The complex heavy-quark potential in an anisotropic QGP – Statics and dynamics

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Speaker and Conference Info

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Abstract

In Ref.[1], we introduced a method for reducing anisotropic heavy-quark potentials to isotropic ones by introducing an effective screening mass that depends on the quantum numbers l and m of a given state.

Introduction

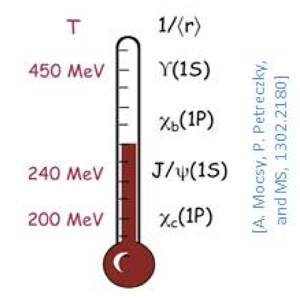
We study the dynamics of heavy quarkonium states ($q\bar{q}$) inside the Quark Gluon Plasma (QGP).

Bottomonium: $b\bar{b}$

$n^{2s+1}L_J$	J^{PC}	Particle	Mass (MeV)
1^3S_1	1^{--}	$\Upsilon(1S)$	9460.30
1^3P_0	0^{++}	$\chi_{b0}(1P)$	9859.44
1^3P_1	1^{++}	$\chi_{b1}(1P)$	9892.78
1^3P_2	2^{++}	$\chi_{b2}(1P)$	9912.21
2^3S_1	1^{--}	$\Upsilon(2S)$	10023.26
2^3P_0	0^{++}	$\chi_{b0}(2P)$	10232.50
2^3P_1	1^{++}	$\chi_{b1}(2P)$	10255.46
2^3P_2	2^{++}	$\chi_{b2}(2P)$	10268.65
3^3S_1	1^{--}	$\Upsilon(3S)$	10355.20

Charmonium: $c\bar{c}$

$n^{2s+1}L_J$	J^{PC}	Particle	Mass (MeV)
1^3S_1	1^{--}	J/ψ	3096.90
1^3P_0	0^{++}	$\chi_{c0}(1P)$	3414.71
1^3P_1	1^{++}	$\chi_{c1}(1P)$	3510.67
1^3P_2	2^{++}	$\chi_{c2}(1P)$	3556.17



Suppression factor,

$$R_{AA} = \frac{\text{number produced in}AA}{\langle N_{\text{bin}}\rangle_b\,(\text{number produced in } pp)}\,,\quad \langle N_{\text{bin}}\rangle_b = \text{number of nucleonic collisions}$$
$$= 1\,,\ \text{No effect}$$
$$> 1\,,\ \text{Enhancement}$$
$$< 1\,,\ \text{Suppression}$$

Full complex potential ⇒ Solve SWE ⇒ Obtain observables such as R_{AA} and elliptic flow (v_2) ⇒ Compare with experimental data available from the ATLAS, ALICE, and CMS collaborations.

Potential Models

Isotropic Potential Model:

$$\operatorname{Re} V_{\text{Iso}}(r) = \alpha m_D\left(\frac{1-e^{-rm_D}}{rm_D}\right) - \alpha m_D - \frac{\sigma}{m_D}(2+rm_D)\,e^{-rm_D} + \frac{2\sigma}{m_D} - \frac{\alpha}{r} - \frac{0.8\sigma}{m_Q^2 r} \tag{1}$$

$$\operatorname{Im} V_{\text{Iso}}(r) = \alpha\lambda\phi_2(rm_D) - \alpha\lambda - \frac{8\sigma\lambda}{m_D^2}\phi_3(rm_D) + \frac{24\sigma\lambda}{m_D^2}\phi_4(rm_D) - \frac{4\sigma\lambda}{m_D^2} \tag{2}$$

3D Anisotropic Potential Model:

$$\operatorname{Re} V_{\text{Aniso}}(r,\xi) = \alpha m_D^A\left(\frac{1-e^{-rm_D^R}}{rm_D^R}\right) - \alpha m_D^A - \frac{\sigma}{m_D^A}\left(2+rm_D^R\right)e^{-rm_D^R} + \frac{2\sigma}{m_D^A} - \frac{\alpha}{r} - \frac{0.8\sigma}{m_Q^2 r} \tag{3}$$

$$\operatorname{Im} V_{\text{Aniso}}(r,\xi) = \alpha\lambda^A\phi_2\left(rm_D^I\right) - \alpha\lambda^A - \frac{8\sigma\lambda^A}{(m_D^A)^2}\phi_3\left(rm_D^I\right) + \frac{24\sigma\lambda^A}{(m_D^A)^2}\phi_4\left(rm_D^I\right) - \frac{4\sigma\lambda^A}{(m_D^A)^2} \tag{4}$$

where,

$$m_D^A = m_D\left(1-\frac{\xi}{6}\right)\,,\quad \lambda^A = \lambda\left(1-\frac{\xi}{6}\right) \tag{5}$$

$$m_D^R = m_D\Big[1+\xi\,(0.108\cos 2\theta - 0.131)\Big]\,,\quad m_D^I = m_D\Big[1+\xi\,(0.026\cos 2\theta - 0.158)\Big] \tag{6}$$

1D Effective Potential Model:

$$\operatorname{Im} V_{\text{Eff}}(r,\xi) = \alpha m_D^A\left(\frac{1-e^{-r\mathcal{M}_{lm}^R}}{r\mathcal{M}_{lm}^R}\right) - \alpha m_D^A - \frac{\sigma}{m_D^A}\left(2+r\mathcal{M}_{lm}^R\right)e^{-r\mathcal{M}_{lm}^R} + \frac{2\sigma}{m_D^A} - \frac{\alpha}{r} - \frac{0.8\sigma}{m_Q^2 r} \tag{7}$$

$$\operatorname{Im} V_{\text{Eff}}(r,\xi) = \alpha\lambda^A\phi_2\left(r\mathcal{M}_{lm}^I\right) - \alpha\lambda^A - \frac{8\sigma\lambda^A}{(m_D^A)^2}\phi_3\left(r\mathcal{M}_{lm}^I\right) + \frac{24\sigma\lambda^A}{(m_D^A)^2}\phi_4\left(r\mathcal{M}_{lm}^I\right) - \frac{4\sigma\lambda^A}{(m_D^A)^2} \tag{8}$$

where,

$$K_{lm} = \frac{2l(l+1)-2m^2-1}{4l(l+1)-3} \tag{9}$$

$$\mathcal{M}_{lm}^R = m_D\Big[1+\xi\,(0.216K_{lm}-0.239)\Big]\,,\quad \mathcal{M}_{lm}^I = m_D\Big[1+\xi\,(0.052K_{lm}-0.184)\Big] \tag{10}$$

Acknowledgements and references

[1] Lihua Dong, Yun Guo, Ajaharul Islam, Alexander Rothkopf, and Michael Strickland. *"The complex heavy-quark potential in an anisotropic quark-gluon plasma - Statics and dynamics"*. arXiv:2205.10349v1[hep-ph] 20 May 2022.

[2] Lihua Dong, Yun Guo, Ajaharul Islam, and Michael Strickland. *"Effective Debye screening mass in an anisotropic quark gluon plasma"*, Phys. Rev. D, 104(9):096017, 2021.

Numerical Solution

To assess the efficacy of using the effective 1D potentials we compared to static and dynamical solutions of the 3D Schrödinger equation. For the static solutions, we used a previously developed 3D eigensolver called quantumFDTD. Using this code, we compared results obtained with the 1D effective potential and the full 3D anisotropic potential. For the real time solutions, we wrote a CUDA-based code to solve the 3D Schrödinger equation in real time. For this purpose, we used a split-step pseudospectral method and once again compared results obtained with the full 3D anisotropic potential to those obtained with the 1D effective potential [1,2].

Static Results

$\Upsilon(1S)$	ReE	δReE	E_{bind}	ImE	δImE
T_o	182.869	0.611	−662.669	11.838	0.027
$1.1T_o$	174.957	0.593	−570.612	14.830	0.031
$1.2T_o$	166.556	0.573	−493.689	18.190	0.034
$1.4T_o$	148.439	0.531	−372.540	26.004	0.039

$\chi_{b0}(1P)$	ReE	δReE	E_{bind}	ImE	δImE
T_o	492.974	1.444	−352.564	35.872	0.132
$1.1T_o$	475.762	1.345	−269.808	44.749	0.131
$1.2T_o$	457.998	1.246	−202.248	54.566	0.123
$1.3T_o$	439.822	1.149	−146.364	65.232	0.107
$1.4T_o$	421.347	1.057	−99.632	76.643	0.085

J/Ψ	ReE	δReE	E_{bind}	ImE	δImE
T_o	439.336	1.230	−406.202	41.980	0.107
$1.1T_o$	422.207	1.163	−323.362	51.467	0.105
$1.2T_o$	404.597	1.095	−255.648	61.698	0.098
$1.3T_o$	386.604	1.028	−199.583	72.564	0.086
$1.4T_o$	368.301	0.963	−152.678	83.958	0.070

$\chi_{b\pm1}(1P)$	ReE	δReE	E_{bind}	ImE	δImE
T_o	461.960	0.997	−383.578	34.996	0.097
$1.1T_o$	446.761	0.935	−298.809	43.448	0.099
$1.2T_o$	431.014	0.872	−229.231	52.752	0.097
$1.3T_o$	414.833	0.810	−171.353	62.831	0.090
$1.4T_o$	398.307	0.750	−122.672	73.599	0.079

Fig. 1: The exact results of the complex eigenenergies (E) and binding energies (E_{bind}) for different quarkonium states at various temperatures with $\xi = 1$. Comparing with the results obtained based on the 1D effective potential model, the corresponding differences as denoted by δE are also listed. The reference temperature T_o is 192 MeV and all the results are given in the units of MeV.

Dynamical Results

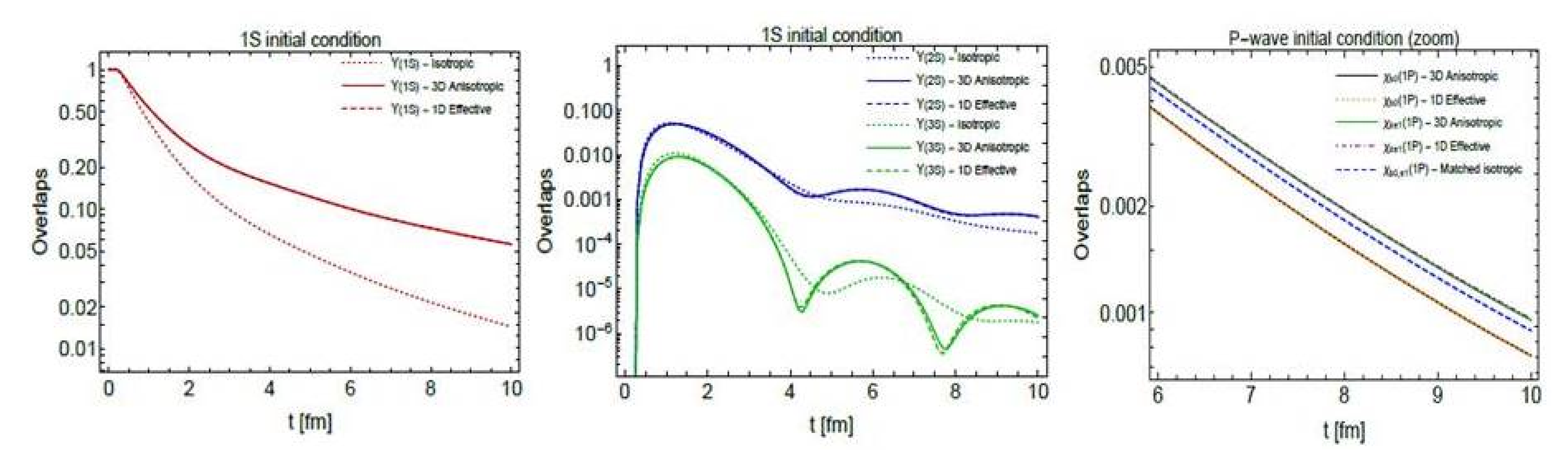


Fig. 2: Left: $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$ overlaps resulting from real-time solution of the SWE. we initialized the wave function as pure $\Upsilon(1S)$. Right: Time evolution of the bottomonium p-wave overlaps resulting from initialization with different p-wave polarizations corresponding to $l = 1$ and $m = 0, \pm1$ labeled as $\chi_{b0}(1P)$ and $\chi_{b\pm1}(1P)$, respectively.

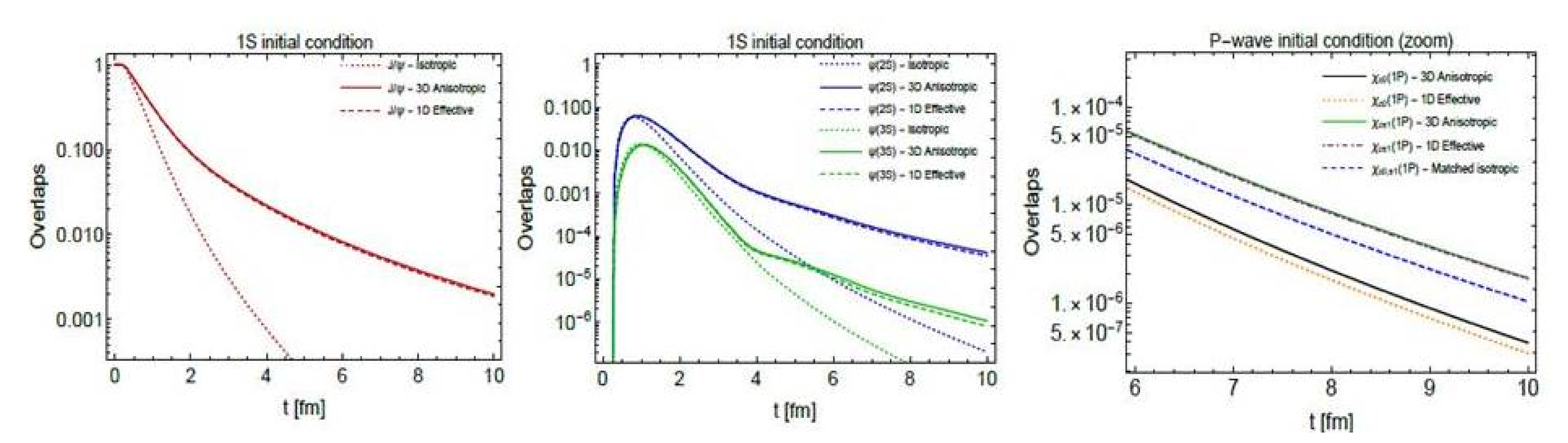


Fig. 3: Left: Time evolution of the J/ψ, $\psi(2S)$, and $\psi(3S)$ overlaps. we initialized the wave function as pure J/ψ. Right: Time evolution of the charmonium p-wave overlaps resulting from initialization with different p-wave polarizations corresponding to $l = 1$ and $m = 0, \pm1$ labeled as $\chi_{c0}(1P)$ and $\chi_{c\pm1}(1P)$, respectively.

Conclusions

- The 1D effective isotropic potential is much easier to solve as compared to the 3D anisotropic potential.
- We demonstrate that, using the resulting 1D effective potential model, one can solve a 1D Schrödinger equation and reproduce the full 3D results for the energies and binding energies of low-lying heavy-quarkonium bound states to relatively high accuracy.
- The resulting 1D effective isotropic potential model includes the splitting of different p-wave polarizations.
- The resulting 1D effective isotropic potential model could provide an efficient method for including momentum anisotropy effects in open quantum system simulations of heavy-quarkonium dynamics in the quark gluon plasma.

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