Progress on stabilisation of complex Langevin for real-time simulations of non-abelian gauge theories

Paul Hotzy with Kirill Boguslavski and David Müller

Complex Langevin equation for Yang-Mills

$$\frac{\partial A^a_\mu(\theta, x)}{\partial \theta} = i \frac{\delta S_{\mathrm{YM}}}{\delta A^a_\mu(\theta, x)} + \eta^a_\mu(\theta, x),$$

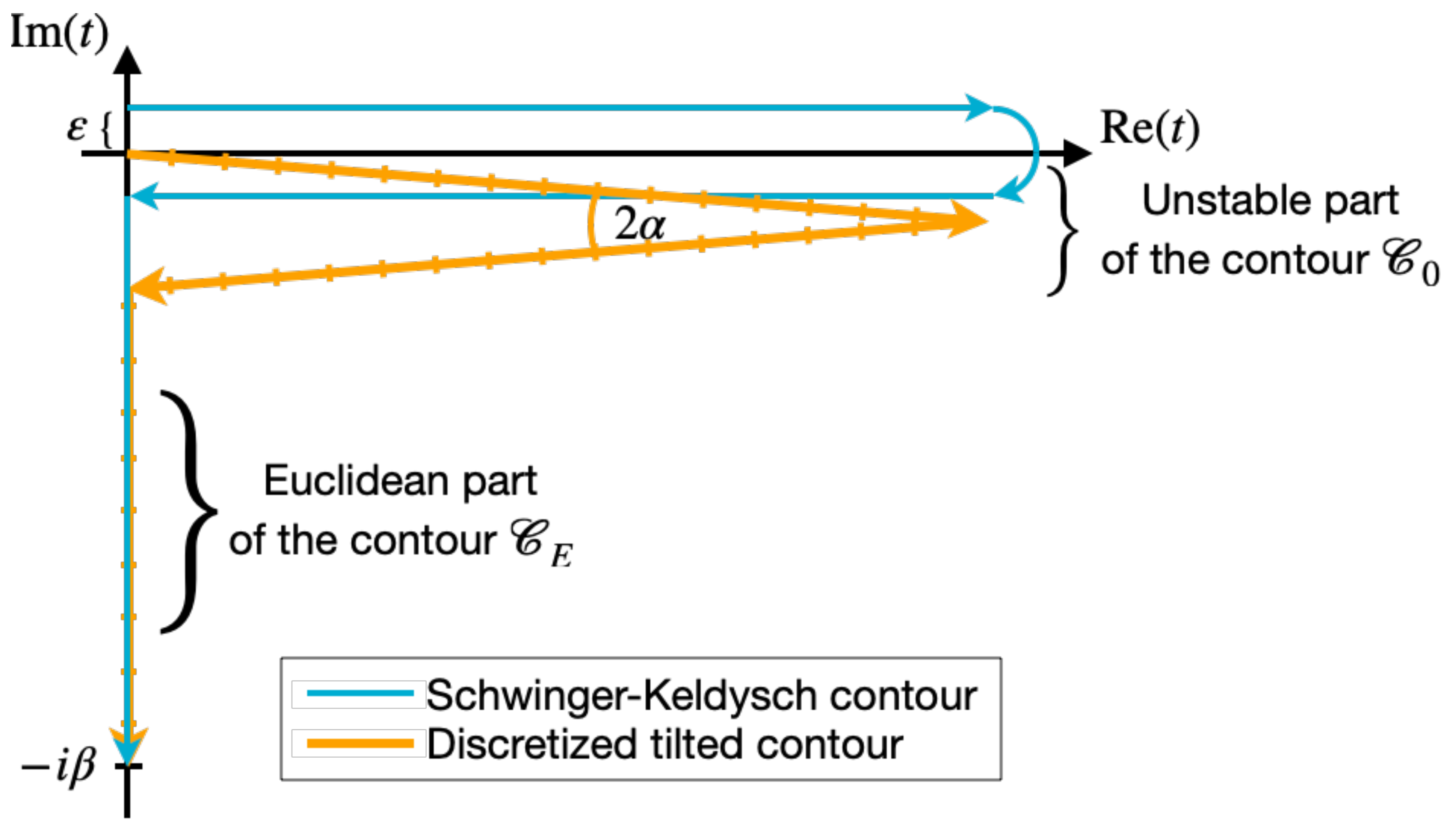
$$S_{\mathrm{YM}} = -\frac{1}{4} \int_{\mathscr{C}} d^4x F^{\mu\nu}_a F^a_{\mu\nu}$$

- Introduction of auxiliary time (Langevin time)
- Complexification of degrees of freedom
- ***Goal: Calculate oscillatory integrals (sign problem)***

$$\langle \mathscr{O} \rangle = \frac{1}{Z} \int dx\, \mathscr{O}(x) \exp\left[iS(x)\right] \approx \lim_{\theta_0 \to \infty} \frac{1}{T} \int_{\theta_0}^{\theta_0 + T} d\theta\, \mathscr{O}[z(\theta)]$$

- Application to Schwinger-Keldysh contour, but:
- Runaway instabilities → adaptive stepsize
- Wrong convergence → gauge cooling & dynamical stabilisation

→ ***We introduce an improved CL step for anisotropic lattices to alleviate instabilities!***



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TECHNISCHE UNIVERSITÄT WIEN Vienna | Austria

Doktoratskolleg Particles and Interactions

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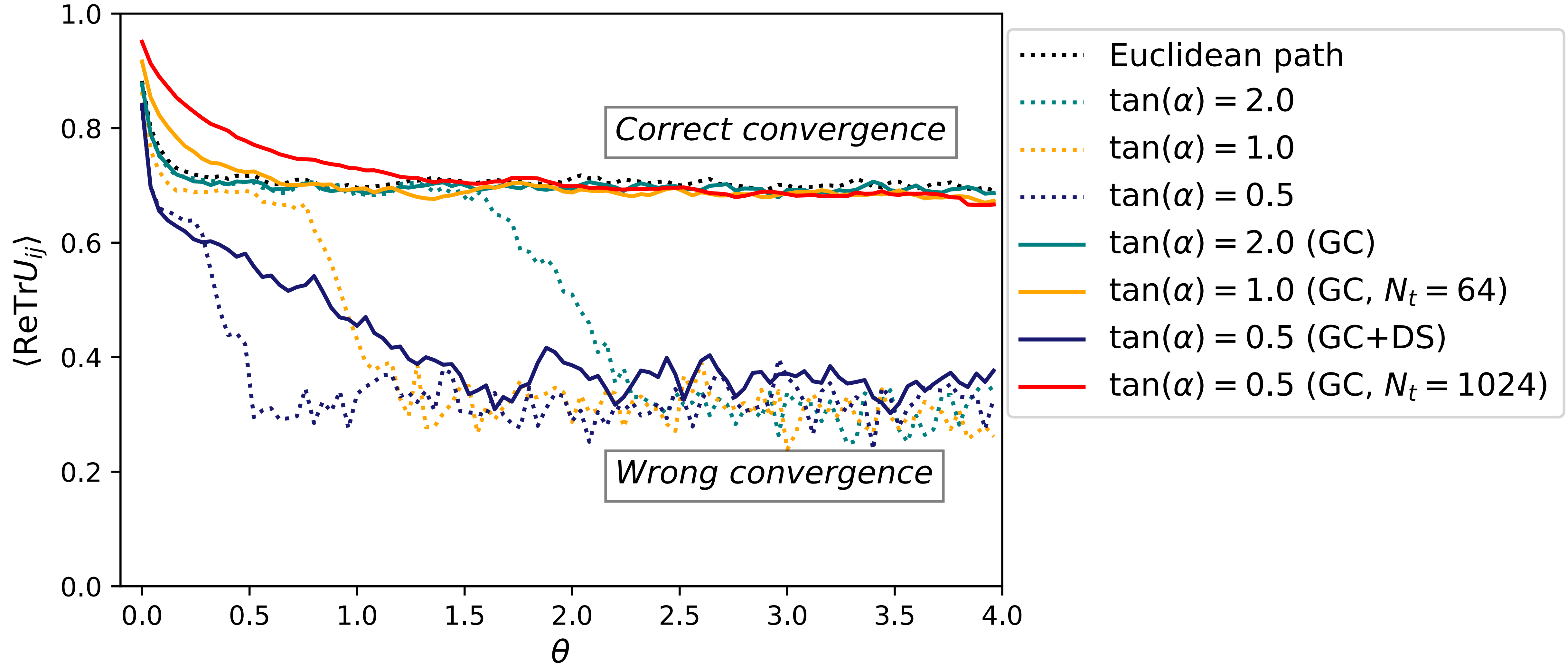
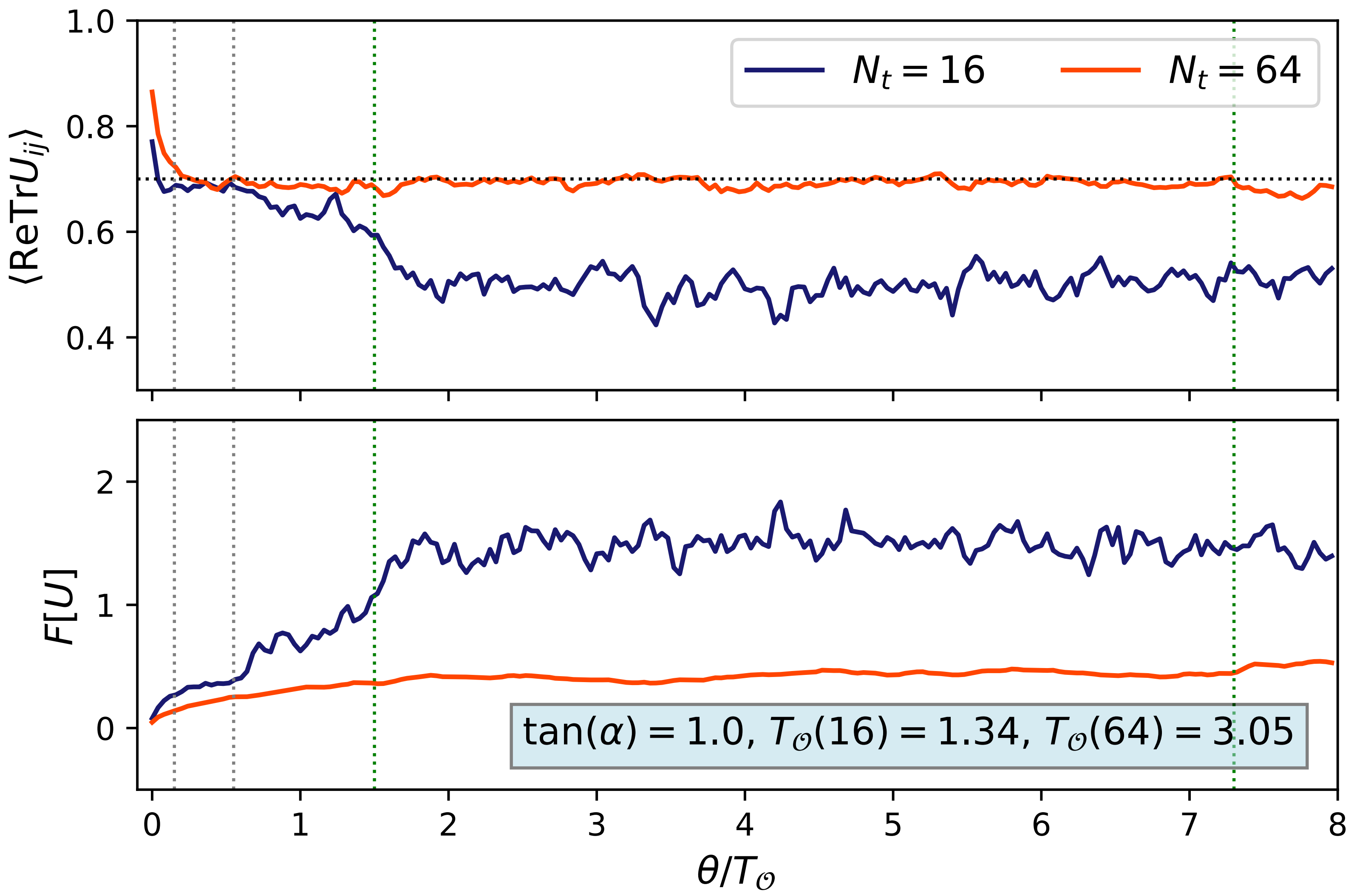
- Stabilization techniques extend the applicability of Complex Langevin by mitigating instabilities
- Our CL step improves convergence (see $\tan(\alpha) = 0.5$)
- *Note:* Autocorrelation length increases with N_t





- Anisotropic lattice discretization can enlarge the stable θ-regions
- Stability region increases faster with respect to N_t than the autocorrelation time $T_{\mathcal{O}}$

→ ***Extrapolation to Schwinger-Keldysh contour may be possible***

→ ***Calculation of real-time observables***