3+1D observables in the dilute Glasma of relativistic heavy ion collisions

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Based on

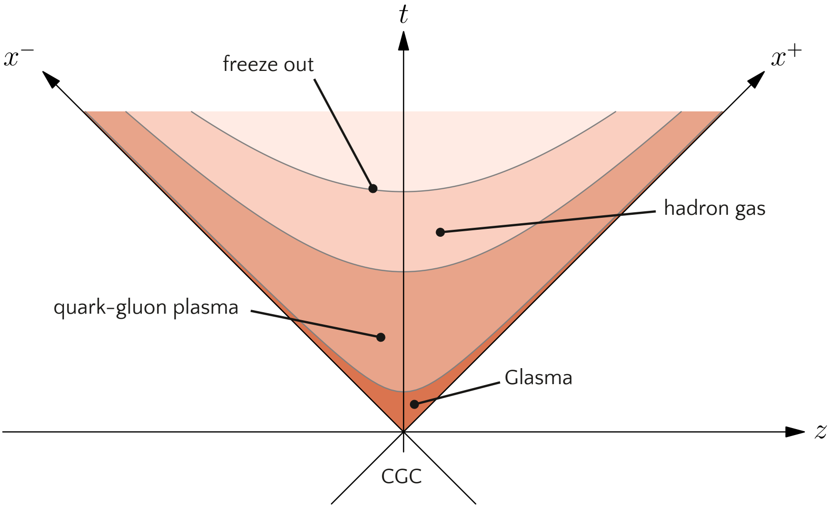
A. Ipp, D. I. Müller, S. Schlichting and P. Singh,
Space-time structure of (3 + 1)D color fields in high energy nuclear collisions,
Phys. Rev. D **104** (2021) no.11, 114040, arXiv:2109.05028.

TECHNISCHE UNIVERSITÄT WIEN

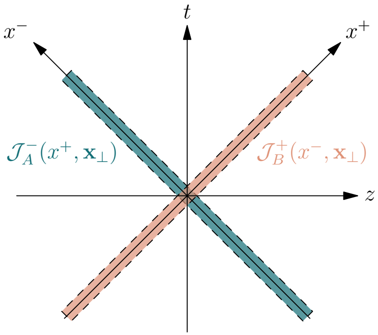
Doktoratskolleg Particles and Interactions

Der Wissenschaftsfonds.

Relativistic heavy ion collisions



Color glass condensate



Effective theory for high energy QCD

Separation of scales into

- Hard partons – color charges $\mathcal{J}^\mu$
- Soft partons – gauge field $\mathcal{A}^\mu$

Nuclei A and B have currents $\mathcal{J}_A^-$ and $\mathcal{J}_B^+$ which are stochastic variables distributed according to functionals $W_{A/B}[\mathcal{J}_{A/B}]$.

Gauge fields are obtained by solving the classical Yang–Mills equations, which reduce to

$$-\Delta_\perp \mathcal{A}^{\mp}_{A/B}\left(x^\pm, \mathbf{x}_\perp\right) = \mathcal{J}^{\mp}_{A/B}\left(x^\pm, \mathbf{x}_\perp\right)$$

in covariant gauge.

Solution strategies

No analytic solution to Yang-Mills equations in the forward lightcone! What now?

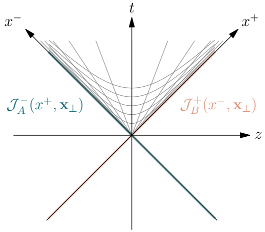
Assume recoilless nuclei and boost invariance

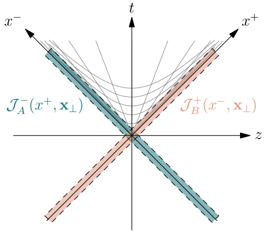
$$\mathcal{J}^{\mp}_{A/B}(x^{\pm}, \mathbf{x}_{\perp}) = \delta(x^{\pm})\,\overline{\mathcal{J}}^{\mp}_{A/B}(\mathbf{x}_{\perp})$$

- analytic initial conditions on forward lightcone [1]
- 2+1D classical lattice simulations, see e.g. [2, 3]
- small τ expansion, see e.g. [4]
- dilute approximation, see e.g. [5]

No recoil, but keep longitudinal strcuture of nuclei

- 3+1D classical lattice simulations [6, 7]
- dilute approximation [8]





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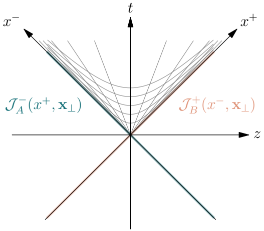
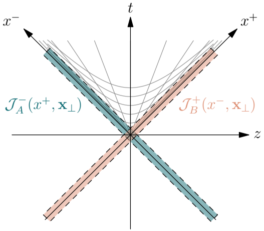
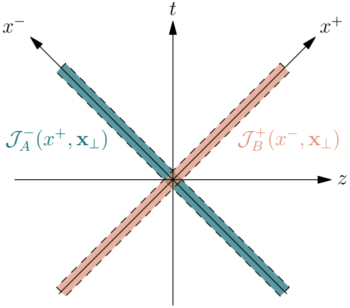


No recoil, but keep longitudinal strcuture of nuclei

- 3+1D classical lattice simulations [6, 7]
- dilute approximation [8] ← this is what we do



Dilute approxmation in 3+1D



Collisions are described by the YM equations

$$D_\mu F^{\mu\nu} = J^\nu,$$
$$D_\mu J^\mu = 0,$$

where

$$A^\mu(x) = \mathcal{A}_A^\mu(x) + \mathcal{A}_B^\mu(x) + a^\mu(x),$$
$$J^\mu(x) = \underbrace{\mathcal{J}_A^\mu(x) + \mathcal{J}_B^\mu(x)}_{\text{background}} + \underbrace{j^\mu(x)}_{\text{perturbation}}.$$

- We expand the YM equations in powers of $\mathcal{J}_A$ and $\mathcal{J}_B$
- Background fields are solutions to all orders $\mathcal{O}(\mathcal{J}_A{}^n)$ and $\mathcal{O}(\mathcal{J}_B{}^n)$
- We solve for j^μ and a^μ at $\mathcal{O}(\mathcal{J}_A \mathcal{J}_B)$ (see arXiv:2109.05028 for details)

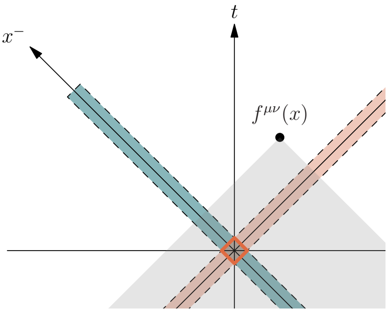
Field strength tensor

Perturbative field strength tensor [9]

$$f^{+-} = -\frac{g}{2\pi}\int\limits_{\mathbf{u}_\perp}\int\limits_{-\infty}^{\infty} \mathrm{d}\eta_z\, V$$

$$f^{\pm i} = \frac{g}{2\pi}\int\limits_{\mathbf{u}_\perp}\int\limits_{-\infty}^{\infty} \mathrm{d}\eta_z\, \left(V^{ij} \mp \delta^{ij} V\right) w^j \frac{\mathrm{e}^{\pm\eta_z}}{\sqrt{2}}$$

$$f^{ij} = -\frac{g}{2\pi}\int\limits_{\mathbf{u}_\perp}\int\limits_{-\infty}^{\infty} \mathrm{d}\eta_z\, V^{ij}$$



with

$$V := f_{abc}t^c \partial^i \mathcal{A}_A^{-a}\left(x^+ - \frac{|\mathbf{x}_\perp - \mathbf{u}_\perp|}{\sqrt{2}}\mathrm{e}^{+\eta_z}, \mathbf{u}_\perp\right)\partial^i \mathcal{A}_B^{+b}\left(x^- - \frac{|\mathbf{x}_\perp - \mathbf{u}_\perp|}{\sqrt{2}}\mathrm{e}^{-\eta_z}, \mathbf{u}_\perp\right)$$

$$V^{ij} := f_{abc}t^c \left(\partial^i \mathcal{A}_A^{-a}(\dots)\partial^j \mathcal{A}_B^{+b}(\dots) - \partial^j \mathcal{A}_A^{-a}(\dots)\partial^i \mathcal{A}_B^{+b}(\dots)\right)$$

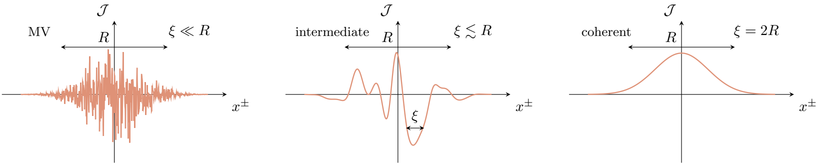
Perturbative energy-momentum tensor $t^{\mu\nu} = 2\,\mathrm{Tr}\left[f^{\mu\rho} f_\rho{}^\nu + \frac{1}{4} g^{\mu\nu} f^{\rho\sigma} f_{\rho\sigma}\right]$

Nuclear model

Inspired by the McLerran-Venugopalan (MV) model we use a probability functional $W_{A/B}[\mathcal{J}^{\mp}_{A/B}]$ with

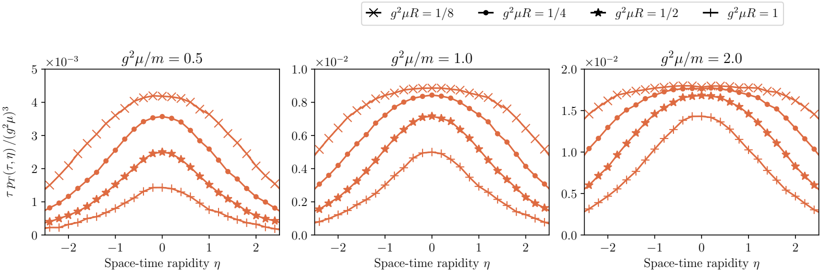
$$\langle \mathcal{J}^{\mp a}_{A/B}(x^{\pm}, \mathbf{x}_{\perp}) \rangle = 0$$

$$\langle \mathcal{J}^{\mp a}_{A/B}(x^{\pm}, \mathbf{x}_{\perp}) \mathcal{J}^{\mp b}_{A/B}(x'^{\pm}, \mathbf{x}'_{\perp}) \rangle = \underbrace{g^2 \mu^2_{A/B}}_{\substack{\text{strength of} \\ \text{color charges} \\ Q_s \propto g^2 \mu}} \delta^{ab} \underbrace{T_R\left(\frac{x^{\pm} + x'^{\pm}}{2}\right)}_{\substack{\text{longitudinal profile} \\ \text{Gaussian of} \\ \text{width } R}} \underbrace{U_{\xi}(x^{\pm} - x'^{\pm})}_{\substack{\text{long. correlations} \\ \text{Gaussian of} \\ \text{width } \xi}} \underbrace{\delta^{(2)}(\mathbf{x}_{\perp} - \mathbf{x}'_{\perp})}_{\text{transverse correlations}}$$



We use an infrared regulator m and ultraviolet regulator Λ when solving the Poisson equation.

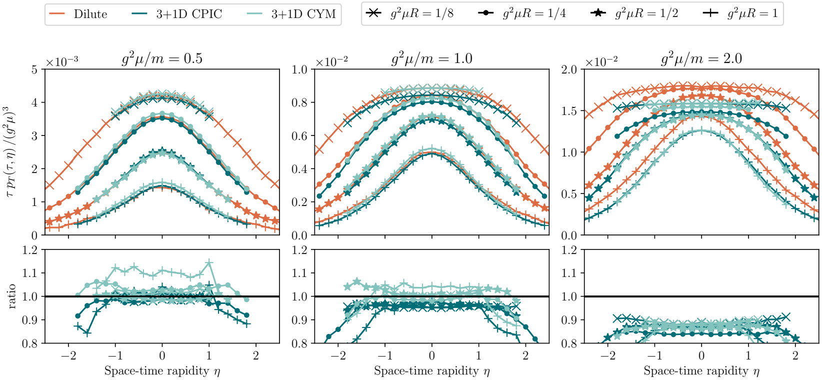
Numerical results



stronger fields

We obtain the non-trivial rapidity profile of the energy-momentum tensor

Numerical results

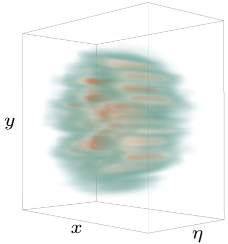


Excellent agreement between dilute approximation [8] and 3+1D simulations [6, 7] for $g^2\mu/m = 0.5$

Results differ by a scale factor for $g^2\mu/m = 2.0$

Conclusions and outlook

We have computed the full 3+1D field strength tensor of the Glasma in the dilute limit

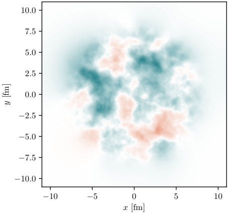


What we can do right now:

- Monte Carlo integration on GPUs
- Large speedup compared to lattice simulations
- Recover boost-invariant limit

What we are currently working on:

- More sophisticated nuclear models (incorporating hot spots or PDFs)
- Angular momentum of the Glasma
- Coupling to kinetic theory and hydrodynamics



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[9] A. Ipp, M. Leuthner, D. I. Müller, S. Schlichting and P. Singh, *in preparation*