Weyl Consistency Conditions:
Predicting the shape of β-functions

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Outline

C. Poole, AET [arXiv:1901.02749, 1906.04625]

1. Motivation and Osborn's Equation
2. Formalism and tensor–graph identification
3. Computation and results
4. Constraining the treatment of γ_5

Outline

Motivation

- β-function are of interest in BSM physics and in AS, but limited by perturbative computations.
- Local RG constrains β functions: 6D ϕ^3, SUSY, 4D Yukawa...
- What can we say about generic, four-dimensional, renormalizable theories?
- We can provide a hint to the treatment of γ_5 in dimensional regularization.

Osborn's equation

Parametrize the anomaly $\Delta_\sigma W = \int \mathrm{d}^d x \sqrt{\gamma} \mathcal{A}_\sigma$ and set

$$[\Delta_\sigma, \Delta_{\sigma'}] W = 0$$

to obtain Osborn's equation

$$\partial_I \hat{A} \equiv \frac{\partial \hat{A}}{\partial g^I} = \hat{T}_{IJ} B^J$$

- Proposed A-function: $\hat{A}$
- Would-be metric: T_{IJ}
- Modified β-function: $B^I = \beta^I - (S\,g)^I$

Osborn '89, '91, Jack and Osborn '90, '13, Baume *et al.* '14.

Weyl consistency conditions

Osborn's Equation, now what?

$$\partial_I \hat{A}(g) = \hat{T}_{IJ}(g) \left[\beta^J(g) - (S(g)\, g)^J\right]$$

Weyl consistency conditions

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Largely unknown

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Jack, Osborn '90 Fortin, Grinstein, Stergiou [1208.3674]

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Expand OE in polynomials of the couplings, e.g.

$$\beta^I = \sum_n c_n P_n(g), \qquad \hat{A} = \sum_n a_n Q_n(g),$$

leads to more constraints than there are unknown coefficients of $\hat{A}$, $\hat{T}_{IJ}$ and S.

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$\Longrightarrow$ Constraints on the coefficients of β^I.

Jack, Osborn '90 Jack, Poole [1411.1301,...]

Outline

β-functions in renormalizable theories

Most general renormalizable theory in 4D (ignoring relevant couplings):

$$\mathscr{L} = -\sum_u \frac{1}{4g_u^2} F^{A_u}_{u,\mu\nu} F^{A_u\mu\nu}_u + \tfrac{1}{2}(D_\mu\phi)_a(D^\mu\phi)_a + i\psi^\dagger_i \bar{\sigma}^\mu (D_\mu\psi)^i \\ - \tfrac{1}{2}\left(Y_{aij}\psi^i\psi^j + \mathrm{H.c.}\right)\phi_a - \tfrac{1}{24}\lambda_{abcd}\phi_a\phi_b\phi_c\phi_d$$

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Present status of the β-functions: 3–2–2

- Gauge β-function to 2.9 (now 3) loop-orders
- Yukawa β-function to 2 loop-orders
- Quartic β-function to 2 loop-orders

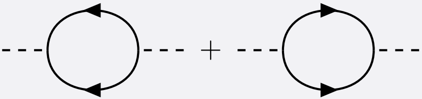
Macachek, Vaughn '83, '84, '85 Luo, Wang, Xiao [hep-ph/0211440]

Pickering, Gracey, Jones [hep-ph/0104247]

Luo, Xiao [hep-ph/0212152]

Fermion sector

Fermions can travel both directions in fermion loops:



$$\sim \mathrm{Tr}[Y_a Y_b^* + Y_a^* Y_b]$$

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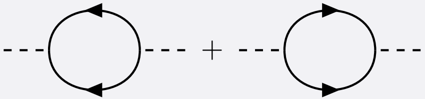

$$\sim \mathrm{Tr}[Y_a Y_b^* + Y_a^* Y_b]$$

Put fermion indices in a "Real representation"

$$\Psi_i = \begin{pmatrix} \psi \\ \psi^\dagger \end{pmatrix}, \qquad y_a = \begin{pmatrix} Y_a & 0 \\ 0 & Y_a^* \end{pmatrix}, \qquad T_u^{A_u} = \begin{pmatrix} T_{\psi,u}^{A_u} & 0 \\ 0 & -(T_{\psi,u}^{A_u})^* \end{pmatrix},$$

so that

$$\mathscr{L} \supset \tfrac{i}{2}\Psi^{\mathrm{T}} \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} D_\mu \Psi - \tfrac{1}{2}\Psi^{\mathrm{T}} y_a \Psi \phi^a$$

$$\mathrm{Tr}[Y_a Y_b^* + Y_a^* Y_b] = \mathrm{Tr}[y_a \tilde{y}_b]$$

Treatment of gauge couplings

Gauge group: $\mathcal{G} = \times_u \mathcal{G}_u = \mathrm{U}(1)_1 \times \ldots \times \mathrm{U}(1)_n \times \mathcal{G}_{n+1} \times \ldots$

$$\mathscr{L} \supset -\sum_{u>n} \frac{1}{4g_u^2} F_{u,\mu\nu}^{A_u} F_u^{A_u\mu\nu} - \sum_{u,v\leq n} \frac{1}{4h_{uv}^2} F_{u,\mu\nu} F_v^{\mu\nu}$$

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Collect gauge couplings in a tensor, by defining collective index:

$$A \in \{(u, A_u) : \; A_u \leq d(\mathcal{G}_u)\} \quad \text{with} \quad \sum_A = \sum_u \sum_{A_u=1}^{d(\mathcal{G}_u)}$$

$$G_{AB}^2 = \begin{cases} h_{uv}^2 & \text{for } A, B \leq n \\ g_u^2 \delta_{uv} \delta^{A_u B_v} & \text{for } A > n \end{cases}$$

Similar for structure constants and generators.

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Similar for structure constants and generators.

Kinetic mixing is included on line with any other gauge coupling

Diagram formalism

A typical tensor structure in the 3-loop gauge β-function:

$$\beta^{(3)}_{AB} \supset (T^A_\phi T^B_\phi T^C_\phi T^D_\phi)_{ab}\,\mathrm{Tr}[y_b \tilde{y}_a]\,G^2_{CD}$$

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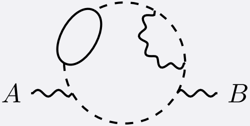
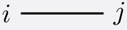
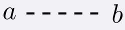


Tensor structures are graphs: Jack, Poole [1411.1301]

$= G^2_{AB},$

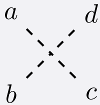


$= \delta_{ij},$

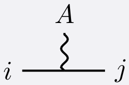


$= \delta_{ab},$

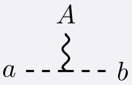
$= y_{aij},$



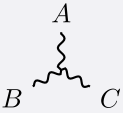
$= \lambda_{abcd},$



$= (T^A)_{ij},$

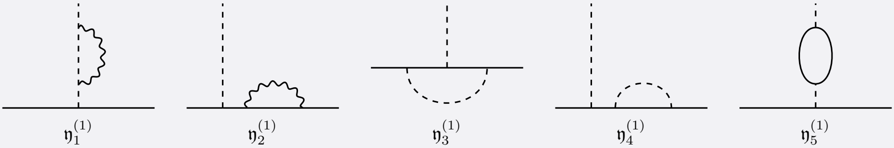


$= (T^A_\phi)_{ab},$



$= G^{-2}_{AD} f^{DBC}$

All tensor structures that can appear in the 1-loop Yukawa β-function



$$\beta_a^{(1)} = +\mathfrak{y}_1^{(1)}\, y_b [C_2(S)]_{ba} \quad +\mathfrak{y}_2^{(1)}\, y_a C_2(F) \quad +\mathfrak{y}_3^{(1)}\, y_b \tilde{y}_a y_b$$
$$+\mathfrak{y}_4^{(1)}\, y_a \tilde{Y}_2(F) \quad +\mathfrak{y}_5^{(1)}\, y_b [Y_2(S)]_{ba},$$

Example: 1-loop Yukawa β-function

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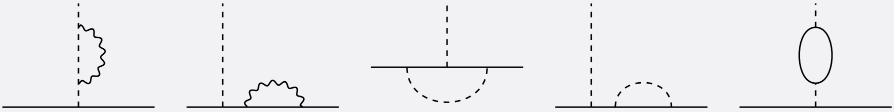

$\mathfrak{y}_1^{(1)}$ $\mathfrak{y}_2^{(1)}$ $\mathfrak{y}_3^{(1)}$ $\mathfrak{y}_4^{(1)}$ $\mathfrak{y}_5^{(1)}$

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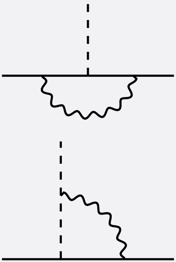
$$+\mathfrak{y}_4^{(1)}\, y_a \tilde{Y}_2(F) \quad +\mathfrak{y}_5^{(1)}\, y_b [Y_2(S)]_{ba},$$

Consulting with literature we recover $\overline{\text{MS}}$ coefficients

$$\mathfrak{y}_1^{(1)} = 0, \quad \mathfrak{y}_2^{(1)} = -6, \quad \mathfrak{y}_3^{(1)} = 2, \quad \mathfrak{y}_4^{(1)} = 1, \quad \mathfrak{y}_5^{(1)} = \frac{1}{2}$$

Employing gauge identities

There seem to be another two tensor structures...



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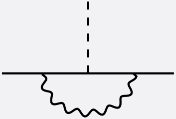
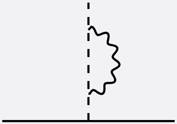
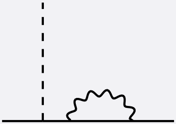
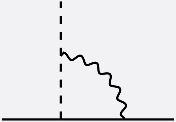
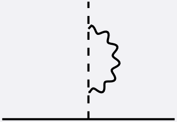
 $=\ \frac{1}{2}$  $-$ 

 $=\ -\frac{1}{2}$ 

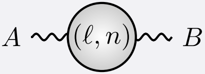
...but they are made redundant by gauge identities.

Outline

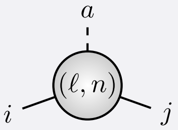
Goal

ℓ-loop β-functions:

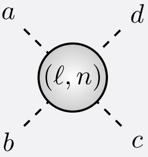
$$\beta^{(\ell)}_{AB} = \sum_n \mathfrak{g}^{(\ell)}_n$$



$$\beta^{(\ell)}_{aij} = \sum_n \mathfrak{y}^{(\ell)}_n$$



$$\beta^{(\ell)}_{abcd} = \sum_n \mathfrak{q}^{(\ell)}_n$$



- Use couplings $g^I = \{G^2_{AB},\ y_{aij},\ \lambda_{abcd}\}$ and $\left\{f^{ABC},\ T^A{}_{ij},\ T^A_{\phi,ab}\right\}$ to parametrize $\hat{A}$, $\hat{T}_{IJ}$, and β^I
- Write up the system of equations resulting from OE (to 5 loops)
- Eliminate unknowns and find constraints on the 4–3–2 β-functions

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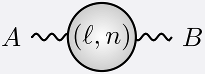
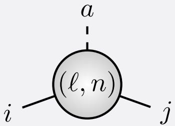


$$\beta^{(\ell)}_{aij} = \sum_n \mathfrak{y}^{(\ell)}_n$$



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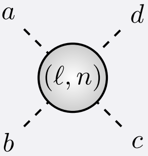

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Thousands of diagrams introduced the need for automation

Strategy

$$\mathrm{d}g^I \partial_I \hat{A} = \mathrm{d}g^I \hat{T}_{IJ} B^J$$

LHS:

1) Generate all unique closed graphs with $\leq \ell$ loops
2) Eliminate redundant TSs using gauge identities
3) Apply differential operator by marking each coupling

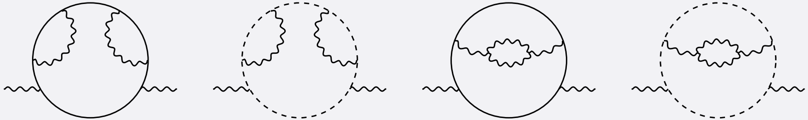
Strategy

$$\mathrm{d}g^I \partial_I \hat{A} = \mathrm{d}g^I \hat{T}_{IJ} B^J$$

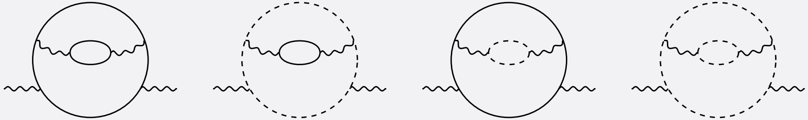
RHS:

1) Determine all unique β TSs by removing corresponding vertices from closed graphs
2) ($\overline{\text{MS}}$) Eliminate non-1PI vertex or fields-strength-type TS
3) Eliminate redundant TSs using gauge identities
4) At sufficiently high loop order, account for the shift $\beta^I \to B^I$ using all possible antisymmetric combinations of 2-point TSs
5) Determine $\hat{T}_{IJ}$ tensors from all closed contractions appearing in $\hat{A}$, marking 2 separate vertices
6) Insert all ℓ_1-loop B^I TSs in the ℓ_2-loop $\hat{T}_{IJ}$, with $\ell_1 + \ell_2 \leq \ell$, and match to the LHS basis

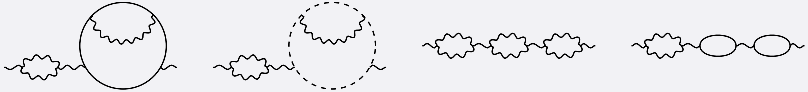
Example: 3-loop gauge β-function



$\mathfrak{g}_1^{(3)}$ $\mathfrak{g}_2^{(3)}$ $\mathfrak{g}_3^{(3)}$ $\mathfrak{g}_4^{(3)}$



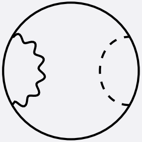
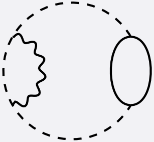
$\mathfrak{g}_5^{(3)}$ $\mathfrak{g}_6^{(3)}$ $\mathfrak{g}_7^{(3)}$ $\mathfrak{g}_8^{(3)}$

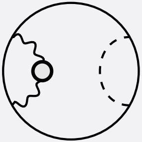
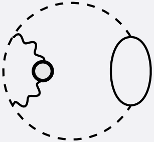


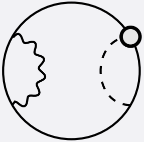
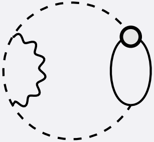
$\mathfrak{g}_9^{(3)}$ $\mathfrak{g}_{10}^{(3)}$ $\mathfrak{g}_{11}^{(3)}$ $\mathfrak{g}_{12}^{(3)}$

And another 21 tensor structures.

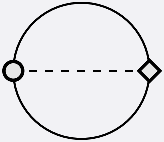
Osborn's equation at 3 loops: LHS

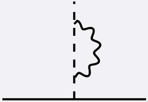
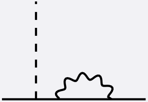
$\hat{A} \supset \quad \mathcal{A}_{10}^{(3)}$  $+\mathcal{A}_{11}^{(3)}$ 

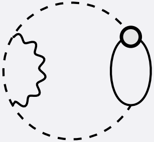
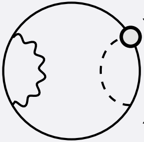
$\mathrm{d}g^I \partial_I \hat{A} \supset \quad \mathcal{A}_{10}^{(3)}$  $+\mathcal{A}_{11}^{(3)}$ 

$+2\mathcal{A}_{10}^{(3)}$  $+2\mathcal{A}_{11}^{(3)}$ 

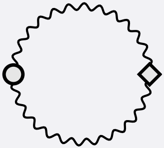
Osborn's equation at 3 loops: RHS

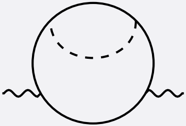
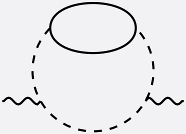
$\mathrm{d}g^I \hat{T}_{IJ} \supset \quad \mathcal{T}^{(2)}_{yy,1}$ 

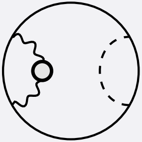
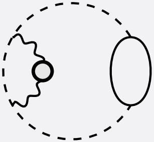
$\beta^J \supset \quad \mathfrak{y}^{(1)}_1$  $+\ \mathfrak{g}^{(1)}_2$ 

$\mathrm{d}g^I \hat{T}_{IJ} B^J \supset \quad \mathcal{T}^{(2)}_{yy,1} \Bigg(\mathfrak{y}^{(1)}_1$  $+\ \mathfrak{y}^{(1)}_2$  $\Bigg)$

Osborn's equation at 3 loops: RHS

$\mathrm{d}g^I \hat{T}_{IJ} \supset$ $\mathcal{T}^{(1)}_{gg,1}$ 

$\beta^J \supset$ $\mathfrak{g}^{(2)}_6$  $+\, \mathfrak{g}^{(2)}_7$ 

$\mathrm{d}g^I \hat{T}_{IJ} B^J \supset$ $\mathcal{T}^{(1)}_{gg,1} \Bigg(\mathfrak{g}^{(2)}_6$  $+\, \mathfrak{g}^{(2)}_7$  $\Bigg)$

Osborn's equation at 3 loops: Consistency condition

Equating the LHS and the RHS of OE gives

$$\mathcal{A}_{10}^{(3)} = \mathcal{T}_{gg,1}^{(1)}\, \mathfrak{g}_6^{(2)} \qquad \mathcal{A}_{11}^{(3)} = \mathcal{T}_{gg,1}^{(1)}\, \mathfrak{g}_7^{(2)}$$

$$2\mathcal{A}_{10}^{(3)} = \mathcal{T}_{yy,1}^{(2)}\, \mathfrak{y}_2^{(1)} \qquad 2\mathcal{A}_{11}^{(3)} = \mathcal{T}_{yy,1}^{(2)}\, \mathfrak{y}_1^{(1)}$$

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We may eliminate $\mathcal{A}_{10}^{(3)}$, $\mathcal{A}_{11}^{(3)}$, $\mathcal{T}_{gg,1}^{(1)}$, and $\mathcal{T}_{yy,1}^{(2)}$ to give

$$\mathfrak{g}_7^{(2)}\, \mathfrak{y}_2^{(1)} = \mathfrak{g}_6^{(2)}\, \mathfrak{y}_1^{(1)}$$

Satisfied by known values: $\mathfrak{y}_1^{(1)} = 0$, $\mathfrak{y}_2^{(1)} = -6$, $\mathfrak{g}_6^{(2)} = -1$, and $\mathfrak{g}_7^{(2)} = 0$.

Findings by loop order

ℓ	No. of coefficients $\hat{A}^{(\ell+1)}$	$\hat{T}^{(\ell)}_{IJ}$	$S^{(\ell-1)}$	$\beta^{(\ell)}_{AB}$	$\beta^{(\ell-1)}_{aij}$	$\beta^{(\ell-2)}_{abcd}$	TS basis	CCs
1	4	1		3			4	
2	14	4		7	5		16	1
3	49	27		33	33	5	91	26
4	257	260	9	198	303	33	703	265
4 (γ_5)	4			4	5		9	5

(green): Known

(yellow): Now determined from previous works

(red): Unknown

Outline

Chiral fermions in d dimensions

The Dirac algebra in Naive(normal) Dimensional Regularization obeys

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}, \qquad g^{\mu\nu}g_{\nu\mu} = d, \qquad \{\gamma_\mu, \gamma_5\} = 0$$

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Trace-cyclicity then implies

$$\mathrm{Tr}[\gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma \gamma_5] = 0$$

in contradiction with the $d = 4$ identity

$$\mathrm{Tr}[\gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma \gamma_5] = 4i\epsilon_{\mu\nu\rho\sigma}$$

γ_5 ambiguities in the SM

For the 3-loop Yukawa β-function a semi-naive treatment suffices

$$\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5] = 4i\tilde{\epsilon}^{\mu\nu\rho\sigma} + \mathcal{O}(\epsilon), \qquad \tilde{\epsilon}^{\mu\nu\rho\sigma}\tilde{\epsilon}_{\alpha\beta\gamma\delta} = g^{[\mu}_{\ [\alpha}g^{\nu}_{\ \beta}g^{\rho}_{\ \gamma}g^{\ \sigma]}_{\ \delta]},$$

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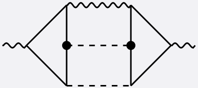
$$\tilde{\epsilon}^{\mu\nu\rho\sigma} \xrightarrow{d\to 4} \epsilon^{\mu\nu\rho\sigma}$$

The 4-loop strong coupling β-function suffers from the first real ambiguity

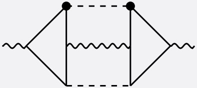
$$\beta^{(4)}_{a_S} \supset R\left(\frac{16}{3} + 32\zeta_3\right) T_F^2 a_S^2 a_t^2, \qquad R \overset{?}{=} 1, 2, \text{or } 3.$$

Bednyakov, Pikelner [1508.02680]

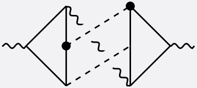
γ_5 β-functions at order 4–3–2



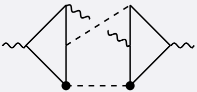
$\mathfrak{g}^{(4)}_{199}$



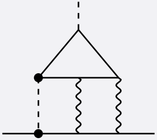
$\mathfrak{g}^{(4)}_{200}$



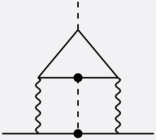
$\mathfrak{g}^{(4)}_{201}$



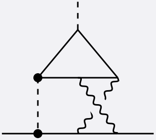
$\mathfrak{g}^{(4)}_{202}$



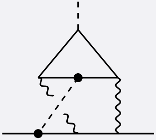
$\mathfrak{y}^{(3)}_{304}$



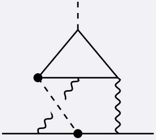
$\mathfrak{y}^{(3)}_{305}$



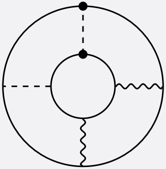
$\mathfrak{y}^{(3)}_{306}$



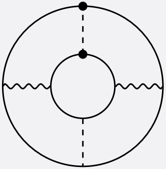
$\mathfrak{y}^{(3)}_{307}$



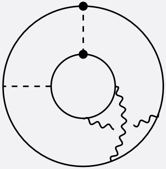
$\mathfrak{y}^{(3)}_{308}$



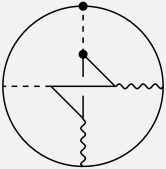
$\mathcal{A}^{(5)}_{258}$



$\mathcal{A}^{(5)}_{259}$



$\mathcal{A}^{(5)}_{260}$



$\mathcal{A}^{(5)}_{261}$

Inserting known LO values for the metric we obtain CCs

$$\mathfrak{g}^{(4)}_{199} = \frac{1}{6}\mathfrak{y}^{(3)}_{304}\,, \quad \mathfrak{g}^{(4)}_{200} = \frac{1}{6}\mathfrak{y}^{(3)}_{305}\,, \quad \mathfrak{g}^{(4)}_{201} = \frac{1}{6}\mathfrak{y}^{(3)}_{306}\,,$$
$$\mathfrak{g}^{(4)}_{202} = \frac{1}{3}\mathfrak{y}^{(3)}_{307}\,, \quad \mathfrak{y}^{(3)}_{307} = \mathfrak{y}^{(3)}_{308}$$

We can reverse-engineer coefficients the from 3-loop SM Yukawa results

Herre, Mihaila, Steinhauser [1712.06614]

$$\mathfrak{y}^{(3)}_{304} = -24, \qquad \mathfrak{y}^{(3)}_{305} = -12, \qquad \mathfrak{y}^{(3)}_{306} = \mathfrak{y}^{(3)}_{307} = \mathfrak{y}^{(3)}_{308} = 8 - 24\zeta_3$$

From which the γ_5 contribution is uniquely determined:

$$\beta^{(4)}_{a_S} \supset (16 + 96\zeta_3)\, T_F^2 a_S^2 a_t^2, \qquad R = 3$$

Summary

- Constructed a fully general framework for all marginal couplings of 4D renormalizable theories.
- Managed the full construction of the 3-loop gauge β-function.
- Determined all Weyl consistency conditions between the gauge coefficients up to order 4–3–2.
- Shown that consistency conditions can resolve the γ_5-related ambiguity in the 4-loop α_s β-function.