NLO quark self-energy and dispersion relation using the hard thermal loop resummation

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Abstract

Using the hard-thermal-loop (HTL) resummation in real-time formalism, we study the next-to-leading order (NLO) quark self-energy and corresponding NLO dispersion laws. In NLO, all the propagators and vertices are replaced with the HTL effective ones in the usual quark self-energy diagram. Additionally, a four-point vertex diagram also contributes to the quark NLO self-energy. The usual quark self-energy diagram and the four-point vertex diagram are calculated separately, and the NLO quark self-energy is expressed in terms of the three- and four-point HTL effective vertex functions. The integrals containing the three- and four-point HTL effective vertex functions are expressed in terms of the solid-angles using the Feynman technique. After completing the solid-angle integrals, the momentum integrals in the transverse part of the NLO quark self-energy has been calculated numerically and plotted as a function of the ratio of momentum and energy. Using the transverse part of the NLO quark self-energy, NLO dispersion laws have been plotted.

Motivation

In gauge theories at high temperature T: physical quantities like the dispersion laws give gauge dependent results.

In order to resolve that issue, effective perturbative expansion came in the picture which sums the so-called hard thermal loops (HTL) into effective propagators and effective vertices.

Because of the HTL theory, at lowest order gT, in effective perturbation, the infrared region is 'safe' since HTL summation dresses the massless quarks and gluons by giving them thermal masses.

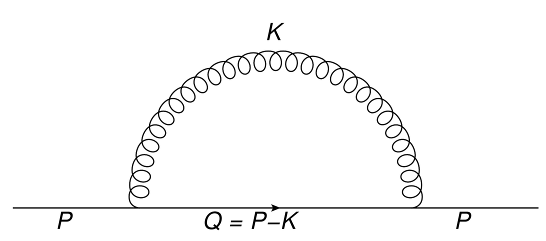
But, static chromomagnetic fields do not screen at this lowest order, they are believed to do so at the next-to-leading order g^2T, the so-called magnetic scale.

Leading order dispersion relations

One-loop effective quark propagator is given by

$$S(P)=\frac{1}{\not P-\Sigma(P)}$$

Feynman diagram for one-loop quark self-energy



Lowest order quark self-energy for both modes is

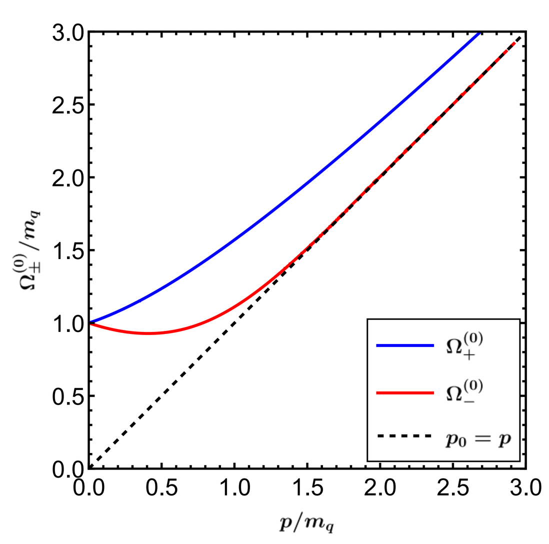
$$\Sigma_\pm(\omega,p)=\frac{m_q^2}{p}\left[\pm1+\frac{1}{2}\left(1\mp\frac{\omega}{p}\right)\ln\frac{\omega+p}{\omega-p}\right]$$

where $m_q=\sqrt{C_F/8}gT$ is thermal mass of a quark with zero chemical potential.

The lowest order dispersion relations are obtained from the equation

$$p_0\mp p-\Sigma_\pm(P)=0$$

The below figure shows the solution of the lowest order dispersion relation



NLO formalism

NLO dispersion relations take the final form as

$$\Omega_\pm^{(1)}(p)=\frac{{\Omega_\pm^{(0)}}^2(p)-p^2}{2m_q^2}\Sigma_\pm^{(1)}\left(\Omega_\pm^{(0)}(p),p\right)$$

Two Feynman diagram contributes at NLO quark self-energy

First Feynman diagram contribution after doing summation over Keldysh indices

$$\Sigma_{1\pm}^{(1)}(P)=\frac{-ig^2C_F}{2}\int\frac{d^4K}{(2\pi)^4}\mathrm{tr}\,\gamma_{\pm p}\left[\left\{\gamma^\mu+I^\mu_{--}(P,Q)\right\}\Delta^{\mathrm{R}}(Q)\gamma^\nu+I^\nu_{--}(Q,P)D^{\mathrm{S}}_{\mu\nu}(K)\right.$$
$$\left.+\left\{\gamma^\mu+I^\mu_{--}(P,Q)\right\}\Delta^{\mathrm{S}}(Q)\left\{\gamma^\nu+I^\nu_{+-}(Q,P)\right\}D^{\mathrm{A}}_{\mu\nu}(K)\right.$$

Four-point vertex diagram contribution comes out to be

$$\Sigma_{2\pm}^{(1)}(P)=\frac{-ig^2C_F}{4}\int\frac{d^4K}{(2\pi)^4}\mathrm{tr}\,\gamma_{\pm p}I^{\mu\nu}_{--}(P,K)D^{\mathrm{S}}_{\mu\nu}(K)$$

The three and four point vertex integrals in NLO quark self-energy diagram are

$$I^\mu_{\eta_1\eta_2}(P,Q)=m_q^2\int\frac{d\Omega_s}{4\pi}\frac{S^\mu\not S}{(PS+i\eta_1\varepsilon)(QS+i\eta_2\varepsilon)};$$
$$I^{\mu\nu}_{\eta_1\eta_2}(P,K)=m_q^2\int\frac{d\Omega_s}{4\pi}\frac{-S^\mu S^\nu\not S}{(PS+i\eta_1\varepsilon)(PS+i\eta_2\varepsilon)}\times\left[\frac{1}{(P+K)S+i\eta_1\varepsilon}+\frac{1}{(P-K)S+i\eta_2\varepsilon}\right]$$

Final result of the NLO quark self-energy in compact form, after doing summation over Lorentz and Dirac indices is

$$\Sigma_\pm^{(1)}(P)=-\frac{ig^2C_F}{2}\int\frac{d^4K}{(2\pi)^4}\left[F^{\mathrm{SR}}_{\pm;0}(P,K)+F^{\mathrm{AS}}_{\pm;0}(P,K)+2F^{\mathrm{SR}}_{\pm;--}(P,K)+F^{\mathrm{AS}}_{\pm;--}(P,K)\right.$$
$$\left.+F^{\mathrm{AS}}_{\pm;-+}(P,K)+F^{\mathrm{SR}}_{\pm;--;--}(P,K)+F^{\mathrm{AS}}_{\pm;--;+-}(P,K)+G^{\mathrm{S}}_{\pm;--}(P,K)\right]$$

Resummed retarded propagator

Effective retarded transverse gluon propagator can be written as

$$D_T^{R(-1)}(k,k_0,\varepsilon)=-\left[\frac{4k_0^2}{k^2}+\left(k^2-k_0^2\right)\left\{1-\frac{k_0}{k^3}\ln\frac{(k_0-k)^2+\varepsilon^2}{(k_0+k)^2+\varepsilon^2}\right\}\right]$$
$$-i\left[\frac{2k_0}{k^3}\left(k_0^2-k^2\right)\left\{\tan^{-1}\left(\frac{\varepsilon}{k_0-k}\right)-\tan^{-1}\left(\frac{\varepsilon}{k_0+k}\right)\right\}-\varepsilon\Theta(k_0)\right]$$

Effective retraded quark propagator comes out to be

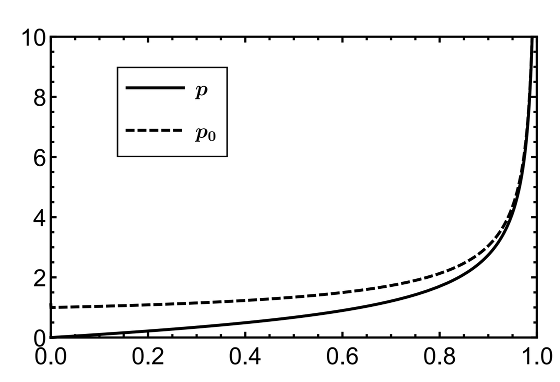
$$\Delta_-^{R(-1)}(q,q_0,\varepsilon)=-\left[\frac{1}{q}+q-q_0-\frac{q_0-q}{4q^2}\ln\frac{(q_0+q)^2+\varepsilon^2}{(q_0-q)^2+\varepsilon^2}+\frac{\varepsilon}{2q^2}\right.$$
$$\left\{\tan^{-1}\left(\frac{\varepsilon}{q_0+q}\right)-\tan^{-1}\left(\frac{\varepsilon}{q_0-q}\right)\right\}-i\left[\varepsilon+\frac{\varepsilon}{4q^2}\ln\frac{(q_0+q)^2+\varepsilon^2}{(q_0-q)^2+\varepsilon^2}\right.$$
$$\left.\left.+\frac{q_0-q}{2q^2}\left\{\tan^{-1}\left(\frac{\varepsilon}{q_0+q}\right)-\tan^{-1}\left(\frac{\varepsilon}{q_0-q}\right)\right\}\right]\right]$$

Evaluation of NLO quark self energy

The scaled quark momentum can be written in terms of the variable $t=\frac{p}{p_0}$ as

$$p(t)/m_q=\sqrt{\frac{t}{1-t}-\frac{1}{2}\ln\left(\frac{1+t}{1-t}\right)}$$

The below figure shows the variation of quark energy and momentum w.r.t. variable t.



The third term of NLO quark self-energy is

$$\Sigma3_\pm^{(1)}(P)=\frac{-ig^2C_F}{2}\int\frac{d^4K}{(2\pi)^4}\left[2F^{\mathrm{SR}}_{\pm;--}(P,K)\right]$$

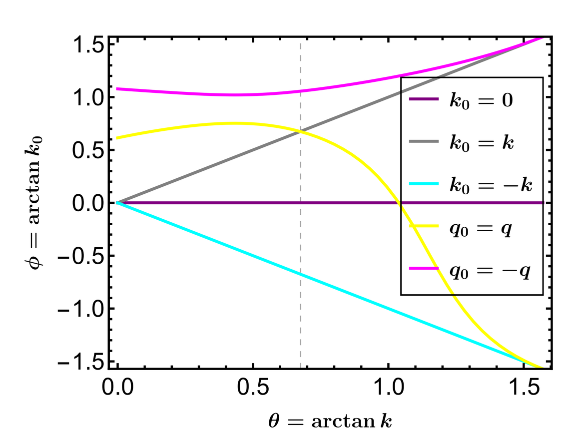
The transverse part of $F^{\mathrm{SR}}_{\pm;--}(P,K)$ is

$$F^{\mathrm{SR}}_{\pm;--}(P,K)=m_q^2\int\frac{d\Omega_s}{4\pi}\frac{1}{(PS-i\varepsilon)(QS-i\varepsilon)}\left[\left(1-\hat p_\epsilon.\hat q_\epsilon-\hat p_\epsilon.\hat s-\hat q_\epsilon.\hat s+\hat p_\epsilon.\hat k\,\hat k.\hat s+\hat q_\epsilon.\hat k\,\hat k.\hat s\right.\right.$$
$$\left.\left.-(\hat k.\hat s)^2+\hat p_\epsilon.\hat q_\epsilon\,(\hat k.\hat s)^2+2\hat p_\epsilon.\hat s\,\hat q_\epsilon.\hat s-\hat p_\epsilon.\hat k\,\hat q_\epsilon.\hat s\,\hat k.\hat s-\hat q_\epsilon.\hat k\,\hat p_\epsilon.\hat s\,\hat k.\hat s\right)D_T^{\mathrm{S}}(K)\Delta^{\mathrm{R}}_\mp(Q)\right]$$

Thus, all the lines of discontinuity are

$$k_0=0;\quad k_0=\pm k;$$
$$k_0=p_0\pm\sqrt{p^2+k^2-2pkx};$$
$$k=k_t\equiv\frac{1}{2}\frac{p_0^2-p^2}{p_0-xp}=\frac{1}{2t}\frac{1-t^2}{1-xt}\sqrt{\frac{t}{1-t}-\frac{1}{2}\ln\left(\frac{1+t}{1-t}\right)}$$

All the above lines of discontinuity are plotted in below figure and momentum integration has been done in each of the domain of the figure.



The eighth term of NLO quark self-energy is

$$\Sigma8_\pm^{(1)}(P)=\frac{-ig^2C_F}{2}\int\frac{d^4K}{(2\pi)^4}\left[G^{\mathrm{S}}_{\pm;--}(P,K)\right]$$

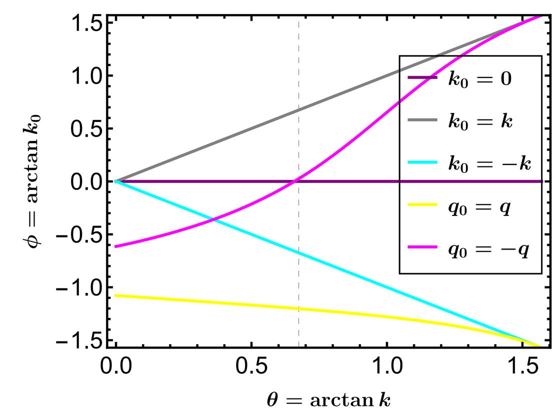
The transverse part of $G^{\mathrm{S}}_{\pm;--}(P,K)$ is

$$G^{\mathrm{S}}_{\pm;--}(P,K)=\int\frac{d\Omega_s}{4\pi}\frac{1}{[PS-i\varepsilon][PS-i\varepsilon]}\left[\frac{1}{(P+K)S-i\varepsilon}+\frac{1}{(P-K)S-i\varepsilon}\right]$$
$$\times\left\{1-\hat p_\epsilon\cdot\hat s-\left(\hat k\cdot\hat s\right)^2+\hat p_\epsilon\cdot\hat s\left(\hat k\cdot\hat s\right)^2\right\}D_T^{\mathrm{S}}(K)$$

The sudden jumps in the above integrand are at

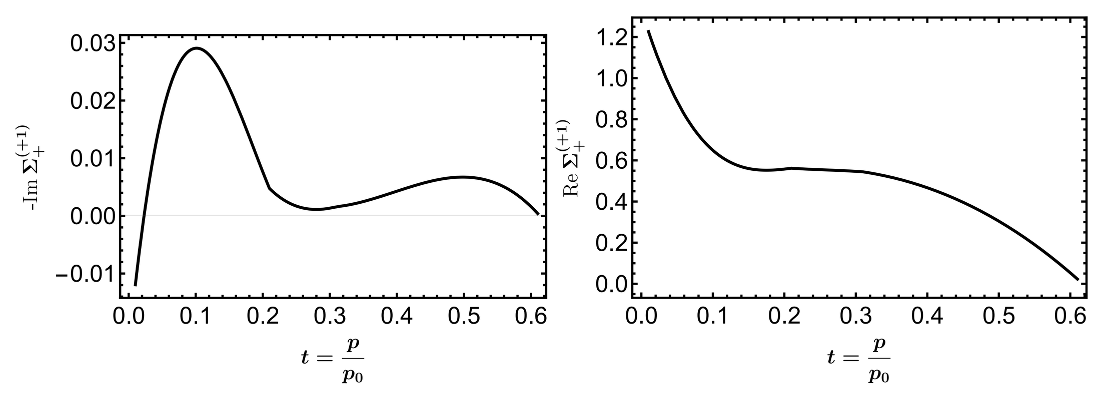
$$k_0=0;\quad k_0=\pm k;$$
$$k_0=-p_0\pm\sqrt{p^2+k^2+2pkx};$$
$$k=k_t\equiv\frac{1}{2}\frac{p_0^2-p^2}{p_0-xp}=\frac{1}{2t}\frac{1-t^2}{1-xt}\sqrt{\frac{t}{1-t}-\frac{1}{2}\ln\left(\frac{1+t}{1-t}\right)}$$

These lines of sudden jumps are plotted in below figure and momentum integrations has been performed in each of the domain of the figure.

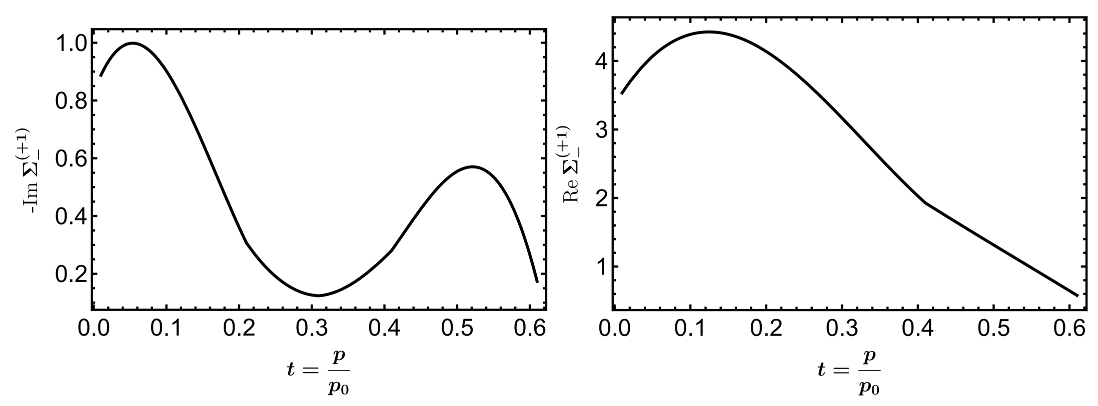


Results

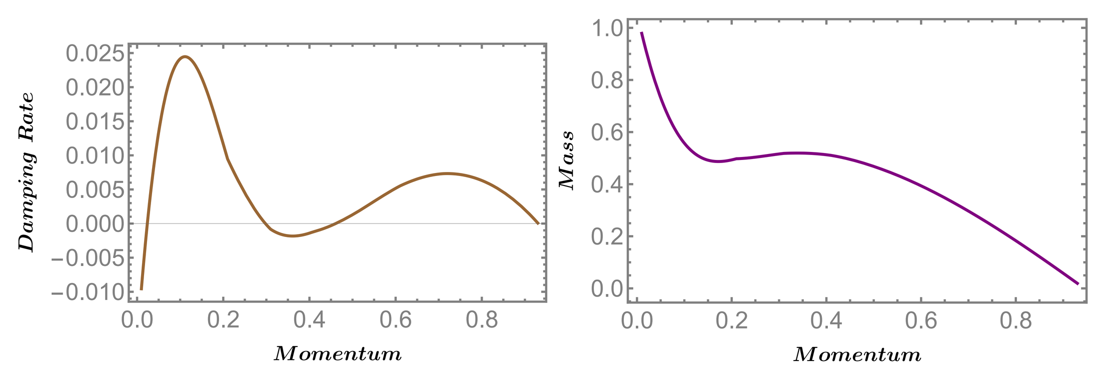
Variation of imaginary part and real part of $\Sigma_+^{(1)}$ with a coefficient $(4\pi g)$ w.r.t variable $t=\frac{p}{p_0}$



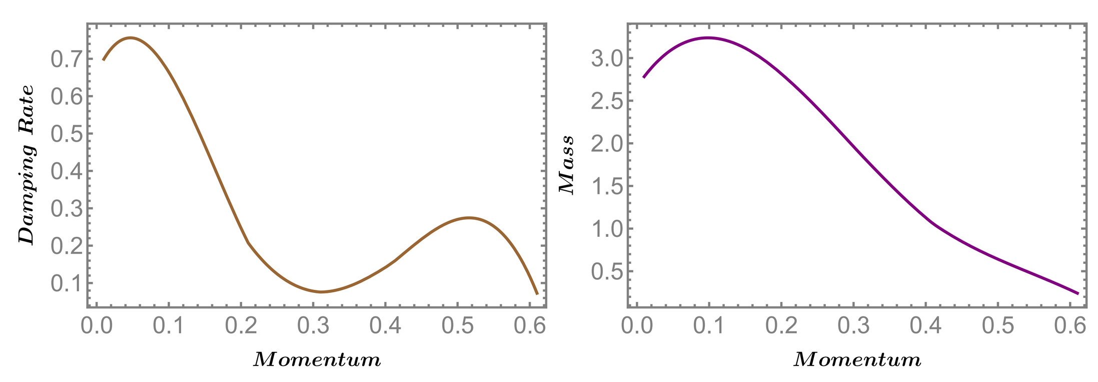
Variation of imaginary part and real part of $\Sigma_-^{(1)}$ with a coefficient $(4\pi g)$ w.r.t variable $t=\frac{p}{p_0}$



Behavior of NLO damping rate and NLO mass with a coefficient $(4\pi g)$ for real quark mode w.r.t their momentum



Behavior of NLO damping rate and NLO mass with a coefficient $(4\pi g)$ for plasmino mode w.r.t their momentum



Conclusion

Next-to-leading order transverse quark self energy has been evaluated numerically.

Using the NLO quark self energy, NLO dispersion relations have been plotted for both quark mode and plasmino mode.

It has been found that at NLO , infrared region is safe atleast for transverse part of the NLO quark self energy.

References

[1] Sumit, N.Haque, B.K.Patra, arxiv:2201.07173[hep-ph].