Pion properties under strong magnetic fields

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We investigate the leptonic decay of charged pions $\pi^- \to l^- \bar{\nu}_l$ in the presence of a static uniform magnetic field. We show that, in this situation, four independent form factors appear when hadronizing one-pion-to-vacuum matrix elements. We obtain a model independent expression for the decay width in terms of the form factors. Interestingly, no helicity suppression is found when the magnetic field is present. Using the Nambu-Jona—Lasinio (NJL) model we estimate the effect of the magnetic field on pion masses, decay constants and on the charged pion leptonic decay width.

Pion-to-vacuum amplitudes

The matrix element of the negatively charged pion hadronic current is

$$H^-_{\mu,L} = H^-_{\mu,V} - H^-_{\mu,A} = \langle 0|\, \bar{\psi}_u\, \gamma^\mu (1-\gamma_5)\, \psi_d |\pi^-\rangle$$

In an external uniform magnetic field, new axial and vector decay constants appear **[1,2]**. In fact, four independent form factors arise when hadronizing the quark currents

$$H^\pm_{\mu,L} = \left[\epsilon^{\mu\nu\alpha\beta} F_{\nu\alpha} D_\beta \frac{f^{(V)}_{\pi^\pm}}{2B} - D^\mu f^{(A1)}_{\pi^\pm} + i F^{\mu\nu} D_\nu \frac{f^{(A2)}_{\pi^\pm}}{B} - F^{\mu\nu} F_{\nu\alpha} D^\alpha \frac{f^{(A3)}_{\pi^\pm}}{B^2} \right] \sqrt{2}\, \langle 0|\Phi_{\pi^\pm}|\pi^\pm\rangle$$

Similar definitions apply to neutral pions.

Discrete symmetries $(\mathcal{C},\mathcal{P},\mathcal{T})$ of the interaction Lagrangian between light quarks and the external magnetic field constrain these form factors.

As a result, all form factors are real, $f^{(V,Ai)}_{\pi^-} = f^{(V,Ai)}_{\pi^+}$ and $f^{(A2)}_{\pi^0} = 0$.

Two-flavor NJL model with B (Schwinger form)

In the NJL model, the Euclidean Lagrangian density in an external uniform $\vec{B} = B\hat{z}$ magnetic field is given in the Landau gauge by

$$\mathcal{L}_{\rm NJL} = \bar{\psi}\,(-i\not{D} + m_0)\,\psi - G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}\, i\gamma_5 \vec{\tau}\,\psi)\right], \quad D_\mu = \partial_\mu - i\hat{Q}Bx_1\delta_{\mu 2}$$

To obtain meson masses, we bosonize and expand mesons fields around their Mean Field (MF) values. The quadratic contribution is given by

$$S^{\rm quad}_{\rm Bos} = \frac{1}{2} \sum_{M=\sigma,\pi^0,\pi^\pm} \int_{x,x'} \delta M(x)^* \left[\frac{1}{2G}\delta^{(4)}(x-x') - J_M(x,x') \right] \delta M(x')$$

$$J_{\pi^0}(x,x') = N_c \sum_f \mathrm{Tr}\left[\mathcal{S}^{MF,f}_{x,x'} \gamma_5 \mathcal{S}^{MF,f}_{x',x} \gamma_5 \right], \quad J_{\pi^\pm}(x,x') = 2N_c \mathrm{Tr}\left[\bar{\mathcal{S}}^u_{x,x'} \gamma_5 \bar{\mathcal{S}}^d_{x',x} \gamma_5 \right]$$

where $\bar{\mathcal{S}}^f_{x,x'} = e^{i\Phi_f(x,x')} \int_p e^{ip(x-x')} \bar{\mathcal{S}}^f_p$ is the MF quark propagator.

Here $\Phi_f(x,x') = q_f B(x_1 + x_1')(x_2 - x_2')/2$ is the Schwinger phase.

Divergent integrals are regulated in the MFIR scheme through a 3D cutoff. We consider three parametrizations, which reproduce empirical values $m_\pi = 138$ MeV and $f_\pi = 92.4$ MeV at $B = 0$.

Pion masses

For π^0, Schwinger phases cancel out and J_{π^0} is diagonal in the Fourier basis. We have $E_{\pi^0} = \sqrt{m^2_{\pi^0} + u^2_{\pi^0} q^2_\perp + q^2_3}$ (anisotropy due to $\vec{B}$).

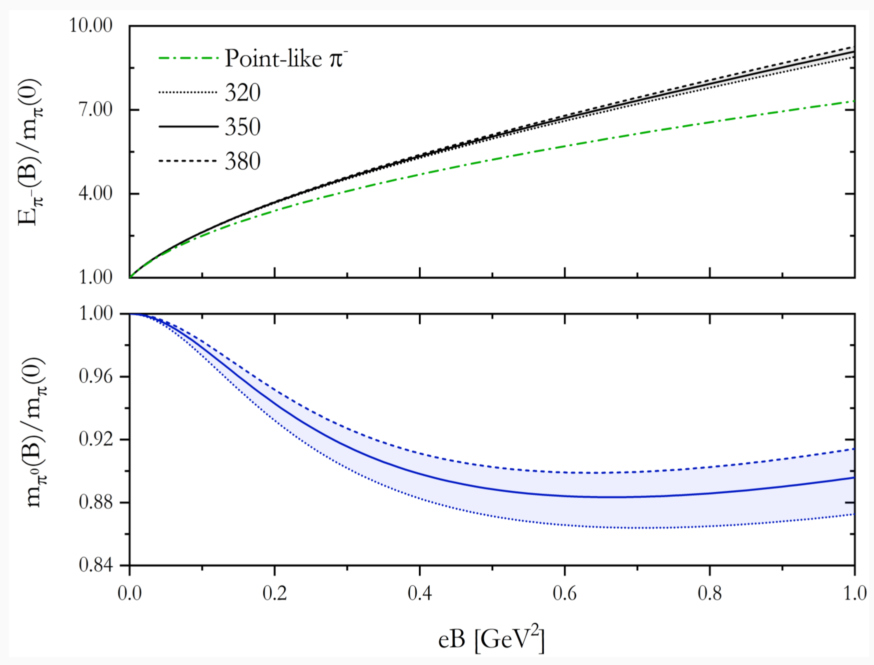
In contrast, charged pions have to be diagonalized in the Ritus basis since Schwinger phases do not cancel. Here $E_{\pi^-} = \sqrt{m^2_{\pi^-} + (2n+1)eB + q^2_3}$

We define the π^- mass as the lowest energy state,

$$E_{\pi^-} = \sqrt{m^2_{\pi^-} + eB}\,.$$

Parametrizations sets are labeled by the effective quark mass: $M = 320, 350, 380$ MeV.

Quark condensates reproduce Magnetic Catalysis in close agreement with Lattice QCD **[3]**.



Pion decay constants

We gauge the effective action $\gamma_\mu \partial_\mu \to \gamma_\mu \partial_\mu - i\frac{\tau^a}{2}\gamma_\mu \left(W^{V,a}_\mu + \gamma_5 W^{A,a}_\mu \right)$

Then (similarly for π^0) $H^\pm_{\mu,C} = -\sqrt{2}\, \frac{\partial S_{\pi W}}{\partial \delta\pi^\mp\, \partial W^\pm_{\mu,C}}$, $C = V, A$

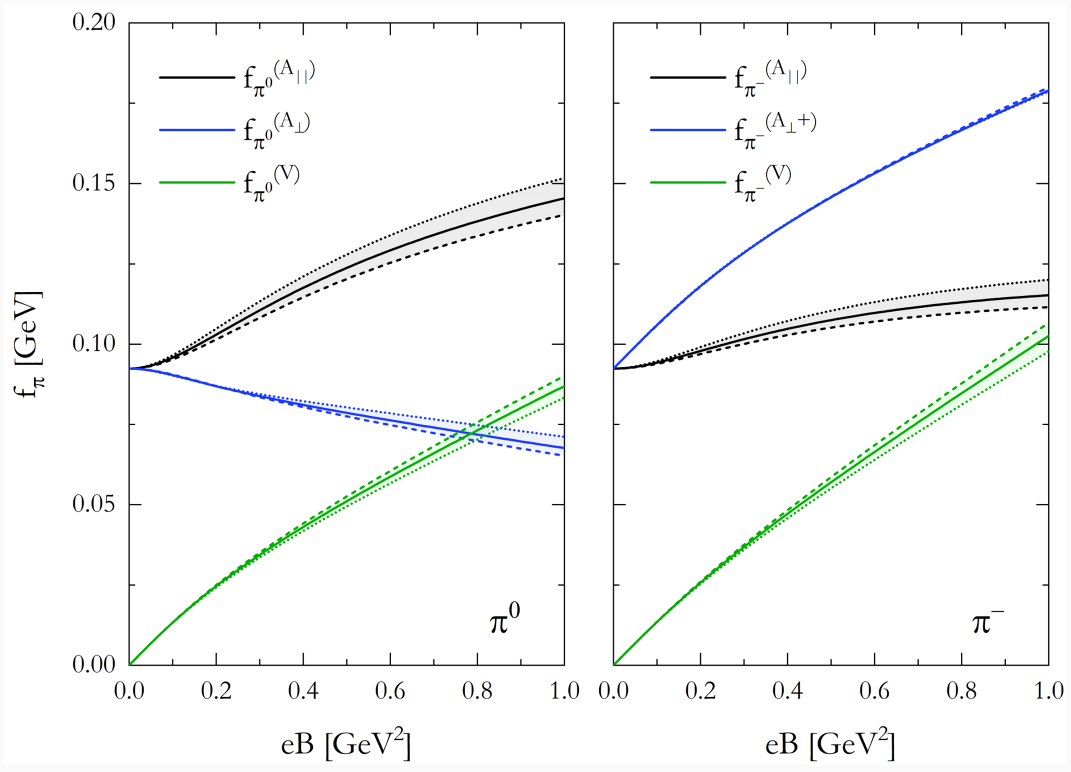
In the direction of $\vec{B}$ only $f^{(V)}_\pi$ and $f^{(A1)}_\pi$ appear. We define $f^{(A_\parallel)}_\pi \equiv f^{(A1)}_\pi$.

Perpendicular to $\vec{B}$, it is convenient to define the following combinations

$$f^{(A_\perp)}_{\pi^0} \equiv f^{(A1)}_{\pi^0} - f^{(A3)}_{\pi^0}\,, \qquad f^{(A_\perp \pm)}_{\pi^-} \equiv f^{(A1)}_{\pi^-} \pm f^{(A2)}_{\pi^-} - f^{(A3)}_{\pi^-}$$

When the charged pion lies on the lowest Landau level ($n = 0$), the hadronic matrix element associated to $f^{(A_\perp -)}_{\pi^-}$ vanishes.

Results are shown for the same three sets within NJL model **[4]**.



Charged pion decay $\pi^- \to l^- \bar{\nu}_l$

We obtain a model independent expresion for the decay width, in both Landau and symmetric gauges **[2,5]**.

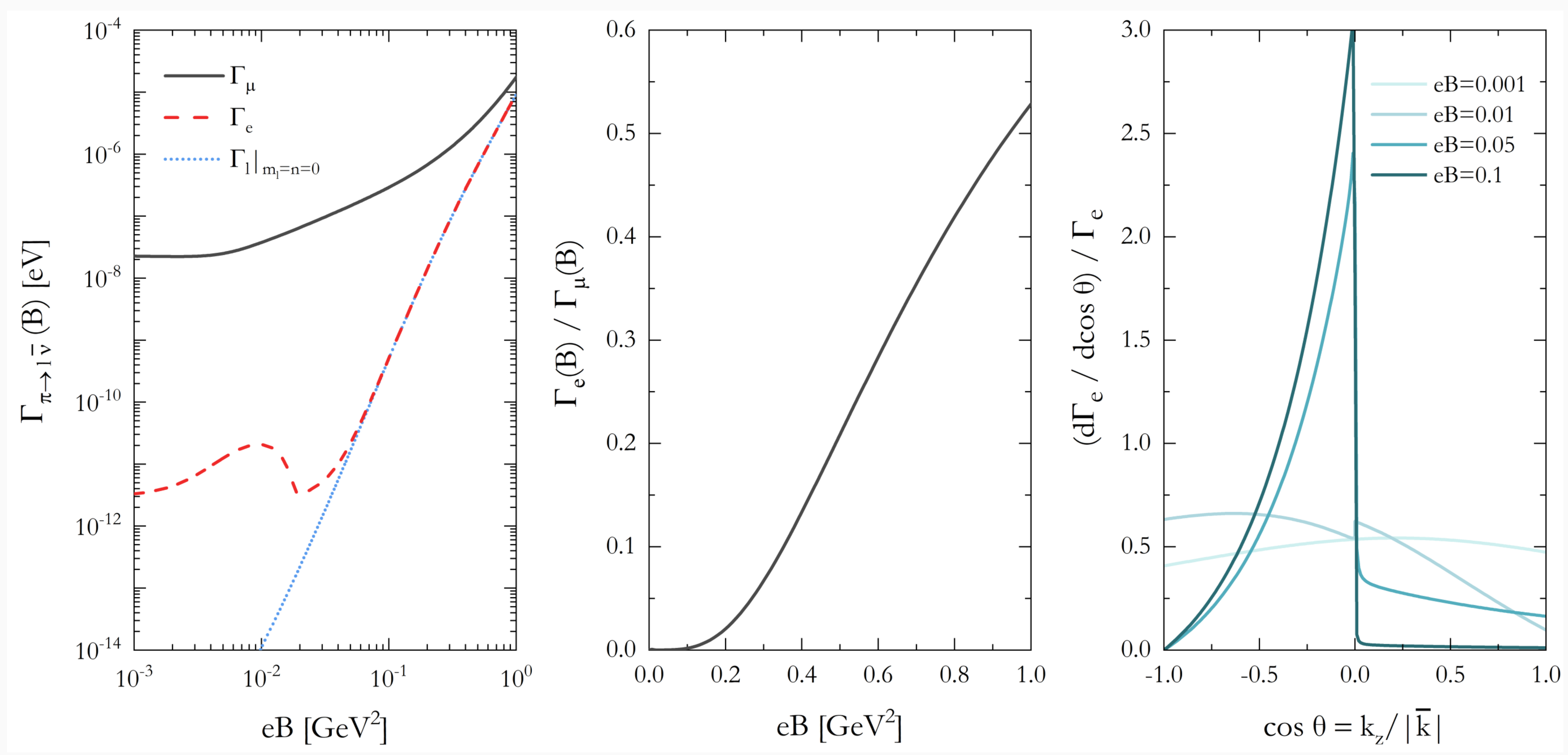
When $B > m^2_{\pi^-} - m^2_l$ and $B \gg m^2_l$, $n = 0$ and $m_l \sim 0$ approximately

$$\Gamma^-_l(B)\Big|_{\substack{n_{\max}=0 \\ m_l=0}} = \frac{G_F^2 \cos^2\theta_c}{\pi} \frac{B_e^2}{E_{\pi^-}} \left[1 - \left(1 + \frac{E^2_{\pi^-}}{2B_e} \right) e^{-E^2_{\pi^-}/(2B_e)} \right] \left| f^{(V)}_{\pi^-} - f^{(A2)}_{\pi^-} + f^{(A3)}_{\pi^-} \right|^2$$

There is no helicity suppression in the $m_l \to 0$ limit when $\vec{B}$ is present.

Chirality, $\gamma_5 |l\rangle_{m_l=n=0} = -\mathrm{sign}(q_z)|l\rangle_{m_l=n=0}$, implies $k_z = -q_z < 0$.

The angular distribution of outgoing $\bar{\nu}_l$ is anisotropic for large B.



We show only Set 350; others sets do not differ by more than 3% **[6]**.

Conclusions

- In the presence of a uniform magnetic field, four form factors appear when hadronizing quark currents in pion-to-vacuum amplitudes.
- Pion properties are estimated using the NJL model. Diagonalization of neutral and charged pions require different quantum basis.
- When B is enhanced, the π^0 mass slightly decreases, while E_{π^-} steadily increases, remaining always larger than in the case of a point-like pion.
- The decay width of charged pions is strengthened by the magnetic field, up to 1000 times for $eB = 1$ GeV^2.
- No helicity-suppression is found. As a consequence, the ratio Γ_e/Γ_μ is dramatically enhanced from $\sim 10^{-4}$ in vacuum up to ~ 0.5 at $eB = 1$ GeV^2.
- For large B, the $\bar{\nu}_l$ angular distribution becomes highly anisotropic.

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