Phenomenology of Asymptotic Safety with Large N_f

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Question 1

How to make the Standard Model UV complete without gravity?

Answer 1

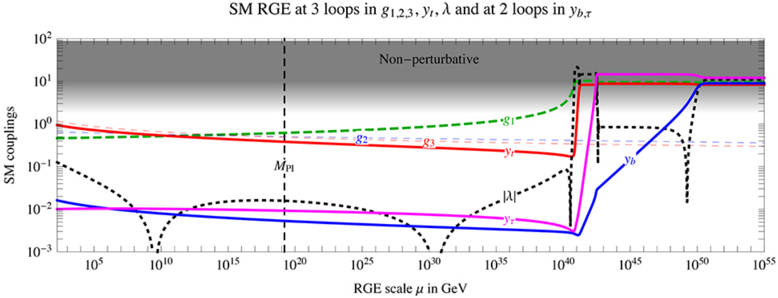
- Free: GUT embedding
- Safe: Veneziano limit

  (D. F. Litim and F. Sannino, “Asymptotic safety guaranteed,” JHEP **1412** (2014) 178.)
- Safe: Large N_f procedure

  (Mann, Meffe, Sannino, Steele, Z.W. Wang and Zhang, “Asymptotically Safe Standard Model via Vectorlike Fermions,” Phys. Rev. Lett. **119** (2017) 261802)

The Standard Model Running Couplings

- Field: Gauge fields + Fermions + Scalars
- Interactions: Gauge ($SU(3) \times SU(2)_L \times U(1)$) + Yukawa (Fermions Mass) + Scalar self-interaction
- Not UV Complete: the theory is not well defined at very high energy scale
- $U(1)$ gauge coupling runs into Landau Pole



G. M. Pelaggi, F. Sannino, A. Strumia and E. Vigiani, Front. in Phys. **5** (2017) 49

Fundamental Theory

- A fundamental theory has an UV fixed point (K. G. Wilson, Phys. Rev. B **4** (1971) 3174.)
- Couplings stop running with the energy scale at the fixed point
- The Standard Model is not a fundamental theory
- Asymptotically Free: non-interacting (Gaussian) fixed point (D. J. Gross and F. Wilczek, Phys. Rev. D **8** (1973) 3633; D. J. Gross and F. Wilczek, Phys. Rev. Lett. **30** (1973) 1343.)
  - non-interacting in the UV
  - coupling runs with logarithmic scale dependence
  - Perturbation theory in UV
- Asymptotically Safe: Interacting fixed point (S. Weinberg(1979). "Ultraviolet divergences in quantum theories of gravitation".)
  - interacting in the UV
  - coupling runs with power law scale dependence
  - Perturbative/Non perturbative theory in UV
  - Smaller critical surface dimension $\Rightarrow$ more IR predictiveness

$U(1)$ Landau Pole Problem

- The Standard Model runs into Landau Pole at UV because of the abelian $U(1)$ gauge group ($SU(2)$ and $SU(3)$ are asymptotically free)
- Two ways to address the $U(1)$ problem
  - Assuming $U(1)$ group is not a fundamental group and should be embedded in a non-abelian group at high energy scale (asymptotically free/safe).
  - $U(1)$ gauge coupling will run into an interacting fixed point. (asymptotically safe)$\Rightarrow$ Highly non-perturbative (without large N_F strategy requiring an extremely large Yukawa coupling)

Game of Asymptotic Safety: One Example

- Consider a special gauge-Yukawa system (Gauge: $SU(3)$ and $SU(2)$). It more or less mimic standard model gauge Yukawa sector.
- Positive and Negative (see also Bond, Hiller, Kowalska and Litim, JHEP **1708** (2017) 004)

$$\begin{aligned}
SU(3): &\quad \beta_3 = -B_3\alpha_3^2 + \left(C_3\alpha_3 + G_3\alpha_2 - D_3\alpha_y\right)\alpha_3^2 \\
SU(2): &\quad \beta_2 = -B_2\alpha_2^2 + \left(C_2\alpha_2 + G_2\alpha_3 - D_2\alpha_y\right)\alpha_2^2 \\
Yukawa: &\quad \beta_y = \left(E\alpha_y - F_2\alpha_2 - F_3\alpha_3\right)\alpha_y
\end{aligned}$$

- At two loop order gauge and one loop order Yukawa coupling, the Yukawa terms are the only negative terms in the gauge RG functions.
- The Yukawa terms occur at next leading order rather than leading order $\Rightarrow$ Non-perturbative Yukawa coupling is required to blance the positive contributions (with lowest dimension of representation) and highly non-perturbative to involve $U(1)$

Non-Perturbative Issue: Veneziaono Limit and Lisa Model

- In Lisa Model, the Veneziano limit is implemented to make the leading order terms as small as possible (D. F. Litim and F. Sannino, “Asymptotic safety guaranteed,” JHEP **1412** (2014) 178.)
- Define $\varepsilon = \frac{N_F}{N_c} - \frac{11}{2}$, the general leading order term of the gauge RG function is:

$$\beta_\alpha = -\frac{4}{3}\varepsilon\alpha^2 + O(\alpha^3)$$

- In the limit $N_F \to \infty$ and $N_c \to \infty$, ε could be as small as possible and the perturbative analysis is under control.
- Large N_c will make it difficult to connect to phenomenologies.
- $U(1)$ Landau pole problem is not addressed

Large N_f Expansion

- $1/N_F$ expansion in Abelian/non-Abelian gauge theory was firstly developed respectively by Pascual and Gracey and later on summarized by Bob Holdom with initial analysis of the pole structure A. Palanques-Mestre and P. Pascual, Commun. Math. Phys. **95** (1984) 277; J. A. Gracey, Phys. Lett. B **373** (1996) 178; B. Holdom, Phys. Lett. B **694**, 74 (2011).

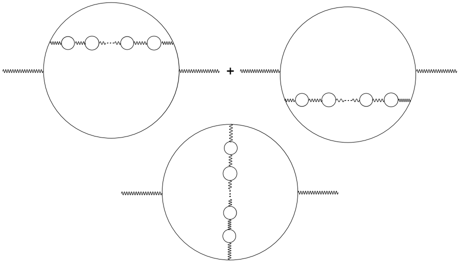


Figure: Higher order photons self-energy diagram

Large N_f Expansion

- Pascual noticed that it is possible to sum up a subset of the diagrams and the resulting power series is so well behaved to provide a closed form expression at $1/N_f$ order
- The resummed U(1) beta function reads:

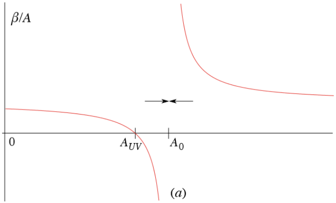
$$\beta_A = \frac{2A^2}{3}\left[1 + \frac{1}{N_f}F_1(A)\right]$$

$$F_1(A) = \frac{3}{4}\int_0^A \mathrm{d}x\,\tilde{F}\left(0, \tfrac{2}{3}x\right), \quad \tilde{F}(0,x) = \frac{(1-x)(1-\frac{x}{3})(1+\frac{x}{2})\Gamma(4-x)}{3\Gamma^2(2-\frac{x}{2})\Gamma(3-\frac{x}{2})\Gamma(1+\frac{x}{2})}$$

- By using the $1/N_f$ expansion, it is possible to involve the loop contributions up to infinity high order.

Pole Structure and UV fixed point

- The summation function $F_1(A)$ has a pole structure at $A = 15/2$.
- $\beta - A$ diagram(B. Holdom, Phys. Lett. B **694**, 74 (2011)).



- The pole structure guarantees the UV fixed point of the gauge coupling
  Mann, Meffe, Sannino, Steele, Z. W. Wang and Zhang, Phys. Rev. Lett. **119** (2017) 261802
- The claim even works in non-perturbative regime (summation function in a closed form).

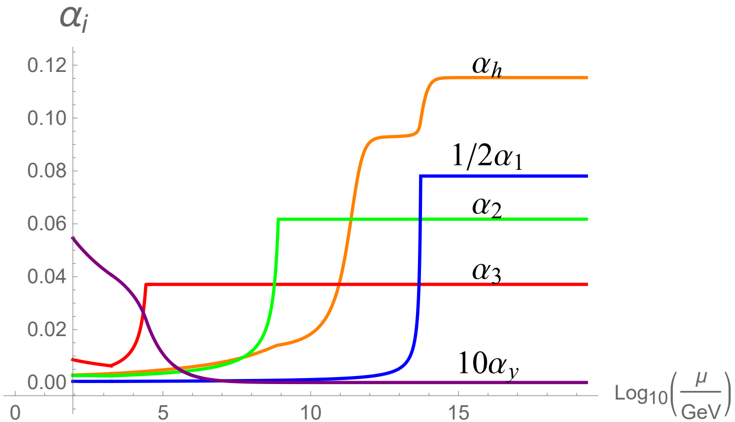
Generalize to the Standard Model: $SU(3) \times SU(2) \times U(1)$

- Holdom's system only involves two kinds fields (gauge field+fermions) and one coupling (gauge coupling g)
- For $U(1)$, it requires $N_F > 16$ while for $SU(3)$, it requires $N_F > 32$ to suppress $1/N_f^2$ contributions.
- Safety can be realized in a more general system: three fields (gauge fields+fermions+scalars) and three couplings (gauge, Yukawa, quartic)
  Mann, Meffe, Sannino, Steele, Z. W. Wang and Zhang, Phys. Rev. Lett. **119** (2017) 261802
- The standard Model can be safe at UV when including reasonably number of vector-like fermions.
- Use vector-like fermions for simplification to not involve extra scalars to generate the mass terms

Safe Standard Model: $SU(3) \times SU(2) \times U(1)$

Mann, Meffe, Sannino, Steele, Z.W. Wang and Zhang, Phys. Rev. Lett. **119** (2017) 261802

- The gauge couplings $\alpha_1, \alpha_2, \alpha_3$ and Higgs quartic coupling α_h are safe while the top Yukawa coupling α_y is free
- The transition scale of the interacting fixed point is dependent on N_f



Large N_F beta Functions: Semi-Simple Gauge

(Antipin, Dondi, Sannino, Thomsen and ZW. Wang, PRD 98 (2018) 016003, arXiv:1803.09770.)

- Vector-like fermions charged under only simple gauge group (PRL **119** (2017) 261802) $\Rightarrow$ semi-simple gauge group (this work)

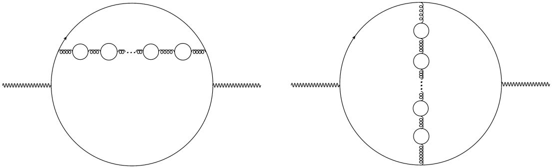


Figure: Feynman diagrams for the 2-point functions giving mixed terms to the beta functions.

- $$\beta_i^{\rm ho} = \frac{2A_i\alpha_i}{3}\left(\frac{d(R_i)H_{1_i}(A_i)}{N_{F_i}\prod_k d\left(R_\psi^k\right)} + \frac{\sum_j d(G_j)\,F_{1_j}(A_j)}{N_{F_i}\prod_k d\left(R_\psi^k\right)}\right)$$

- $U(1)$ problem can not be addressed with a semi-simple gauge $U(1)\times SU(N)$

$U(1)$ Landau Pole Problem

- Two ways to address the $U(1)$ problem
  - Embedding in a non-abelian group
  - $U(1)$ safety with large N_f
- $U(1)$ problem is not successfully addressed in the large N_F framework
  - the mass anomalous dimension blows up at the Abelian pole place (Antipin and Sannino, Phys. Rev. D **97** (2018) 116007, arXiv:1709.02354.)
  - the quartic coupling will also blow up at the abelian pole (Antipin, Dondi, Sannino, Thomsen and ZW. Wang, PRD 98 (2018) 016003.)
  - Semi-simple gauge does not help (Antipin, Dondi, Sannino, Thomsen and ZW. Wang, PRD 98 (2018) 016003.)
  - Yukawa summation does not help (T. Alanne and S. Blasi, Phys. Rev. D **98** (2018) 116004, arXiv:1808.03252.)
- The above flaws motivate the study of a safe GUT theory where U(1) is embedded in a non-abelian group.

Question 2

Can we make the Standard Model UV safe through GUT embedding?

Answer 2

- Safe Pati-Salam: $G_{\text{PS}} = SU(4) \otimes SU(2)_L \otimes SU(2)_R$
  (Emiliano, Francesco, ZW. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.)
- Safe Trinification: $G_{\text{TR}} = SU(3)_C \otimes SU(3)_L \otimes SU(3)_R$
  (ZW. Wang, Balushi, Mann and Jiang, Phys. Rev. D **99** (2019) 115017, arXiv:1812.11085 [hep-ph].)

Safety of Grand Unified Theory: Pati-Salam Model

Emiliano, Francesco, ZW. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- Pati-Salam model is under gauge symmetry group G_{PS}

  J. C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974) Erratum: [Phys. Rev. D **11**, 703 (1975)].

$$G_{\text{PS}} = SU(4) \otimes SU(2)_L \otimes SU(2)_R \,.$$

- The SM quark and lepton fields are unified into the G_{PS} irreducible representations

$$\psi_{Li} = \begin{pmatrix} u_L & u_L & u_L & \nu_L \\ d_L & d_L & d_L & e_L \end{pmatrix}_i \sim (4,2,1)_i \,,$$
$$\psi_{Ri} = \begin{pmatrix} u_R & u_R & u_R & \nu_R \\ d_R & d_R & d_R & e_R \end{pmatrix}_i \sim (4,1,2)_i \,,$$

- Symmetry breaking pattern: $G_{\text{PS}} \xrightarrow{v_R} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$

Pati-Salam Model: Gauge Field Content

Emiliano, Francesco, ZW. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- The gauge fields of G_{PS} can be written as follows:

$$\hat{W}_{L\mu} \equiv \frac{1}{2}\begin{pmatrix} W^0_{L\mu} & \sqrt{2}W^+_{L\mu} \\ \sqrt{2}W^-_{L\mu} & -W^0_{L\mu} \end{pmatrix},$$

$$\hat{W}_{R\mu} \equiv \frac{1}{2}\begin{pmatrix} W^0_{R\mu} & \sqrt{2}W^+_{R\mu} \\ \sqrt{2}W^-_{R\mu} & -W^0_{R\mu} \end{pmatrix},$$

$$\hat{G}_{\mu} \equiv \frac{1}{2}\begin{pmatrix} G_{3\mu}+\frac{G_{8\mu}}{\sqrt{3}}+\frac{B_\mu}{\sqrt{6}} & \sqrt{2}G^+_{12\mu} & \sqrt{2}G^+_{13\mu} & \sqrt{2}X^+_{1\mu} \\ \sqrt{2}G^-_{12\mu} & -G_{3\mu}+\frac{G_{8\mu}}{\sqrt{3}}+\frac{B_\mu}{\sqrt{6}} & \sqrt{2}G^+_{23\mu} & \sqrt{2}X^+_{2\mu} \\ \sqrt{2}G^-_{13\mu} & \sqrt{2}G^-_{23\mu} & -\frac{2G_{8\mu}}{\sqrt{3}}+\frac{B_\mu}{\sqrt{6}} & \sqrt{2}X^+_{3\mu} \\ \sqrt{2}X^-_{1\mu} & \sqrt{2}X^-_{2\mu} & \sqrt{2}X^-_{3\mu} & -\frac{3B_\mu}{\sqrt{6}} \end{pmatrix}$$

- $W^0_{L\mu}$ and $W^\pm_{L\mu}$ correspond to the electroweak (EW) gauge bosons, $G_{3\mu}$, $G_{8\mu}$, $G^\pm_{12\mu}$, $G^\pm_{13\mu}$ and $G^\pm_{23\mu}$ are the $SU(3)_C$ gluons, B_μ is the $B-L$ gauge field, and $X^\pm_{1\mu}$, $X^\pm_{2\mu}$ and $X^\pm_{3\mu}$ are leptoquarks.

Pati-Salam Model: Field Content and Couplings

Emiliano, Francesco, ZW. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- Scalar field $\phi_R \sim (4,1,2)$ triggers Pati-Salam symmetry breaking:

$$\phi_R = \begin{pmatrix} \phi_R^u & \phi_R^0 \\ \phi_R^d & \phi_R^- \end{pmatrix}, \qquad G_{\rm PS} \xrightarrow{v_R} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

- Scalar bi-doublet $\Phi \sim (1,2,2)$ triggers electroweak symmetry breaking:

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} \equiv \begin{pmatrix} \Phi_1 & \Phi_2 \end{pmatrix}$$

- All the couplings:

Gauge Couplings	Yukawa Couplings	Scalar Couplings
$SU(4):\ g_4$	$\psi_{L/R}:\ y,\ y_c$	$\phi_R:\ \lambda_{R1},\ \lambda_{R2}$
$SU(2)_L:\ g_L$	$N_L:\ y_\nu$	portal: $\lambda_{R\Phi_1},\ \lambda_{R\Phi_2},\ \lambda_{R\Phi_3}$
$SU(2)_R:\ g_R$	$F:\ y_F$	$\Phi:\ \lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4$

Table: Gauge, Yukawa and scalar quartic couplings of the Pati-Salam model.

The Attractive Features of Pati-Salam Model

J. C. Pati, Int. J. Mod. Phys. A **32** (2017) 1741013

- Unification of all 16 members of a family (SM matter content + Right Handed neutrino) within one left-right self-conjugate multiplet
- Quantization of electric charge i.e. $Q_{e^-} + Q_P = 0$
- Quark-Lepton unification through $SU(4)$ colour
- The Right Handed neutrino as a compelling member of each family
- Universality for the weak interactions with respect to quarks and leptons
- conservation of parity at a fundamental level
- B-L as a local symmetry
- No proton decay issue
- Possible to be embedded in a simple group $SO(10)$ to address gauge coupling unification

Vector-like Fermions Charges & Large N_F Gauge Beta

Emiliano, Francesco, ZW. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- We consider three sets of vector-like fermions charged under $G_{\rm PS}$, with the charge assignment:

$$N_{F_4}\,(4,1,1) \oplus N_{F_{2L}}\,(1,3,1) \oplus N_{F_{2R}}\,(1,1,2)$$

  where the $N_{F_{2L}}$ vector-like fermions are chosen in the adjoint representation of $SU(2)_L$ to avoid fractional electrical charges.
- The charge assignments are chosen to avoid the extra contributions in the summation of semi-simple group
- The large N_F gauge beta functions are given by:

$$\beta^{tot}_{\alpha_{2L}} = \beta^{1loop}_{\alpha_{2L}} + \beta^{\rm ho}_{\alpha_{2L}} = -6\alpha^2_{2L} + \frac{2A_{2L}\alpha_{2L}}{3}\frac{H_{1_{2L}}\,(A_{2L})}{N_{F_{2L}}}$$

$$\beta^{tot}_{\alpha_{2R}} = \beta^{1loop}_{\alpha_{2R}} + \beta^{\rm ho}_{\alpha_{2R}} = -\frac{14}{3}\alpha^2_{2R} + \frac{2A_{2R}\alpha_{2R}}{3}\frac{H_{1_{2R}}\,(A_{2R})}{N_{F_{2R}}}$$

$$\beta^{tot}_{\alpha_4} = \beta^{1loop}_{\alpha_4} + \beta^{\rm ho}_{\alpha_4} = -18\alpha^2_4 + \frac{2A_4\alpha_4}{3}\frac{H_{1_4}\,(A_4)}{N_{F_4}}\,.$$

Gauge Coupling Unification

Emiliano, Francesco, ZW. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- A sample case of gauge unification with $N_{F_{2L}} = 35,\ N_{F_{2R}} = N_{F_4} = 140$:

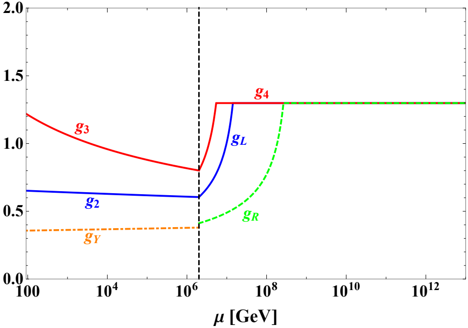
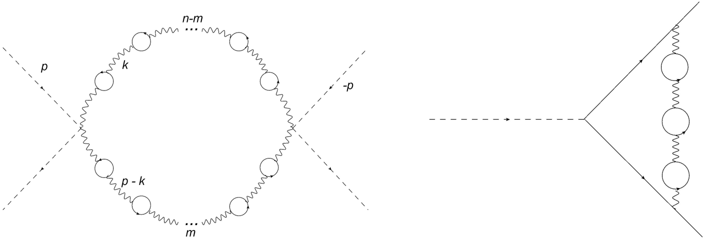


Figure: The dashed line represents the Pati-Salam symmetry breaking scale at $2000\,\mathrm{TeV}$ where all the vector-like fermions are introduced. The three couplings $g_Y,\ g_2,\ g_3$ at the left hand side of the dashed line are determined by the running of the SM gauge couplings.

Large N_F beta Functions: Quartic and Yukawa

(Antipin, Dondi, Sannino, Thomsen and ZW. Wang, PRD 98 (2018) 016003, arXiv:1803.09770.)

- Bubble chain insertion only for gauge couplings (PRL **119** (2017) 261802)
  $\Rightarrow$ all gauge, Yukawa and Quartic couplings (PRD 98 (2018) 016003)



Recipe of Bubble Improved RG Function: Yukawa

(Antipin, Dondi, Sannino, Thomsen and ZW. Wang, PRD 98 (2018) 016003, arXiv:1803.09770.)

See also (Kowalska, Sessolo, JHEP 1804 (2018) 027; Phys.Rev. D97 (2018) 095013)

- Recipe for Yukawa

$$\beta_y = c_1 y^3 + y \sum_\alpha c_\alpha K_\alpha I_y\left(K_\alpha\right), \text{ where}$$

$$I_y\left(K_\alpha\right) = H_\phi\left(0, \tfrac{2}{3}K_\alpha\right)\left(1 + K_\alpha \frac{C_2\left(R_\phi^\alpha\right)}{6\left(C_2\left(R_\chi^\alpha\right) + C_2\left(R_\xi^\alpha\right)\right)}\right),$$

$$H_\phi(0,x) = \frac{(1-\frac{x}{3})\Gamma(4-x)}{3\Gamma^2(2-\frac{x}{2})\Gamma(3-\frac{x}{2})\Gamma(1+\frac{x}{2})}.$$

- The summation function H_ϕ has a pole at $x = 5$ corresponding to $K = 15/2$.
- For models where c_1 and c_α are known, we can immediately obtain the bubble diagram contributions following this recipe.

Recipe of Bubble Improved RG Function: Quartic

(Antipin, Dondi, Sannino, Thomsen and ZW. Wang, PRD 98 (2018) 016003, arXiv:1803.09770.

G. M. Pelaggi, A. D. Plascencia, A. Salvio, F. Sannino, J. Smirnov and A. Strumia, PRD **97** (2018) 095013.)

- Recipe for Quartic Coupling

$$\beta_\lambda = c_1\lambda^2 + \lambda\sum_\alpha c_\alpha K_\alpha I_{\lambda g^2}\left(K_\alpha\right) + \sum_\alpha c'_\alpha K_\alpha^2 I_{g^4}\left(K_\alpha\right) + \sum_{\alpha<\beta} c_{\alpha\beta}K_\alpha K_\beta I_{g_1^2 g_2^2}\left(K_\alpha,\, K_\beta\right)\,,$$

$$I_{\lambda g^2}\left(K_\alpha\right) = H_\phi\left(0,\, \tfrac{2}{3}K_\alpha\right)$$

$$I_{g^4}\left(K_\alpha\right) = H_\lambda\left(1,\, \tfrac{2}{3}K_\alpha\right) + K_\alpha \frac{dH_\lambda\left(1,\, \frac{2}{3}K_\alpha\right)}{dK_\alpha}$$

$$I_{g_1^2 g_2^2}\left(K_\alpha,\, K_\beta\right) = \frac{1}{K_\alpha - K_\beta}\left[K_\alpha H_\lambda\left(1,\, \tfrac{2}{3}K_\alpha\right) - K_\beta H_\lambda\left(1,\, \tfrac{2}{3}K_\beta\right)\right]$$

$$H_\lambda(1,x) = \frac{(1-\frac{x}{3})\Gamma(4-x)}{6\Gamma^3(2-\frac{x}{2})\Gamma(1+\frac{x}{2})}\,.$$

Pole Structure

(Antipin, Dondi, Sannino, Thomsen and ZW. Wang, PRD 98 (2018) 016003, arXiv:1803.09770.

G. M. Pelaggi, A. D. Plascencia, A. Salvio, F. Sannino, J. Smirnov and A. Strumia, PRD **97** (2018) 095013.)

- Pole in the summation functions:

$$H_\phi\left(0, \tfrac{2}{3}K_\alpha\right) \sim \frac{1}{\frac{15}{2} - K_\alpha}, \; H_\lambda\left(1, \tfrac{2}{3}K_\alpha\right) \sim \frac{1}{K_\alpha - \frac{15}{2}},$$
$$\frac{\partial}{\partial K_\alpha} H_\lambda\left(1, \tfrac{2}{3}K_\alpha\right) \sim -\frac{1}{\left(K_\alpha - \frac{15}{2}\right)^2}$$

- Pole structure of Yukawa coupling

$$\beta_y = c_1 y^3 + y K_\alpha \left(\frac{1}{K_\alpha - \frac{15}{2}}\right)(c_2 + c_3 K_\alpha)$$

- Pole structure of Quartic Coupling

$$\beta_\lambda = c_1\lambda^2 + c_2\lambda K_\alpha \left(\frac{1}{K_\alpha - \frac{15}{2}}\right) + c_3 K_\alpha^2 \left(\frac{1}{K_\alpha - \frac{15}{2}} - \frac{1}{\left(K_\alpha - \frac{15}{2}\right)^2}\right)$$

Pole Structure: Advantage of GUT

Emiliano, Francesco, ZW. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- The beta functions of Yukawa and Quartic couplings have the poles only at the Abelian pole where $K = \frac{15}{2}$
- When Abelian gauge coupling reaches a fixed point, the Yukawa coupling will be asymptotically free (negative pole contributions and multiplicative proportional to the Yukawa coupling itself).
- The quartic coupling will blow up (very negative) due to the negative pole contributions and not multiplicative proportional to the quartic coupling itself for the g^4 term.
- Higher order singular structure is involved in the quartic beta. When the gauge coupling logarithmically approaching the pole, the pole contribution in the quartic beta will blow up.
- In certain GUT (only Non-abelian Gauge group involved), the UV fixed point at $K = 3$ in gauge sector is away from the pole in the quartic and Yukawa couplings allowing the existence of UV fixed points in all couplings.
- Pati-Salam model has the potential to be asymptotically safe

Classification of UV Fixed Point: Relevant & Irrelevant

Emiliano, Francesco, ZW. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- Vacuum stability condition

$$\lambda_{R1} + \lambda_{R2} > 0 \qquad \lambda_1 - \lambda_2 + \lambda_4 > 0, \qquad \lambda_1 > 0$$

λ_1	λ_2	λ_3	λ_4	$\lambda_{R\Phi_1}$	$\lambda_{R\Phi_{2,3}}$	λ_{R1}	λ_{R2}	y	y_c	y_ν	y_F
0.12	0.05	0	0.13	0.02	0	0.13	-0.01	0.78	0.78	0.84	0
Irev	Rev	0	Irev	Irev	0	Irev	Rev	Irev	Irev	Irev	0

λ_1	λ_2	λ_3	λ_4	$\lambda_{R\Phi_1}$	$\lambda_{R\Phi_{2,3}}$	λ_{R1}	λ_{R2}	y	y_c	y_ν	y_F
0.05	0.02	0	0.01	0.04	0	0.02	0.08	0.24	0.24	0.57	0.74
Irev	Rev	0	Irev	Irev	0	Irev	Irev	Irev	Irev	Irev	Irev

Table: These tables summarize the sample UV fixed point solution with two sample values ($N_{F_{2L}} = 40,\ N_{F_{2R}} = 150,\ N_{F_4} = 200$; $N_{F_{2L}} = 40,\ N_{F_{2R}} = 130,\ N_{F_4} = 130$) involving the bubble diagram contributions in the Yukawa and quartic RG beta functions. The UV fixed point solutions of the couplings are classified with relevant (Rev) and irrelevant (Irev) characteristics. "0" denotes Gaussian Fixed points.

RG Flow: Gauge and Yukawa

Emiliano, Francesco, ZW. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- RG running of the gauge and Yukawa couplings by using the UV to IR approach.

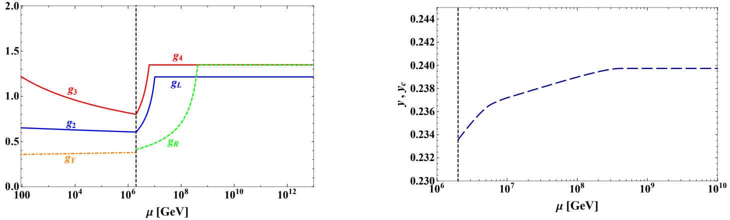
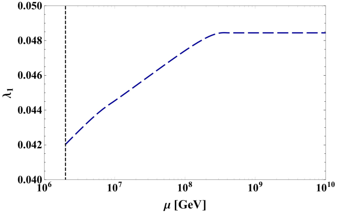


Figure: We have chosen $N_{F2} = 40$, $N_{F3} = 130$, $N_{F4} = 130$. We have used the matching conditions at IR to set the initial conditions of g_L, g_R, g_4 at IR. For simplification, we have assumed that the vector-like fermions under different symmetry group are exactly introduced at the symmetry breaking scale of the Pati-Salam group at $2000\,\mathrm{TeV}$ shown with a dashed line.

Emiliano, Francesco, ZW. Wang, arXiv:1807.03669, accepted by PRD

- RG running of the Quartic Coupling by using the UV to IR approach.



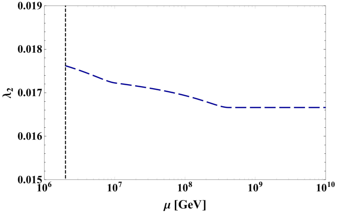


Figure: We have chosen $N_{F2} = 40,\ N_{F3} = 130,\ N_{F4} = 130$. For simplification, we have assumed that the vector-like fermions under different symmetry group are exactly introduced at the symmetry breaking scale of the Pati-Salam group at $2000\,\mathrm{TeV}$ shown with a dashed line.

RG Flow: Quartic Coupling of ϕ_R

Emiliano, Francesco, ZW. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- RG running of the Quartic Coupling by using the UV to IR approach.

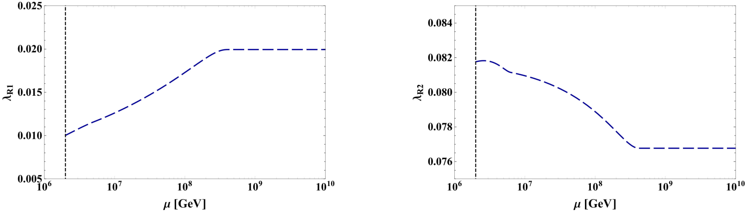


Figure: We have chosen $N_{F2} = 40,\ N_{F3} = 130,\ N_{F4} = 130$. For simplification, we have assumed that the vector-like fermions under different symmetry group are exactly introduced at the symmetry breaking scale of the Pati-Salam group at $2000\,\mathrm{TeV}$ shown with a dashed line.

Matching the Standard Model: Top Yukawa Coupling

Emiliano, Francesco, ZW. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- The top Yukawa mass term is given by (with CP symmetry $y = y_c$ and $\tan\beta = 1$):

$$m_{\text{top}} = (y \sin\beta + y_c \cos\beta)v \rightarrow \sqrt{2}yv = m_{\text{top}}$$

- Thus at electroweak scale, y is smaller than the conventional SM top Yukawa coupling value $\sim \frac{0.93}{\sqrt{2}} \sim 0.66$
- It can be shown that by choosing $N_{F2} = 32,\ N_{F3} = 108,\ N_{F4} = 56$, we obtain $y \sim 0.614$ as required.

Matching the Standard Model: Higgs Quartic Coupling

Emiliano, Francesco, ZW. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- The low energy scalar sector of the Pati-Salam model is the two Higgs doublet model

$$\begin{aligned} V_H = & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left(m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{H.c.} \right) \\ & + \frac{1}{2} \bar{\lambda}_1 \left(\Phi_1^\dagger \Phi_1 \right)^2 + \frac{1}{2} \bar{\lambda}_2 \left(\Phi_2^\dagger \Phi_2 \right)^2 + \bar{\lambda}_3 \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_2^\dagger \Phi_2 \right) \\ & + \bar{\lambda}_4 \left(\Phi_1^\dagger \Phi_2 \right) \left(\Phi_2^\dagger \Phi_1 \right) + \frac{1}{2} \bar{\lambda}_5 \left[\left(\Phi_1^\dagger \Phi_2 \right)^2 + \left(\Phi_2^\dagger \Phi_1 \right)^2 \right] . \end{aligned}$$

- The matching conditions (by comparing with the UV bi-doublet scalar potential) are (at the Pati-Salam symmetry breaking scale):

$$\bar{\lambda}_1 = \lambda_1 , \quad \bar{\lambda}_2 = \lambda_1 , \quad \bar{\lambda}_3 = 2\lambda_1 , \quad \bar{\lambda}_4 = 4 \left(-2\lambda_2 + \lambda_4 \right) , \quad \bar{\lambda}_5 = 4\lambda_2$$

Matching the Standard Model: Higgs Quartic Coupling

Emiliano, Francesco, ZW. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- The mass matrix (neutral scalar fields) of the two Higgs doublet model is given by:

$$M^2_{\text{neutral}} = \begin{bmatrix} \frac{m_{12}^2 v_2}{v_1} + 2\bar{\lambda}_1 v_1^2 & -m_{12}^2 + \left(\bar{\lambda}_3 + \bar{\lambda}_4 + \bar{\lambda}_5\right) v_1 v_2 \\ -m_{12}^2 + \left(\bar{\lambda}_3 + \bar{\lambda}_4 + \bar{\lambda}_5\right) v_1 v_2 & \frac{m_{12}^2 v_2}{v_1} + 2\bar{\lambda}_2 v_2^2 \end{bmatrix}$$

- This matrix is defined at the electroweak scale. By using the two Higgs double beta functions and the matching condtions, we obtain the quartic couplings $\lambda_i \ (i = 1, \cdots 5)$ at the electroweak scale.
- The phenomenological constraint are: both of the eigenvalues of the mass matrix should be positive and the lighter one should close to the $125\,\mathrm{GeV}$ Higgs mass.

Matching the Standard Model

Emiliano, Francesco, ZW. Wang, Phys.Rev. D98 (2018) 115007, arXiv:1807.03669.

- It can be shown that by choosing $N_{F2} = 32,\ N_{F3} = 108,\ N_{F4} = 56$, we obtain:

$$\bar{\lambda}_1 = 0.135,\ \bar{\lambda}_2 = 0.360,\ \bar{\lambda}_3 = 0.25,\ \bar{\lambda}_4 = -0.379,\ \bar{\lambda}_5 = 0.259,\ y = 0.614\,.$$

- Matching of the scalar quartic coupling: two neutral scalar mass with $M_{H1} \sim 125\,\mathrm{GeV}$ (lighter Higgs) and the heavier one $M_{H2} > 238\,\mathrm{GeV}$ with $m_{12} > 150\,\mathrm{GeV}$
- Matching of the top Yukawa coupling: the IR value of y is around 0.66 as required
- Asymptotic Safe Pati-Salam model can roughly match the SM at IR.
- In this minimal model, most of the RG flows lead to much lighter Higgs mass and Pati-Salam symmetry breaking scale above $10000\,\mathrm{TeV}$ is required.

Safety of Grand Unified Theory: Trinification Model

Z. W. Wang, A. Al Balushi, R. Mann and H. M. Jiang, Phys. Rev. D **99** (2019) 115017, arXiv:1812.11085.

- Trinification model is under gauge symmetry group G_{TR}
  K. S. Babu, X. G. He and S. Pakvasa, Phys. Rev. D **33** (1986) 763.

$$G_{\text{TR}} = SU(3)_C \otimes SU(3)_L \otimes SU(3)_R$$

- The coloured fermions are given by $\psi_{Q_L} \sim (3, \bar{3}, 1)$ & $\psi_{Q_R} \sim (3, 1, \bar{3})$:

$$\psi_{Q_L} = \begin{pmatrix} u_L^1 & u_L^2 & u_L^3 \\ \mathscr{D}_L^1 & \mathscr{D}_L^2 & \mathscr{D}_L^3 \\ \mathscr{D}_L'^1 & \mathscr{D}_L'^2 & \mathscr{D}_L'^3 \end{pmatrix}, \psi_{Q_R} = \begin{pmatrix} u_R^1 & u_R^2 & u_R^3 \\ \mathscr{D}_R^1 & \mathscr{D}_R^2 & \mathscr{D}_R^3 \\ \mathscr{D}_R'^1 & \mathscr{D}_R'^2 & \mathscr{D}_R'^3 \end{pmatrix},$$

- The lepton content in this minimal Trinification model is given by:

$$\psi_E = \begin{pmatrix} \bar{\nu}_L' & e_L' & e_L \\ \bar{e}_L' & \nu_L' & \nu_L \\ \bar{e}_R & \bar{\nu}_R & \nu' \end{pmatrix} \sim (1, 3, \bar{3}),$$

The Symmetry Breaking of Trinification Model

Z. W. Wang, A. Al Balushi, R. Mann and H. M. Jiang, Phys. Rev. D **99** (2019) 115017, arXiv:1812.11085.

- To induce the breaking of $G_{\rm TR}$ to the SM gauge group, we introduce two scalar triplet fields $\Phi_1,\ \Phi_2$ which transform under the $G_{\rm TR}$ as $(1,3,3)$:

$$\Phi_a = \begin{pmatrix} \phi_1^a & \phi_2^a & \phi_3^a \\ S_1^a & S_2^a & S_3^a \end{pmatrix}, \qquad (a=1,2)\ ,$$

  where $\phi_i^a,\ (i=1,2,3)$ denotes the Higgs doublets while $S_i^a,\ (i=1,2,3)$ denotes the singlets.

- The vacuum configuration of the scalar triplet is given as:

$$\langle\Phi_1\rangle = \begin{pmatrix} u_1 & 0 & 0 \\ 0 & u_2 & 0 \\ 0 & 0 & v_1 \end{pmatrix}, \qquad \langle\Phi_2\rangle = \begin{pmatrix} n_1 & 0 & n_3 \\ 0 & n_2 & 0 \\ v_2 & 0 & v_3 \end{pmatrix}.$$

- G_{TR} to left right model (through v_3, v_1):
  $SU(3)_C \times SU(3)_L \times SU(3)_R \to SU(3)_C \times SU(2)_R \times SU(2)_L \times U(1)_{B-L}$

The Yukawa Sector of Trinification Model

Z. W. Wang, A. Al Balushi, R. Mann and H. M. Jiang, Phys. Rev. D **99** (2019) 115017, arXiv:1812.11085.

- The Yukawa terms for the quark sector is given by:

$$\mathcal{L}^{Q}_{\text{Yuk}} = m_{d'}\overline{d}'_L d'_R + \sum_{a=1}^{2} y_{\psi_{Qa}} \Big[\overline{Q}\left(-s_\alpha d_R + c_\alpha d'_R\right)\phi_1^a + \overline{Q} u_R \phi_2^a + \overline{Q}\left(c_\alpha d_R + s_\alpha d'_R\right)\phi_3^a \Big] + \text{h.c.}$$

- The Yukawa terms $\mathcal{L}^{E}_{\text{Yuk}}$ for the lepton sector is given by:

$$m_E \overline{E}_L E_R + \sum_{a=1}^{2} y_{\psi_{Ea}} \Big\{ -\left[\left(-c_\beta \nu_R - s_\beta \nu'\right) E_R - \left(c_\beta \overline{E}_L + s_\beta \overline{L}_L\right) e_R\right]\phi_1^a + \left(\overline{E}_L \nu' - \overline{L}_L \nu_R\right)\phi_2^a + \left[\left(s_\beta \nu_R - c_\beta \nu'\right) E_R - \left(-s_\beta \overline{E} + c_\beta \overline{L}\right) e_R\right]\phi_3^a \Big\} + \text{h.c.}$$

- The quark and lepton masses are given by:

$$m_t = y_{\psi_{Q1}} u_2, \quad m_b = y_{\psi_{Q1}} u_1 s_\alpha\,, \quad m_e = y_{\psi_{E1}} u_1 s_\beta\,, \quad m_{\nu_L,\nu_R} = y_{\psi_{E1}} u_2\,.$$

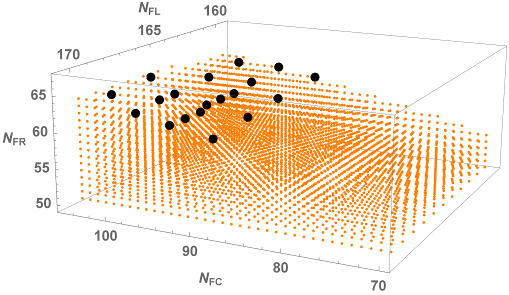
The Matching of the Standard Model

Z. W. Wang, A. Al Balushi, R. Mann and H. M. Jiang, Phys. Rev. D **99** (2019) 115017, arXiv:1812.11085.

- By choosing $N_{FC} = 95,\ N_{FL} = 165,\ N_{FR} = 62$, we obtain:

$$M_{\text{Higgs}}^{\text{Pre}} = 125\,\text{GeV} \qquad y_{\text{top}}^{\text{Pre}} = 0.806, \qquad y_{\text{bottom}}^{\text{Pre}} = 0.019, \qquad y_{\text{tau}}^{\text{Pre}} = 0.011$$
$$M_{\text{Higgs}}^{\text{SM}} = 126\,\text{GeV} \qquad y_{\text{top}}^{\text{SM}} = 0.780, \qquad y_{\text{bottom}}^{\text{SM}} = 0.019, \qquad y_{\text{tau}}^{\text{SM}} = 0.008\,.$$

- 3D scan of the parameter space



Conclusion So Far

- Asymptotically Safe Standard Model is feasible through GUT embedding with large N_f.
- U(1) problem is addressed.
- The requirement of safety at UV has strong predictive power at IR (selecting the parameter space at IR)
- Both Safe Pati-Salam model and the Safe Trinification Model can roughly match the SM at IR.

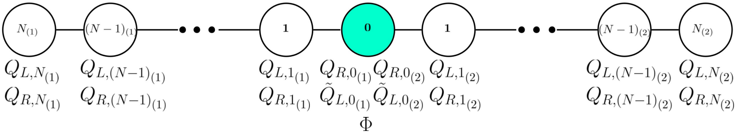
Question 3

Can we address the Yukawa Hierarchies in the Safe GUT?

Answer 3: Safe Clockwork

(Sannino, Smirnov and ZW. Wang, arXiv:1902.05958, accepted by PRD.

- We combine the clockwork mechanism with the Safe Pati-Salam model



- The large number of vector-like fermions play the role of clockwork gears
- We introduce $N_{(i)}$, $(i = 1, 2)$ pair of vector like fermions $\left(Q_{L,1_{(i)}}, Q_{R,1_{(i)}}\right), \cdots, \left(Q_{L,N_{(i)}}, Q_{R,N_{(i)}}\right)$ with one extra chiral fermion $Q_{R,0_{(i)}}$ (i.e. each generation (i) of the PS fermions is associated with a clockwork chain with $N_{(i)}$ nodes).
- For any number of $N_{(i)}$, the chiral fermions $Q_{L,N_{(i)}}$ and $Q_{R,N_{(i)}}$ are charged respectively under the fundamental representation $(4, 2, 1)$ and $(4, 1, 2)$ of PS gauge group $G_{PS} = SU(4) \times SU(2)_L \times SU(2)_R$.

Safe Clockwork

(Sannino, Smirnov and ZW. Wang, arXiv:1902.05958, accepted by PRD.

- We also introduce $\tilde{Q}_{L,0_{(1)}}$ and $\tilde{Q}_{L,0_{(2)}}$ which will only interact respectively with the zero node fields $Q_{R,0_{(1)}}$ and $Q_{R,0_{(2)}}$ through the Yukawa contributions i.e.

$$\mathcal{L}^{Q}_{\rm Yuk} = y_1 \bar{\tilde{Q}}_{L,0_{(1)}} \Phi \, Q_{R,0_{(1)}} + y_2 \bar{\tilde{Q}}_{L,0_{(2)}} \Phi \, Q_{R,0_{(2)}} \, ,$$

- The clockwork mechanism is realized by the following clockwork chain interaction:

$$\mathcal{L}^{Q_R}_{\rm clock} = -m_{(1)} \sum_{j=1}^{N_{(1)}} \left(\bar{Q}_{L,j_{(1)}} Q_{R,j_{(1)}} - q_{(1)} \bar{Q}_{L,j_{(1)}} Q_{R,j-1_{(1)}} \right)$$
$$- m_{(2)} \sum_{j=1}^{N_{(2)}} \left(\bar{Q}_{L,j_{(2)}} Q_{R,j_{(2)}} - q_{(2)} \bar{Q}_{L,j_{(2)}} Q_{R,j-1_{(2)}} \right) \, ,$$

- For simplicity, we set $q_{(1)} = q_{(2)} = q$ and $m_{(1)} = m_{(2)} = m$ (i.e. all the clockwork vectorlike fermions are introduced at one scale). Actually, when turning on the difference between q and m, we will have more freedom and bigger parameter space to explore.

Safe Clockwork

(Sannino, Smirnov and ZW. Wang, arXiv:1902.05958, accepted by PRD.

- After diagonalizing the mass matrix, we obtain $M_{Q_{(i)}} = diag\left(0, M_{1_{(i)}}, \cdots, M_{N_{(i)}}\right), (i = 1, 2)$ where there is always one massless mode $\psi_{R,0_{(i)}}, (i = 1, 2)$.
- The massless modes $\psi_{R,0_{(i)}}, (i = 1, 2)$ overlaps with the fields at the zero node of the chain with a suppression factor i.e. $\psi_{R,0_{(i)}} = 1/q^{N_{(i)}} Q_{R,0_{(i)}}$.
- The Yukawa coupling of the $i-$th generations of the PS fermions which originates from the Yukawa interaction terms between $\tilde{Q}_{L,0_{(i)}}$ and the massless mode $\psi_{R,0_{(i)}}$ will also be suppressed by $1/q^{N_{(i)}}$ leading to:

$$
\begin{aligned}
\mathcal{L}_{\rm Yuk}^{\rm eff} &= y_1^{\rm eff} \bar{\tilde{Q}}_{L,0_{(1)}} \Phi\, \psi_{R,0_{(1)}} + y_2^{\rm eff} \bar{\tilde{Q}}_{L,0_{(2)}} \Phi\, \psi_{R,0_{(2)}} \\
&= \frac{y_1}{q^{N_{(1)}}} \bar{\tilde{Q}}_{L,0_{(1)}} \Phi\, \psi_{R,0_{(1)}} + \frac{y_2}{q^{N_{(2)}}} \bar{\tilde{Q}}_{L,0_{(2)}} \Phi\, \psi_{R,0_{(2)}}
\end{aligned}
$$

Safe Clockwork

(Sannino, Smirnov and ZW. Wang, arXiv:1902.05958, accepted by PRD.

- The clockwork vector-like fermions are charged under G_{PS} with the following charge assignment:

$$N_F\,(4,1,2) \oplus N_F\,(4,2,1)\ , \quad N_F = N_{(1)} + N_{(2)}\,.$$

- We searched the full parameter space in the range of $N_F \in (10,\, 200)$ and find for $N_F = 13$ we can match both the Higgs mass and the top Yukawa coupling at the electroweak scale.
- The relations among $q^{N_{(1)}}$, $q^{N_{(2)}}$ and the light quark masses are

$$q^{N_{(1)}} = \frac{m_{top}}{m_u},\ q^{N_{(2)}} = \frac{m_{top}}{m_c},\ N_{(1)} + N_{(2)} = 13\,,$$

where $m_{top} = 173\,\mathrm{GeV}$, $m_c = 1.29\,\mathrm{GeV}$ and $m_u = 2.3\,\mathrm{MeV}$.
- By solving above Eq., we find

$$N_{(1)} = 9, \quad N_{(2)} = 4, \quad q = 3.46\,.$$

Question 4

Is asymptotic safety compatible with the Coleman-Weinberg symmetry breaking?

Answer 4

- No Coleman-Weinberg with only simple gauge group
  (Abel and Sannino, Phys. Rev. D **96** (2017) 055021.)
- Coleman-Weinberg in semi-simple gauge theory (Large N_c, N_s, N_f)
  (Abel, Molgaard and Sannino, Phys. Rev. D **99** (2019) 035030.)
- Coleman-Weinberg in Safe Pati-Salam and Trinification with large N_f. There exists strong first order phase transition, which might provide interesting gravitational wave signals. (in preparation with Francesco Sanino and Wei-Chih Huang)

Coleman-Weinberg in Safe Pati-Salam with large N_f

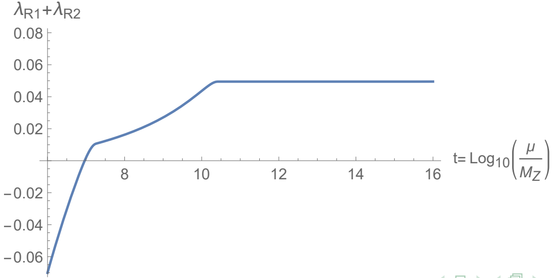
(in preparation with Francesco Sanino and Wei-Chih Huang)

- Scalar field $\phi_R \sim (4,1,2)$ triggers Pati-Salam symmetry breaking:

$$\phi_R = \begin{pmatrix} \phi_R^u & \phi_R^0 \\ \phi_R^d & \phi_R^- \end{pmatrix}, \qquad G_{\rm PS} \xrightarrow{v_R} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

- Above the Pati-Salam symmetry breaking scale, the scalar potential of ϕ_R is written as:

$$V(\phi_R) = \lambda_{R1} \mathrm{Tr}^2 \left(\phi_R^\dagger \phi_R\right) + \lambda_{R2}\, \mathrm{Tr}\left(\phi_R^\dagger \phi_R \phi_R^\dagger \phi_R\right)$$



Coleman-Weinberg in Safe Pati-Salam with large N_f

(in preparation with Francesco Sanino and Wei-Chih Huang)

- The vacuum is UV stable and IR symmetry breaking
- The Pati-Salam symmetry is dynamically broken at $10^7 \mathrm{GeV}$ which aligns with the phenomenological lower bounds of $2000\,\mathrm{TeV}$ from the Kaon decay $\mathrm{Br}\,(K_L \to \mu^{\pm} e^{\mp}) < 4.7 \times 10^{-12}$.
- The Coleman-Weinberg coupling solutions also lead to strong first order phase transition.
- Other safe solutions without Coleman-Weinberg can only lead to second order phase transition.
- Interesting Gravitational Wave signals?

Question 5

Can we construct an asymptotically safe Standard Model satisfying the naturalness criterion/ without fundamental scalars?

Answer 5

- Towards a fundamental safe theory of composite Higgs and Dark Matter
  (G. Cacciapaglia, S. Vatani, T. Ma and Y. Wu, arXiv:1812.04005 [hep-ph].)

Question 6

How to test asymptotic safety?

Answer 6

- Safety versus triviality on the lattice
  (V. Leino, T. Rindlisbacher, K. Rummukainen, F. Sannino and K. Tuominen, arXiv:1908.04605 [hep-lat].
- Any model independent phenomenological signatures?

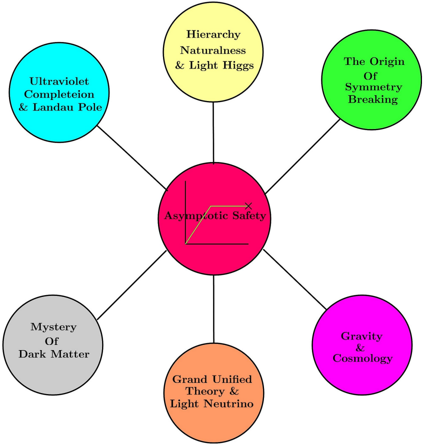
Question 7

How gravity will change this picture?

Conclusion

- Asymptotically Safe Standard Model is feasible through GUT embedding with large N_f.
- U(1) problem is addressed.
- The requirement of safety at UV has strong predictive power at IR (selecting the parameter space at IR)
- Both Safe Pati-Salam model and the Safe Trinification Model can roughly match the SM at IR.
- The Yukawa Hierarchies can be addressed through the Clockwork Mechanism.
- Both Pati-Salam symmetry group and Trinification group can be broken dynamically through Coleman-Weinberg mechanism and strong first order phase transition occurs.

The Power of Asymptotic Safety



Acknowledgements

Thank You

Bottom-Top Mass Splitting

Emiliano, Francesco, ZW. Wang, arXiv:1807.03669

- To obtain the bottom-top mass splitting, we introduce new vector-like fermion $F \sim (10,1,1)$ with mass M_F and Yukawa interactions:

$$\mathcal{L}^F_{\text{Yuk}} \;=\; y_F \,\mathrm{Tr}\left(\overline{F_L}\,\phi_R^T\, i\tau_2\,\psi_R\right) + \text{h.c.} \quad F \;=\; \begin{pmatrix} S & B\sqrt{2} \\ B^T\sqrt{2} & E \end{pmatrix}$$

- The bottom quark Dirac mass term:

$$\mathcal{L}^b_{\text{mass}} \;=\; \begin{pmatrix} \overline{b_L} & \overline{B_L} \end{pmatrix} \begin{pmatrix} m_t & 0 \\ m_B & M_F \end{pmatrix} \begin{pmatrix} b_R \\ B_R \end{pmatrix} + \text{h.c.}\,,$$

- We obtain:

$$m_b \;=\; \sqrt{2}\, m_\tau \;\approx\; \frac{M_F\, m_t}{\sqrt{2}\, m_B} \qquad m_B \equiv y_F\, v_R/\sqrt{2}$$

Neutrino Mass

Emiliano, Francesco, ZW. Wang, arXiv:1807.03669

- In order to split the top and neutrino mass, we implement the inverse seesaw mechanism by adding a new chiral fermion singlet $N_L \sim (1,1,1)$, which has Yukawa interaction

$$\mathcal{L}^{N}_{\rm Yuk} \;=\; -y_\nu\, \overline{N_L} {\rm Tr}\left[\phi^\dagger_R\, \psi_R\right] \;+\; {\rm h.c.}$$

- It generates a Dirac mass term $M_R \overline{N_L} \nu_R$, with $M_R \equiv y_\nu v_R \sim\; 10000\,{\rm GeV}$.
- The Majorana mass term for the neutral fermion fields reads:

$$\mathcal{L}^{\nu}_{\rm mass} \;=\; -\frac{1}{2} \begin{pmatrix} \overline{\nu^c_R} & \overline{\nu_R} & \overline{N^c_R} \end{pmatrix} \begin{pmatrix} 0 & m_t & 0 \\ m_t & 0 & M_R \\ 0 & M_R & 0 \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu^c_L \\ N_L \end{pmatrix} + {\rm h.c.}$$

- By introducing very light Majorana mass term for $-\frac{1}{2} M_N \overline{N^c_R}\, N_L$, we obtain one light active Majorana neutrino ν_τ with mass

$$m_{\nu_\tau} = M_N \frac{m_t^2}{m_D^2}; \quad m_D = \sqrt{m_t^2 + M_R^2}$$

3D Phase Diagrams

- 3D Stream Plot in $\alpha - \lambda_1 - \lambda_2$ parameter space.