Why should we study Yang-Mills gauge theories in 5d?

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Bridging perturbative and non-perturbative physics
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Motivation:
uncovering the universality class
of quantum gravity

A look at Weinberg's proposal

Asymptotic safety in QG $\iff$ UV universality class

Weinberg proposes that the FP does not appear out of the blue but comes from continuing asymptotically free $2d$ quantum gravity above its critical dimension to $d = 2 + \epsilon$

Weinberg 1979

However a lot can happen from from $d = 2$ to $d = 4$ ...

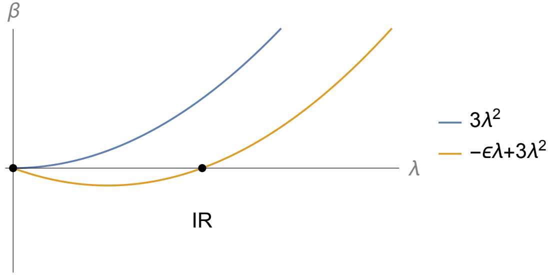
YM above $d = 4$ is certainly a great toy model for this

Take the continuation for granted, what are the obstructions?

Beta functions and ϵ-expansion

or what to look for when moving away from the critical dimension

Example: φ^4 in $d = 4 - \epsilon$

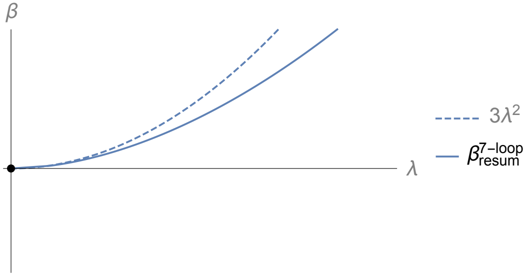


- One loop $\beta > 0$ for $d_c = 4$
- Well-defined ϵ-expansion
- Exactly solved CFT in 2d
- Numerical evidence in 3d

Bridging to non-perturbative physics: triviality in $d = 4$

For example approximate Borel transform with hypergeometric in $d = d_c$:

$$\beta_\lambda = 3\lambda^2 \left\{ 1 - 10^{-15} G^{4,1}_{3,4}\left(\frac{1.2}{\lambda} \middle| \begin{array}{c} 1, 3, 0.058 \\ 1, 1, 18.85, 0.063 \end{array} \right) \right\}$$

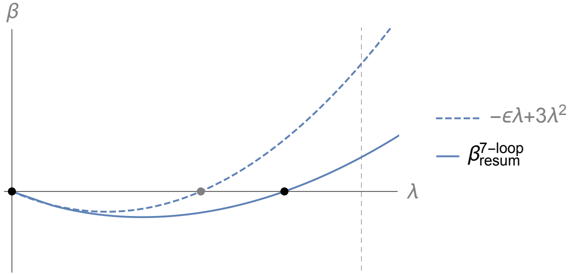


beta from **Antipin, Maiezza, Vasquez 2018**

Bridging to non-perturbative physics: IR FP

Include the scaling term for $d < d_c$:

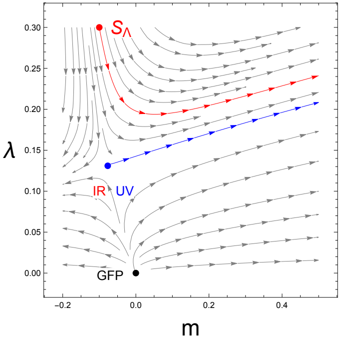
$$\beta_\lambda = -\epsilon\lambda + 3\lambda^2 \left\{ 1 - 10^{-15} G^{4,1}_{3,4}\left(\frac{1.2}{\lambda} \middle| \begin{array}{c} 1, 3, 0.058 \\ 1, 1, 18.85, 0.063 \end{array} \right) \right\}$$



- Does an IR FP exist for each $d < 4$ when $\beta > 0$?

Example: $O(N)$ sigma model

$$V(\phi) = \frac{m^2}{2}(\phi_i\phi_i) + \frac{\lambda}{4!}(\phi_i\phi_i)^2$$



Blue trajectory is nonlinear $O(N)$ sigma model!

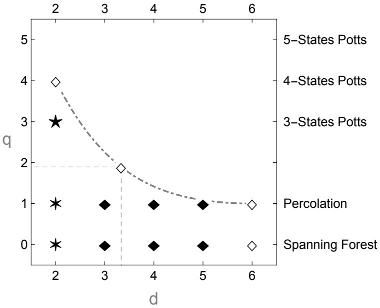
- same FP $\equiv$ same universality class
- asymptotically free in $d = 2$ (Gaussian), nontrivial in $d = 2 + \epsilon$ **Mermin-Wagner**
- Does it always end in Gaussian?

Example: q-states Potts-Landau field theory

Potential invariant under permutations:

$$V(\phi_i) = \lambda \sum_{\alpha=1}^{q} e_i^\alpha e_j^\alpha e_k^\alpha \phi_i \phi_j \phi_k$$

Critical dimension based on power counting is $d_c = 6$
but univ. class nontrivial for $d < d_{ec}(q) < 6$ (critical Potts model)

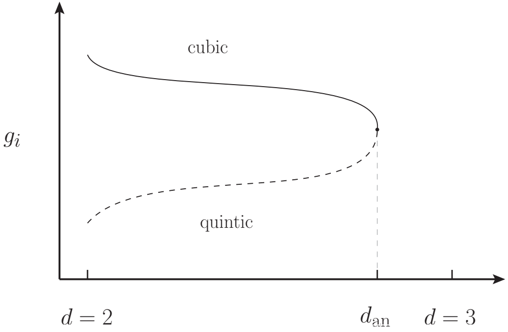


Codello, Safari, Vacca, Z 2019? but several others too

Mechanism: fixed points annihilation

FP annihilates with a multicritical friend for $d > d_{an}$
Therefore $d_{ec} = d_{an}$

For $q = 3$ we have $d_{an} \approx 3 \ll 6$



Very difficult mechanism to see perturbatively
but we have evidence in $d = \frac{10}{3} - \epsilon$ (almost perturbative in $d = 3$)

Recap

The ϵ-expansion:

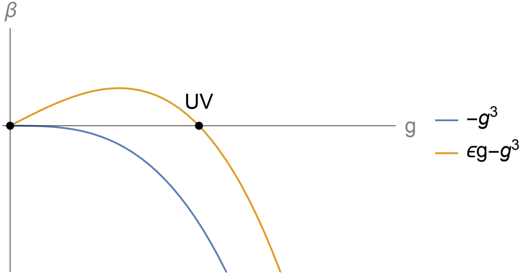
- Perfect for perturbative exploration/classification
- May bridge to non-perturbative (e.g. resum estimates)
- Has interplay with other non-perturbative (e.g. CFT)

However non-trivial things can happen:

- Same universality for different exp. (e.g. LSM vs NLSM)
- Fixed points might collide (Potts)

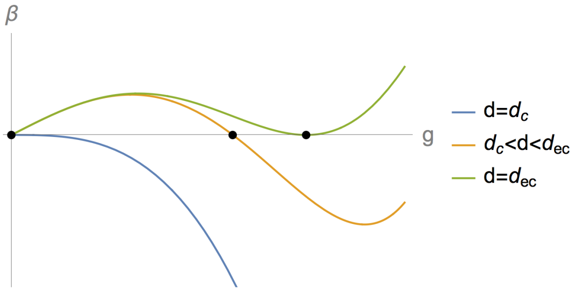
Yang-Mills in more than 4 dimensions

Yang-Mills pure gauge



- $\beta < 0$ for $d_c = 4$, asymptotic freedom
- Continuation to $d = 4 + \epsilon$ already by **Peskin 1980**
- "Confined" vs "deconfined" phases separated by g^* for $\epsilon > 0$?
- Testable on the lattice? **Kawai, Nio, Okamoto 1992**

UV completion for any $d > 4$? Unlikely



Here's a conundrum for you:

Theoretical evidence $d_{ec} \approx 5$ or 6 includes

- $\alpha \simeq 1$ exponent **Peskin 1980**
- Functional RG **Gies 2003**
- Properties of the β series **Morris 2005**

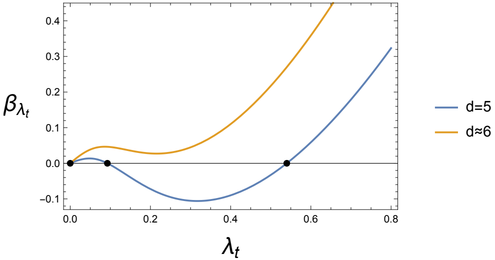
Numerical evidence against $\epsilon > 0$ is

- Only first-order from Wilson plaquette action!

Creutz 1979; Nishimura 1996; Ejiri et al. 2000; Farakos et al. 2003

Theory side: resumming 5-loop YM β

λ_t is t'Hooft coupling. Beta from **Herzog et al. 2017**



Bonati, Z 2020 in prep.

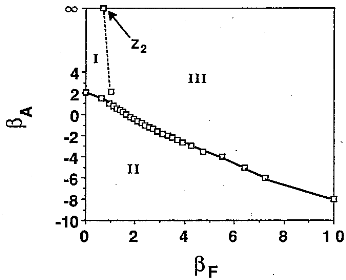
Qualitative features:

- UV FP always present for $d = 5$
- $d_{ec} \approx 6$, higher resums favor $d_{ec} \gtrsim 6$
- Necessity to circumvent Borel singularity or $\mathrm{Im}\beta \neq 0$:(
- Mechanism: collision with an unidentified IR FP

Well actually there is something on the lattice...

Change plaquette action ($N = 2$)

$$S = \sum_{\Box \in \mathcal{L}} \left\{ \beta_F S_\Box^F + \beta_A S_\Box^A \right\} \qquad \frac{4}{g^2} = \beta_F + \frac{8}{3}\beta_A$$



- 1st order for $\beta_A = 0$
- Line of 1st order varying $\beta_A \lesssim 0$
- Indications that $\Delta E \to 0$ along the line

plot from **Kawai et al. 1992**

How does the new action differ from the traditional one?

Expansion in lattice spacing

$$
\begin{aligned}
S &= \sum_{\square} \left\{ \beta_F S^F_{\square} + \beta_A S^A_{\square} \right\} \\
&\simeq \int \mathrm{d}^5 x \left\{ c_2 F^a_{\mu\nu} F^a_{\mu\nu} + c_{3,1} F^a_{\mu\nu} \square F^a_{\mu\nu} + c_{3,2} f_{abc} F^a_{\mu\nu} F^b_{\nu\rho} F^c_{\rho\mu} + \dots \right\}
\end{aligned}
$$

We deduce that higher derivative interactions must play a role

Let's apply the ideas on classification using the ϵ-expansion to infer which could be the active players here:

there are in principle infinitely many higher derivative (non-unitary) generalizations of YM with $d_c = 2n = 4, 6, 8, \dots$

$$
S = \frac{1}{4g^2_{2n}} \int \mathrm{d}^{2n} x \left\{ F^a_{\mu\nu} \square^{n-2} F^a_{\mu\nu} + \dots \right\}
$$

Perturbative $d = 6$ higher derivative YM

The one in $d = 4$ we know already, however

$$S = \frac{1}{2g_6} \int \mathrm{d}^6 x \left\{ (D^\nu F^a_{\mu\nu})^2 + \lambda f_{abc} F^a_{\mu\nu} F^b_{\nu\rho} F^c_{\rho\mu} \right\}$$

is perturbatively renormalizable in $d = 6$

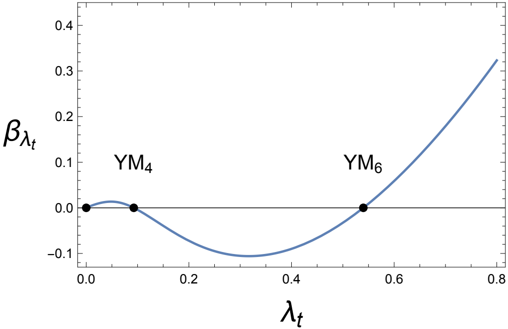
Gracey 2015; Casarin, Tseytlin 2019

- Asymptotically free in g_6
- Need to set λ for freedom, one tunable parameter
- F^2 is relevant deformation hence $\frac{1}{g^2} = 0$ in $\overline{\mathrm{MS}}$ at $d = 6$

Two conjecture(s)

- Either the FP of $d = 4$ YM collides with the one of $d = 6$ like in Potts-Landau
- Or FP of $d = 4$ YM interpolates with the one of $d = 6$ like in $O(N)$ NLSM

In the first case:



Implications for gravity?

If asymptotic safety is asymptotic freedom extended to $d = 2 + \epsilon$
Unlikely continuation to $d = \infty$

my opinion which might differ from **Litim 2004**

because FRG displays same problem for NLSM

again my opinion which might differ **Codello, Percacci 2008**

We need to determine the active players!

Critical dimensions:

$$d_c = 2n = 2, 4, 6, \dots$$

So which models are we talking about?

- $d_c = 2$ controlled by $\int R$ or $\int \left\{ R \frac{1}{\Delta} R \right\}$
- $d_c = 4$ is Stelle's perturbative $\int \left\{ R^2 + C^2_{\mu\nu\rho\theta} + \dots \right\}$
- $d_c = 6$ would include $\int \left\{ R \Box R + C_{\mu\nu\rho\theta} \Box C^{\mu\nu\rho\theta} + \dots \right\}$

Quadratic gravity is distinct universality from $2 + \epsilon$

Codello, Percacci 2006; Rechenberger et al. 2011

Related to original proposal of Smolin with large-N

Smolin 1982

Cubic gravity might pose a threat to continuation
or it could provide a brand new model (but was never studied)

Indications that terms $\mathcal{R}^3$ might play a role

Cubic relevant deformation **Falls et al. 2011**

Estimate of $d_c \approx 5$ to 6 **Gies et al. 2015**

The universality class of $d = 2 + \epsilon$ quantum gravity

or a tale of two central charges

Linearized fluctuations

We want to construct the perturbative series in G

$$S[g] = -\frac{1}{G}\int \mathrm{d}^2x\sqrt{g}R[g]$$

Perform the expansion:

$$g_{\mu\nu} = \overline{g}_{\mu\nu} + h_{\mu\nu}$$

Gauge-fixed propagator:

$$G(p^2)^{\rho\theta}_{\mu\nu} = \frac{1}{p^2}\left\{\delta^{\rho\theta}_{\mu\nu} + \frac{1}{d-2}g_{\mu\nu}g^{\rho\theta}\right\}$$

Conformal mode problem

The pole in the conformal part signals change in d.o.f.s.
Locally

$$g_{\mu\nu} = \mathrm{e}^{2\sigma} \hat{g}_{\mu\nu}$$

In $d = 2$ there is strictly only σ and no spin-2

- Discontinuity at $d = 2$
- Kawai-Ninomija: need to reproduce $d = 2$ in $\epsilon \to 0$ limit
- Jack-Jones: new poles cannot be countered beyond 1-loop

Solution by Aida-Kitazawa: separate conformal mode
and quantize nonlinearly the conformal sector

My point of view on AK's approach

General diff $\quad g_{\mu\nu} \to g_{\mu\nu} + \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$

Aida-Kitazawa suggest to split $g_{\mu\nu} = \mathrm{e}^{2\sigma} \hat{g}_{\mu\nu}$; preserve Weyl

$$\hat{g}_{\mu\nu} \to \hat{g}_{\mu\nu} + \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu - \frac{2}{d} \nabla \cdot \xi \, \hat{g}_{\mu\nu}$$

and then break Weyl by $\langle \sigma \rangle \neq 0$

Diff group is nonlinearly realized

$$\mathit{TDiff} \ltimes \mathit{Weyl} \to \mathit{Diff}^* \simeq \mathit{Diff}$$

realization discussed in **Gielen, de Leon Ardon, Percacci 2018**

Natural expansion

$$g_{\mu\nu} \to \mathrm{e}^{2\sigma} [\mathrm{e}^{\hat{h}}]_\mu{}^\rho g_{\rho\nu} = g_{\mu\nu} + h_{\mu\nu} + \frac{1}{2} h_\mu{}^\rho h_{\rho\nu} + \dots$$

AK gravity: functional perspective

Renormalized to two loops by Aida and Kitazawa
(with counterterms on-shell)

In passing we briefly discussed how to *best* renormalize it.
Consider unimodular dilaton gravity
with unimodular metric and normalized dilaton ψ:

$$S[\psi, \hat{g}] = -\frac{1}{G} \int \mathrm{d}^d x \sqrt{\hat{g}} \left\{ L[\psi, \phi] R[\hat{g}] + \frac{1}{2} (\partial \psi)^2 \right\}$$

Impose $\langle \psi \rangle = 1$ and $L[1, 0] \equiv 1$ along the RG flow

$$\beta_G = \epsilon G - \frac{25 - c}{24\pi} G^2 \qquad G^* = \frac{24\pi\epsilon}{25 - c}$$

brief discussion in **Martini, Z 2019**

procedure reminiscent of yesterday's talk by **Orlando**

My point of view on JJ gravity

Diff is now linearly realized

Inconsistency at $d = 2$, but $\epsilon = 2$ at $d = 4$...
Solution: give different names to the poles in $d - 2$

Gauge-fixed propagator in $d = 2 + \bar{\epsilon}$

$$G(p^2)_{\mu\nu}^{\rho\theta} = \frac{1}{p^2} \left\{ \delta_{\mu\nu}^{\rho\theta} + \frac{1}{\bar{\epsilon}}\, g_{\mu\nu} g^{\rho\theta} \right\}$$

$\overline{\mathrm{MS}}$ poles in $d = 2 + \epsilon$

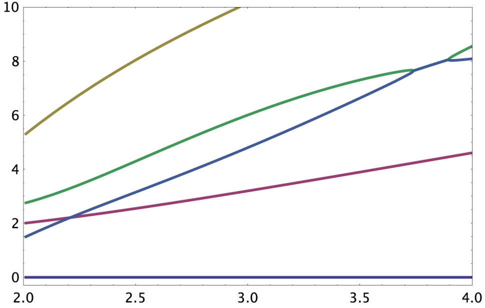
Traces of the metric $g^{\mu}{}_{\mu} = d$

JJ gravity: some result

Setting $\epsilon = \bar{\epsilon} = d - 2$

$$\beta_G = \varepsilon G - \frac{19 - c}{24\pi} G^2 \qquad G^* = \frac{24\pi\varepsilon}{19 - c}$$

Leading spectrum of scaling dimensions Δ up to $\mathcal{R}^2$:



Complex conjugate pairs might danger unitarity **Martini, Z 2020 in prep**

Conclusion

- $5d$ YM can be interesting case study for asymptotic safety besides the naive arguments that the asymptotically free theory can be continued above $d = 4$
- Considerations on universality and universality classes might help pinning where to look for more evidence (and which evidence) of asymptotically safe quantum gravity

Thank you!