Crawling Technicolor

Oscar Catà

BP&nPP Workshop, Primošten, October 8th, 2019

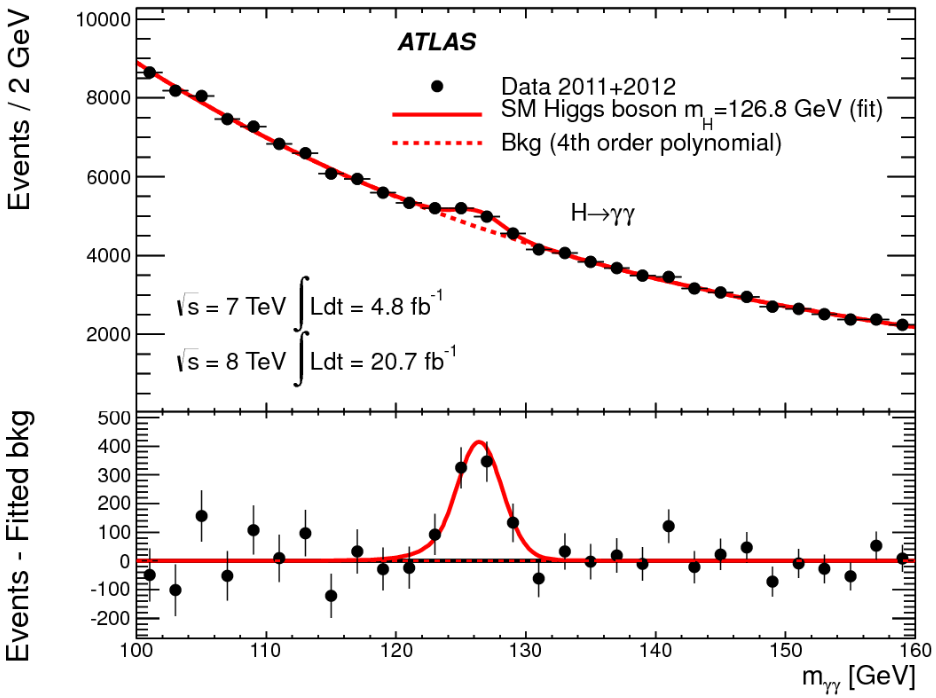
(based on collaboration with R. Crewther and L. Tunstall)

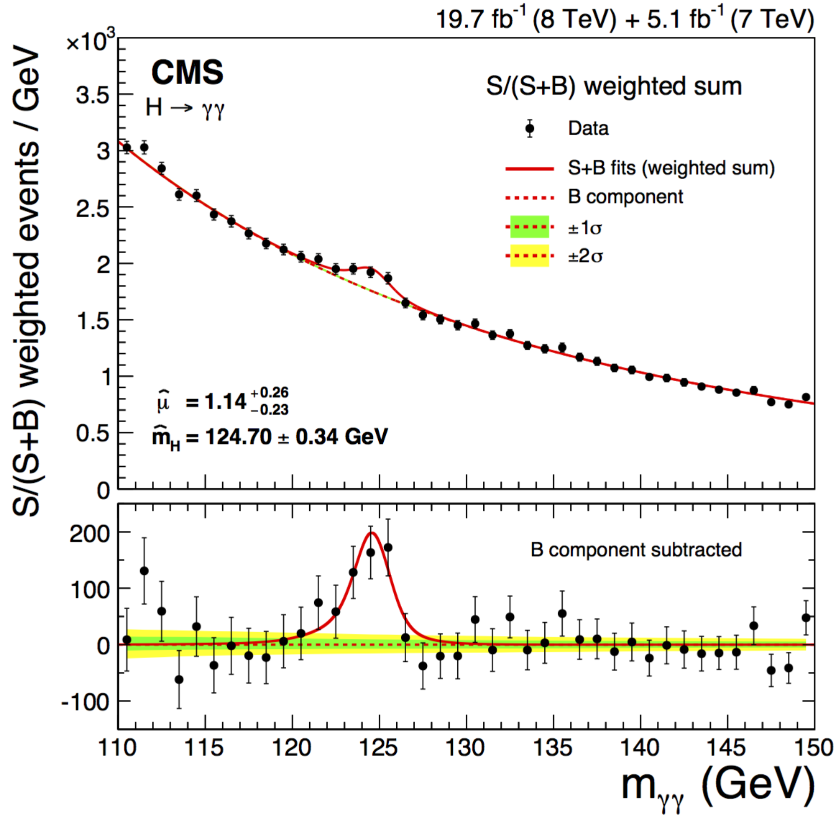
Outline

- Motivation
- Crawling TC: a theory for a dilaton Higgs
- EFT representation
- The dilaton Higgs potential
- Tests of Crawling TC
- Conclusions

A Higgs discovery

- Back in 2012:





- Higgs discovery and tests at the LHC confirm the Standard Model as an excellent low-energy approximation to the electroweak interactions (within the current precision). Higgsless alternatives ruled out.

Is the Higgs fundamental?

- First elementary scalar? Nature has so far chosen a different path to introduce scalar modes (fermion condensation, Goldstone bosons). Naturalness issues avoided.

- Old idea: EWSB from a new strong confining interaction at the TeV scale. Flagship: Technicolor. [Weinberg'76; Susskind'79]

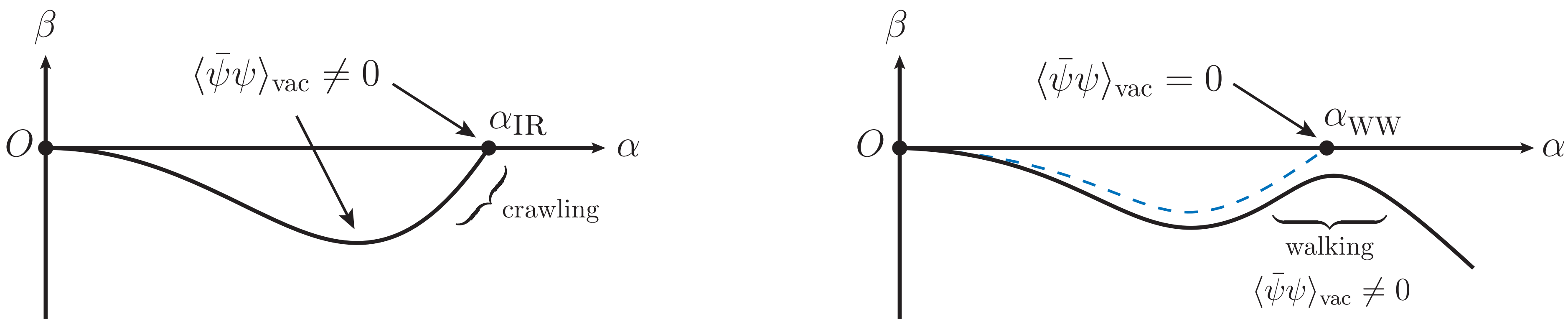
- Technicolor 1.0 a higgsless theory, in analogy with QCD. Nowadays ruled out.

- *However*, a light scalar can be present, typically as a pseudo-NG boson. Explicit breaking at the EW scale will give it a mass, but protected by the Goldstone symmetry from effects at the dynamical scale and above. The mechanism is viable but not many realizations of it.

1. Composite Higgs models: [e.g., Agashe et al'04]
   (i) hard to make them realistic (fermions);
   (ii) typical v/f deviations from the SM are rather constrained.

2. Walking TC with dilatonic extensions: [e.g., Dietrich et al' 05; Appelquist et al'10]
   (i) large number of technipions;
   (ii) hard to prove whether a light scalar actually exists.

Crawling TC in a nutshell



Alternative to WTC with the following ingredients:

- New interactions (e.g. $SU(3)$) strong at the TeV scale.

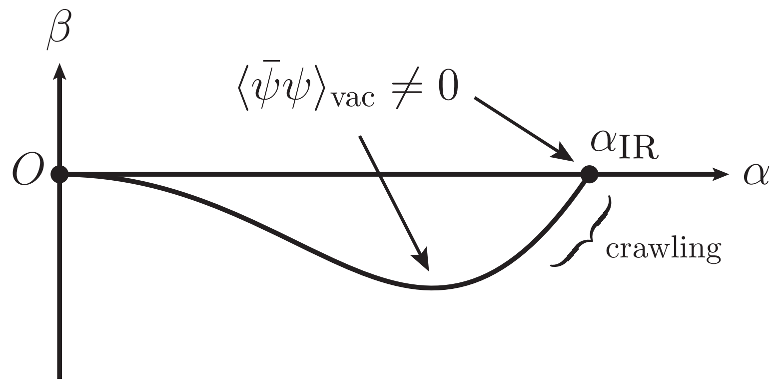
- An infrared fixed point (IRFP) exists, α_{IR}. Scale-invariant limit exists, but hidden. At α_{IR}, Higgs as a (massless) dilaton. (Explicit) scale breaking like $\alpha_{IR} - \alpha$ or better m_h^2.

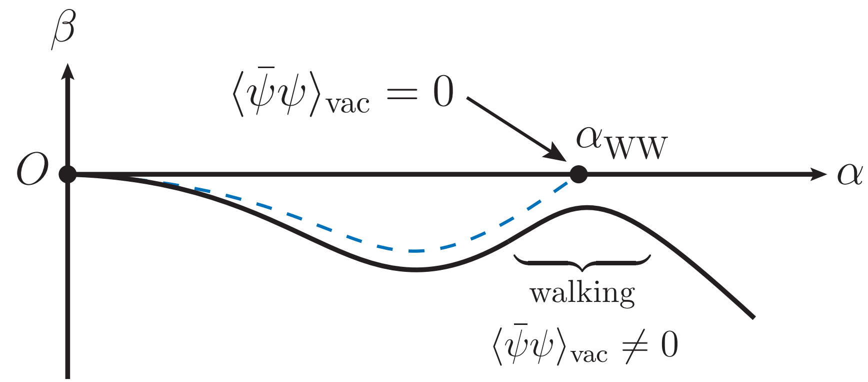
- *Both* electroweak and scale symmetries spontaneously broken by the same object, e.g. a chiral condensate:

$$\langle\bar{\psi}\psi\rangle_{\rm TC} \neq 0$$

- To avoid technipions, $SU(2) \times SU(2)$ global symmetry (not ruled out!). Only light modes: Higgs and EW Goldstones.

Crawling TC vs other TC's





Main features:

- Key observation: in crawling, scales can (and do) exist at α_{IR}.
- Higgs as a dilaton (true Goldstone mode of scale breaking).
- EW scale: confining phase, nonperturbative regime.
- Technicolor 1.0 put in serious trouble by EWPO. Subsequent sophistications of the theory, mostly to address flavour issues:

$$\mathbf{TC} \quad \rightarrow \quad \mathbf{ETC} \quad \rightarrow \quad \mathbf{Walking\ (E)TC}$$

Crawling not worse than Walking in accommodating EWPO and flavour.

Scale invariance and its breaking

- A scale transformation on coordinates, $x^\mu \to e^\lambda x^\mu$, induces the scale transformation on fields:

$$\varphi(x) \to e^{\lambda d}\varphi(e^\lambda x)$$

d (scaling dimension) depends on the field. Infinitesimally,

$$\delta\varphi(x) = (d + x^\mu \partial_\mu)\varphi(x)$$

- The associated Noether current is the dilatation current:

$$\mathcal{D}_\mu(x) = x^\nu \theta_{\mu\nu}(x)$$

where $\theta_{\mu\nu}$ is **symmetric**, **gauge invariant** and **improved**.

- For a general operator $\mathcal{O}$ in a Lagrangian, one finds

$$\delta\mathcal{O} = \partial_\mu(x^\mu \mathcal{O}) + (d_\mathcal{O} - 4)\mathcal{O}$$

and

$$\partial_\mu \mathcal{D}^\mu(x) = \theta^\mu_\mu(x) = \delta\mathcal{L} = \sum_j (d_j - 4)\mathcal{O}_j$$

The current is conserved as long as operators have overall $d = 4$, i.e., if no scales are present.

The conformal anomaly

Example:

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a} + \sum_j \bar{\psi}_j (i\gamma^\mu D_\mu - m_j)\psi_j$$

- Classically, one finds that

  $$\theta^\mu_\mu(x) = \sum_j m_j \bar{\psi}_j \psi_j$$

  signalling explicit breaking by (mass) scales.

- At the quantum level, we know that g_s is scale-dependent. Even in the absence of masses, the Lagrangian has a separate source of breaking:

  $$\theta^\mu_\mu = \beta(g) \frac{\delta \mathcal{L}}{\delta g} = \frac{\beta(\alpha_s)}{4\alpha_s} G^a_{\mu\nu} G^{\mu\nu a}$$

  referred to as the **trace anomaly**.

- Anomalous breaking can be avoided at fixed points, i.e., points where $\beta(\alpha_s^*) = 0$. At those points scale symmetry is exact.

- As with any continuous symmetry, scale invariance might be manifest (WW mode) or hidden (NG mode). *Strictu sensu*, a dilaton only appears with hidden scale invariance. With WW mode, one gets scalons.

Scalons vs dilatons

SCALON	DILATON
WW (manifest)	NG (hidden)
scaleless at α_{IR}	scaleful at α_{IR}
perturbative	nonperturbative (condensates)
dilatonic WTC	crawling TC
scalars with CW potentials	?

Decoupling vs Nondecoupling:

Light scalars in WW mode might exist but not protected by symmetry (no genuine dilaton):

$$m_\sigma^2 f_\sigma = -\langle 0|\theta^\mu_\mu|\sigma\rangle = O(\alpha - \alpha^*)$$

Compare:

$$f_\sigma \sim 0 \ \text{ for } \ \alpha^* \sim \alpha_{\rm WW} \qquad\qquad m_\sigma \text{ arbitrary}$$

$$f_\sigma \to \ \text{constant } \neq 0 \ \text{ as } \ \alpha^* \to \alpha_{\rm IR} \qquad\qquad m_\sigma \text{ small}$$

An example: the Nambu-Freund model

- Consider two real scalar fields ϕ and φ with potential:

$$V(\phi,\varphi) = \frac{1}{2}f^2\phi^2\varphi^2 + \frac{\tau}{8g^4}(g^2\varphi^2 - 1)^2$$

- Field redefinition $\chi = (2g)^{-1}(g^2\varphi^2 - 1)^2$ brings it to

$$V(\phi,\chi) = \frac{1}{2}m_\phi^2(1+2g\chi) + \frac{1}{2}m_\chi^2\chi^2, \qquad\qquad m_\phi^2 = f^2\varphi_0^2; \quad m_\chi^2 = \tau\varphi_0^2$$

  where $\varphi_0 = g^{-1}$ is τ-independent.

In the scale-invariant limit ($\tau \to 0$):

- Scales are generated, $m_\phi \neq 0$.

- There is a dilaton, $m_\chi \to 0$.

The Callan-Symanzik NG-mode equation

- CS equation for an arbitrary operator:

$$\left\{\mu\frac{\partial}{\partial\mu}+\beta(\alpha)\frac{\partial}{\partial\alpha}+\gamma_{\mathcal{O}}(\alpha)\right\}\langle\mathcal{O}\rangle=0$$

- The β term is equivalent to a $-i\theta^{\mu}_{\mu}$ insertion at zero momentum:

$$\begin{aligned}\left\{\mu\frac{\partial}{\partial\mu}+\gamma_{\mathcal{O}}(\alpha)\right\}\langle\mathcal{O}\rangle &= -i\lim_{q\to 0}\int d^4x\, e^{iq\cdot x}\,\langle 0|T\left\{\theta^{\mu}_{\mu}(x)\mathcal{O}(0)\right\}|0\rangle \\ &= -i\lim_{q\to 0}\left[\langle 0|\theta^{\mu}_{\mu}|\sigma\rangle\frac{i}{q^2-m_\sigma^2}\langle\sigma|\mathcal{O}|0\rangle\right]=f_\sigma\langle\sigma|\mathcal{O}|0\rangle\end{aligned}$$

  using PCDC:

$$\langle 0|\theta^{\mu}_{\mu}|\sigma\rangle=-f_\sigma m_\sigma^2$$

- In WW mode one finds instead

$$\left\{\mu\frac{\partial}{\partial\mu}+\gamma_{\mathcal{O}}(\alpha)\right\}\langle\mathcal{O}\rangle=0$$

- Soft-dilaton theorem vs hyperscaling

Two important results from the Callan-Symanzik equation

- Consider the CS equation for a RG-invariant amplitude $\mathcal{A}$:

$$\left\{\mu\frac{\partial}{\partial\mu} + \beta(\alpha)\frac{\partial}{\partial\alpha}\right\}\mathcal{A} = 0$$

- Apply $\alpha\partial/\partial\alpha$:

$$\left\{\mu\frac{\partial}{\partial\mu} + \beta(\alpha)\frac{\partial}{\partial\alpha} + \beta'(\alpha) - \frac{\beta(\alpha)}{\alpha}\right\}\alpha\frac{\partial\mathcal{A}}{\partial\alpha} = 0$$

- But $\alpha\partial\mathcal{A}/\partial\alpha$ is a $\hat{G}^2$ insertion, so

$$\left\{\mu\frac{\partial}{\partial\mu} + \beta(\alpha)\frac{\partial}{\partial\alpha} + \gamma_{\hat{G}^2}(\alpha)\right\}\mathcal{A}_{\hat{G}^2} = 0$$

  and the anomalous dimension of $\hat{G}^2$ can be read off:

$$\gamma_{\hat{G}^2}(\alpha) = \beta'(\alpha) - \frac{\beta(\alpha)}{\alpha}$$

- The breaking of scale invariance due to the gluonic anomaly is driven by

$$\boxed{\gamma_{\hat{G}^2}(\alpha_{IR}) = \beta'(\alpha_{IR})}$$

Two important results from the Callan-Symanzik equation

- Take the CS equation for $\hat{G}^2$ itself in the NG mode:

$$\frac{\beta(\alpha)}{4\alpha}\left\{\mu\frac{\partial}{\partial\mu}+\gamma_{\hat{G}^2}(\alpha)\right\}\left\langle\hat{G}^2\right\rangle_{\text{vac}}=f_\sigma\left\langle\sigma\middle|\theta^\mu_\mu\middle|\text{vac}\right\rangle$$

- For an IR expansion in the physical region, $\epsilon=\alpha_{IR}-\alpha\gtrsim 0$,

$$\frac{\beta(\alpha)}{4\alpha}\left\{\mu\frac{\partial}{\partial\mu}+\gamma_{\hat{G}^2}(\alpha_{IR})\right\}\left\langle\hat{G}^2\right\rangle_{\text{vac}}=-\frac{\epsilon\beta'(4+\beta')}{4\alpha_{IR}}\left\langle\hat{G}^2\right\rangle_{\text{vac}}+O(\epsilon^2)=-m_\sigma^2 f_\sigma^2$$

- Prediction for the mass:

$$\boxed{m_\sigma^2=\frac{\epsilon\beta'(4+\beta')}{4\alpha_{\text{IR}}f_\sigma^2}\left\langle\hat{G}^2\right\rangle_{\text{vac}}+O(\epsilon^2)}$$

- The formula can be easily generalized if technifermion masses are present:

$$m_\sigma^2 f_\sigma^2=\frac{\epsilon\beta'(4+\beta')}{4\alpha_{IR}}\langle\hat{G}^2\rangle_{\text{vac}}-(3-\gamma_m)(1+\gamma_m)m_\psi\langle\bar{\psi}\psi\rangle_{\text{vac}}+O(\epsilon^2,\epsilon m_\psi,m_\psi^2)$$

Effective Field Theories

- EFTs are the most efficient way of describing the physics at a certain scale μ, if
  - There is a mass gap between typical scales, such that μ/Λ can be a good expansion parameter.
  - The particle content (φ) and symmetries at μ are known
- One can then generally write

$$\mathcal{L}_{\text{eff}}(\varphi) = \mathcal{L}_0(\varphi) + \underbrace{\mathcal{L}_1(\varphi)}_{\mathcal{O}(\mu^2/\Lambda^2)} + \ldots$$

Each term satisfies in turn

$$\mathcal{L}_j = \sum_n c_n^{(j)} \mathcal{O}_n^{(j)}(\varphi)$$

- $\mathcal{O}_n(\varphi)$, IR-sensitive (φ and symmetries); c_n, UV-sensitive (Λ physics).
- EFTs right template (general, QFT-based, improvable) to parametrize deviations of the SM.

How many EFTs?

- Toy model: SM extended with a real scalar S endowed with a Z_2 symmetry: [e.g., Buchalla et al' 16]

$$\mathcal{L}_{\Phi S} = (D^\mu \Phi)^\dagger (D_\mu \Phi) + \partial^\mu S \partial_\mu S - V(\Phi, S)$$

with

$$V(\Phi, S) = -\frac{\mu_1^2}{2}\Phi^\dagger\Phi - \frac{\mu_2^2}{2}S^2 + \frac{\lambda_1}{4}(\Phi^\dagger\Phi)^2 + \frac{\lambda_2}{4}S^4 + \frac{\lambda_3}{2}\Phi^\dagger\Phi S^2$$

- Expanding around the vacuum

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}\langle G_{\mu\nu}G^{\mu\nu}\rangle - \frac{1}{2}\langle W_{\mu\nu}W^{\mu\nu}\rangle - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \sum_f \bar{f} i \not{D} f + \frac{1}{2}\partial_\mu h \partial^\mu h \\ &+ \frac{1}{2}\partial_\mu H \partial^\mu H + \frac{v^2}{4}\langle D_\mu U^\dagger D^\mu U\rangle \left(1 + \frac{2c}{v}h + \frac{2s}{v}H + \frac{c^2}{v^2}h^2 + \frac{s^2}{v^2}H^2 + \frac{2sc}{v^2}hH\right) \\ &- v\left(\bar{q}Y_u U P_+ r + \bar{q} Y_d U P_- r + \bar{\ell} Y_e U P_- \eta + \text{h.c.}\right)\left[1 + \frac{c}{v}h + \frac{s}{v}H\right] - V(h, H)\end{aligned}$$

with

$$\begin{aligned}V(h, H) = &\frac{1}{2}m^2 h^2 + \frac{1}{2}M^2 H^2 - d_1 h^3 - d_2 h^2 H - d_3 h H^2 - d_4 H^3 \\ &- z_1 h^4 - z_2 h^3 H - z_3 h^2 H^2 - z_4 h H^3 - z_5 H^4\end{aligned}$$

How many EFTs?

- Assume $M \gg m$ and integrate out H.

$$(-\partial^2 - M^2 + 2J_2)H + J_1 + 3J_3H^2 + 4J_4H^3 = 0$$

- The solution depends on the relative magnitude of J_i/M^2. Two scenarios:

$$\begin{array}{rlr} (i) & M \sim v_s \gg v, m, \chi & \lambda_i \sim \mathcal{O}(1) \\ (ii) & M \gg v_s, v, m, \chi & \lambda_i \lesssim 32\pi^2 \end{array}$$

- Take for example:

$$d_4 = -\frac{M^2}{2vv_s}[c^3 v + s^3 v_s] \quad \rightarrow \quad \left\{ \begin{array}{l} \sim M \\ \sim M^2 \end{array} \right.$$

- The two scenarios correspond to decoupling and nondecoupling limits.

Linear vs nonlinear EFT

- In the different limits:

$$H(h) \sim \frac{h^2}{M} \qquad\qquad H(h) \sim \sum_{n\geq 2}^{\infty} a_n \frac{h^{n+1}}{v^n}, \qquad a_n \sim \mathcal{O}(1)$$

- The corresponding EFTs look like:

$$\mathcal{L} \supset \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{v^2}{4}\mathrm{tr}\left[D_\mu U^\dagger D^\mu U\right] f(h) - v\left[\bar{\psi} f_\psi(h) U P_\pm \psi + \mathrm{h.c.}\right] - V(h)$$

with

$$f(h) = 1 + (2-\alpha^2)\frac{h}{v} + (1-2\alpha^2)\left(\frac{h}{v}\right)^2 - \frac{4}{3}\alpha^2\left(\frac{h}{v}\right)^3 - \frac{\alpha^2}{3}\left(\frac{h}{v}\right)^4, \qquad \alpha^2 \sim \mathcal{O}(M^{-2})$$

or

$$f(h) = 1 + 2c\left(\frac{h}{v}\right) + \left[c^4 - s^3 c\frac{v}{v_s}\right]\left(\frac{h}{v}\right)^2 - \frac{4}{3v_s^2}s^2c^3(vs+v_s c)^2\left(\frac{h}{v}\right)^3 + \mathcal{O}(h^4)$$

- Both canonical and loop expansions arise naturally depending on the nature of the UV.

A tale of two EFTs

Assuming:

- observed particle content.
- a mass gap: $\dfrac{v^2}{M_{NP}^2} \ll 1$
- known symmetries valid up to probed scales, $SU(3)_c \times SU(2)_L \times U(1)_Y$.

then the corresponding EFT at LO is either

$$\mathcal{L}_{\rm SM} = -\frac{1}{4} X^a_{\mu\nu} X^{\mu\nu\, a} + i \sum_j \bar{\psi}_j \not{D} \psi_j + D_\mu H^\dagger D^\mu H - V(H)$$
$$- \left[y_d \bar{Q}_L H d + y_u \bar{Q}_L \tilde{H} u + y_e \bar{E}_L H e + \text{h.c.} \right]$$

or

$$\mathcal{L}_{\rm LO} = -\frac{1}{4} X^a_{\mu\nu} X^{\mu\nu\, a} + i \sum_j \bar{\psi}_j \not{D} \psi_j + \frac{1}{2} \partial_\mu h \partial^\mu h + \frac{v^2}{4} \text{tr} \left[D_\mu U D^\mu U^\dagger \right] f(h)$$
$$- v \left[\bar{\psi} f_\psi(h) U P_\pm \psi + \text{h.c.} \right] - V(h)$$

Every model compatible with the assumptions looks at low energies like $\mathcal{L}_{\rm SM}$ or $\mathcal{L}_{\rm LO}$.

How to build scale-invariant EFTs for NG modes

- Old wisdom (60s-70s), starting with Salam et al.; Wess et al; Ellis.

- Given a Lagrangian operator $\mathcal{O}$, weight it by the conformal compensator χ to

$$\mathcal{O} \;\to\; \mathcal{O}_\chi = \mathcal{O} \times \left\{ c_{\mathcal{O}} \chi^{4-d_{\mathcal{O}}} + (1-c_{\mathcal{O}})\chi^{4-d_{\mathcal{O}}+\beta'} \right\} = c_{\mathcal{O}} \mathcal{O}_{\mathrm{inv}} + (1-c_{\mathcal{O}})\mathcal{O}_{\beta'}$$

- $\mathcal{O}_{\beta'}$ has dimension $4+\beta'$ (explicit scale breaking by the gluonic trace anomaly near α_{IR})

- $c_{\mathcal{O}} = 1 + O(\epsilon)$, implied by

$$\theta^\mu{}_\mu\big|_{\mathrm{eff}} = \sum_j (d_j - 4)\left\{ \mathcal{O}^j_\sigma - \left\langle \mathcal{O}^j_\sigma \right\rangle_{\mathsf{vac}} \right\} = \beta' \sum_j (1 - c_{\mathcal{O}j})\left\{ \mathcal{O}^j_{\beta'} - \langle \mathcal{O}^j_{\beta'} \rangle_{\mathrm{vac}} \right\}$$

  which vanishes in the scale-invariant limit.

- Example (χPT in the chiral limit):

$$\frac{v^2}{4}\mathrm{tr}(D_\mu U D^\mu U^\dagger) \to \frac{v^2}{4}\mathrm{tr}(D_\mu U D^\mu U^\dagger)\chi^2 + \frac{f_\sigma^2}{2}\partial_\mu \chi \partial^\mu \chi$$

- Common to use the representation

$$\chi = e^{\sigma/f_\sigma}$$

EFT for crawling TC

- General description valid below the TeV scale, which is the natural cutoff of the EFT:

$$\Lambda \sim 4\pi f_\sigma \sim 4\pi v$$

  Agnostic about the explicit UV theory.

- Assume minimal setup: $SU(2)_L \times SU(2)_R$ global symmetry, spontaneously broken to give 3 Goldstones; Higgs as a dilaton; only SM fields.

- Main advantage of the effective Lagrangian formalism: radiative corrections are easily computed; contact can be made with the SM Lagrangian.

- Final result can be constructed from higgsless EFT with conformal weights, with Goldstones of electroweak and scale symmetry breaking parametrized by

$$U = e^{i\varphi^a \tau^a / v}; \qquad \chi = e^{\sigma / f_\sigma}$$

Crawling TC as an EFT

- LO EFT:

$$\mathcal{L}_{\text{LO}} = \frac{1}{2} e^{2\sigma/f_\sigma} \partial_\mu \sigma \partial^\mu \sigma - V(\sigma) - \frac{1}{4} G^A_{\mu\nu} G^{A\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$
$$+ \bar{q}_L i \not{D} q_L + \bar{u}_R i \not{D} u_R + \bar{d}_R i \not{D} d_R + \bar{\ell}_L i \not{D} \ell_L + \bar{e}_R i \not{D} e_R$$
$$+ \frac{v^2}{4} \mathrm{tr}(D_\mu U D^\mu U^\dagger) e^{2\sigma/f_\sigma} - v \Big\{ \bar{q}_L \hat{Y}_u U \mathcal{U}_R + \bar{q}_L \hat{Y}_d U \mathcal{D}_R + \bar{\ell}_L \hat{Y}_e U \mathcal{E}_R + \text{h.c.} \Big\} e^{\sigma/f_\sigma}$$

- Power counting: LO Lagrangian dictated by homogeneity in chiral dimensions $[\mathcal{L}_{\text{LO}}] = 2$:

$$[G_\mu, W_\mu, B_\mu, \sigma, \varphi^a] = 0 \ , \quad [\psi] = \tfrac{1}{2} \ , \quad [g_s, g_w, g'_w, \hat{y}_{u,d,e}, \partial_\mu] = 1 \ , \quad [m^2_\sigma \sim \epsilon] = 2$$

  Equivalent to loop counting.

- Subleading terms given by

$$\mathcal{L}_{\text{EFT}} = \sum_{\ell \geqslant 0} \mathcal{L}_{\text{N}^\ell\text{LO}} \ \text{with} \ \big[\mathcal{L}_{\text{N}^\ell\text{LO}}\big] = 2\ell + 2$$

The dilaton potential

- When Goldstones come from the breaking of an internal symmetry (χPT), the potential is clearly proportional to the explicit symmetry breaking:

$$\mathcal{L}_{\chi PT} = \frac{f_0^2 B_0}{2} \mathrm{tr}\left[\chi U^\dagger + \chi^\dagger U\right], \qquad \chi = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$$

- With scale invariance, it seems that a scale-invariant term is possible:

$$\mathcal{L}_\chi = \lambda f_\sigma^4 \chi^4$$

Common lore: [exceptions: Zumino'73; Bardeen et al'86]

A. If symmetry allows an operator, it should be there

Information on the symmetry realization actually depends on coefficients:

$$V(\Phi) = +\frac{\mu^2}{2}\Phi^2 + \frac{\lambda}{2}\Phi^4; \qquad V(\Phi) = -\frac{\mu^2}{2}\Phi^2 + \frac{\lambda}{2}\Phi^4$$

The NG realization for scale symmetry corresponds to $\lambda = 0$.

B. A potential without explicit breaking means that dilatons do not exist

A quadratic term is generated for σ, but the potential has no minimum (not a mass!).

The dilaton potential

- $V(\chi)$ has to be of first order in ϵ to have a well-defined propagator and to satisfy Goldstone theorem:

$$V(\chi) = c_1 \chi^4 + c_2 \chi^{4+\beta'}; \quad c_1, c_2 = O(\epsilon)$$

- Assume $c_1 < 0$ and $c_2 > 0$ (minimum exists) and $\langle \sigma \rangle_{\text{vac}} = 0$ (convenient).
- c_1, c_2 determined from $V'(0) = 0$ (no tadpole condition) and $V''(0) = m_\sigma^2$.
- One can write the dilaton potential in closed form as

$$V(\sigma) = \frac{m_\sigma^2 f_\sigma^2}{\beta'} \left[-\frac{1}{4} e^{4\sigma/f_\sigma} + \frac{1}{4+\beta'} e^{(4+\beta')\sigma/f_\sigma} \right]$$

- In NG mode of scale breaking, the Higgs potential is largely fixed by symmetry.
- As expected for Goldstones, interactions with any power of σ.
- Cross-check: this potential should reproduce the CS results.

Dilaton mass (again)

- Expression of the dilaton mass found from the CS equation for $\hat{G}^2$ using:

$$\frac{\beta(\alpha)}{4\alpha}\left\{\mu\frac{\partial}{\partial\mu}+\gamma_{\hat{G}^2}(\alpha)\right\}\langle\hat{G}^2\rangle_{\text{vac}} = F_\sigma\langle\sigma|\theta^\mu_\mu|\text{vac}\rangle$$

  with

$$\gamma_{\hat{G}^2}(\alpha)=\beta'(\alpha)-\frac{\beta(\alpha)}{\alpha}$$

- Alternatively, from the EFT by matching the trace anomaly at the fundamental and EFT levels:

$$\theta^\mu_\mu\big|_{\text{eff}} = -\frac{M_\sigma^2F_\sigma^2}{4+\beta'}\left\{\left(1+\frac{h}{F_\sigma}\right)^{4+\beta'}-1\right\}; \qquad \theta^\mu_\mu = -\frac{\epsilon\beta'}{4\alpha_{\text{IR}}}\{\hat{G}^2-\langle\hat{G}^2\rangle_{\text{vac}}\}+O(\epsilon^2)$$

- End result:

$$\boxed{M_\sigma^2=\frac{\epsilon\beta'(4+\beta')}{4\alpha_{\text{IR}}F_\sigma^2}\langle\hat{G}^2\rangle_{\text{vac}}+O(\epsilon^2)}$$

Effective Field Theory

- Simplification: perform the field redefinition

$$h = \int_0^\sigma e^{\sigma'/f_\sigma} d\sigma' = f_\sigma (e^{\sigma/f_\sigma} - 1)\,, \qquad h \geq -f_\sigma$$

which brings the dilaton kinetic term into canonical form.

- The LO Lagrangian takes the form:

$$\begin{aligned}
\mathcal{L}_{\rm LO} &= \frac{1}{2}(\partial h)^2 - V(h) - \frac{1}{4} G^A_{\mu\nu} G^{A\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \\
&+ \bar{q}_L i \not{D} q_L + \bar{u}_R i \not{D} u_R + \bar{d}_R i \not{D} d_R + \bar{\ell}_L i \not{D} \ell_L + \bar{e}_R i \not{D} e_R + \frac{v^2}{4} {\rm tr}(D_\mu U D^\mu U^\dagger) \left(1 + \frac{h}{f_\sigma}\right)^2 \\
&- v \left\{ \bar{q}_L \hat{Y}_u U \mathcal{U}_R + \bar{q}_L \hat{Y}_d U \mathcal{D}_R + \bar{\ell}_L \hat{Y}_e U \mathcal{E}_R + {\rm h.c.} \right\} \left(1 + \frac{h}{f_\sigma}\right)
\end{aligned}$$

with

$$V(h) = \frac{m_\sigma^2 f_\sigma^2}{\beta'} \left[-\frac{1}{4}\left(1 + \frac{h}{f_\sigma}\right)^4 + \frac{1}{4+\beta'} \left(1 + \frac{h}{f_\sigma}\right)^{4+\beta'} \right]$$

Phenomenological highlights

- Higgs couplings of the LO Lagrangian behave like the SM with $v \to f_\sigma$.

- In Crawling TC no significant deviations expected: electroweak and scale invariance get spontaneously broken by the same condensate, so $v \sim f_\sigma$. (Compare with Goldstone Higgs of internal symmetries)

- Higgs self-interactions differ. They are affected by β':

$$V(h) - V(0) = m_\sigma^2 f_\sigma^2 \left\{ \frac{1}{2}\left(\frac{h}{f_\sigma}\right)^2 + \frac{5+\beta'}{3!}\left(\frac{h}{f_\sigma}\right)^3 + \frac{11+\beta'(\beta'+6)}{4!}\left(\frac{h}{f_\sigma}\right)^4 + O(h^5) \right\}$$

- Even with small β', triple vertex at least twice as big. However, β' nonperturbative, so not necessarily small.

- At the LHC: constraints on β' from Higgs double production. Challenging but feasible.

Testing CTC on the lattice

Different possibilities:

- Freezing of α outside the conformal window in the deep infrared. Hard but a large window for N_f.

- Light scalar mass, $m_\sigma^2 \propto m_\psi$ of the form

$$\tilde{m}_\sigma^2 = \frac{\epsilon\beta'(4+\beta')}{4\tilde{f}_\sigma^2}\langle\hat{G}^2\rangle_{\text{vac}} - (3-\gamma_m)(1+\gamma_m)m_\psi\frac{\langle\bar{\psi}\psi\rangle_{\text{vac}}}{\tilde{f}_\sigma^2} + O(\epsilon^2, \epsilon m_\psi, m_\psi^2)$$

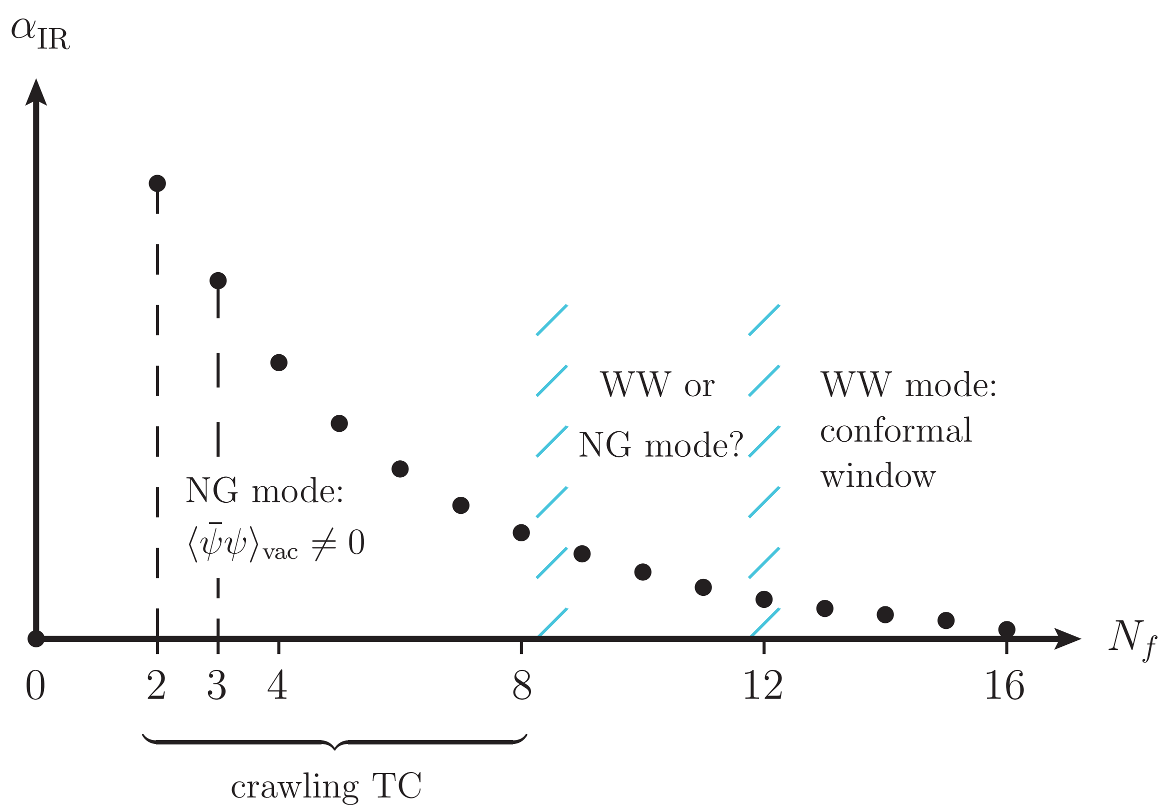
- Promising candidates: $N_f = 8$ (triplet fermions) [Aoki et al'14]; $N_f = 2$ (sextet fermions) [Fodor et al'14]. Search methods have to be adapted: hyperscaling relations do not hold, soft-dilaton theorems do.

- f_σ from matrix element residue with dilatons, and γ_m from the soft-dilaton theorem:

$$3 - \gamma_m = f_\sigma \frac{\langle\sigma|\bar{\psi}\psi|0\rangle}{\langle 0|\bar{\psi}\psi|0\rangle} + O(m_\psi)$$

Testing CTC on the lattice

- Summary chart for lattice searches on IRFPs relevant for EWSB:



- Crawling TC opens up a broader range of phenomenologically relevant IRFPs.

Conclusions

- CTC vs WTC: NG-mode vs WW-mode implementation of conformal breaking in dynamical EWSB. No hierarchy problem, no technipions, similar mechanism to address FCNCs and fermion masses.

- Genuine Goldstone mode of hidden scale symmetry:

$$m_\sigma^2 = \frac{\epsilon\beta'(4+\beta')}{4\alpha_{\mathrm{IR}} f_\sigma^2} \left\langle \hat{G}^2 \right\rangle_{\text{vac}} + O(\epsilon^2)$$

- NG-mode IR fixed points for small N_f are not excluded: power-law scaling of Green's functions tests only WW-mode IRFPs.

- Phenomenologically interesting: no expected deviations for Higgs couplings to fermions and gauge bosons. Deviations in Higgs potential can be written down in closed form in terms of $\beta'(\alpha_{IR})$. Typically, couplings larger than the SM.

- Tests at the LHC (Higgs double production) and with lattice simulations ($f_\sigma, \gamma_m, \beta', \langle \hat{G}^2 \rangle$).