Stability of Classical Chromodynamic Fields

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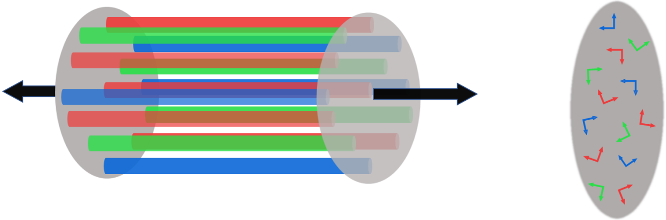
Overview

- Motivation
- Linearized Classical Chromodynamics
- Stability of constant and uniform fields: Abelian vs. nonAbelian configurations
- Gauge dependence & Energy-Momentum Tensor
- Summary & outlook

Motivation

- The earliest phase of heavy-ion collisions is described in terms of classical fields.



- Early configuration is unstable, but the character of the instabilities is not clear (P. Romatschke and R. Venugopalan, Phys. Rev. Lett. 96, 062302 (2006)).
- We plan to study the problem systematically.

Yang-Mills equations & linearized QCD

Yang-Mills equations in adjoint representation

$$\partial_\mu F_a^{\mu\nu} + g f^{abc} A_\mu^b F_c^{\mu\nu} = J_a^\nu, \qquad F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g f^{abc} A_b^\mu A_c^\nu$$

Linearized QCD

$A_a^\mu(t,\mathbf{r}) = \bar{A}_a^\mu(t,\mathbf{r}) + a_a^\mu(t,\mathbf{r}),$ where $|\bar{A}(t,\mathbf{r})| \gg |a(t,\mathbf{r})|$

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Background gauge condition

$$\bar{D}_{ab}^\mu a_\mu^a = \partial^\mu a_\mu^a + g f^{abc} \bar{A}_b^\mu a_\mu^c = 0$$

Yang-Mills equations & linearized QCD

Yang-Mills equations in adjoint representation

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Linearized QCD

$$A_a^\mu(t,\mathbf{r}) = \bar{A}_a^\mu(t,\mathbf{r}) + a_a^\mu(t,\mathbf{r}), \qquad \text{where } |\bar{A}(t,\mathbf{r})| \gg |a(t,\mathbf{r})|$$

Background gauge condition

$$\bar{D}^\mu_{ab} a^a_\mu = \partial^\mu a^a_\mu + g f^{abc} \bar{A}^\mu_b a^c_\mu = 0$$

Linearized Yang-Mills equations in the background gauge

$$\left[g^{\mu\nu} \left(\bar{D}_\rho \bar{D}^\rho \right)_{ac} + 2 g f^{abc} \bar{F}_b^{\mu\nu} \right] a^c_\nu = J^\mu_a$$

Uniform chromoelectric and chromomagnetic fields & assumpions

Generation of constant fields in one direction is possible in two ways:

- Abelian configuration - single color potential linearly dependent on coordinates,
- nonAbelian configuration - multicolor, constant potential.

Calculations done in SU(2) group: $f^{abc} \to \epsilon^{abc}$.

Chromoelectric and chromomagnetic fields along x-axis.

Stability of Abelian chromomagnetic configuration

S. J. Chang and N. Weiss, Phys. rec. D **20**, 869 (1979), P. Sikivie, Phys. Rev. D **22**, 877 (1979)

Constant homogeneous chromomagnetic field

$\bar{A}^\mu_a(t,\mathbf{r}) = (0,0,0,yB)\delta^{a1}$

Potential $\bar{A}^\mu_a(t,\mathbf{r})$ satisfies YM equations with vanishing current

The color component a_1 satisfies Abelian equation and decouples from the remaining two components: $\Box a_1^\nu = 0$

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$a_a^\mu(t, x, y, z) = e^{-i(\omega t - k_x x - k_z z)} a_a^\mu(y)$

Mixing

- colors 2 and 3 $\rightarrow$ $\mathsf{T}^\pm(y) = a_2^0(y) \pm i a_3^0(y)$, $X^\pm(y) = a_2^x(y) \pm i a_3^x(y)$,
  $Y^\pm(y) = a_2^y(y) \pm i a_3^y(y)$, $Z^\pm(y) = a_2^z(y) \pm i a_3^z(y)$
- coordinates y and z $\rightarrow$ $\mathsf{U}^\pm(y) = Y^+(y) \pm i Z^+(y)$,
  $W^\pm(y) = Y^-(y) \pm i Z^-(y)$

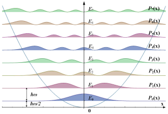
Stability of Abelian chromomagnetic configuration

$$\Big(-\omega^2 + k_x^2 - \frac{d^2}{dy^2} + (k_z + gBy)^2 \mp 2gB \Big) W^{\pm} = 0$$

Stability of Abelian chromomagnetic configuration

$$\Big(- \omega^2 + k_x^2 - \frac{d^2}{dy^2} + (k_z + gBy)^2 \mp 2gB \Big) W^{\pm} = 0$$

Non-relativistic Schrödinger equation of harmonic oscillator
$\Big(- 2m\mathcal{E} + m^2 \bar{\omega}^2 (y_0 - y)^2 - \frac{d^2}{dy^2} \Big) \varphi(y) = 0$



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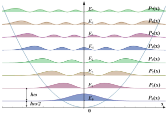
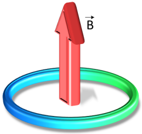


Unstable solution

$\omega_-^2 = k_x^2 - gB < 0$ for $gB > k_x^2 \quad \longrightarrow \quad a \sim e^{\sqrt{gB - k_x^2}\,t}$

Nielsen Olesen instability

The result is purely classical!



Stability of nonAbelian chromomagnetic configuration

T. N. Tudron, Phys. Rev. D **22**, 2566 (1980)

Constant homogeneous chromomagnetic field

$$\bar{A}_a^\mu = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{B/g} \\ 0 & 0 & \sqrt{B/g} & 0 \end{bmatrix}, \quad J_a^\nu = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{gB^3} \\ 0 & 0 & \sqrt{gB^3} & 0 \end{bmatrix}.$$

Assumption

$a_a^\mu(t, \mathbf{x}) = a_a^\mu e^{-i(\omega t - \mathbf{kx})}$

Matrix equations

12x12 matrix in block form $\longrightarrow$ 2 equal matrices 3x3 and one 6x6

Homogeneous equations $\longrightarrow$ solutions exist if determinant of the matrix vanishes.

Stability of nonAbelian chromomagnetic configuration

$$\hat{M}_{B_t} = \hat{M}_{B_x} = \begin{bmatrix} -\omega^2 + \mathbf{k}^2 + 2g^2A^2 & -2igAk_y & 2igAk_z \\ 2igAk_y & -\omega^2 + \mathbf{k}^2 + g^2A^2 & 0 \\ -2igAk_z & 0 & -\omega^2 + \mathbf{k}^2 + g^2A^2 \end{bmatrix}$$

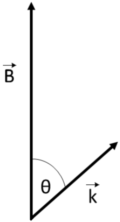
$$\det \hat{M}_{B_t} = \left(-\omega^2 + \mathbf{k}^2 + g^2A^2\right)\Big(\omega^4 - \omega^2(2\mathbf{k}^2 + g^2A^2) + g^2A^2(3\mathbf{k}^2 - 2k_T^2) + \mathbf{k}^4 + 2g^4A^4\Big) = 0,$$

$$k_T = \sqrt{k_y^2 + k_z^2}, \qquad k_T = k \sin\theta$$

Solutions

- $\omega^2 = \mathbf{k}^2 + gB$
- $\omega_\pm^2 = \mathbf{k}^2 + \frac{3}{2}gB \pm \frac{1}{2}\sqrt{16gBk_T^2 + g^2B^2}$

The solutions are stable.



Stability of nonAbelian chromomagnetic configuration

$$\hat{M}_{B_{y,z}} = \begin{bmatrix} -\omega^2+\mathbf{k}^2+2g^2A^2 & -2igAk_y & 2igAk_z & 0 & 0 & 0 \\ 2igAk_y & -\omega^2+\mathbf{k}^2+g^2A^2 & 0 & 0 & 0 & -2g^2A^2 \\ -2igAk_z & 0 & -\omega^2+\mathbf{k}^2+g^2A^2 & 0 & 2g^2A^2 & 0 \\ 0 & 0 & 0 & -\omega^2+\mathbf{k}^2+2g^2A^2 & -2igAk_y & 2igAk_z \\ 0 & 0 & 2g^2A^2 & 2igAk_y & -\omega^2+\mathbf{k}^2+g^2A^2 & 0 \\ 0 & -2g^2A^2 & 0 & -2igAk_z & 0 & -\omega^2+\mathbf{k}^2+g^2A^2 \end{bmatrix}$$

$$\begin{aligned} \det \hat{M}_{B_{y,z}} &= \Big(-\omega^6 + (3\mathbf{k}^2 + 4gB)\omega^4 - (3\mathbf{k}^4 + 8gB\mathbf{k}^2 - 4gBk_T^2 + g^2B^2)\omega^2 \\ &+ (\mathbf{k}^6 + 4gB\mathbf{k}^4 + g^2B^2\mathbf{k}^2 - 4gB\mathbf{k}^2k_T^2 - 4g^2B^2k_T^2 - 6g^3B^3)\Big)^2 \end{aligned}$$

Cubic equation

$$x^3 + a_2 x^2 + a_1 x + a_0 = 0,$$

where a_2, a_1, a_0 are real numbers.

Character of the equation's roots depends on discriminant's value.

$$\Delta = 18 a_0 a_1 a_2 - 4 a_2^3 a_0 + a_1^2 a_2^2 - 4 a_1^3 - 27 a_0^2$$

There are three possibilities:

- if $\Delta > 0$, the roots are real and distinct;
- if $\Delta = 0$, the roots are real and at least two coincide;
- if $\Delta < 0$, one root is real and remaining two are complex.

Cubic equation

For $\Delta > 0$ the solutions can be written in a Viète's trigonometric form:

$$x_n = 2\sqrt{-\frac{p}{3}} \cos\left[\frac{1}{3} \arccos\left(\frac{3q}{2p}\sqrt{\frac{-3}{p}}\right) - \frac{2\pi(n-1)}{3}\right] - \frac{a_2}{3},$$

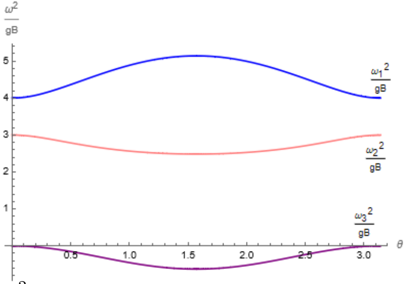
where $n = 1, 2, 3$ and $p \equiv \frac{3a_1 - a_2^2}{3}$, $q \equiv \frac{2a_2^3 - 9a_2 a_1 + 27a_0}{27}$.

When $\Delta < 0$ the solutions can be found according to Cardano formula.

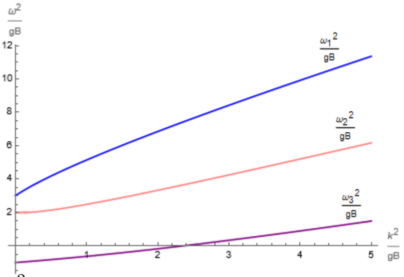
$$x_1 = -\tfrac{1}{2}(u+v) + \tfrac{i\sqrt{3}}{2}(u-v) - \tfrac{1}{3}a_2,$$
$$x_2 = -\tfrac{1}{2}(u+v) - \tfrac{i\sqrt{3}}{2}(u-v) - \tfrac{1}{3}a_2,$$
$$x_3 = u + v - \tfrac{1}{3}a_2,$$

where $u \equiv \sqrt[3]{-\tfrac{q}{2} + \sqrt{\tfrac{q^2}{4} + \tfrac{p^3}{27}}}, \qquad v \equiv \sqrt[3]{-\tfrac{q}{2} - \sqrt{\tfrac{q^2}{4} + \tfrac{p^3}{27}}}.$

Stability of nonAbelian chromomagnetic configuration

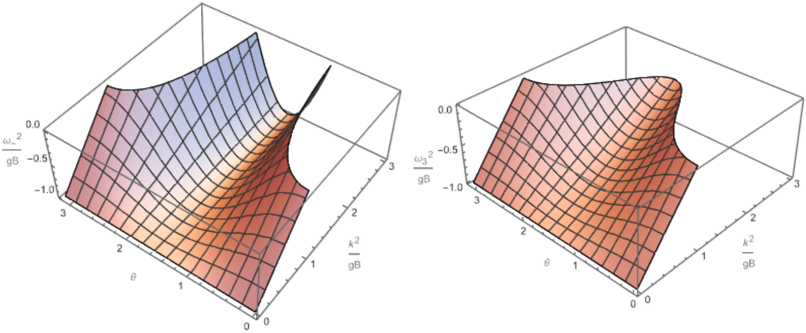


$\frac{\omega_n^2}{gB}$ as a function of θ for $\mathbf{k}^2 = gB$



$\frac{\omega_n^2}{gB}$ as a function of $\mathbf{k}^2$ for $\theta = \frac{\pi}{2}$

Instability: Abelian vs. nonAbelian chromomagnetic configuration



Domain of instability $\omega^2 < 0$ of Abelian (left) and nonAbelian (right) configurations

Stability of Abelian chromoelectric configuration

S. J. Chang and N. Weiss, Phys. rec. D **20**, 869 (1979), P. Sikivie, Phys. Rev. D **22**, 877 (1979)

Constant homogeneous chromoelectric field

$\bar{A}^{\mu}_{a}(t,\mathbf{r}) = (-xE,0,0,0)\delta^{a1}$
Potential $\bar{A}^{\mu}_{a}(t,\mathbf{r})$ satisfies YM equations with vanishing current

The color component a_1 satisfies Abelian equation and decouples from the remaining two components: $\Box a^{\nu}_{1} = 0$

$a^{\mu}_{a}(t,x,y,z) = e^{-i(\omega t - k_y y - k_z z)} a^{\mu}_{a}(x)$

Mixing

- colors 2 and 3 $\rightarrow$ $\mathsf{T}^{\pm}(x) = a^{0}_{2}(x) \pm i a^{0}_{3}(x)$, $X^{\pm}(x) = a^{x}_{2}(x) \pm i a^{x}_{3}(x)$, $Y^{\pm}(x) = a^{y}_{2}(x) \pm i a^{y}_{3}(x)$, $Z^{\pm}(x) = a^{z}_{2}(x) \pm i a^{z}_{3}(x)$
- coordinates t and x $\rightarrow$ $\mathsf{G}^{\pm}(x) = T^{+}(x) \pm X^{+}(x)$
  $H^{\pm}(x) = T^{-}(x) \pm X^{-}(x)$,

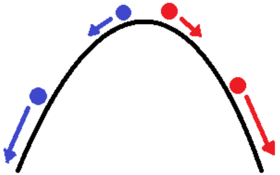
Stability of Abelian chromoelectric configuration

$$\Big(- g^2 x^2 E^2 + k_y^2 + k_z^2 - \frac{d^2}{dx^2} \Big) Y^{\pm}(x) = 0$$

Stability of Abelian chromoelectric configuration

$$\left(-g^2x^2E^2+k_y^2+k_z^2-\frac{d^2}{dx^2}\right)Y^{\pm}(x)=0$$

The equation coincides with the non-relativistic Schrödinger equation of inverted harmonic oscillator



Solutions

run-away solutions $\longrightarrow$ unstable

Stability of nonAbelian chromoelectric configuration

Constant homogeneous chromoelectric field

$$\bar{A}_a^\mu = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \sqrt{E/g} & 0 & 0 & 0 \\ 0 & \sqrt{E/g} & 0 & 0 \end{bmatrix}, \qquad J_a^\nu = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \sqrt{gE^3} & 0 & 0 & 0 \\ 0 & -\sqrt{gE^3} & 0 & 0 \end{bmatrix}.$$

Assumption

$a_a^\mu(t, \mathbf{x}) = a_a^\mu e^{-i(\omega t - \mathbf{kx})}$

Matrix equations

12x12 matrix in block form $\longrightarrow$ 2 equal matrices 3x3 and one 6x6

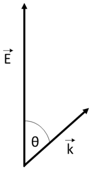
Homogeneous equations $\longrightarrow$ solutions exist if determinant of the matrix vanishes.

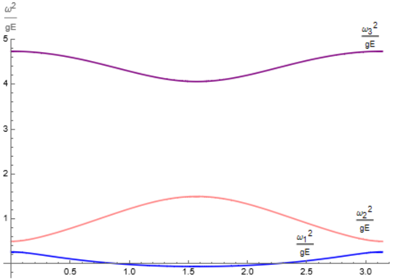
Stability of nonAbelian chromoelectric configuration

$$\hat{M}_{E_y} = \hat{M}_{E_z} = \begin{bmatrix} -\omega^2 + \mathbf{k}^2 & -2igAk_x & -2igA\omega \\ 2igAk_x & -\omega^2 + \mathbf{k}^2 + g^2A^2 & 0 \\ 2igA\omega & 0 & -\omega^2 + \mathbf{k}^2 - g^2A^2 \end{bmatrix}$$

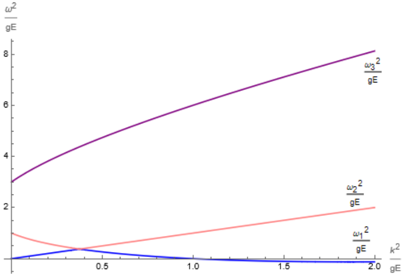
$$\begin{aligned} \det \hat{M}_{E_y} = \det \hat{M}_{E_z} \; &= \; -\omega^6 + (4gE + 3k^2)\omega^4 - (3g^2E^2 + 4gB(k^2 - k_x^2) + 3k^4)\omega^2 \\ &- \; 4gEk^2k_x^2 + 4g^2E^2k_x^2 - g^2B^2k^2 + k^6 = 0 \end{aligned}$$

Stability of nonAbelian chromoelectric configuration



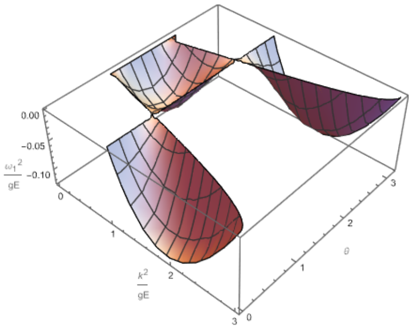


$\frac{\omega_n^2}{gE}$ as a function of θ for $\mathbf{k}^2 = \frac{gE}{2}$



$\frac{\omega_n^2}{gE}$ as a function of $\mathbf{k}^2$ for $\theta = 0$

Stability of nonAbelian chromoelectric configuration



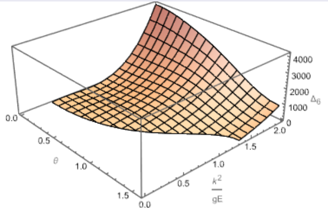
Domain of instability $\omega^2 < 0$

Stability of nonAbelian chromoelectric configuration

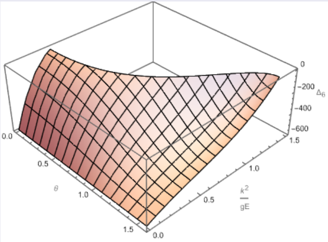
$$\hat{M}_{E_{t,x}} = \begin{bmatrix} -\omega^2+\mathbf{k}^2 & -2igAk_x & -2igA\omega & 0 & 0 & 0 \\ 2igAk_x & -\omega^2+\mathbf{k}^2+g^2A^2 & 0 & 0 & 0 & -2g^2A^2 \\ 2igA\omega & 0 & -\omega^2+\mathbf{k}^2-g^2A^2 & 0 & 2g^2A^2 & 0 \\ 0 & 0 & 0 & -\omega^2+\mathbf{k}^2 & -2igAk_x & -2igA\omega \\ 0 & 0 & -2g^2A^2 & 2igAk_x & -\omega^2+\mathbf{k}^2+g^2A^2 & 0 \\ 0 & 2g^2A^2 & 0 & 2igA\omega & 0 & -\omega^2+\mathbf{k}^2-g^2A^2 \end{bmatrix}$$

$$\det \hat{M}_{E_{t,x}} = \left(-\omega^6+(4gE+3k^2)\omega^4-(7g^2E^2+4gE(k^2-k_x^2)+3k^4)\omega^2+3g^2E^2+4g^2E^2k_x^2-4gEk^2k_x^2+k^6\right)^2 = 0$$

Discriminant

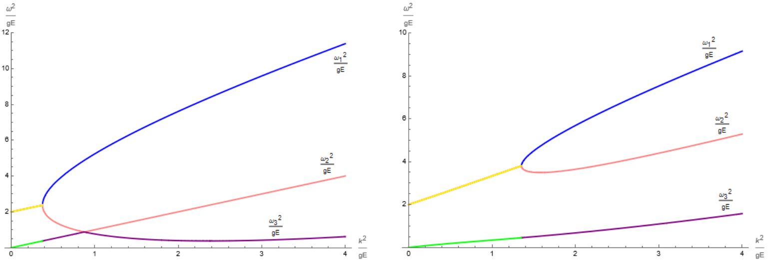


$\Delta > 0$



$\Delta < 0$

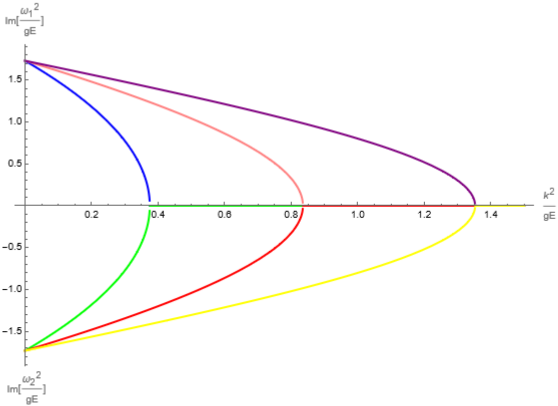
Stability of nonAbelian chromoelectric configuration



(a) (b)

$\frac{\omega_n^2}{gE}$ as a function of $\frac{k^2}{gE}$ for $\theta = 0$ (a) and $\theta = \frac{\pi}{2}$ (b)

Stability of nonAbelian chromoelectric configuration



Imaginary part of ω_1^2 and ω_2^2 as a function of k^2 for three values of θ

Chromoelectric and chromomagnetic fields

The configuration of parallel chromoelectric and chromomangetic fields can be generated only in Abelian way.

Constant homogeneous chromoelectric and chromomagnetic fields

$\bar{A}^{\mu}_{a}(t, \mathbf{r}) = (-xE, 0, 0, yB)\delta^{a1}$
Potential $\bar{A}^{\mu}_{a}(t, \mathbf{r})$ satisfies YM equations with vanishing current

$\Box a^{\nu}_{1} = 0$

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Potential $\bar{A}^{\mu}_{a}(t,\mathbf{r})$ satisfies YM equations with vanishing current

$\Box a^{\nu}_{1} = 0$

$a^{\mu}_{a}(t,x,y,z) = e^{-i(\omega t - k_z z)} a^{\mu}_{a}(x,y)$

Mixing

- colors 2 and 3 $\rightarrow$ $$\begin{aligned} T^{\pm}(x,y) &= a^{0}_{2}(x,y) \pm i a^{0}_{3}(x,y), \\ X^{\pm}(x,y) &= a^{x}_{2}(x,y) \pm i a^{x}_{3}(x,y), \\ Y^{\pm}(x,y) &= a^{y}_{2}(x,y) \pm i a^{y}_{3}(x,y), \\ Z^{\pm}(x,y) &= a^{z}_{2}(x,y) \pm i a^{z}_{3}(x,y) \end{aligned}$$
- coordinates t and x $\rightarrow$ $$\begin{aligned} G^{\pm}(x,y) &= T^{+}(x,y) \pm X^{+}(x,y) \\ H^{\pm}(x,y) &= T^{-}(x,y) \pm X^{-}(x,y) \end{aligned}$$
- coordinates y and z $\rightarrow$ $$\begin{aligned} U^{\pm}(x,y) &= Y^{+}(x,y) \pm i Z^{+}(x,y) \\ W^{\pm}(x,y) &= Y^{-}(x,y) \pm i Z^{-}(x,y) \end{aligned}$$

Chromoelectric and chromomagnetic fields

$$\Big(- \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - (\omega + gEx)^2 + (k_z - gBy)^2 \pm 2gB \Big) U^{\pm}(x,y) = 0$$

Chromoelectric and chromomagnetic fields

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The equation can be solved by the variable separation method

$$U^{\pm}(x,y) = U_E^{\pm}(x) U_B^{\pm}(y)$$

Chromoelectric and chromomagnetic fields

$$\Big(-\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - (\omega + gEx)^2 + (k_z - gBy)^2 \pm 2gB \Big) U^{\pm}(x,y) = 0$$

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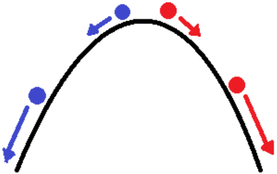
The chromoelectric field determines the system's behaviour



Solutions

run-away solutions $\longrightarrow$ unstable

Energy-Momentum Tensor

Energy-Momentum Tensor is gauge invariant

$$T^{\mu\nu} = F_a^{\mu\rho} F_\rho{}^\nu{}_a + \frac{1}{4} g^{\mu\nu} F_a^{\sigma\tau} F_{\sigma\tau\, a}$$

The diagonal elements

- energy density $\varepsilon = T^{00} = \frac{1}{2}(\mathbf{E}_a \cdot \mathbf{E}_a + \mathbf{B}_a \cdot \mathbf{B}_a)$
- longitudinal pressure $p_L = T^{xx} = -E_a^x E_a^x - B_a^x B_a^x + \varepsilon$
- transverse pressure $p_T = T^{yy} = -E_a^y E_a^y - B_a^y B_a^y + \varepsilon$

Gauge dependence

Gauge transformations

- $F^{\mu\nu} \longrightarrow U F^{\mu\nu} U^\dagger$
- $A^\mu = \bar{A}^\mu + a^\mu \longrightarrow U \bar{A}^\mu U^\dagger + U a^\mu U^\dagger + \frac{i}{g} U \partial^\mu U^\dagger$

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The strength tensor with neglected terms quadratic in a^μ still transforms as $F^{\mu\nu} \longrightarrow UF^{\mu\nu}U^\dagger$.

Energy density and pressure for stable and unstable modes

We considered modes from the Abelian configuration of chromomagnetic field.

Stable mode

$$\varepsilon = \frac{1}{2}B^2 + gB\delta^2(3 + 2gy^2B)e^{-gBy^2}$$

$$p_L = -\frac{1}{2}B^2 + gB\delta^2(3 - 2gy^2B)e^{-gBy^2}$$

$$p_T = \frac{1}{2}B^2 + 2g^2y^2B^2\delta^2e^{-gBy^2}$$

Unstable mode

$$\varepsilon = \frac{1}{2}B^2 + gB\delta^2e^{-gBy^2}e^{2\sqrt{gB}t}$$

$$p_L = -\frac{1}{2}B^2 + gB\delta^2e^{-gBy^2}e^{2\sqrt{gB}t}$$

$$p_T = \frac{1}{2}B^2$$

Summary & outlook

- We found complete spectra of eigenmodes for Abelian and nonAbelian constant and uniform chromoelectric and chromomagnetic fields.
- The spectra of Abelian and nonAbelian fields configurations are rather different, but there are everywhere unstable modes.
- The Energy-Momentum Tensor ($T^{\mu\nu}$) with fields linearized in a^{μ}_{a} is gauge invariant.

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Outlook

We plan to perform the stability analysis of the boost invariant background fields with the fluctuations breaking the Lorenz invariance.

Thank you for attention!