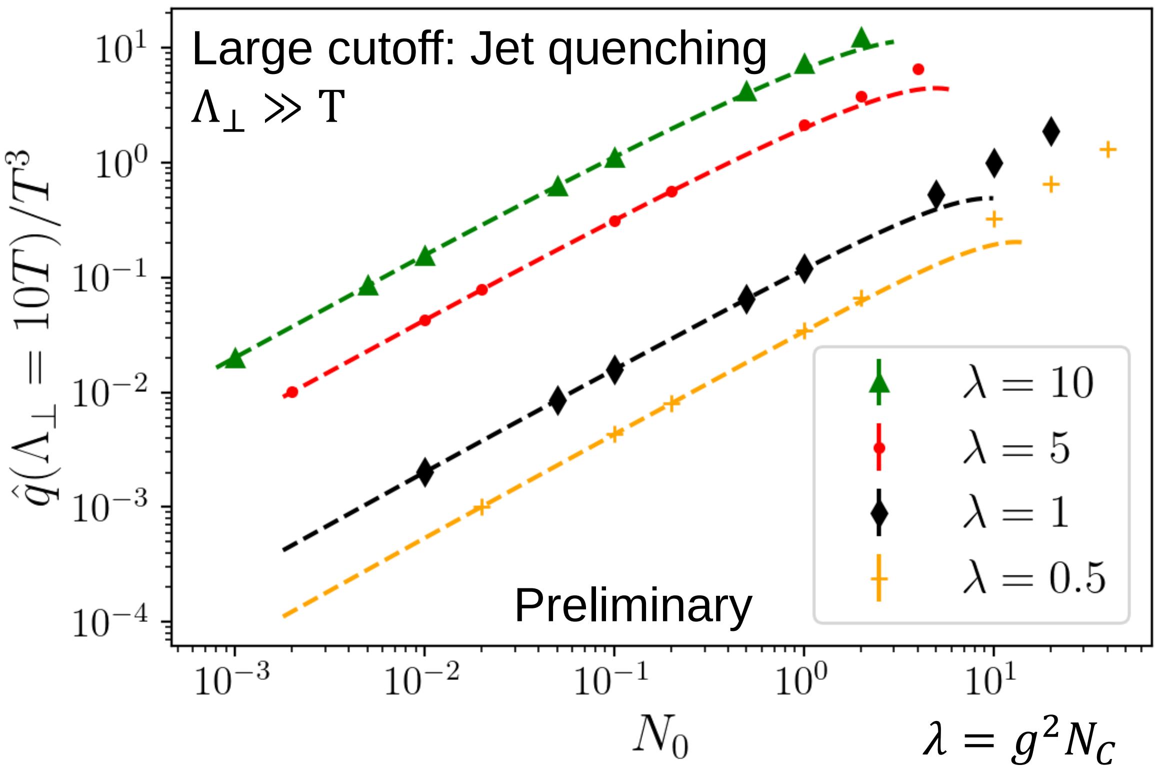
- Momentum cutoff $\Lambda_\perp$ grows for larger jet energy

- Thermalization: Plasma gluon as "jet", splitting rates calculated using $\Lambda_\perp \ll T$
  Phys.Rev.D 78 (2008) 065008 [Arnold, Dogan]
  Ann.Rev.Nucl.Part.Sci. 69 (2019) 447-476 [Schlichting, Teaney]

- For highly energetic jets we need to use $\Lambda_\perp \gg T$
  Phys.Rev.D 78 (2008) 125008 [Arnold, Xiao]

Over- and underoccupied: Scaled thermal distribution





$\lambda = g^2 N_C$

- Dashed lines: Analytical expressions, points: numerical data (full HTL screening)
- Observations:
  - $\hat{q}$ starts at $\sim 10T^3$, decreases during evolution (faster than T)
  - For thermalization: smaller values of $\hat{q}$

Thermalization in heavy-ion collisions

- Initial condition, with $\lambda = g^2 N_C$,

$$f(p_\perp, p_z) = \frac{2}{\lambda} A \frac{\langle p_T \rangle}{\sqrt{p_\perp^2 + (\xi p_z)^2}} \times \exp\left(\frac{-2}{3\langle p_T \rangle^2} \left(p_\perp^2 + (\xi p_z)^2\right) \right)$$

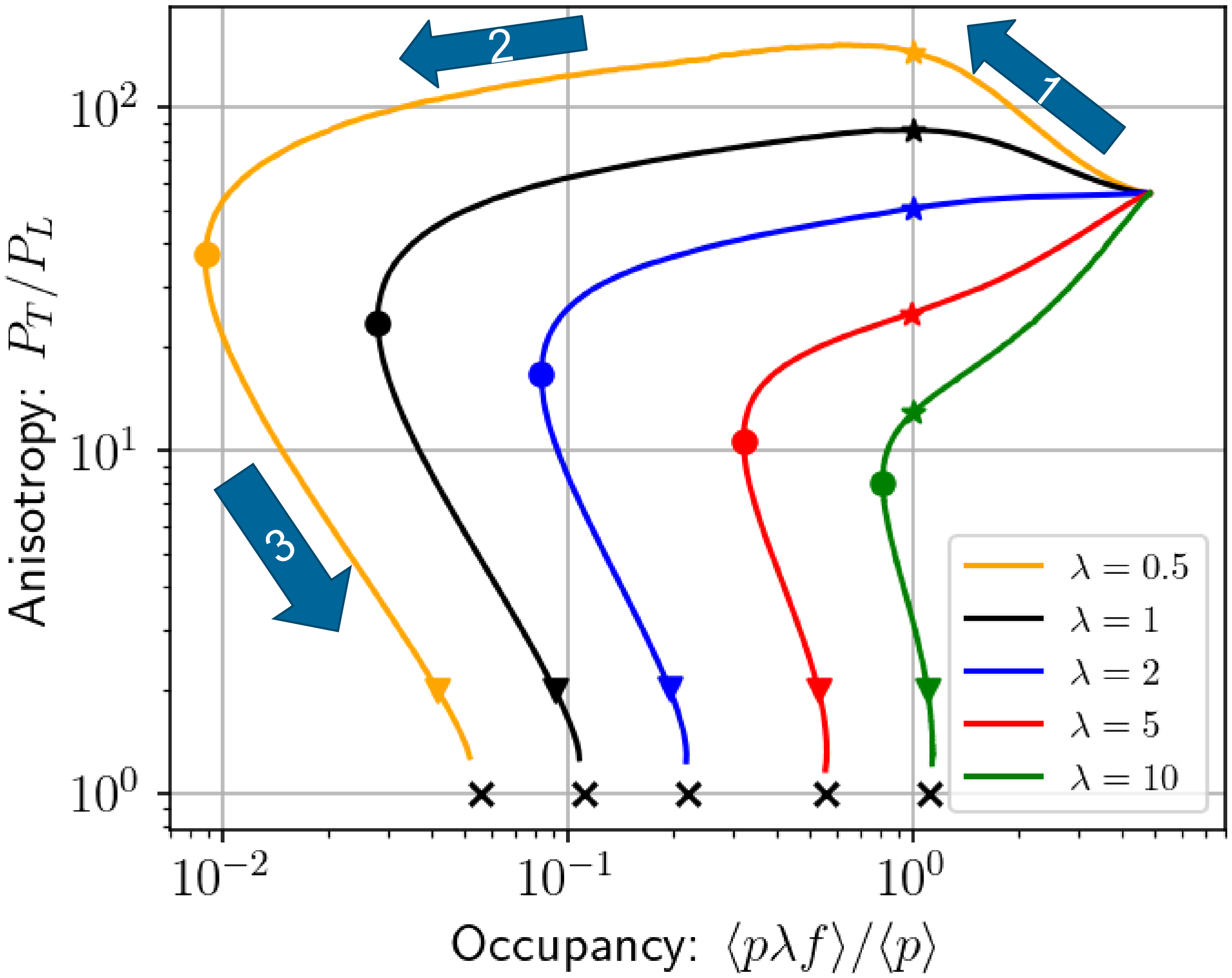
Phys.Rev.Lett. 115 (2015) 18, 182301 [Kurkela, Zhu]

- Phase 1: Anisotropy increases
- Phase 2: Occupancy decreases
- Phase 3: System thermalizes

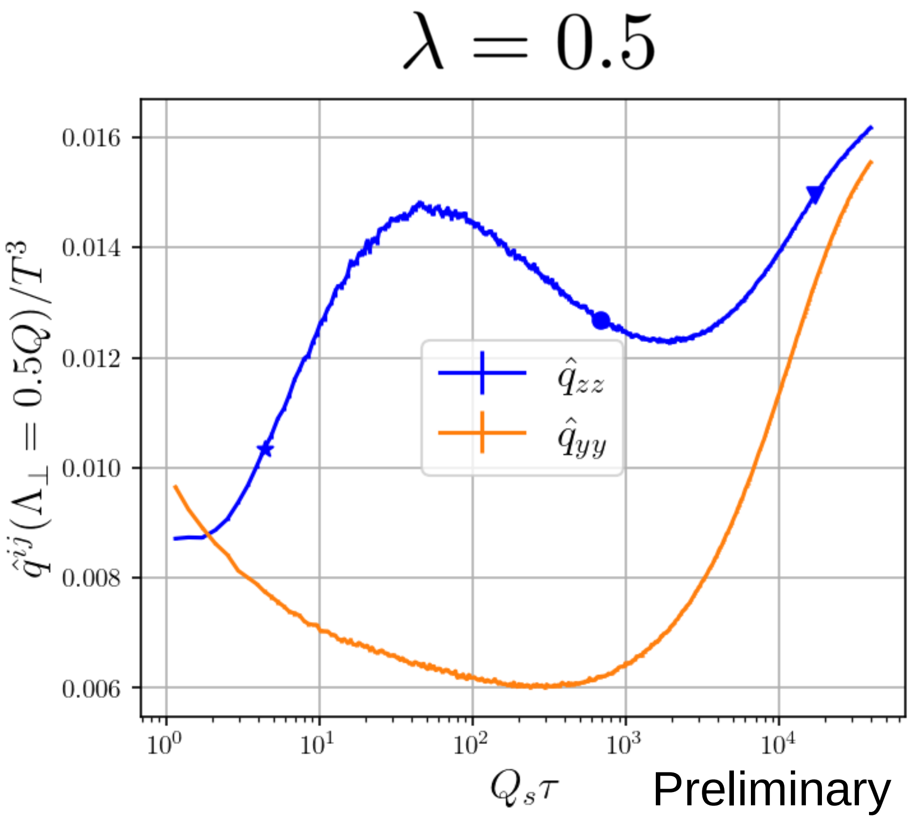
Phys.Lett.B502:51-58,2001 [Bayer, Mueller, Schiff, Son]

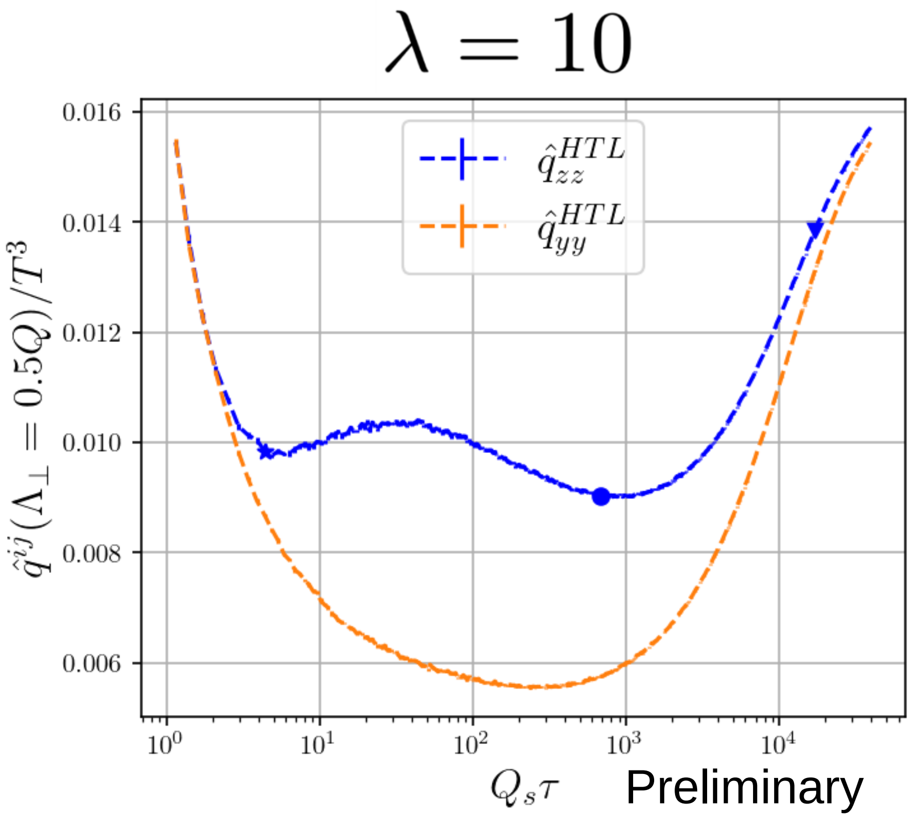
Markers represent different stages

Time evolution of a purely gluonic plasma



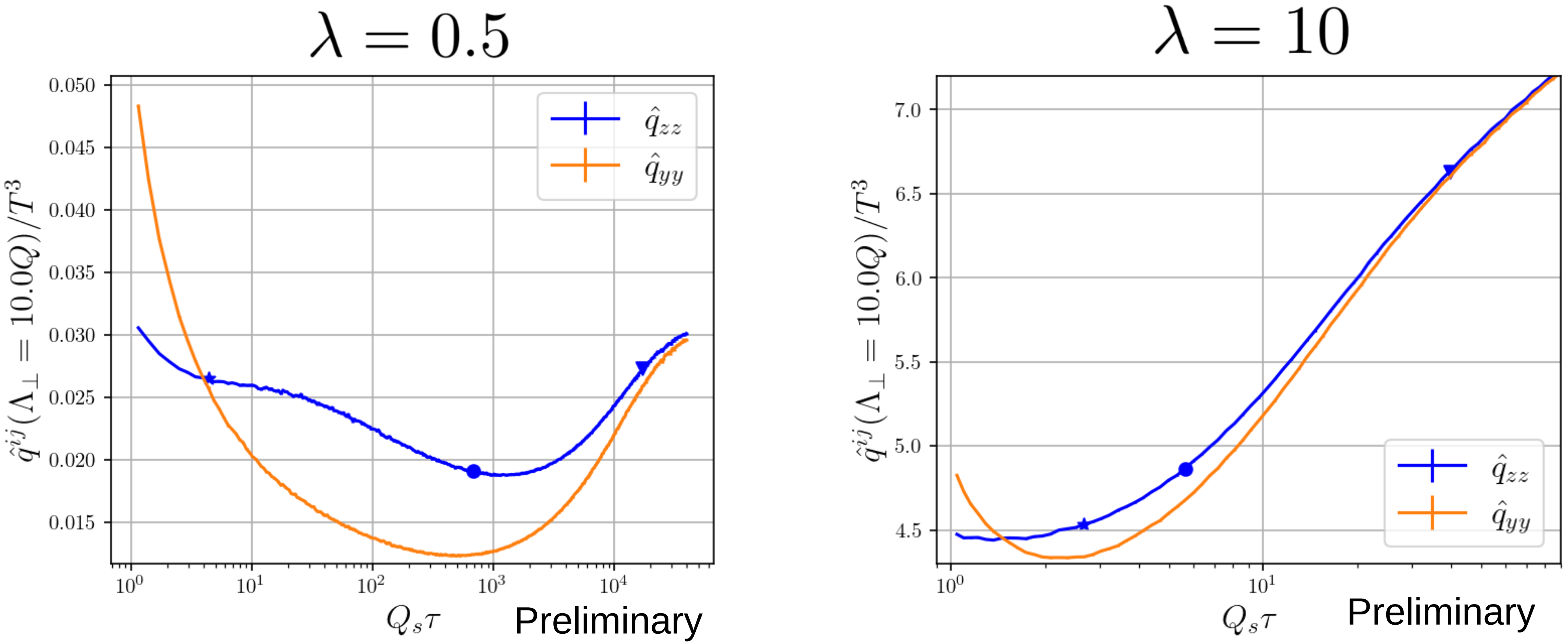
Time evolution – small cutoff





- For small cutoff: Qualitative behavior depends strongly on coupling
- Mostly $\hat{q}^{zz} > \hat{q}^{yy}$, anisotropy up to factor 2

Time evolution – large cutoff



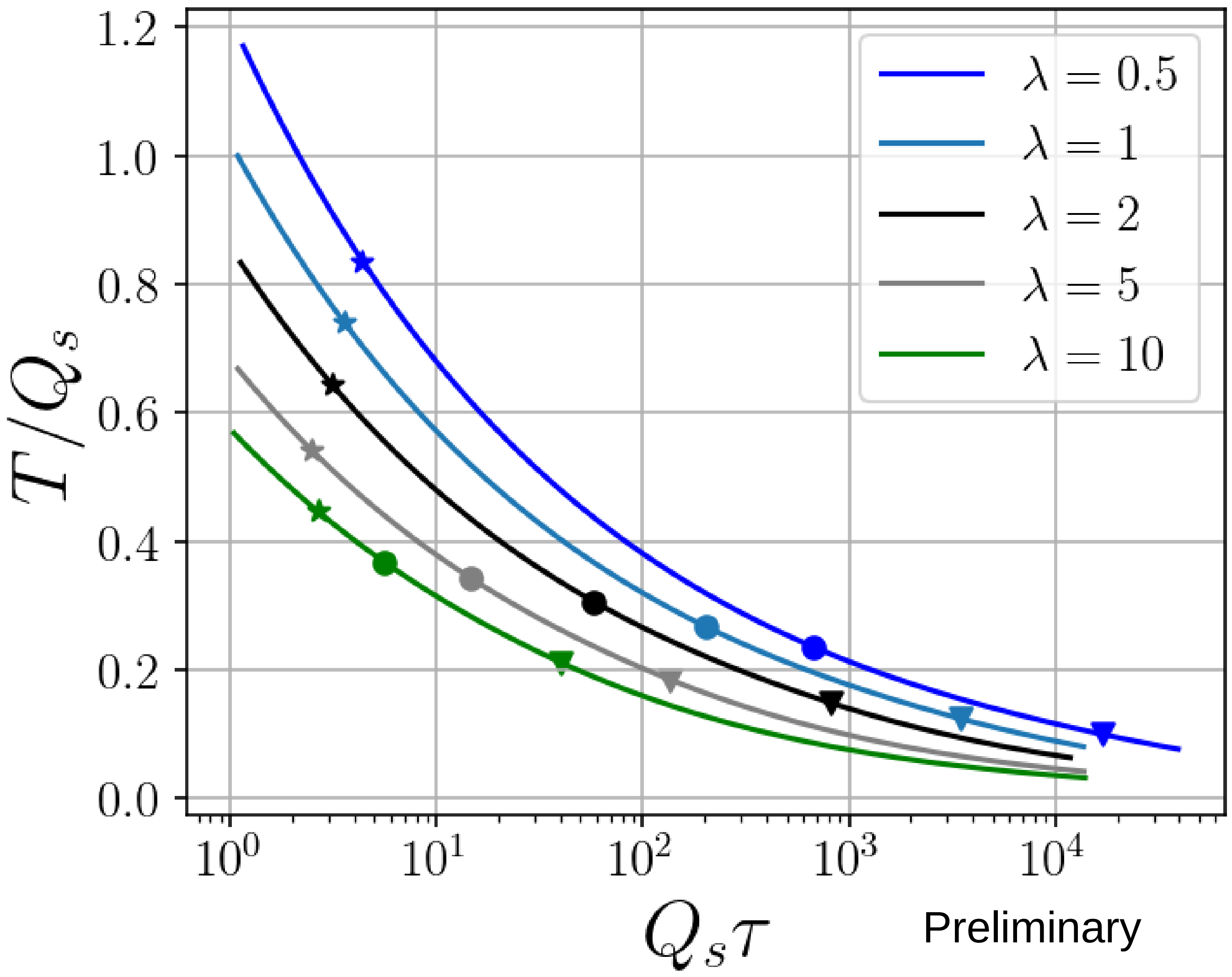
- For large cutoff: Qualitatively similar behavior
- After overoccupied phase: Same ordering $\hat{q}^{zz} > \hat{q}^{yy}$ as in glasma

Temperature evolution

- We have used a constant cutoff $\Lambda_\perp$
- Comparison with temperature T, as extracted from the energy density via

$$\epsilon = \int \frac{d^3p}{(2\pi)^3} p\, f(\vec{p}) =: \frac{\pi^2 T^4}{30}$$

- T decreases throughout evolution: factor 3 for $\lambda = 10$ (realistic coupling)

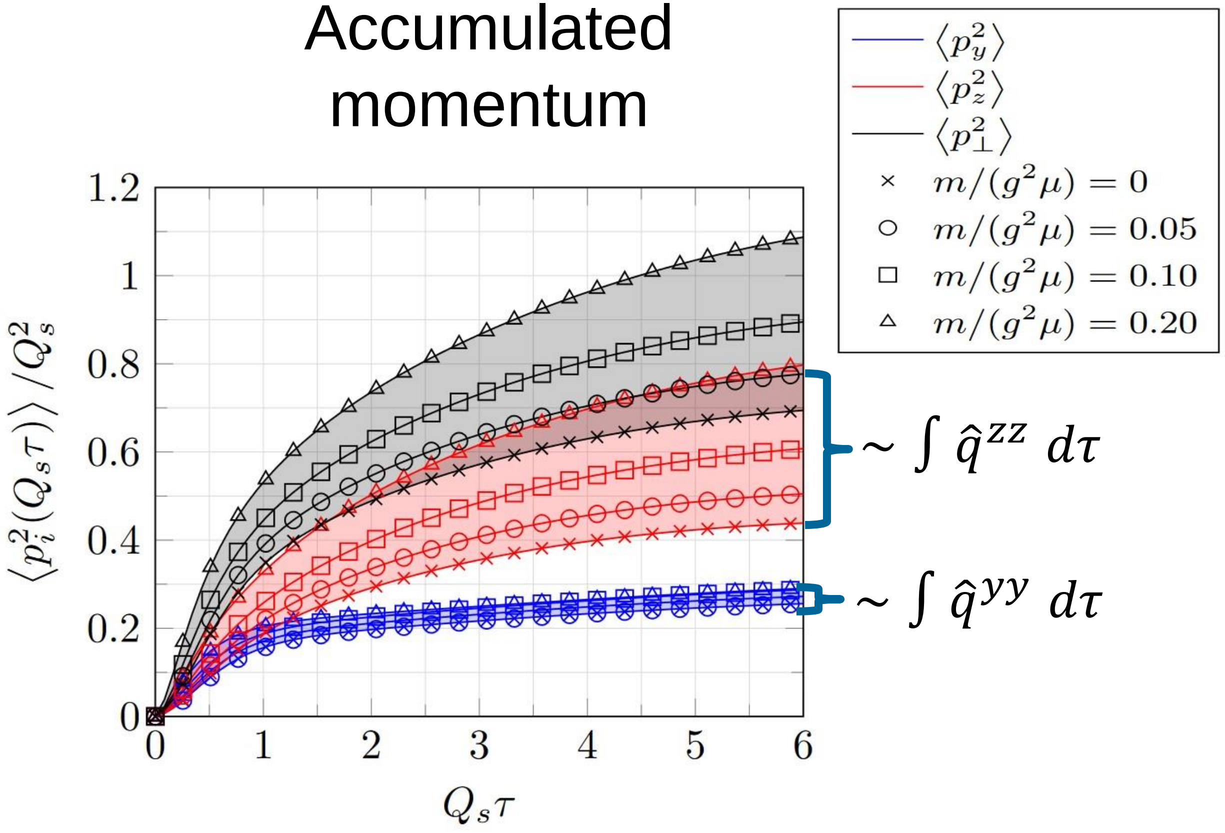


Preliminary

Comparison with the glasma

- Different ordering at beginning
- Same ordering (after overoccupied phase)

Accumulated momentum



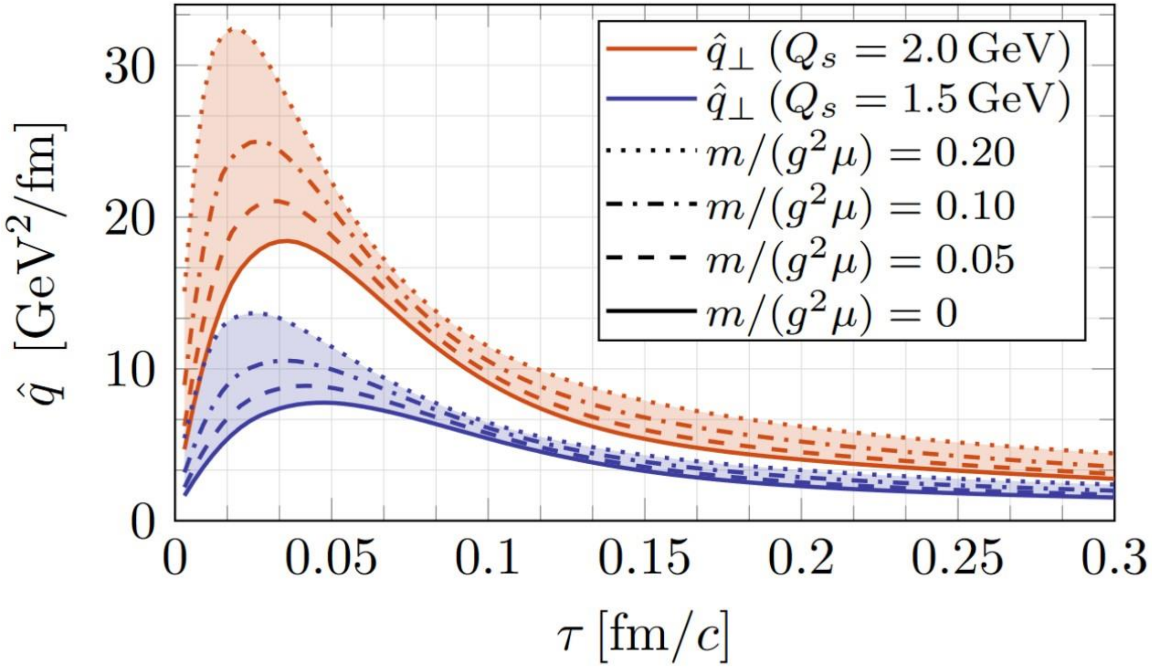
Phys.Rev.D 102 (2020) 7, 074001 [Ipp, Müller, Schuh]

Comparison with the glasma

- Consistent with value for $\hat{q}$ extracted from glasma
  Phys.Lett.B 810 (2020) 135810 [Ipp, Müller, Schuh]
  arXiv 2112.06812 [Carrington, Czajka, Mrowczynski]

- We obtain for $Q_s \sim \mathcal{O}(1)\ \mathrm{GeV}$ a value of $\hat{q} \sim \mathcal{O}(10)\,\frac{\mathrm{GeV}^2}{\mathrm{fm}}$ at beginning

(e.g. $\lambda = 10, \Lambda_\perp = 10\ Q_s$ and our initial conditions:
$T \approx 0.5\ Q_s, \hat{q} \approx 9\ T^3 \approx Q_s^3$ at beginning)



Phys.Lett.B 810 (2020) 135810 [Ipp, Müller, Schuh]

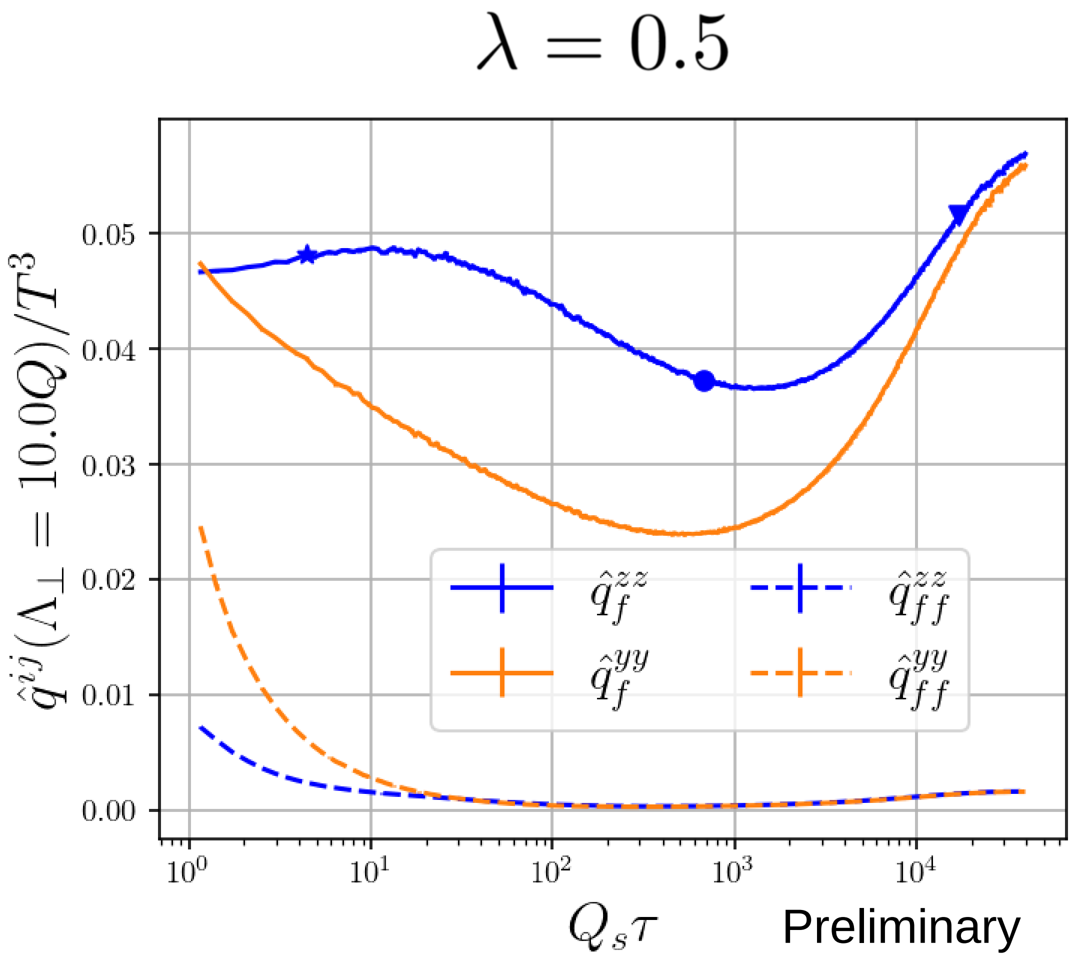
Individual components

$$\hat{q}^{ij}(\Lambda_\perp) = \int_{q_\perp < \Lambda_\perp} \mathrm{d}\Gamma_{PS}\, q^i q^j \, |\mathcal{M}|^2 \, f(\mathbf{k})(1 + f(\mathbf{k}'))$$

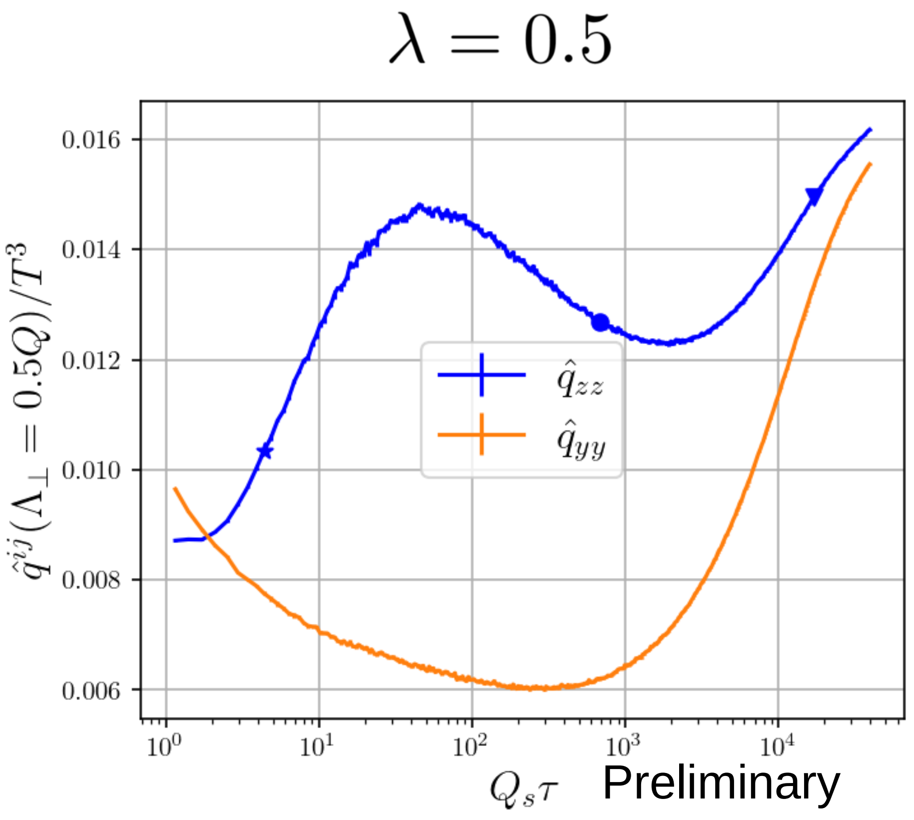
- Separate $\hat{q}$ into

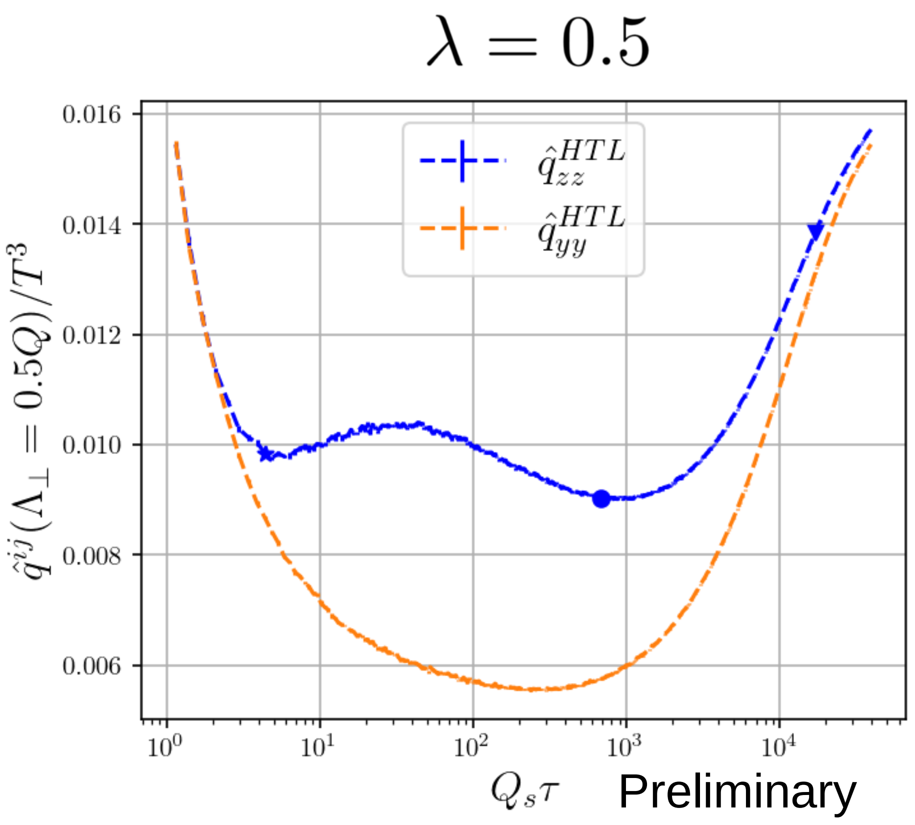
  $$\hat{q} = \hat{q}_{ff} + \lambda \hat{q}_f$$

- In overoccupied phase: Large contribution from $\hat{q}_{ff}$, especially $\hat{q}_{ff}^{yy}$



Comparison with isotropic HTL matrix element





- Largest effects for small coupling and small cutoff
- Others: Qualitatively similar

Approach to attractor

- Time evolution of $\hat{q}^{yy}/\hat{q}^{zz}$ for a large cutoff

- Approach to thermal: Kinetic relaxation time

$$\tau_R = \tau_R(\lambda, \tau) = \frac{4\pi\eta/s(\lambda)}{T(\tau)}$$

  should determine the time when hydro starts (hydrodynamical attractor)

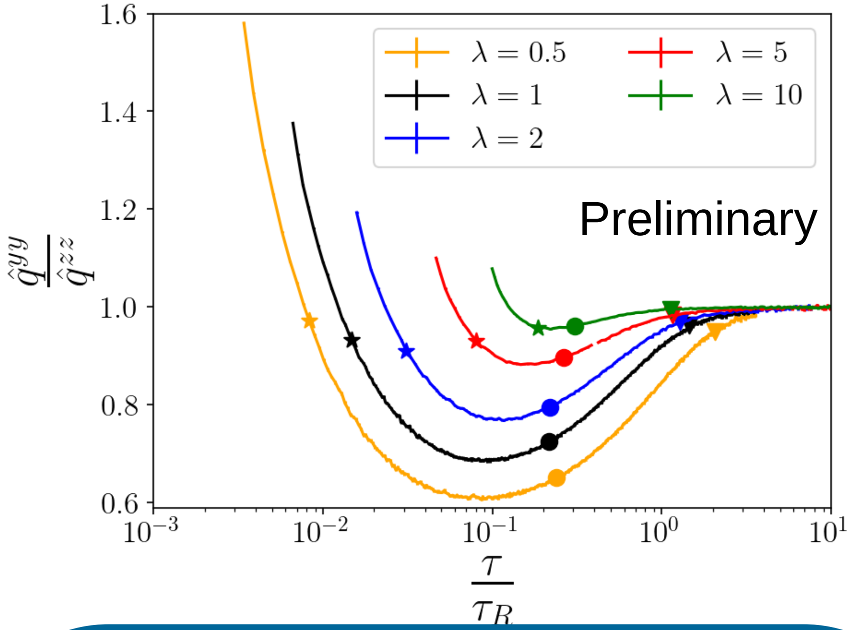
  Phys.Rev.Lett. 115 (2015) 7, 072501 [Heller, Spalinski]

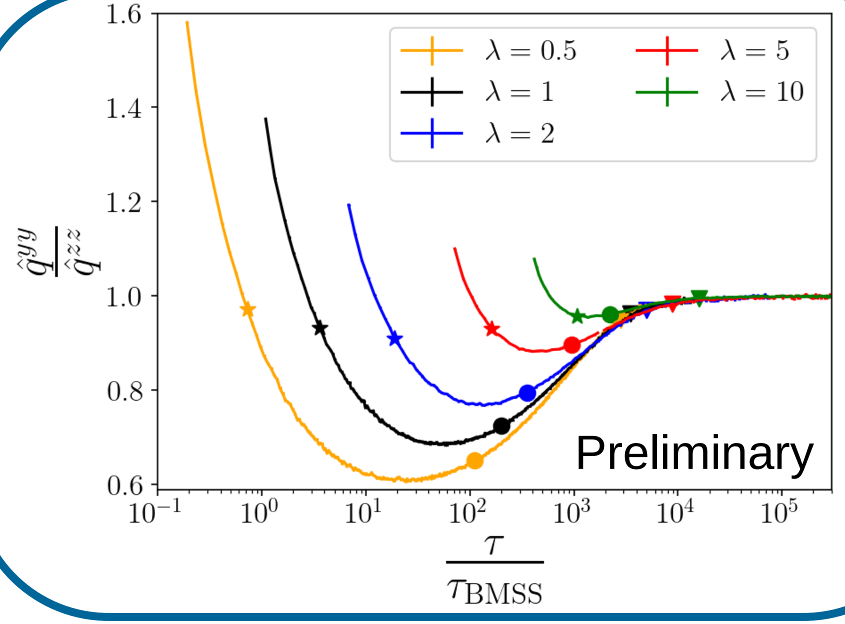
  Phys.Rev.D 99 (2019) 5, 054018 [Kurkela, Mazeliauskas]

- In bottom-up thermalization the thermalization scale should be $\tau_{\text{BMSS}} = \lambda^{-13/5}/Q_s$

  Phys.Lett.B 502:51-58,2001 [Bayer, Mueller, Schiff, Son]

- **Here: Bottom-up thermalization scale works for all couplings!**



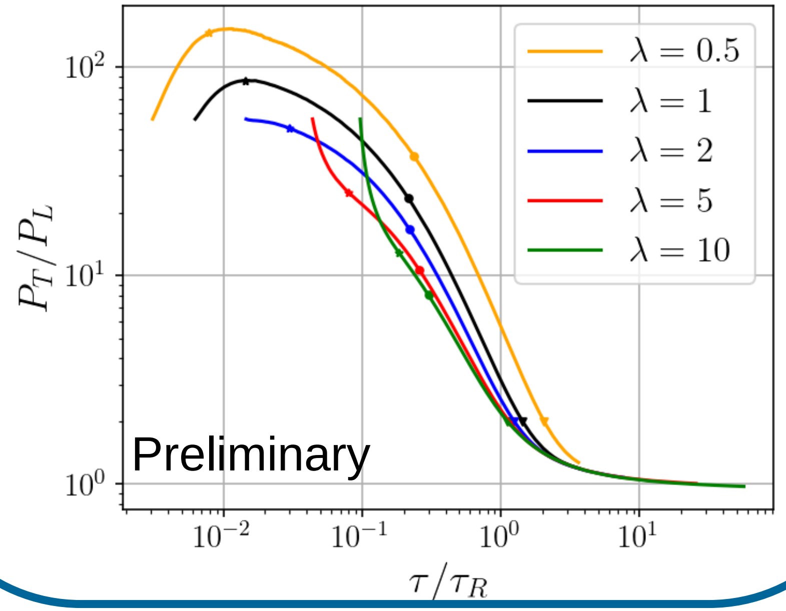


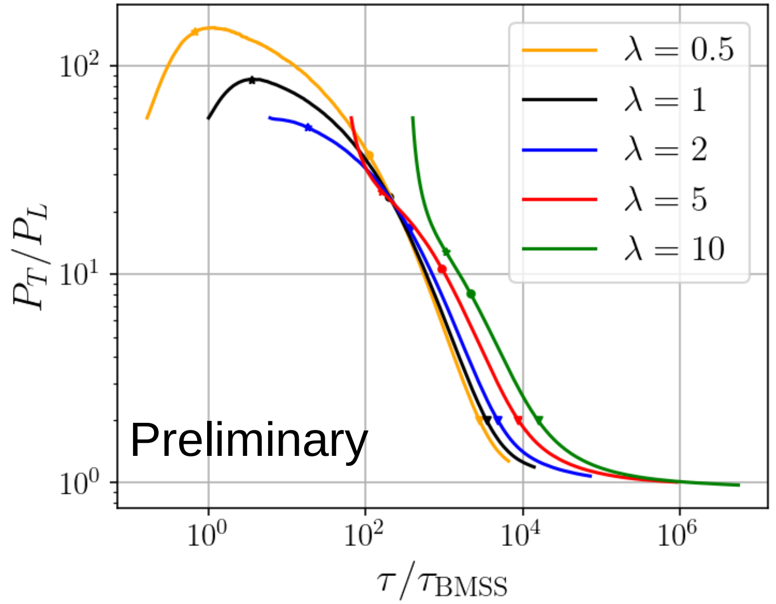
Approach to attractor

- Previously, hydrodynamical attractor found for pressure ratio $\frac{P_T}{P_L}$:

- Attractor property in $\frac{\tau}{\tau_R}$.

$$\tau_R = \frac{4\pi \frac{\eta}{s}(\lambda)}{T(\tau)}$$





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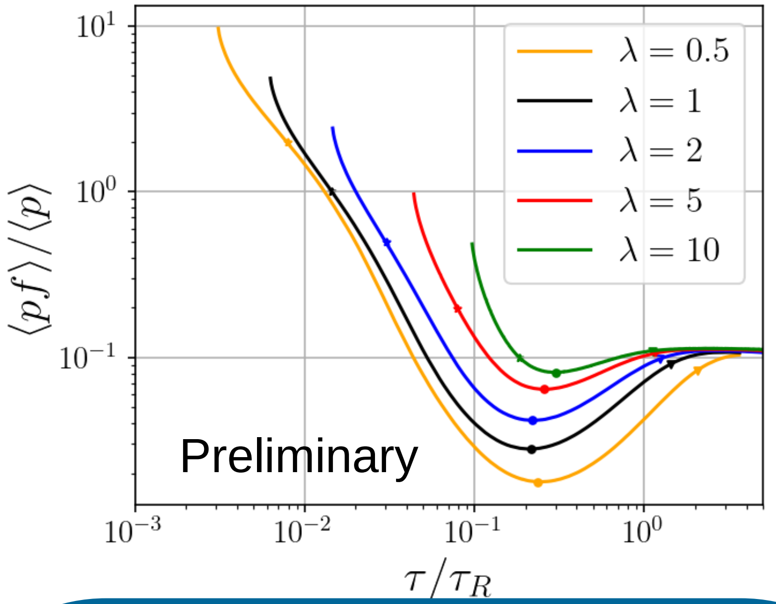
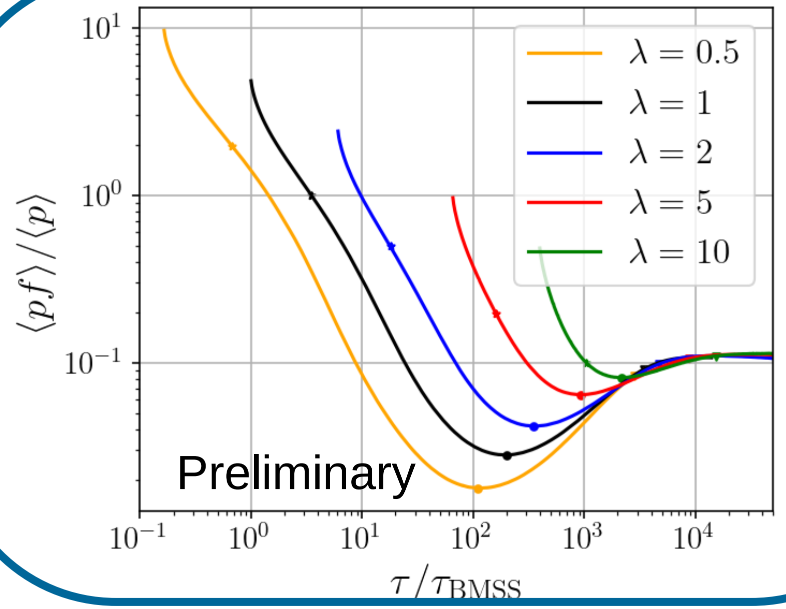
- Attractor property in $\frac{\tau}{\tau_R}$.

$$\tau_R = \frac{4\pi \frac{\eta}{s}(\lambda)}{T(\tau)}$$

- However, occupation follows bottom-up attractor

$$\tau_{\text{BMSS}} = \lambda^{-13/5}/Q_s$$





Approach to attractor for heavy-quark diffusion

- Heavy-quark diffusion coefficient

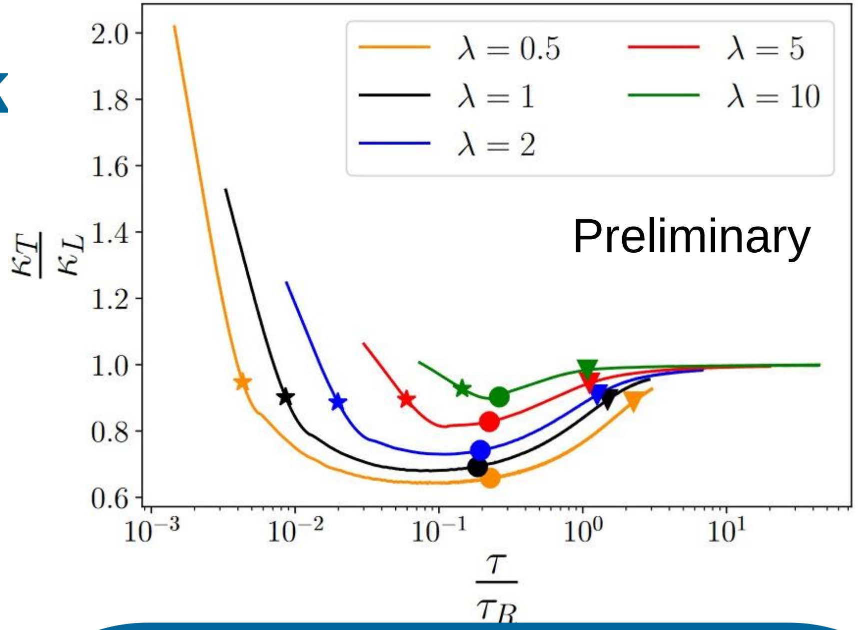
$$\kappa_i = \int_{\boldsymbol{k}\boldsymbol{k}'\boldsymbol{p}'} (2\pi)^3 \, \delta^3 \left(\boldsymbol{p} + \boldsymbol{k} - \boldsymbol{p}' - \boldsymbol{k}'\right) \times 2\pi\delta\left(k' - k\right) q_i^2 \left[\left|\mathcal{M}_g\right|^2 f(\boldsymbol{k})(1 + f(\boldsymbol{k}')) \right].$$

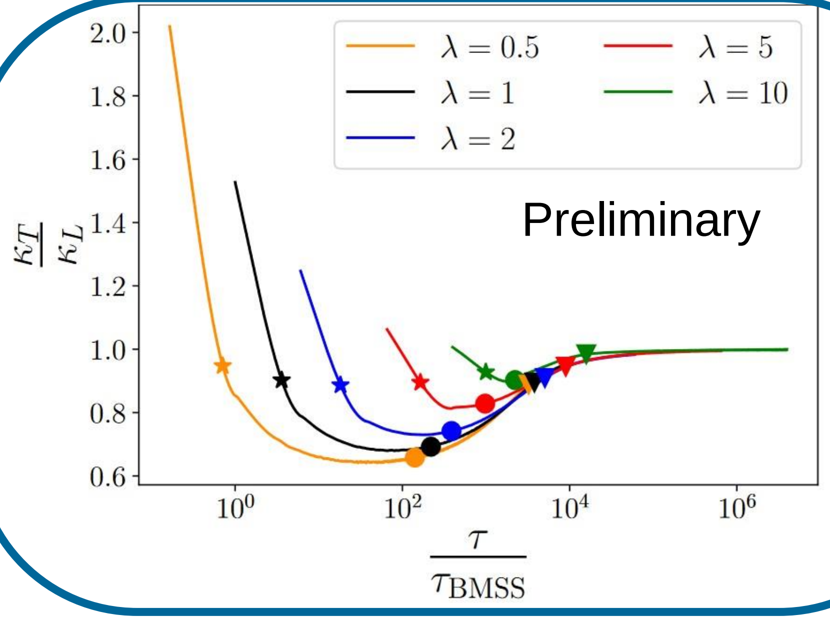
Phys.Rev.C 71 (2005) 064904 [Moore, Teaney]

- Momentum transferred to heavy-quark

- Also ratio κ_T/κ_L follows bottom-up attractor

$$\tau_{\mathrm{BMSS}} = \lambda^{-13/5}/Q_s$$





Conclusion

- $\hat{q}$ decreases with time during kinetic evolution, being initially of the order of magnitude of the glasma value
- $\hat{q}$ components depend on coupling λ and cutoff $\Lambda_\perp$
- $\hat{q}^{zz} > \hat{q}^{yy}$ during most of the evolution
- Different screening approximations provide qualitatively similar results
- $\hat{q}^{yy}/\hat{q}^{zz}$ does not follow the hydrodynamical attractor, but BMSS attractor
  - Like other observables: $\frac{\kappa_T}{\kappa_L}$, occupancy
  - This shows limitations of universality of hydro attractor

Future directions

- Different initial conditions, dependence on jet angle and momentum
- Different screening approximations
  - Relax isotropic screening approximation
- Experimental signature of $\hat{q}$ from thermalization stages, impact

Backup slides

Over- and underoccupied: Scaled thermal distribution

- For $\Lambda_\perp \ll$ T:

$$f_g(k; N_0, \beta) = \frac{N_0}{\exp(\beta k) - 1}$$

$$\hat{q}(N_0) = \frac{N_0 g^4 T^3 C_R}{24\,\pi^3} \ln\left(1 + \frac{\Lambda_\perp^2}{m_D^2}\right)\left(N_0 \pi^2 (2N_C + n_f) + \zeta(3)(1 - N_0)(12\,N_C + 9n_f)\right)$$

- For $\Lambda_\perp \gg$ T and $g\sqrt{\frac{N_0}{3}\left(N_C + \frac{n_f}{2}\right)} \ll 1$

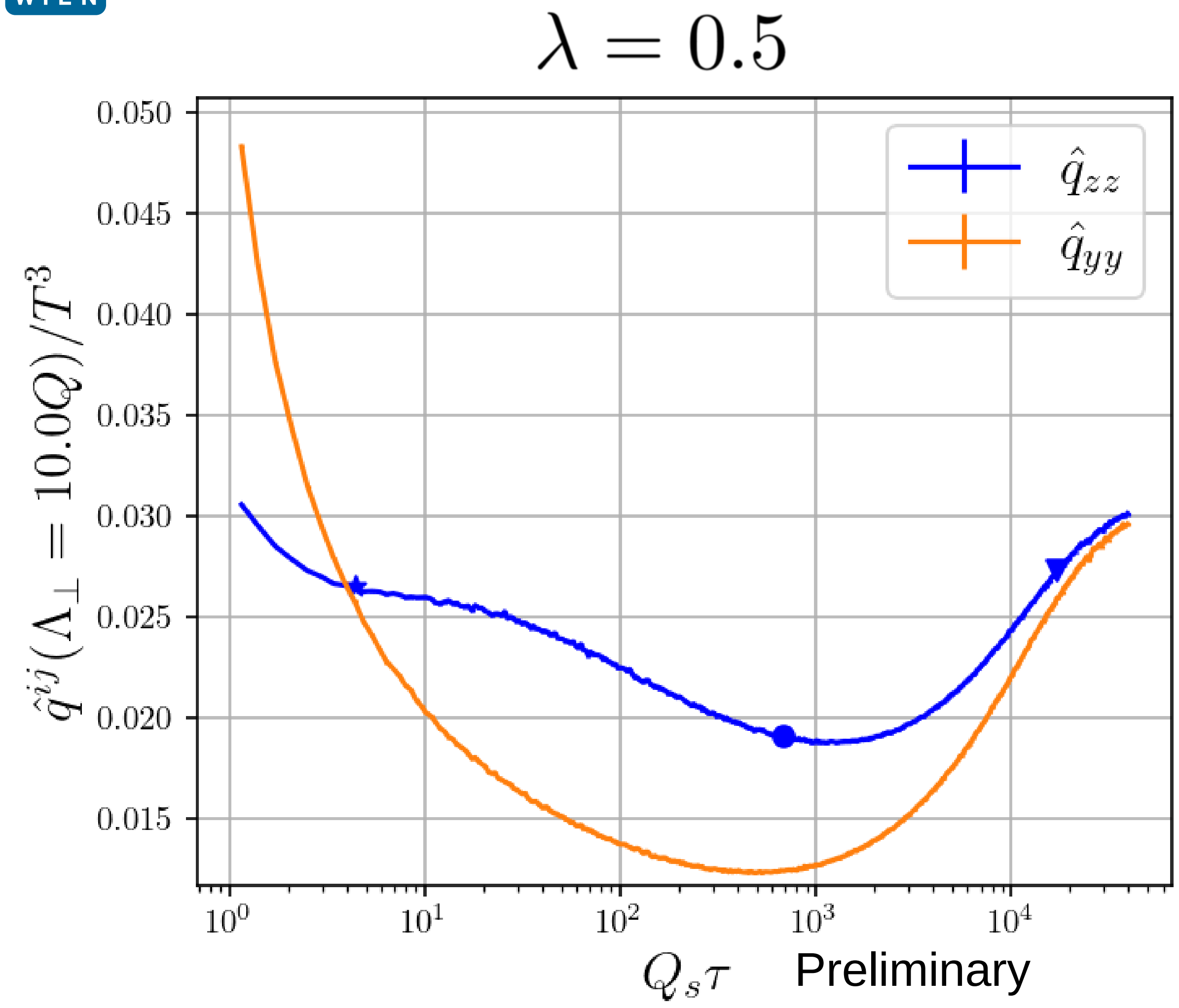
$$\hat{q}(N_0) = \frac{C_R g^4 T^3}{\pi^2}\left(\Xi_b\left(N_0^2 \mathcal{I}_+(\Lambda_\perp) + \frac{N_0(1-N_0)\zeta(3)}{4\pi}\ln\left(1 + \frac{\Lambda_\perp^2}{m_D^2}\right)\right) + \Xi_f\left(N_0^2 \mathcal{I}_-(\Lambda_\perp) + \frac{N_0(1-N_0)\zeta_-(3)}{4\pi}\ln\left(1 + \frac{\Lambda_\perp^2}{m_D^2}\right)\right)\right)$$

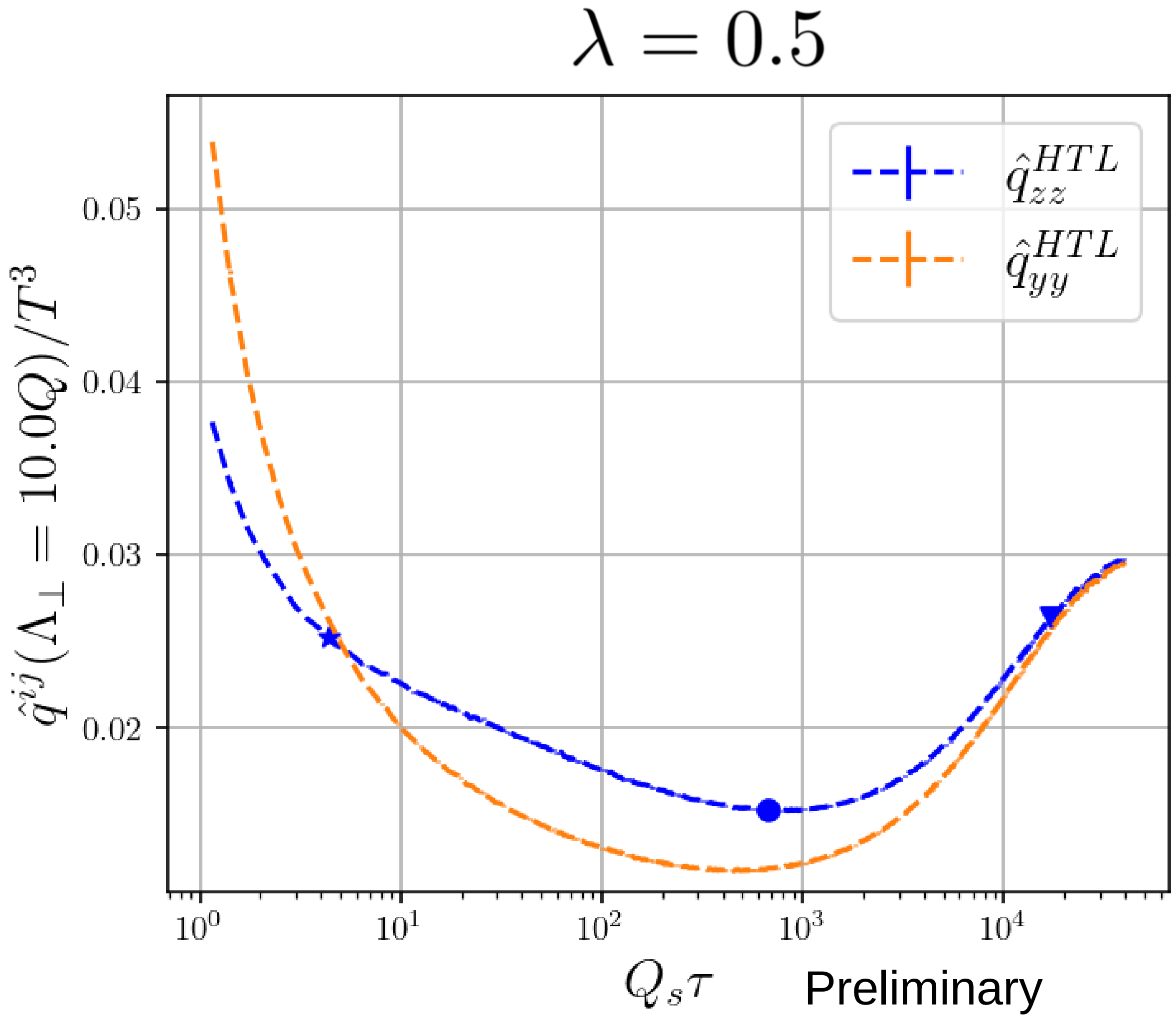
$$\mathcal{I}_\pm(\Lambda_\perp) = \frac{\zeta_\pm(3)}{4\pi}\ln\left(1 + \frac{\Lambda_\perp^2}{m_D^2}\right) + \frac{\zeta_\pm(2) - \zeta_\pm(3)}{2\pi}\left[\frac{1}{2}\ln\left(1 + \frac{T^2}{m_D^2}\right) + \frac{1}{2} - \gamma_E + \ln 2\right] - \frac{\sigma_\pm}{2\pi}$$

$$\zeta_+(s) = \zeta(s)\,,\ \zeta_-(s) = (1 - 2^{1-s})\zeta(s),\ \sigma_+ = 0.3860438,\ \sigma_- = 0.0112168$$

Extending a calculation from Phys.Rev.D 78 (2008) 125008 [Arnold, Xiao]

Comparison with isotropic HTL matrix element





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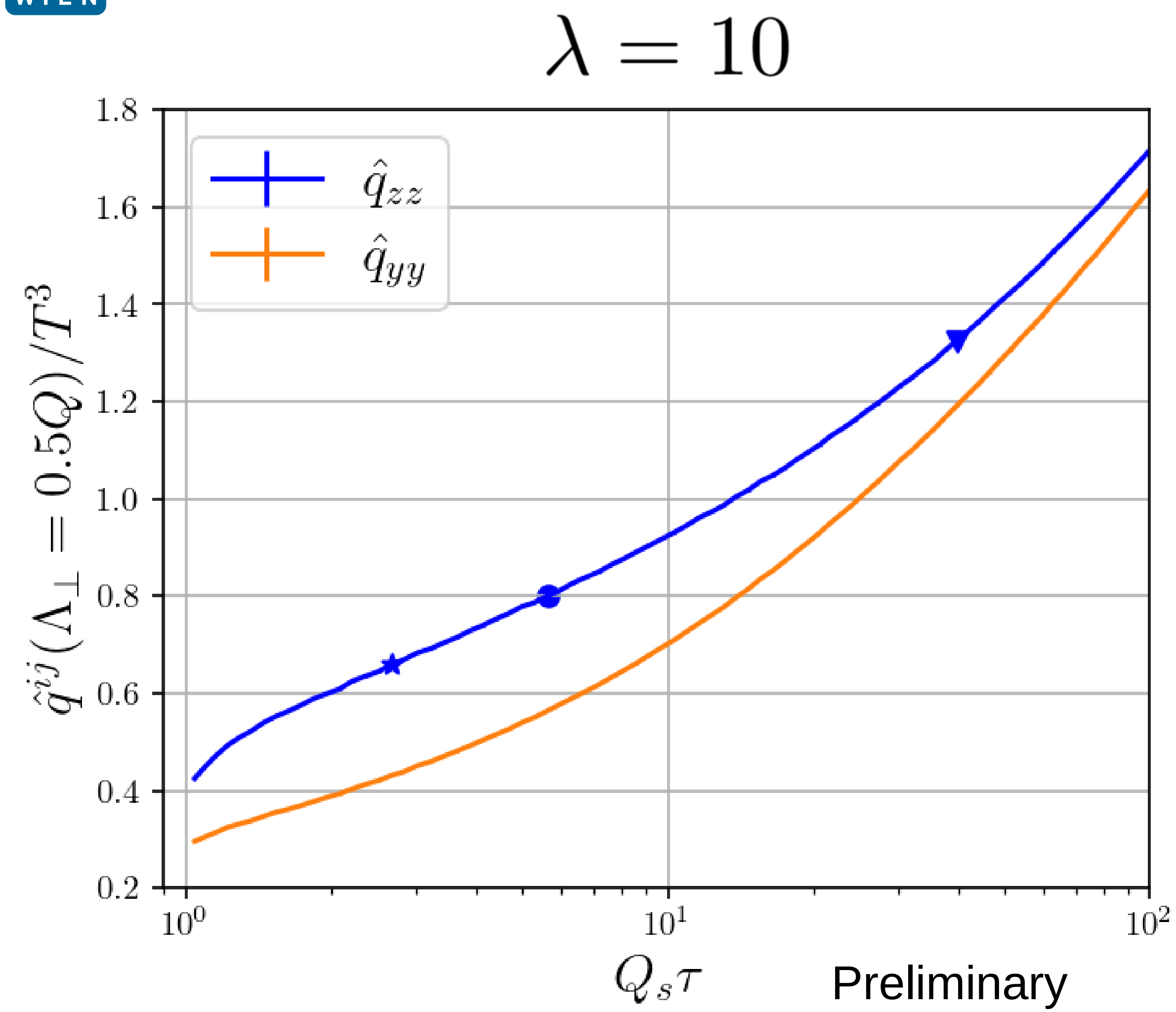

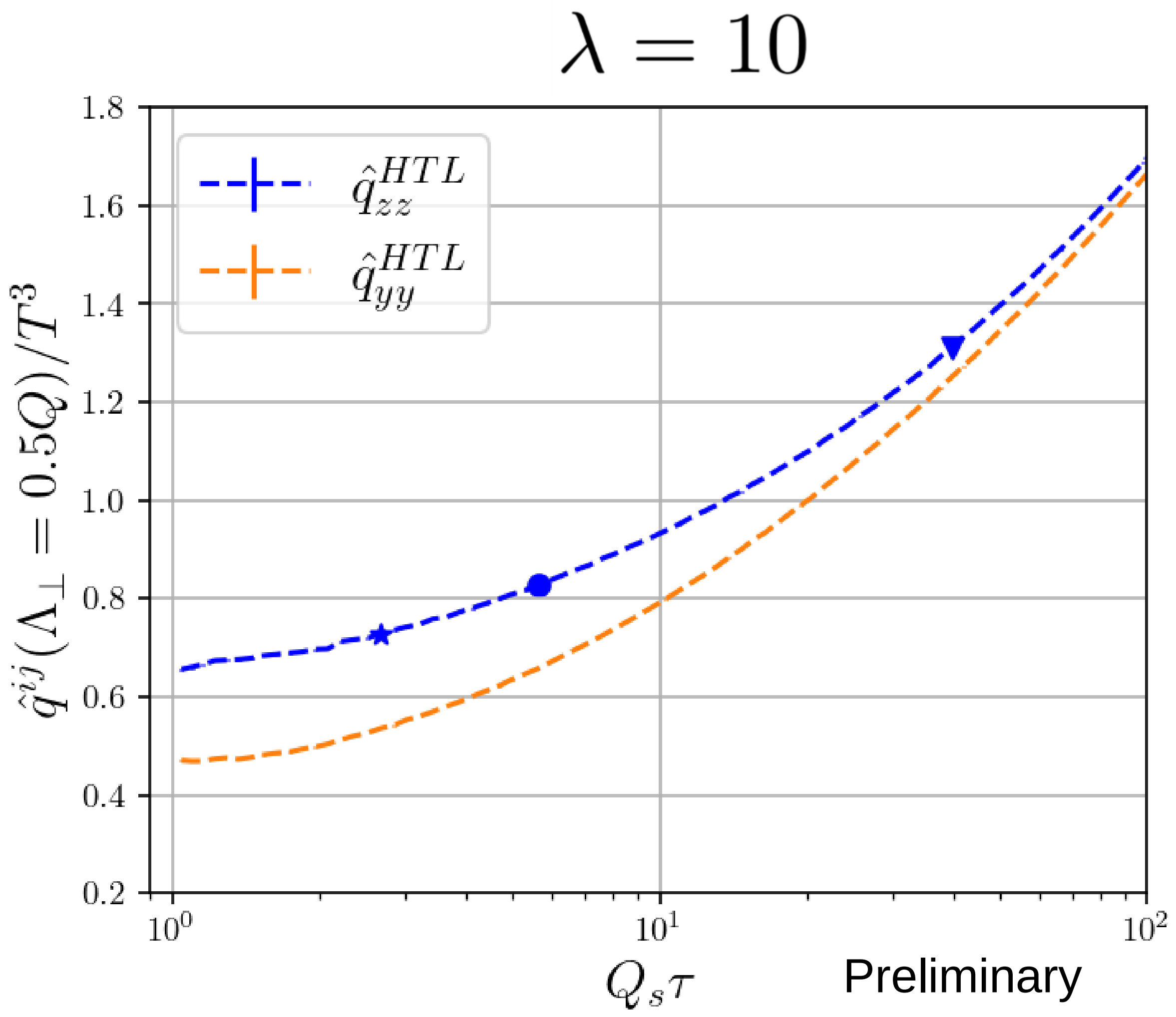

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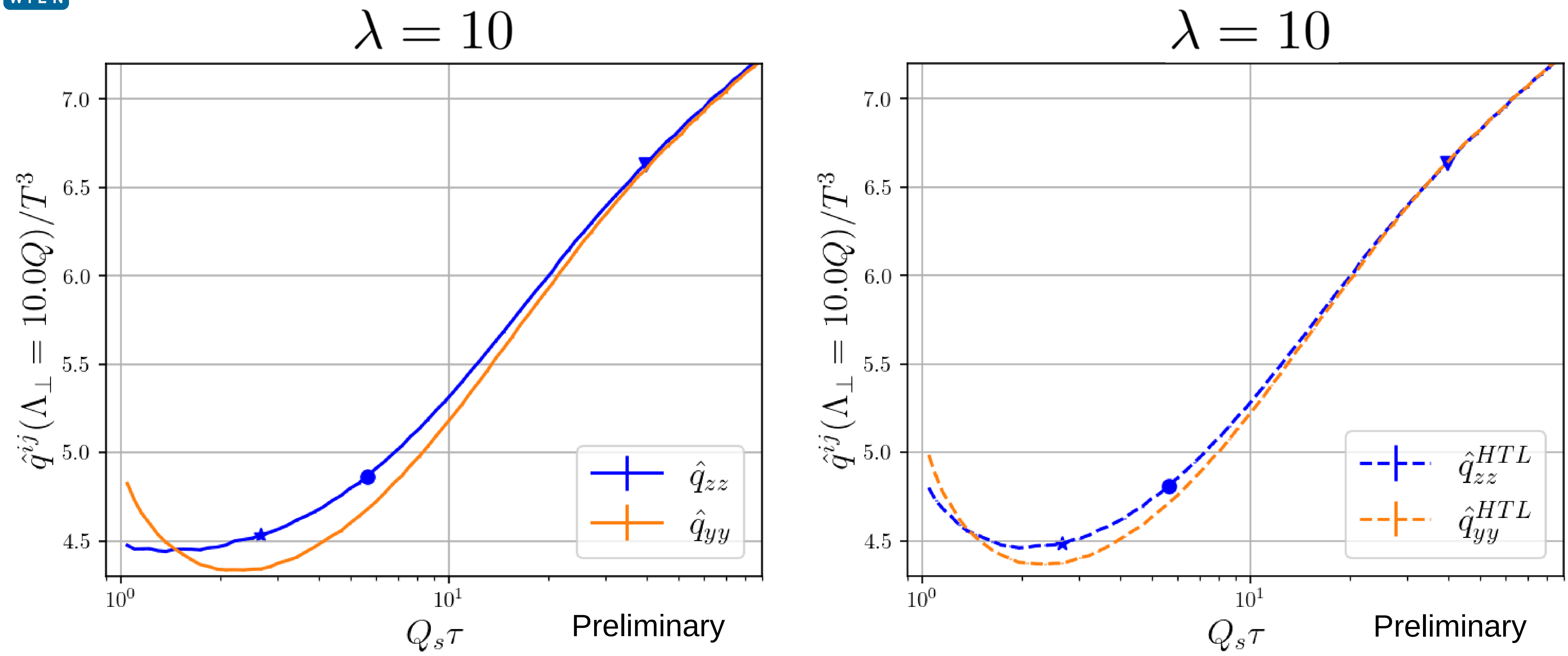
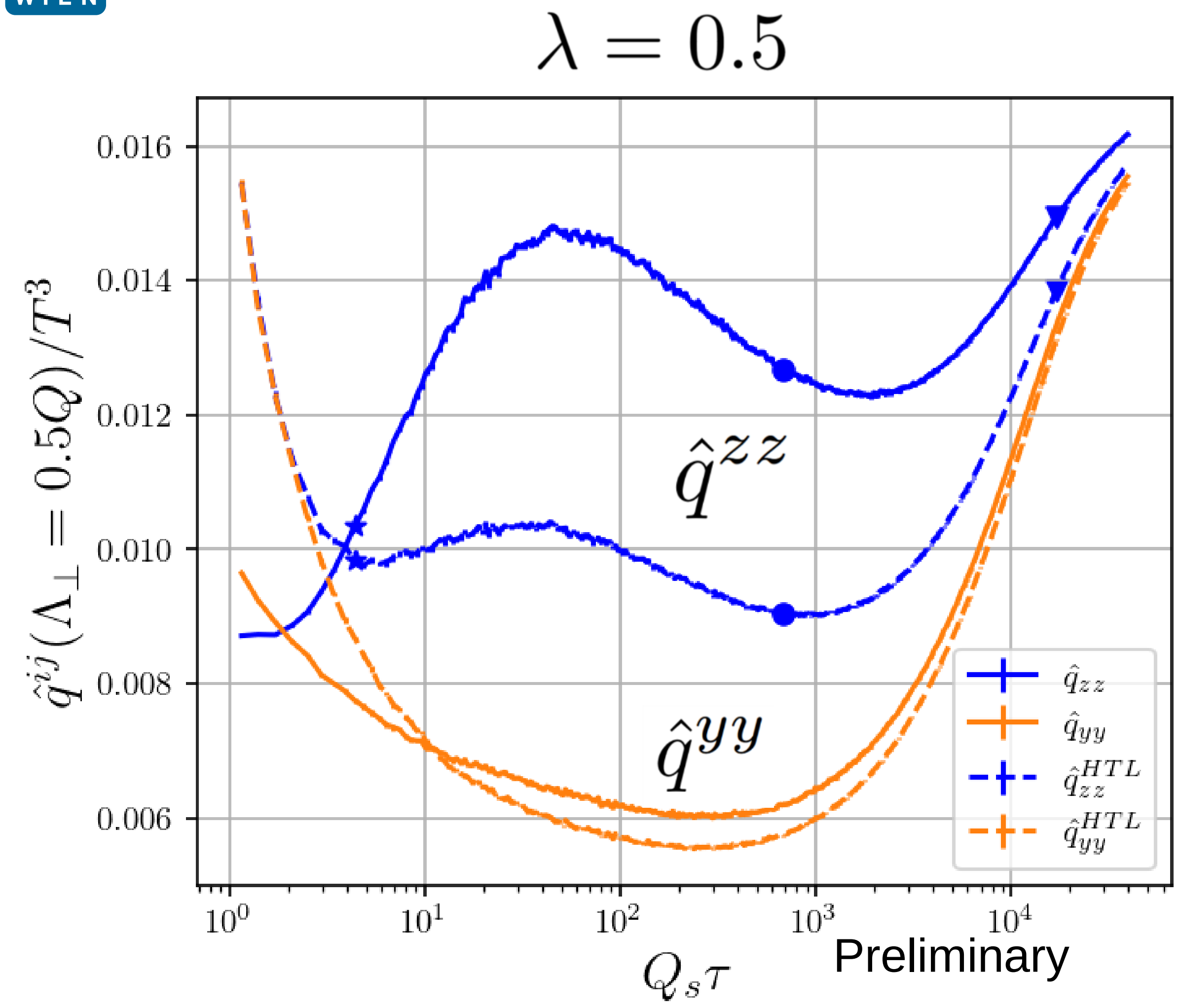
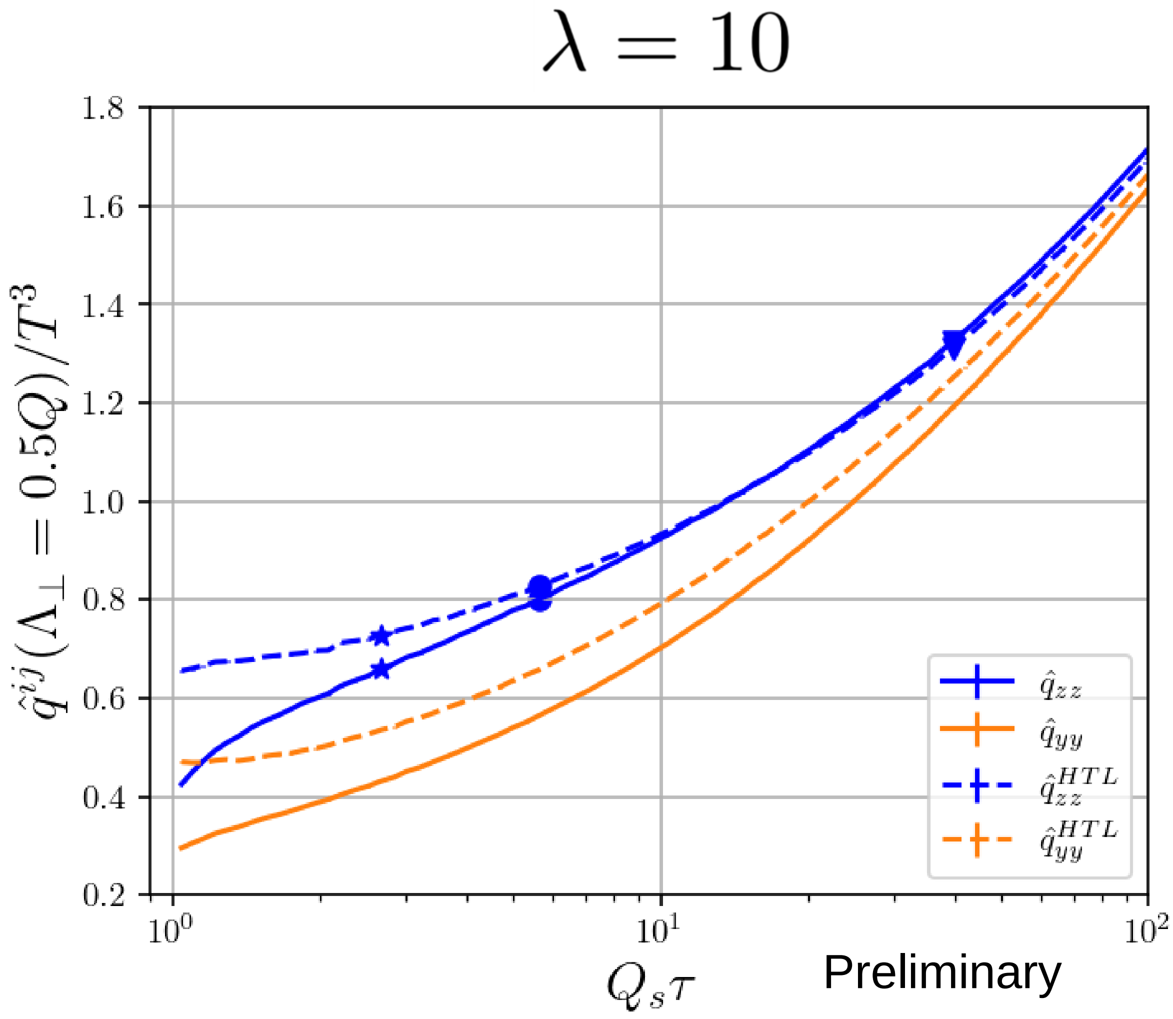


Time evolution – small cutoff





Time evolution – large cutoff