Equilibration of QCD plasma and Non-hydrodynamic modes

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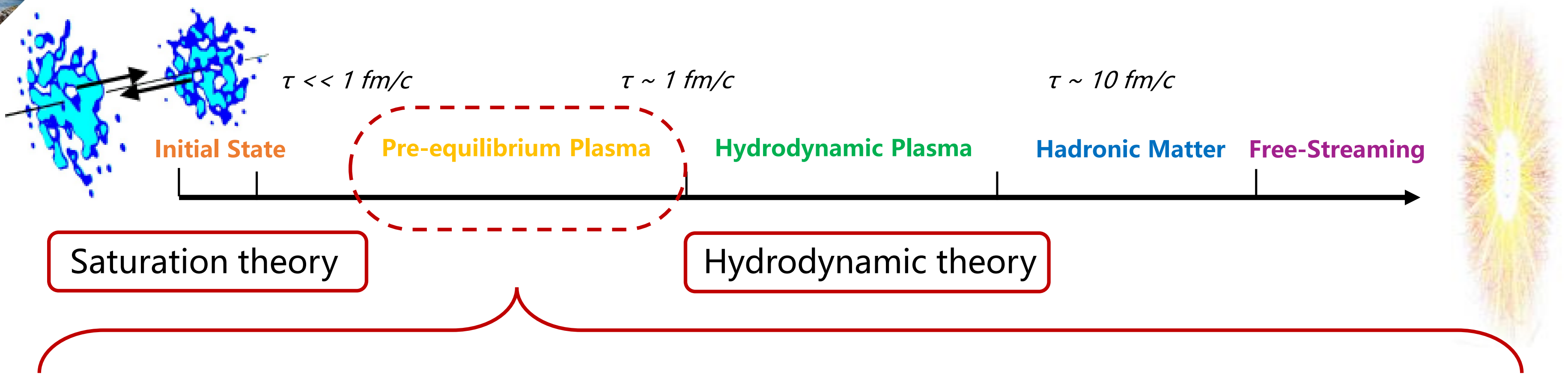
CRC–TR 211
Strong-interaction matter under extreme conditions

Pre-equilibrium stage in HICs



τ << 1 fm/c *τ ~ 1 fm/c* *τ ~ 10 fm/c*

Initial State | Pre-equilibrium Plasma | Hydrodynamic Plasma | Hadronic Matter | Free-Streaming

Saturation theory

Hydrodynamic theory

QGP equilibration/hydrodynamization in HICs

- Connects initial condition to equilibrium states
  - Off-thermal initial states into near-thermal hydrodynamic states (kinetics)
  - Saturated gluon fields into quark-gluon plasma (chemistry)

Kinetic Theory description of QGP equilibration

- Mechanism to thermalize states (kinetic equilibration)
- Include both gluon + quark degrees of freedom (chemical equilibration)

Effective Kinetic Theory

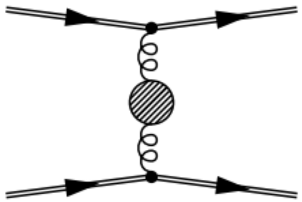
QCD Effective Kinetic Theory (EKT)

Arnold, Moore, Yaffe, JHEP01 (2003) 030
Arnold, Moore, Yaffe, JHEP0206 (2002) 030
Kurkela, Mazeliauskas, PRD99 (2019) 054018

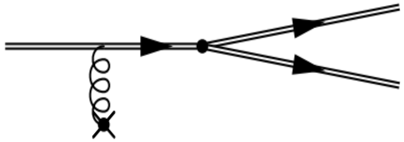
$$\left(\frac{\partial}{\partial\tau}-\frac{p_\parallel}{\tau}\frac{\partial}{\partial p_\parallel}\right)f_a(\tau,p_T,p_\parallel) = -C_a^{2\leftrightarrow2}[f](\tau,p_T,p_\parallel) - C_a^{1\leftrightarrow2}[f](\tau,p_T,p_\parallel)$$

Solving a set of coupled Boltzmann equations

- LO 2↔2 elastic scatterings & 1↔2 inelastic scatterings



2↔2: Color screening by Debye mass fit to Hard Thermal Loop (HTL) calculation



1↔2: Collinear radiation including Landau-Pomeranchuk-Migdal (LPM) effect via effective vertex resummation

- Gluon + all light quarks/antiquarks (finite net-baryon density) $a = g, u, \bar{u}, d, \bar{d}, s, \bar{s}$

Yang-Mills Effective Kinetic Theory

- Gluon only

XD, Schlichting, PRD104(2021)054011
XD, Schlichting, PRL127(2021)122301

Hydrodynamization of YM theory

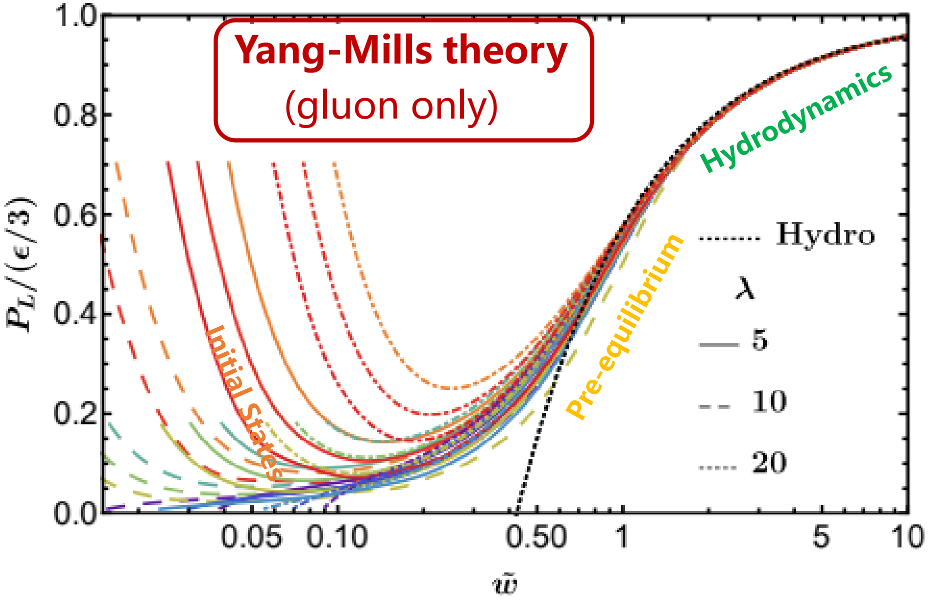
Isotropization

- First-order hydrodynamics

$$\frac{p_L}{e} = \frac{1}{3} - \frac{4}{9\pi\tilde{\omega}} \qquad \tilde{\omega} = \frac{\tau\, T}{4\pi\eta/s}$$

Universal time scale

Evolution of pressure



Yang-Mills theory (gluon only)

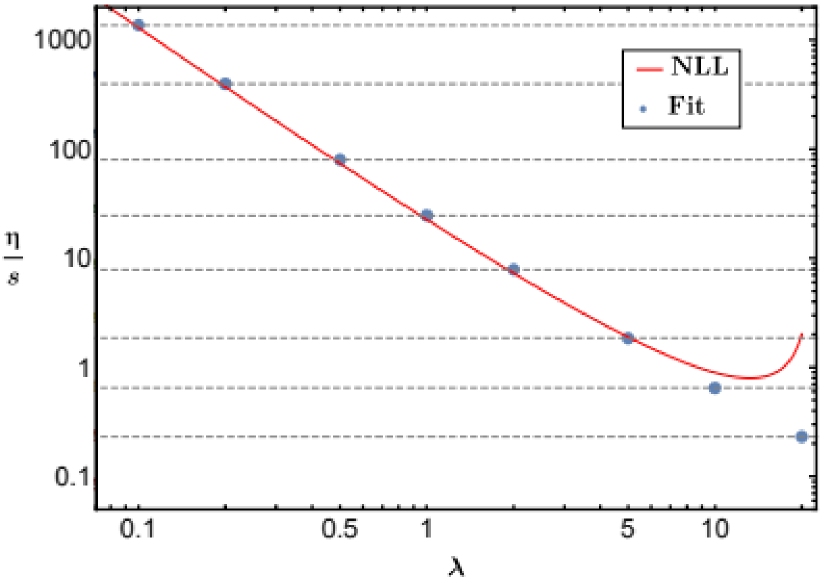
Hydrodynamics

Pre-equilibrium

Initial States

Viscosity

- Quick memory loss regardless of **initial states** (η/s not relevant to init. cond.)
- Different isotropization speed (by η/s) towards **hydrodynamics** at different coupling (by λ)



XD, Heller, Schlichting, Svensson, PRD106(2022)014016

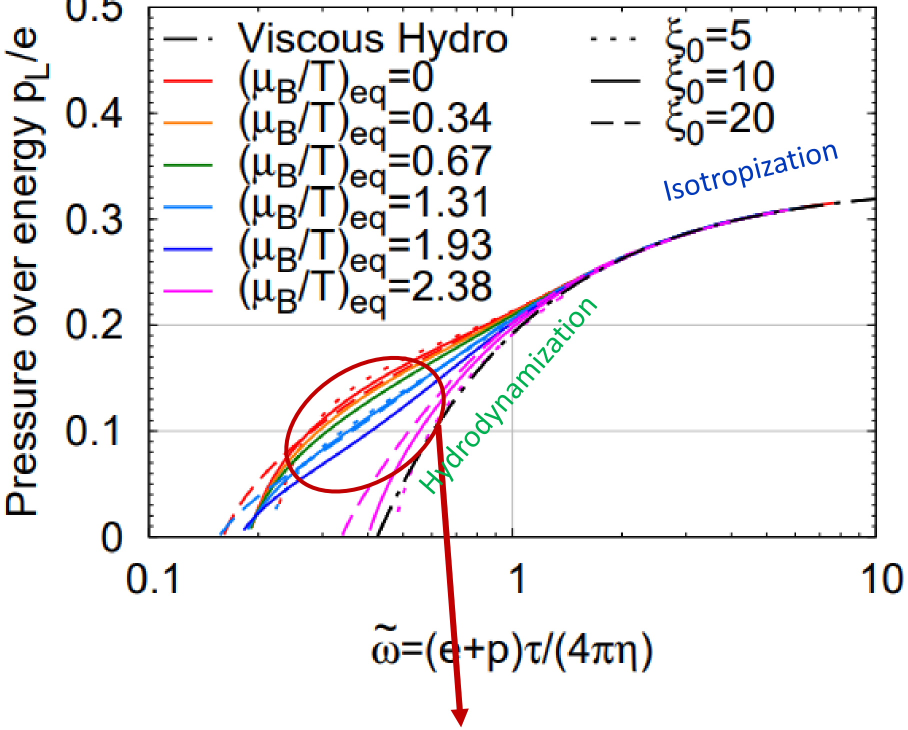
Hydrodynamization of QCD theory

QCD Plasma Isotropization

- Overall similar to Yang-Mills theory
- Quarks interact slower than gluon (chemical equilibration)

Evolution of pressure

QCD theory (gluon + quark/antiquark)



Early timeQuarks slow down equilibration

Viscosity

- Larger η/s at higher baryon density

QCD vs Yang-Mills

- Different isotropization speed (by η/s) towards **hydrodynamics** at QCD and Yang-Mills theory

For ‘t Hooft coupling $\lambda=10$

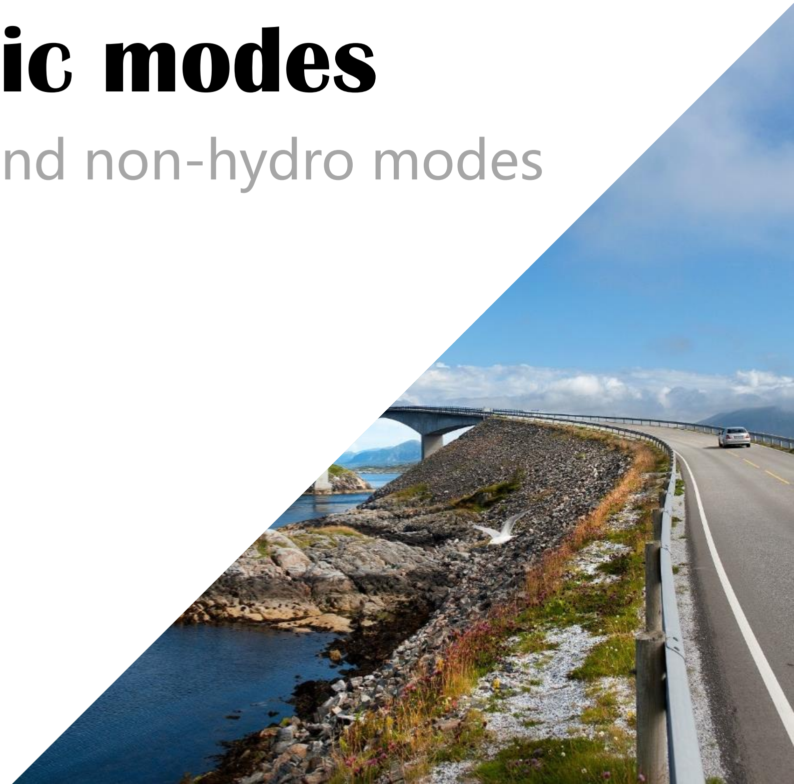
$\eta/s \approx 0.6$ (Yang-Mills), $\eta/s \approx 1.0$ (QCD)

η/s is a measure of the speed of equilibration

XD, Schlichting, PRD104(2021)054011
XD, Schlichting, PRL127(2021)122301

Non-hydrodynamic modes

Response to perturbation, hydro and non-hydro modes



Hydrodynamics and more

Hydrodynamics

- Mesoscopic theory at long-wavelength, low-frequency limit of any system

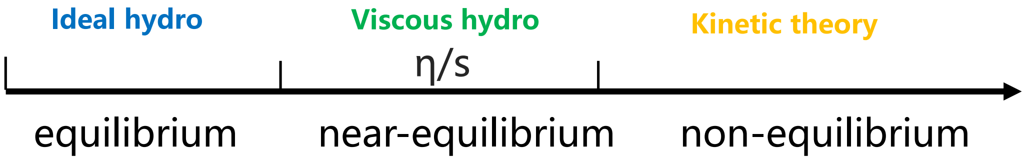
- Faster equilibration: smaller η/s, stronger coupling → ideal hydrodynamics (η/s →0)

- Slower equilibration: larger η/s, weaker coupling → viscous hydrodynamics

η/s is a measure of the speed of equilibration

Non-hydrodynamic modes

- Anything not hydrodynamic…
- Hydro with very large η/s ? → not hydro anymore: non-equilibrium kinetics



Ideal hydro | Viscous hydro | Kinetic theory

η/s

equilibrium | near-equilibrium | non-equilibrium

- How to study non-hydro modes → Linear response near thermal equilibrium

Perturbation

Linearized QCD Effective Kinetic Theory

- Consider a boost-invariant Effective Kinetic Theory with transverse expansion

$$\left(\frac{\partial}{\partial\tau}+\frac{ip}{p^{\tau}}\cdot\nabla_{x_T}-\frac{p_\parallel}{\tau}\frac{\partial}{\partial p_\parallel}\right)f_a(\tau,x_T,p_T,p_\parallel)=-C_a^{LO\;2\leftrightarrow2,1\leftrightarrow2}[f](\tau,x_T,p_T,p_\parallel)$$

- Split distribution into a transversely symmetric background **f(τ,p)** and asymmetric linearized perturbation **δf(τ,x_T,p)** labelled by a transverse wavenumber **k_T**

$$\boxed{f_a(\tau,x_T,p_T,p_\parallel)}=\boxed{f_a(\tau,p_T,p_\parallel)}+\boxed{\delta f_a(\tau,x_T,p_T,p_\parallel)}$$

Background Perturbation

$$\delta f_a(\tau,k_T,p_T,p_\parallel)=\int\frac{d^2k_T}{(2\pi)^2}e^{ix_T\cdot k_T}\delta f_a(\tau,x_T,p_T,p_\parallel)$$

Gradient → k-modes

- Results in a Linearized Effective Kinetic Theory with a transverse wavenumber **k_T**

$$\left(\frac{\partial}{\partial\tau}+\frac{ip_T\cdot k_T}{p^{\tau}}-\frac{p_\parallel}{\tau}\frac{\partial}{\partial p_\parallel}\right)\delta f_a(\tau,k_T,p_T,p_\parallel)=-\delta C_a^{LO\;2\leftrightarrow2,1\leftrightarrow2}[f](\tau,k_T,p_T,p_\parallel)$$

Keegan, Kurkela, Mazeliauskas, Teaney, JHEP09(2016)171

Response to perturbation

Energy-momentum tensor

- Background

$$\boxed{T^{\mu\nu}(\tau\,)} = \int \frac{d^3p}{(2\pi)^3}\frac{p^\mu p^\nu}{p}\sum_a v_a f_a(\tau, p_T, p_\parallel)$$

- Perturbation

$$\boxed{\delta T^{\mu\nu}(\tau, k_T)} = \int \frac{d^3p}{(2\pi)^3}\frac{p^\mu p^\nu}{p}\sum_a v_a \delta f_a(\tau, k_T, p_T, p_\parallel)$$

Response function

- Simple relation to calculate response function

$$G^{\mu\nu}_{\alpha\beta}(\tau-\tau_0, k_T) = \frac{\delta T^{\mu\nu}(\tau, k_T)}{T^{\tau\tau}(\tau\,)} \Big/ \frac{\delta T^{\alpha\beta}(\tau_0, k_T\,)}{T^{\tau\tau}(\tau_0\,)}$$

- For Yang-Mills theory & relaxation time approximation, see

Kurkela, Mazeliauskas, Paquet, Schlichting, Teaney, PRL122(2019)122302, PRC99(2019)034910
Kamata, Martinez, Plaschke, Ochsenfeld, Schlichting, PRD102(2020)056003

Sound and non-hydrodynamic modes

Scalar-Perturbation

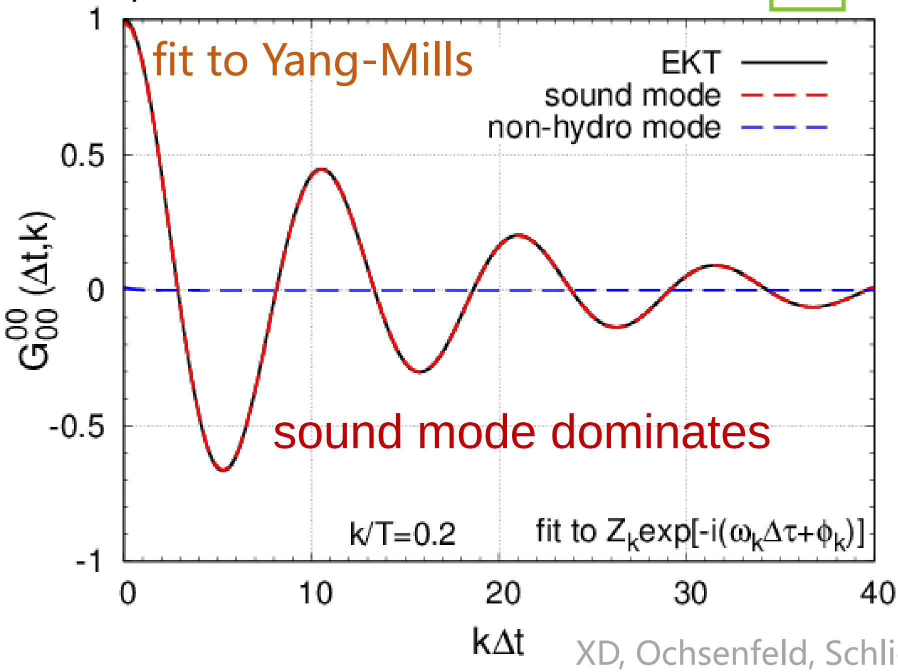
- Initial conditions

$$\boxed{f_a(\tau,p_T,p_\parallel)} = \frac{1}{e^{p/T} \mp 1} \qquad \boxed{\delta f_a(\tau,k_T,p_T,p_\parallel)} = -\frac{\delta T}{T}\partial_p f_a(\tau,p_T,p_\parallel)$$

Fitting response from perturbation to wave-modes

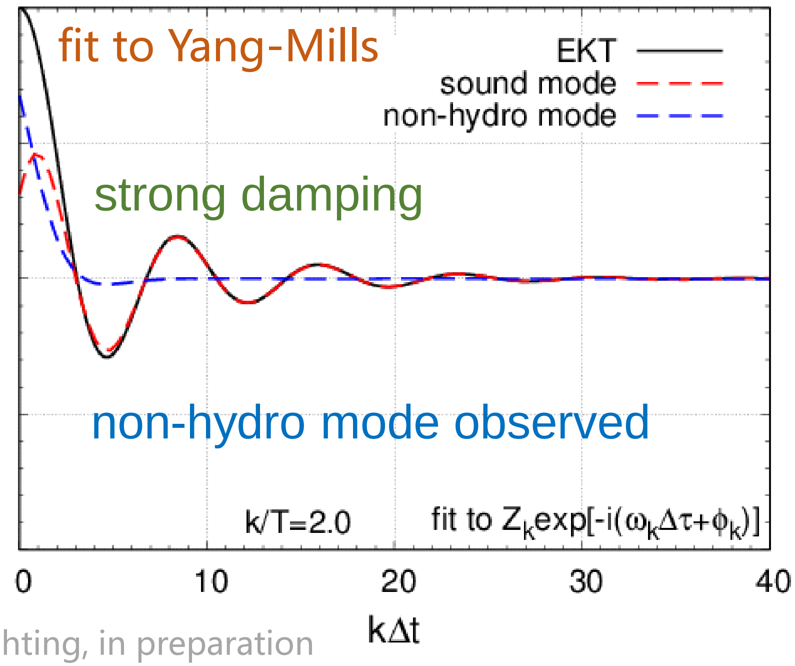
- With real (oscillation) and imaginary (damping) frequencies

$$G^{\mu\nu}_{\alpha\beta}(\Delta\tau = \tau - \tau_0, k_T) \sim Z_k \exp[-i(\boxed{\omega_k}\Delta\tau + \phi_k)] \qquad \omega_k = Re[\omega_k] + iIm[\omega_k]$$



fit to Yang-Mills

sound mode dominates



fit to Yang-Mills

strong damping

non-hydro mode observed

XD, Ochsenfeld, Schlichting, in preparation

- One can fit to QCD as well

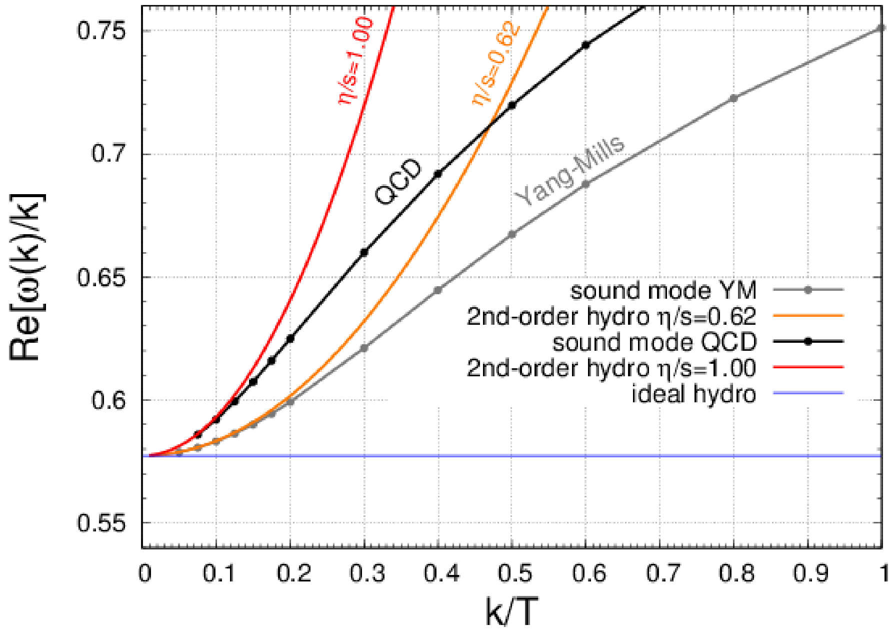
Comparing to hydrodynamics

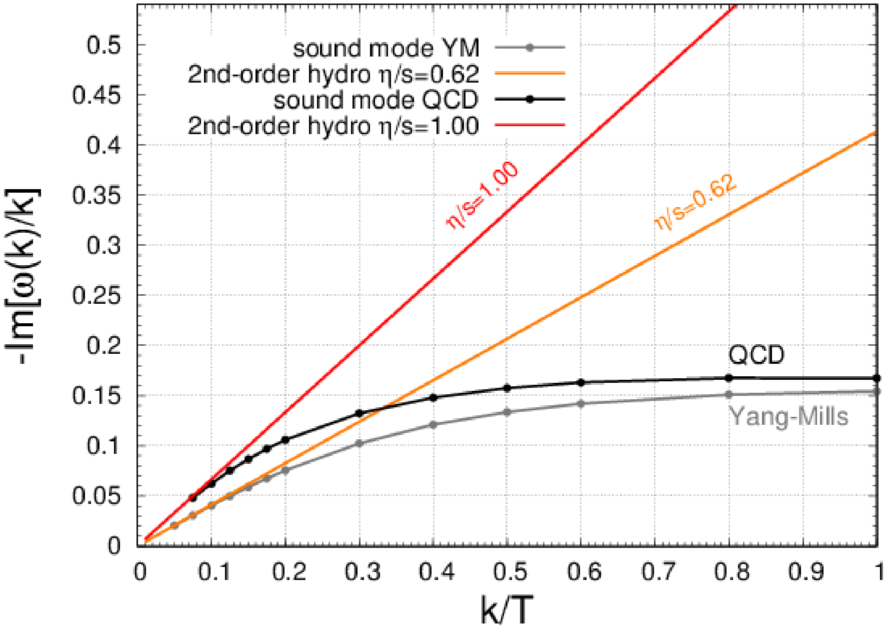
Dispersion relations in Yang-Mills & QCD EKT

- Fit both for various k-wave modes

$$Z_k\exp[-i(\boxed{\omega_k}\Delta\tau+\phi_k)] \qquad \omega_k = Re[\omega_k]+iIm[\omega_k]$$

- At small k, YM&QCD converge to hydro with different η/s





XD, Ochsenfeld, Schlichting, in preparation

- 2^{nd}-order hydro has

$$\omega_{1,2}=\pm c_s k - i\Gamma k^2 \pm \frac{\Gamma}{c_s}\left(c_s^2\tau_\Pi-\frac{\Gamma}{2}\right)k^3+\mathcal{O}(k^4)\,,\quad \Gamma=\frac{d-2}{d-1}\frac{\eta}{\varepsilon+P}$$

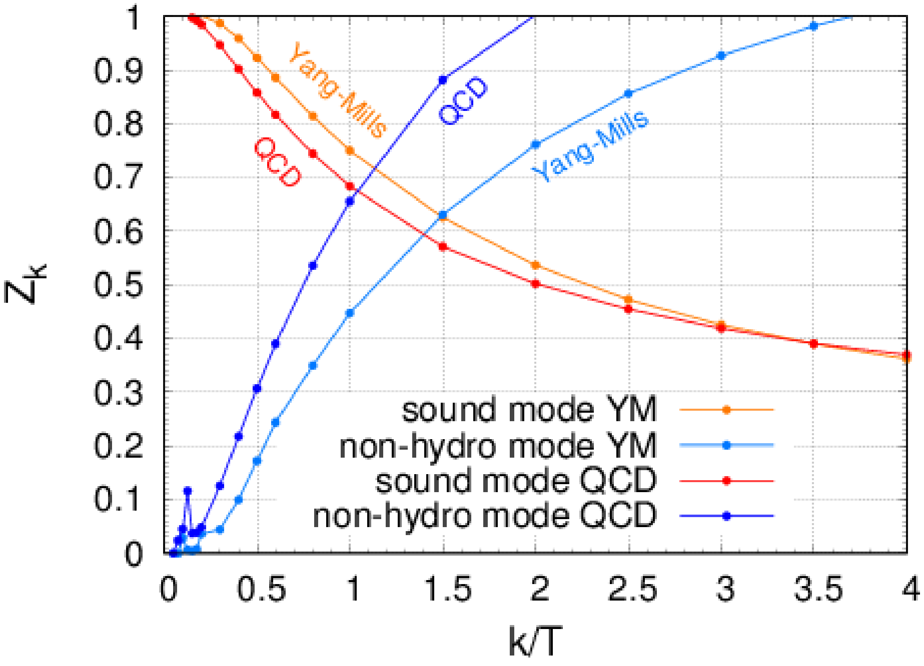
Baier, Romatschke, Son, Starinets, Stephanov, JHEP04(2008)100
Keegan, Kurkela, Mazeliauskas, Teaney, JHEP08(2016)171

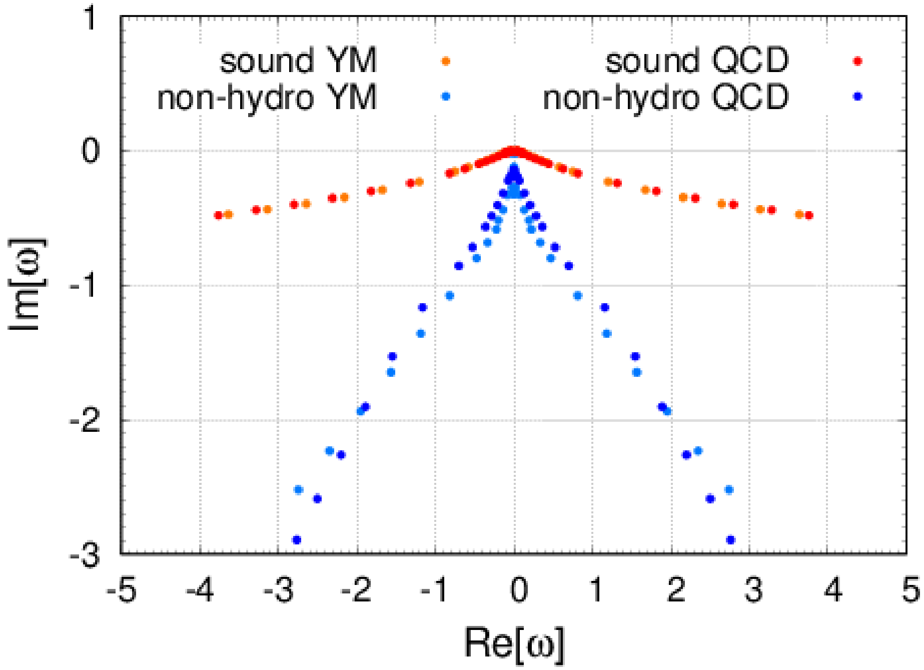
Yang-Mills vs QCD

Residue and poles in the complex plane

- Pre-equilibrium plasma described by a sound mode + a non-hydro mode

$$\boxed{Z_k}\exp[-i(\boxed{\omega_k}\Delta\tau+\phi_k)]$$





XD, Ochsenfeld, Schlichting, in preparation

- More discussions:

RTA: Romatschke, EPJC76(2016)352, Kurkela, Wiedeman, EPJC79(2019)776

AdS/CFT: Buchel, Heller, Noronha, PRD94(2016)106011

Conclusions

- **Hydrodynamization of QCD plasmas**
  - Viscosity/entropy density is a measure of the speed of equilibration
  - QCD plasma equilibrates slower than Yang-Mills plasma due to quarks

- **Non-hydrodynamic modes**
  - Can be studied with linear response to fluctuation
  - Yang-Mills/QCD plasma described by a sound mode + a non-hydro mode
  - QCD plasma has larger non-hydro residue than Yang-Mills plasma (due to quarks!)