Thermalization of spatially homogeneous systems of gluons

Sergio Barrera Cabodevila, Carlos A. Salgado, Bin Wu

Boltzmann Equation in Diffusion Aproximation

- In the early stages after a heavy ion collision, a dense system of gluons is produced.
- We use the **Boltzmann Equation in the Diffusion Approximation** (BEDA) in order to study this process in a homogeneous and non-expanding system.

$$\partial_t f = C_{el}[f] + C_{inel}[f]$$

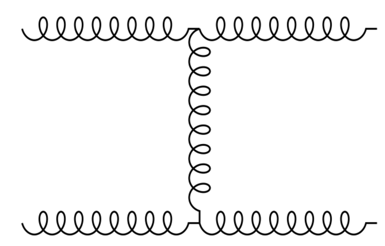
- The f is the function distribution in momentum and allows us to compute some characteristic parameters of the system such as the jet-quenching parameter, the Debye mass and the temperature of the system.

$$m_D^2 = 8\pi\alpha_s N_c \int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{f}{|\boldsymbol{p}|} \qquad \hat{q} = 8\pi N_c^2 \alpha_s^2 \ln\frac{\langle p_t^2\rangle}{m_D^2}\int \frac{d^3\boldsymbol{p}}{(2\pi)^3} f\,(1+f)$$

$$T_*(t) \equiv \frac{\hat{q}}{2\alpha_s N_C m_D^2 \ln\frac{\langle p_t^2\rangle}{m_D^2}}$$

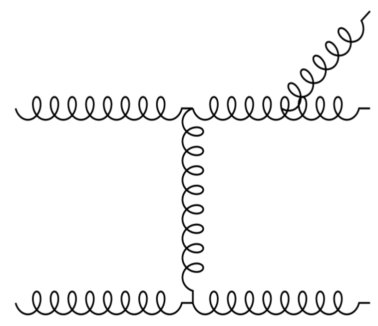
- **Elastic kernel**. The 2 ↔ 2 processes in QCD are dominated by small-angle scatterings [1].

$$C_{el}[f] = \frac{1}{4}\hat{q}(t)\nabla_{\boldsymbol{p}}\cdot\left[\nabla_{\boldsymbol{p}} f + \frac{\boldsymbol{v}}{T_*(t)} f(1+f)\right]$$



- **Inelastic kernel**. The 1 ↔ 2 scattering due to the LPM effect is the most efficient mechanism to drive the system towards full thermalization [2].

$$C_{inel}[f_{\boldsymbol{p}}] = \int d^3\boldsymbol{p}' \int_0^1 dx \frac{d^2I(\boldsymbol{p}')}{dx\,dt}\Big\{\delta(\boldsymbol{p}-x\boldsymbol{p}')\Big[f_{\boldsymbol{p}'}(1+f_{x\boldsymbol{p}'})\Big(1+f_{(1-x)\boldsymbol{p}'}\Big) - f_{x\boldsymbol{p}'}f_{(1-x)\boldsymbol{p}'}(1+f_{\boldsymbol{p}'})\Big] - \frac{1}{2}\delta(\boldsymbol{p}-\boldsymbol{p}')\Big[f_{\boldsymbol{p}}(1+f_{x\boldsymbol{p}})(1+f_{(1-x)\boldsymbol{p}}) - f_{x\boldsymbol{p}}f_{(1-x)\boldsymbol{p}}(1+f_{\boldsymbol{p}})\Big]\Big\}$$



Splitting rate computed with the LPM effect [3]

$$\frac{d^2I(p)}{dx\,dt} = \frac{\alpha_s N_c}{\pi}\frac{(1-x+x^2)^{5/2}}{(x-x^2)^{3/2}}\sqrt{\frac{\hat{q}}{p}}$$

Rapid thermalization in the soft sector

- At early times, the evolution of the soft sector is dominated by the inelastic kernel.

$$C_{inel}[f_{\boldsymbol{p}}] \approx \frac{\alpha_s N_c}{\pi} I_a \sqrt{\frac{\hat{q}}{p}}\frac{1}{p^3}\left(1 - \frac{p f_{\boldsymbol{p}}}{T_*}\right)$$

- Neglecting the elastic contribution, we solve (approximately) the Boltzmann equation and find that the soft gluons quickly thermalize with temperature T* at a characteristic scale given by p*.

$$p f_{\mathrm{p}}(t) \approx T_*\left[1 - \left(1 - \frac{p f_p(t=0)}{T_*}\right)e^{-\frac{1}{2}\left(\frac{p_*}{p}\right)^{\frac{5}{2}}}\right] = p f_p(t=0) + \frac{T_*}{2}\left(\frac{p_*}{p}\right)^{\frac{5}{2}} + \cdots$$

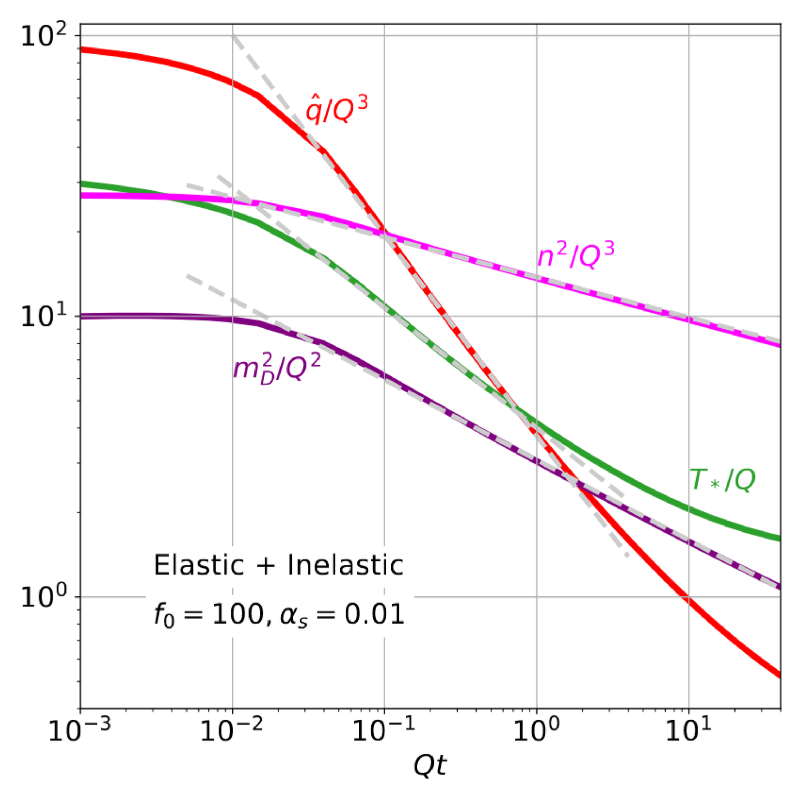
$$p_* \equiv (\hat{q} m_D^4 t^2)^{\frac{1}{5}}$$

Initially over-populated system $f_0 \gg 1$

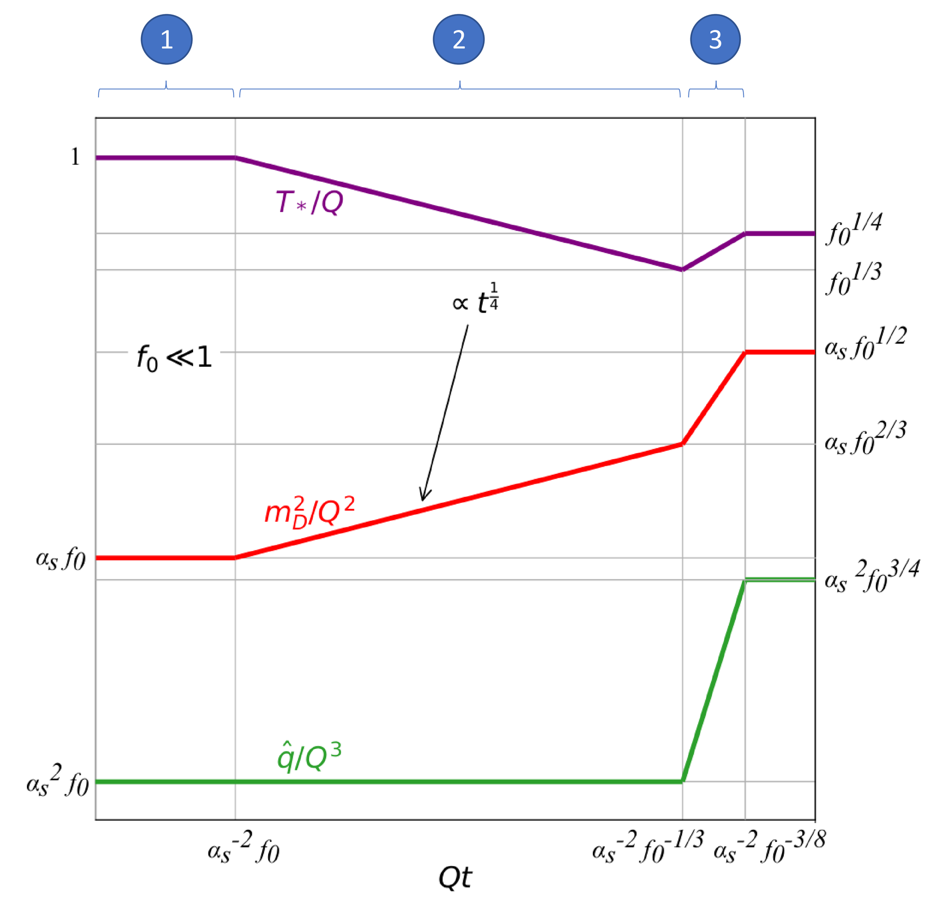
Thermalization occurs in only two stages:

1. The soft sector increases its population due to radiation. This gluons quickly thermalize, avoiding the formation of a Bose-Einstein condensate.
2. The typical momentum of gluons is mainly determined by multiple elastic scatterings, which is traduced as a **broadening in momentum** and **cooling of the system**.

We also confirm the universality of the non-thermal fixed-point solution in systems with or without the inelastic scattering.



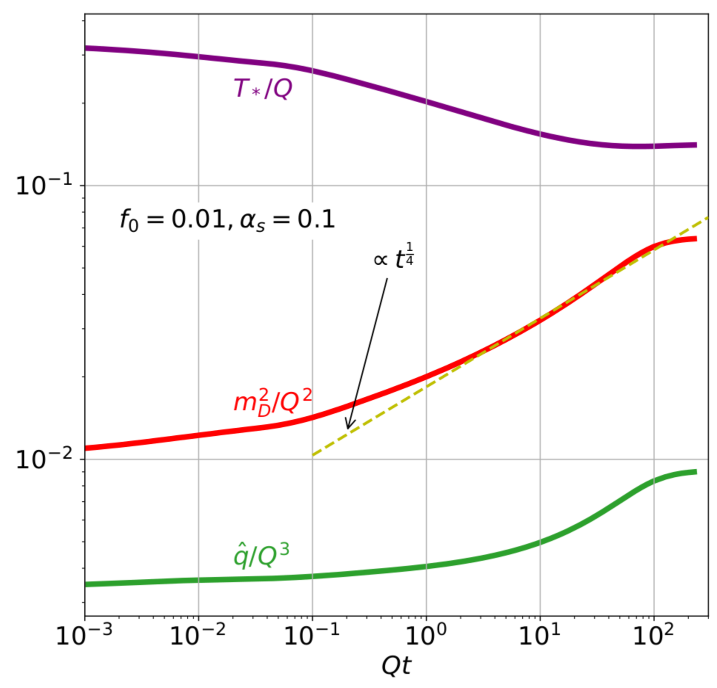
Initially under-populated system $f_0 \ll 1$



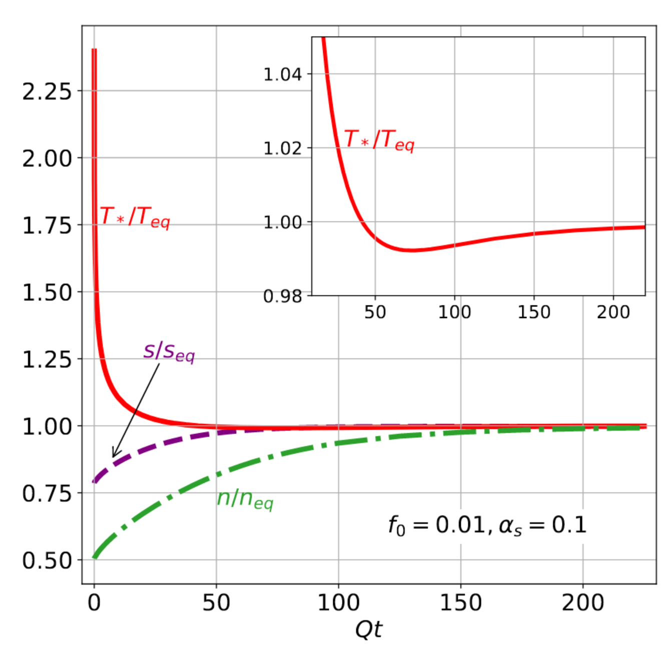
We distinguish three different stages in this case just by doing a parametrical study of some characteristic quantities:

1. Hard gluons **radiate soft gluons**, which quickly thermalizes with a momentum scale p_* and a **higher temperature** than the equilibrium one, T_{eq}.
2. **Soft sector undercools** T_{eq}, and its typical momentum is pushed by elastic scatterings. Debye mass higher contribution is due to the soft sector.
3. **Reheating of soft gluons** and mini-jet quenching. The Debye mass and the jet quenching parameter are dominated by the soft sector.

A numerical simulation confirms the parametrical prediction.



A zoom-in shows explicitly the overcooling and reheating of the soft gluons, as well as the convergence of the T_* to the equilibrium value of the thermal distribution.



Summary

- The Boltzmann Equation in Diffusion Approximation (BEDA) provides a framework to study the thermalization evolution of a system of gluons.
- The qualitative features of thermalization described by the BEDA agree with previous studies using effective kinetic theory, as summarized in [4].
- **The soft sector quickly acquires a thermal distribution** due to inelastic processes.
- In an initially under-populated system, **the soft sector is undercooled** before the full system thermalization is achieved.
- The inelastic contribution helps the soft sector to acquire a thermal equilibrium and **avoid Bose-Einstein condensation**.
- Check our paper for more detailed information: [arXiv:2206.12376]

References

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Outlook

- Quarks will be included in future calculations.
- Generalization of the problem to more arbitrary geometries, such as a heavy-ion collision expanding system.
- Connect our results with some heavy-ion observables, such as collective flow.

Acknowledgments

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