Multiplicity Fluctuations and Temperature Fluctuations

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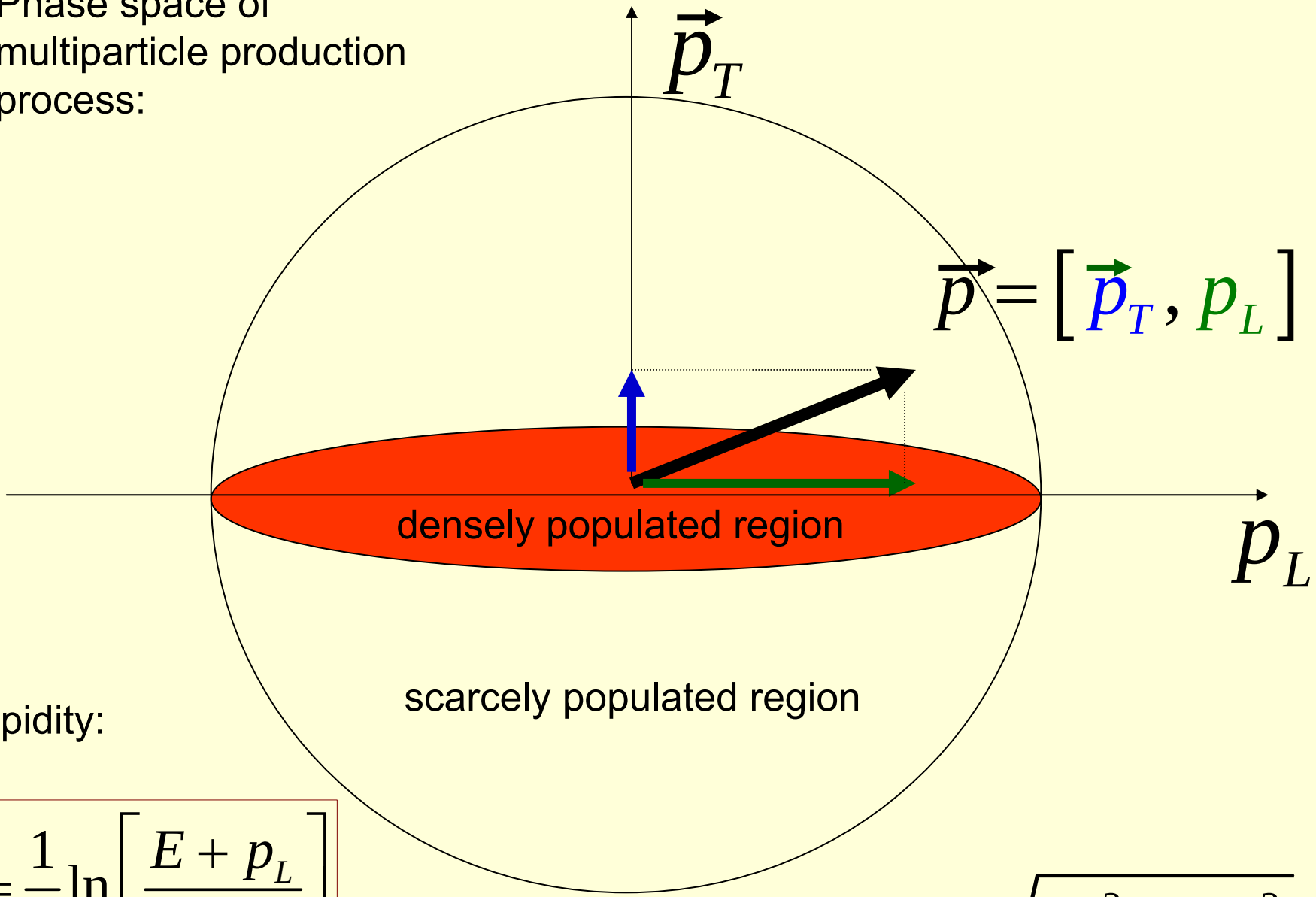
Statistical Particle Production: Beyond the first moment
25-28 April, Bad Liebenzell

Phase space of multiparticle production process:



$$\vec{p} = [\vec{p}_T, p_L]$$

Rapidity:

$$y = \frac{1}{2}\ln\left[\frac{E + p_L}{E - p_L}\right]$$

$$E = \sqrt{m^2 + \vec{p}_2^{\,2}}$$

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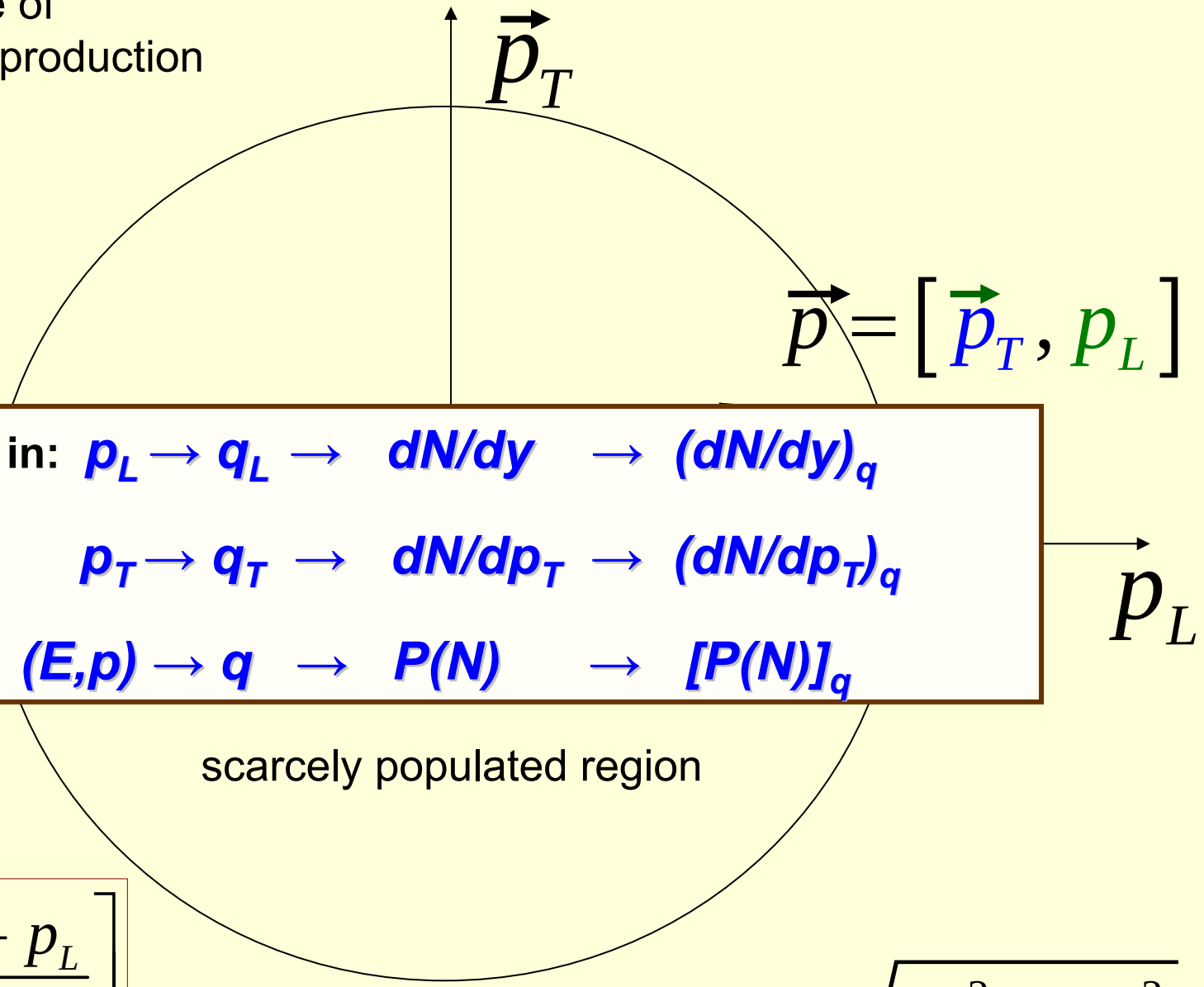

Fluctuations in:

$p_L \rightarrow q_L \rightarrow dN/dy \rightarrow (dN/dy)_q$

$p_T \rightarrow q_T \rightarrow dN/dp_T \rightarrow (dN/dp_T)_q$

$(E,p) \rightarrow q \rightarrow P(N) \rightarrow [P(N)]_q$

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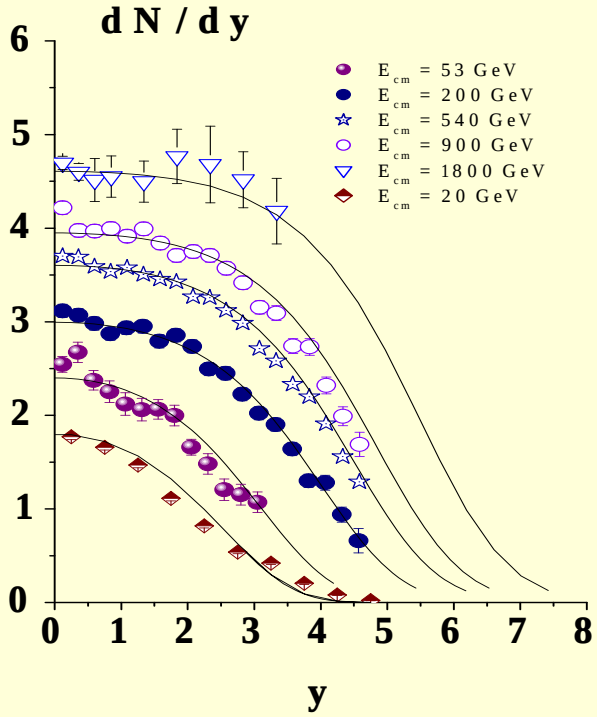

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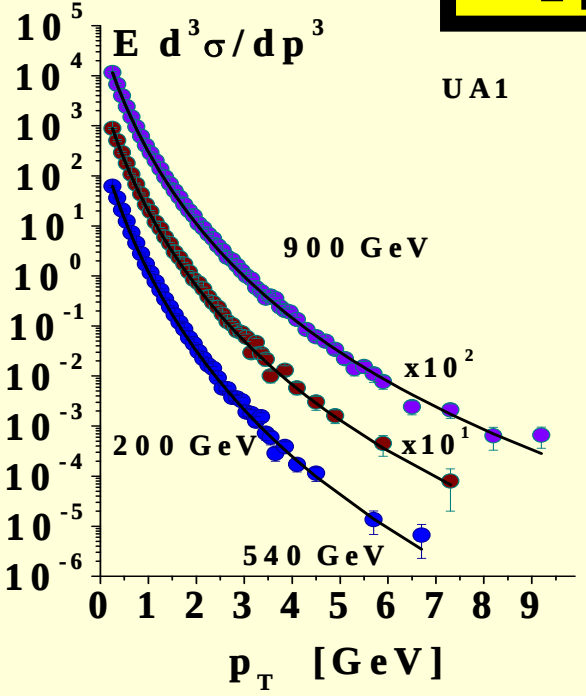

$S^{1/2}$ [GeV]	q_L	$T_L=1/\beta$ [GeV]
200	1.203	12.12
546	1.262	22.38
900	1.291	29.47

$$q = \frac{q_L T_L^2 + q_T T_T^2}{T^2} - \frac{T_L^2 + T_T^2}{T^2} + 1$$

$$\Rightarrow \quad q \approx q_L \quad because \quad T = \sqrt{T_L^2 + T_T^2} \approx T_L$$

$S^{1/2}$ [GeV][GeV]	q_T	$T_T=1/\beta_T$
200	1.095	0.134
546	1.105	0.135
900	1.110	0.140

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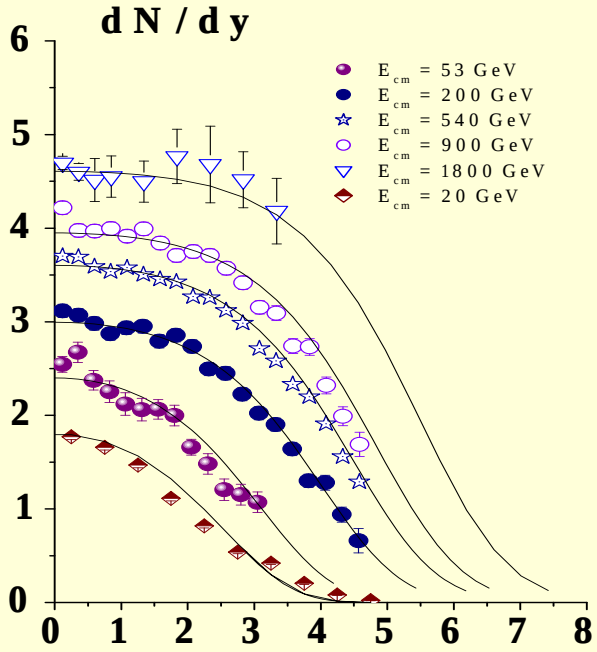

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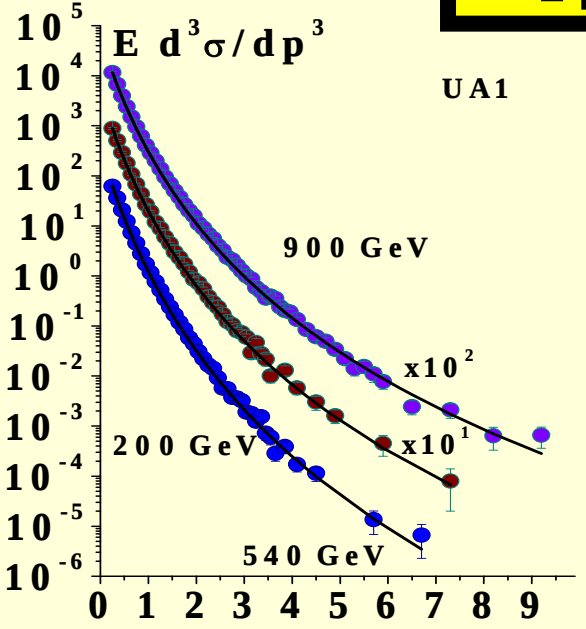

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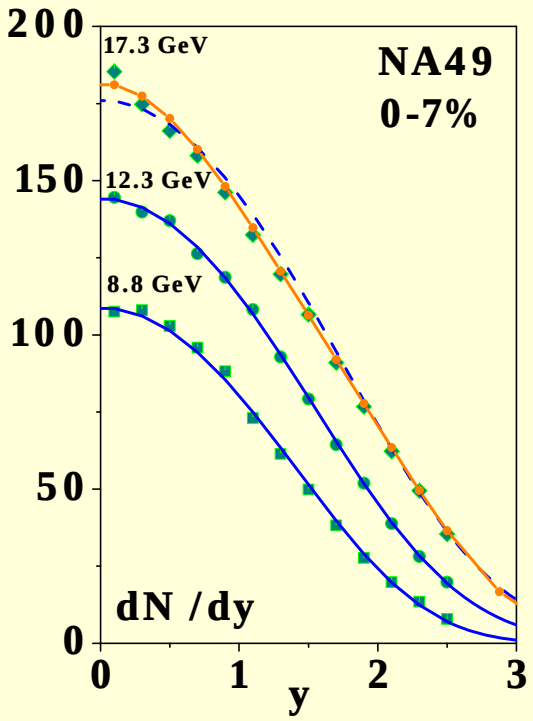
$q_l - 1 \approx 0.2\text{-}0.3$

$q_T - 1 \approx 0.05\text{-}0.1$

Example of application to AA collisions



Example of use of MaxEnt method applied to some NA49 data for π^- production in PbPb collisions (centrality 0-7%):

- **(blue lines)**

$S^{1/2}$	q	K_q
8.8	1.040	0.22
12.3	1.164	0.30
17.3	1.200	0.33

- **(orange line)** **q=1, two sources of mass M=6.34 GeV located at |y|=0.83**

this is example of adding new dynamical assumption

Possible meaning of parameter q_L in rapidity distributions

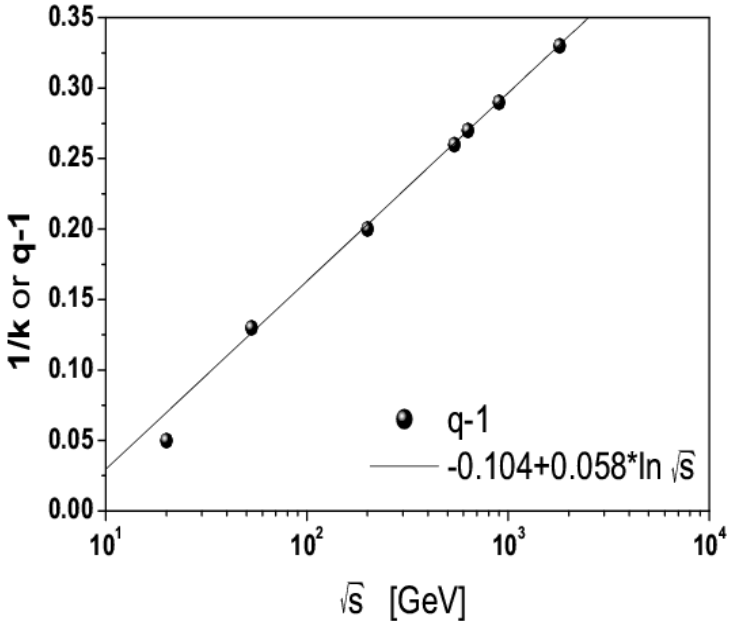


Figure 6:

The values of the nonextensivity parameter q obtained in fits shown here and listed in Table I compared with the values of the parameter k of Negative Binomial distribution fit to the corresponding multiplicity distributions (as given by C.Geich-Gimbel, *Int. J. Mod. Phys.* A4 (1989) 1527).

NUWW PRD67 (2003) 114002

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(*) From fits to rapidity distribution data one gets systematically q>1 with some energy dependence

(*) What is now behind this q?

(*) y-distributions ⇔ 'partition temperature' $T \approx K \cdot \sqrt{s}/N$ (N=multiplicity)

(*) $q_L \approx q$ ⇔ fluctuating T ⇔ fluctuating N

(*) Conjecture: q-1 should measure amount of fluctuation in P(N)

(*) It does so, indeed, see Fig. where data on q obtained from fits are superimposed with fit to data on parameter k in Negative Binomial Distribution!

Parameter q measures dynamical fluctuations in P(N)

(*) **Experiment: P(N) is adequately described by NBD depending on <N> and k (k≥1) affecting its width:**

$$\frac{1}{k} = \frac{\sigma^2(N)}{< N >^2} - \frac{1}{< N >}$$

(*) If 1/k is understood as a measure of fluctuations of <N>, then

$$P(N) = \int_0^\infty d\bar{n} \frac{\bar{n}^n \exp(-\bar{n})}{n!} \cdot \frac{\gamma^k \bar{n}^{k-1} \exp(-\gamma\bar{n})}{\Gamma(k)}$$

$$= \frac{\Gamma(k+n)}{\Gamma(1+n)\Gamma)\Gamma(} \cdot \frac{\gamma^k}{(\gamma+1)^{k+n}} \qquad \text{with} \qquad \gamma = \frac{k}{<\bar{n}>}$$

(P.Carruthers,C.C.Shih, Int.J.Phys. A4 (1989)5587)

$$\frac{1}{k} = D(\bar{n}) = \frac{\sigma^2(\bar{n})}{<\bar{n}>^2} = q - 1$$

(*) → one expects: q=1+1/k what indeed is observed

[G.Wilk, Z.Włodarczyk,EPJA40(2009)299; F.Navarra,O.Utyuzh,WW, PRD67(2003)114002]

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Digression:

C.Vignat and A.Plastino in *Estimation in a fluctuating medium and power-law distributions , PlA360(2007)415,* demanded to give credit to

R.A.Fisher, Ann.Eugenics 11(1941)182, available at

http://digital.library.adelaide.edu.au/coll/special/fisher/stat_math.html

as the first person to propose and discus such connection between the Poisson and NB distributions.

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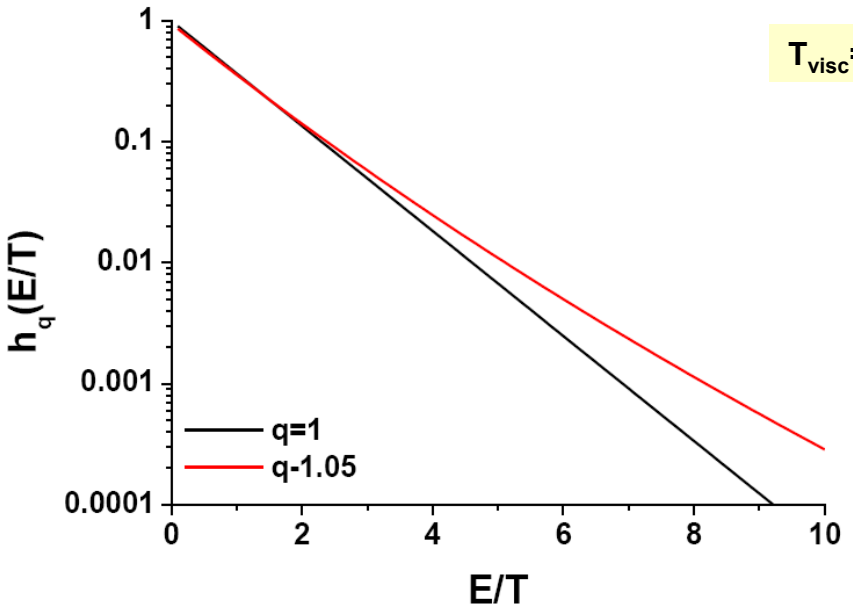
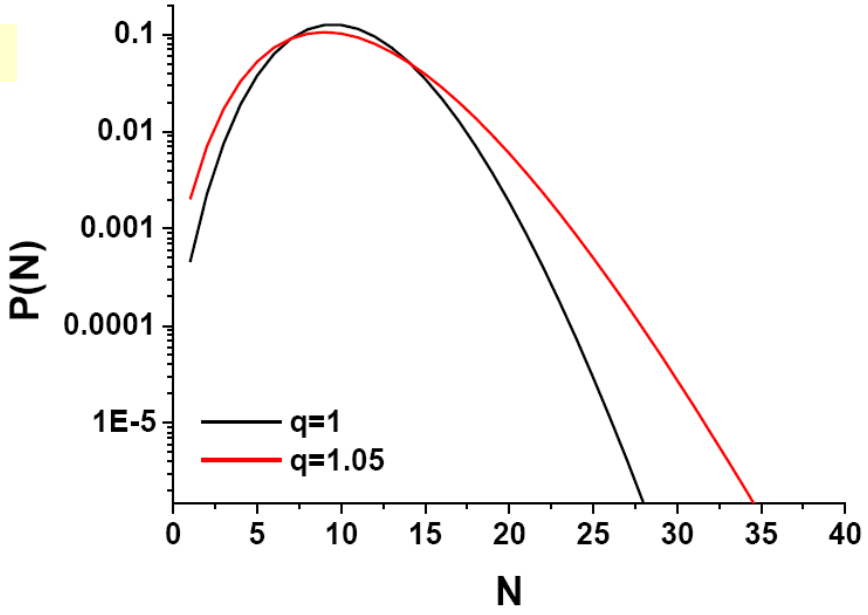
Temperature fluctuations vs. Multiplicity fluctuations





$$g(E_i) = C\exp\left(-\frac{E_i}{T}\right) \quad \Rightarrow$$

$$h_q(E_i) = C_q\left[1-(1-q)\frac{E_i}{T_{eff}}\right]^{\frac{1}{1-q}}$$

where $\quad q = 1 + \dfrac{Var(T)}{\langle T\rangle^2}$

and $\quad T_{eff} = T_0 + (q-1)T_{visc}$

$$P(N) = \frac{\langle N\rangle^N}{N!}\exp\left(-\langle N\rangle\right); \quad \langle N\rangle = \frac{E}{T} \quad \Rightarrow$$

$$P(N) = \frac{\Gamma(N+k)}{\Gamma(N+1)\Gamma(k)}\frac{\left(\frac{\langle N\rangle}{k}\right)^N}{\left(1+\frac{\langle N\rangle}{k}\right)^{N+k}}$$

where $\quad k = \dfrac{1}{q-1}$ *and* $\quad Var(N) = \langle N\rangle + (q-1)\langle N\rangle^2$

[G.Wilk, Z.Włodarczyk, PRC79(2009)054903; EPJA40(2009)299; Physica A376(2007)279]

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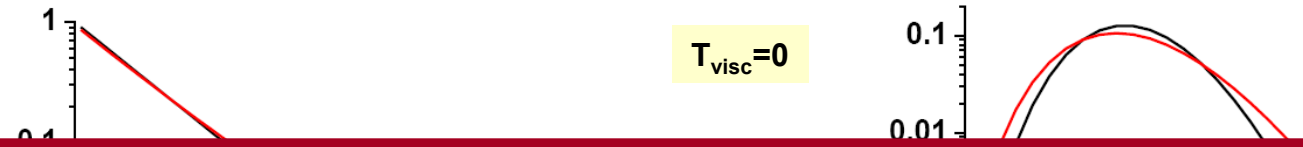

Whenever N variables $E_{1,\ldots,N}$ follow exponential distribution and satisfy condition

$$\sum_{i=0}^{N} E_i \le E \le \sum_{i=0}^{N+1} E_i$$

then the corresponding multiplicity N has a Poissonian distribution.

If, instead, they follow the q-exponential distribution,
then N has negative binomial distribution

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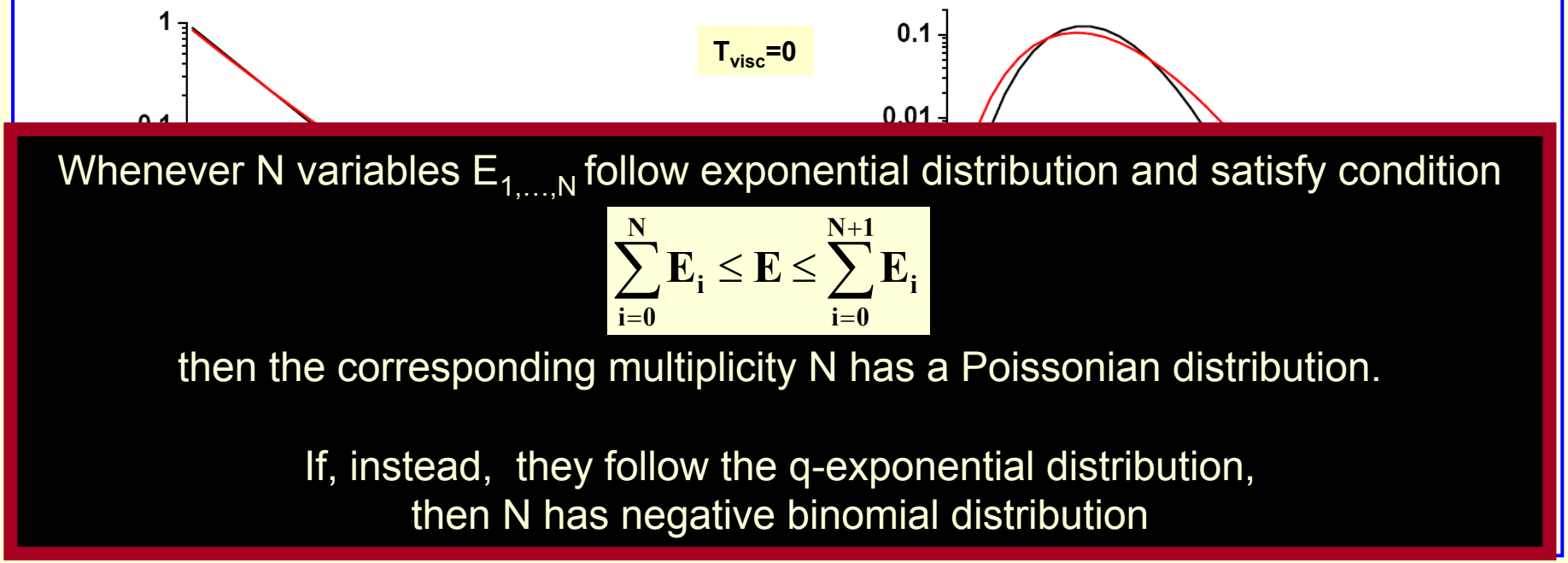

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where $k = \frac{1}{q-1}$ *and* $\mathrm{Var}(N) = \langle N \rangle + (q-1)\langle N \rangle^2$

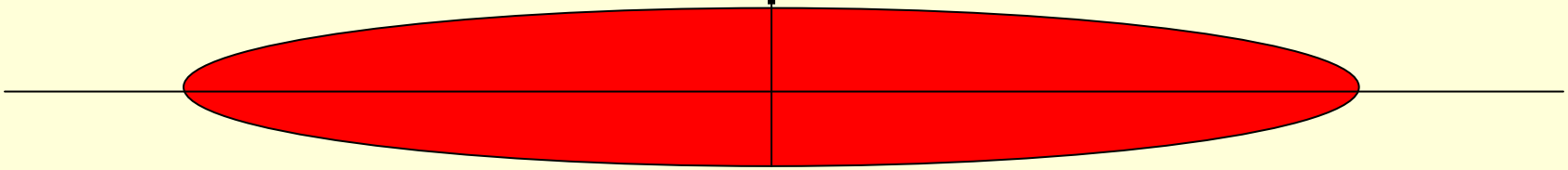
or

$$\frac{\mathrm{Var}\left(\frac{1}{T}\right)}{\left\langle \frac{1}{T} \right\rangle^2} = q - 1 = \frac{\frac{\mathrm{Var}(N)}{\langle N \rangle} - 1}{\langle N \rangle}$$

q_T

Let us now proceed to the problem of description of transverse momentum spectra, which are important because they provide us with information on thermal properties of the hadronizing system

$p_T \leftrightarrow T_T = T$



M.Biyajima, T.Mizoguchi, N.Nakamija, N.Suzuki and G.Wilk, EPJ C48 (2006) 597

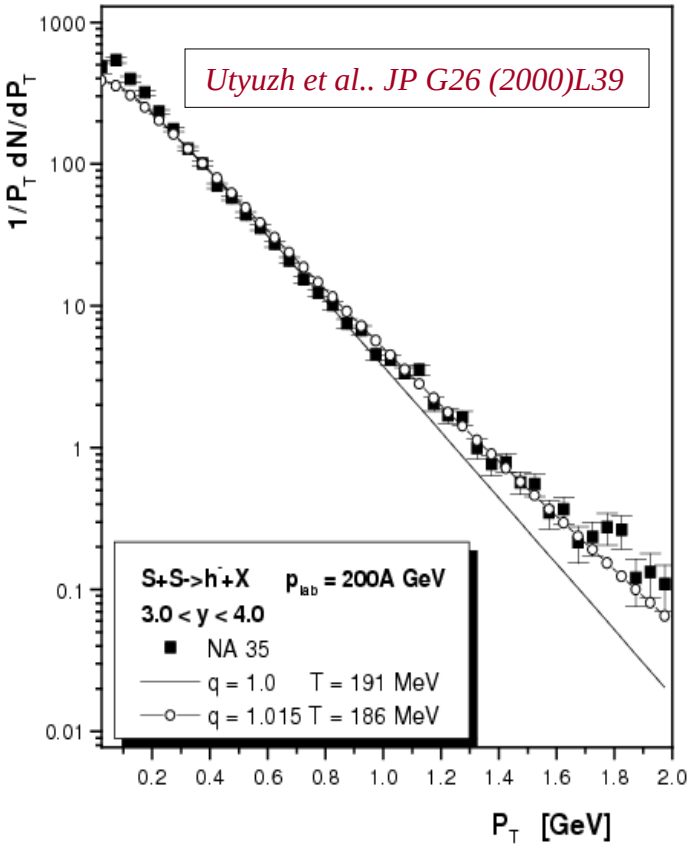


Fig.2

Some comments on T-fluctuations:

(*) Common expectation: slopes of p_T distributions $\Rightarrow$ information on T

(*) Only true for q=1 case, otherwise it is <T>, |q-1| provides us additional information

(*) Example: $|q-1|=0.015 \Rightarrow \Delta T/T \approx 0.12$

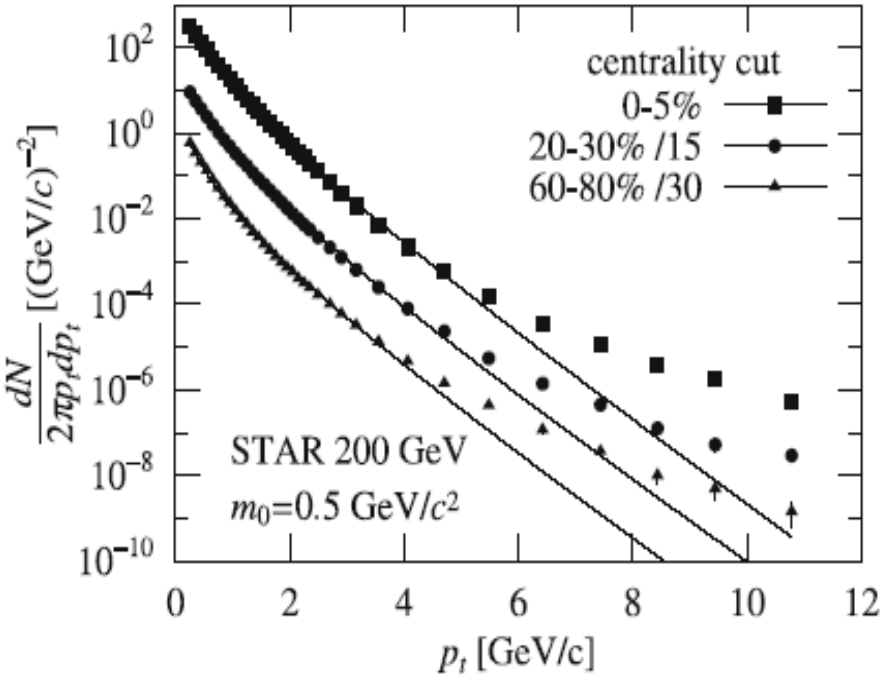
(*) Important: these are fluctuations existing in small parts of the hadronic system with respect to the whole system rather than of the event-by-event type for which $\Delta T/T = 0.06/\sqrt{N} \to 0$ for large N

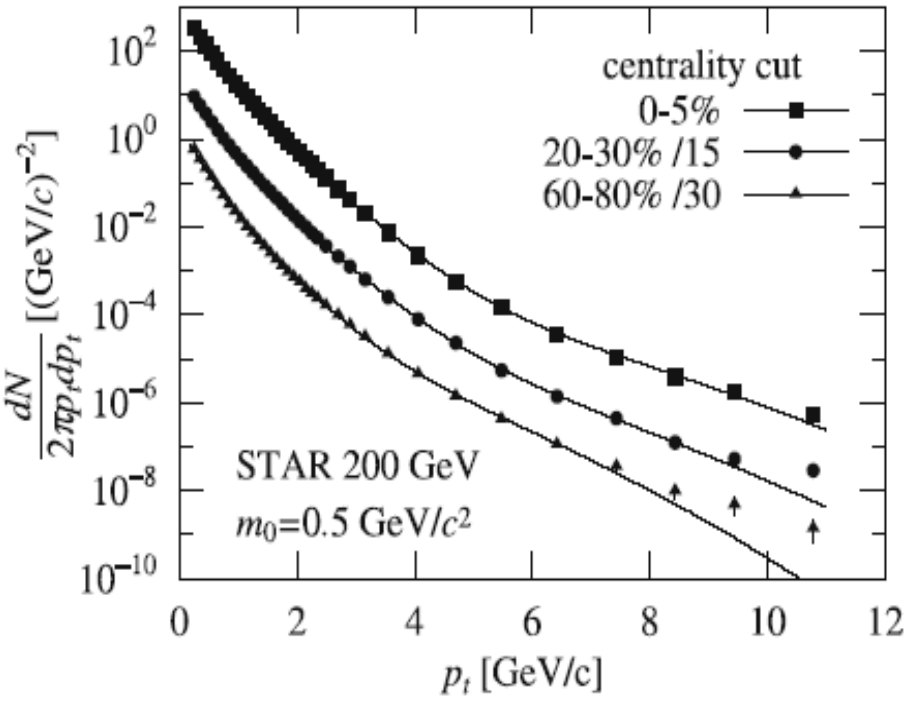
Such fluctuations are potentially very interesting because they provide a direct measure of the total heat capacity of the system

$$\frac{\sigma^2(\beta)}{<\beta>^2} = \frac{1}{C} = \omega = q - 1$$

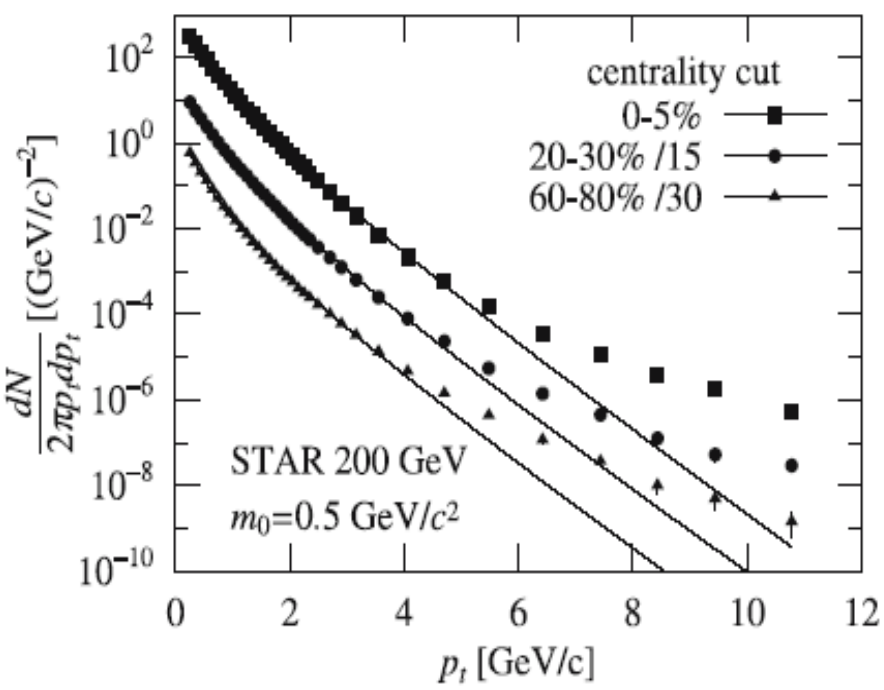
Prediction: C ~ volume of reaction V, therefore q(hadronic)>>q(nuclear)

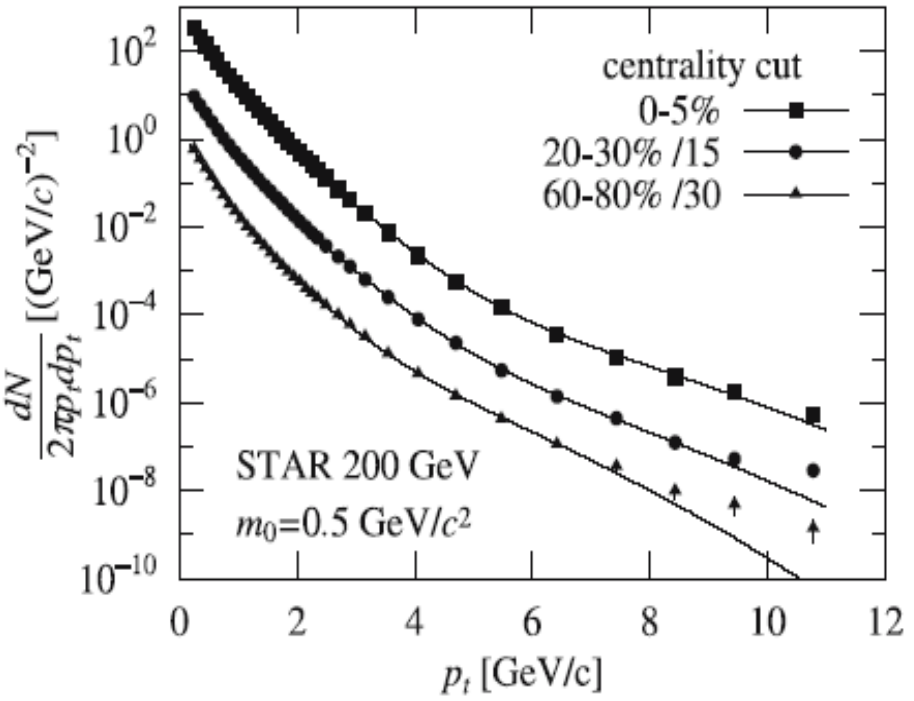
Experiments on Au+Au → π + X collisions at energy 200 GeV per nucleon (in CMS system) performed at RHIC accelerator in Brookhaven NL (USA) show nonexponential, power-like tail for large transverse momenta. These experiments are important because from such distributions one expects to deduce the temperature T of the hadronizing system and to learn about possible phase transition from the hypothetical Quark-Gluon Plasma to hadronic matter.





There are many attempts to explain these results using some kind of nonequilibrium approach like, for example, flow of prehadronic and hadronic matter and/or decay of hadronic resonances produced in such collisions. Rather than excluding them we have investigated the possibility that observed nonexponential spectra could result from some kind of equilibrium characteristic of nonextensive thermodynamics. One can then successfully account for the whole range of the observed transverse momenta.





Hagedorn bootstrap model: ***gas of noninteracting resonances which consists of resonances etc., with charecteristic exponential mass spectrum ρ(m)***

(*) limiting temperature: hadronic phase can live only at $T<T_H=1/\beta_H$

(*) energy distributions are governed by temperature $T_0=1/\beta_0$

$$\frac{d^3\sigma}{dp^3} = \int_0^\infty dm\,\rho(m)\exp\left(-\sqrt{p_T^2 + p_L^2 + m^2}\cdot\beta_0\right)$$

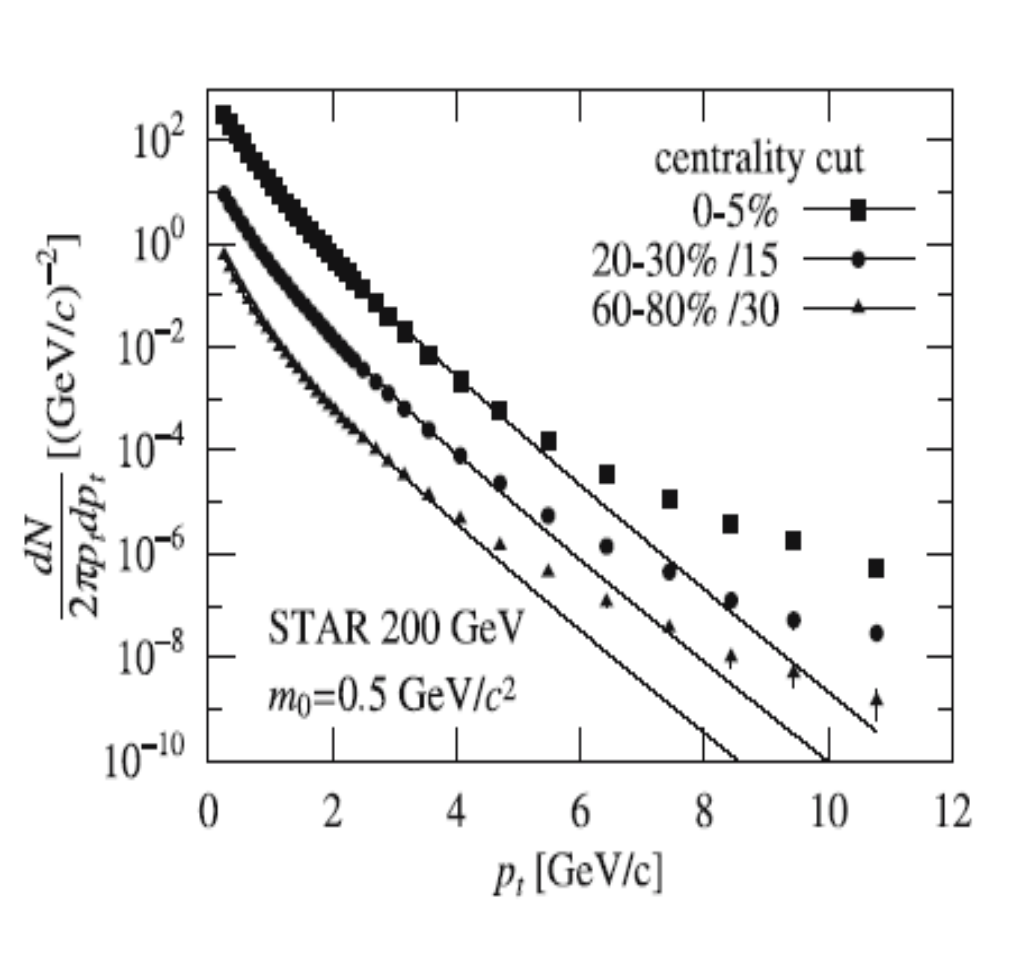
$$f(p_T) = \frac{d^2\sigma}{2\pi p_T dp_T} = C\int_{m_0}^\infty dm\rho(m)m_T K_1(\beta_0 m_T)$$

$$\rho(m) = \frac{\exp(\beta_H m_T)}{\left(m^2 + m_0^2\right)^{5/4}};\quad m_T = \sqrt{m^2 + p_T^2}$$

However: the formula

$$\frac{d^3\sigma}{dp^3} = \int_0^\infty dm\, \rho(m) \exp\left(-\sqrt{p_T^2 + p_L^2 + m^2} \cdot \beta_0\right)$$

can describe data only in limited range of transverse momenta



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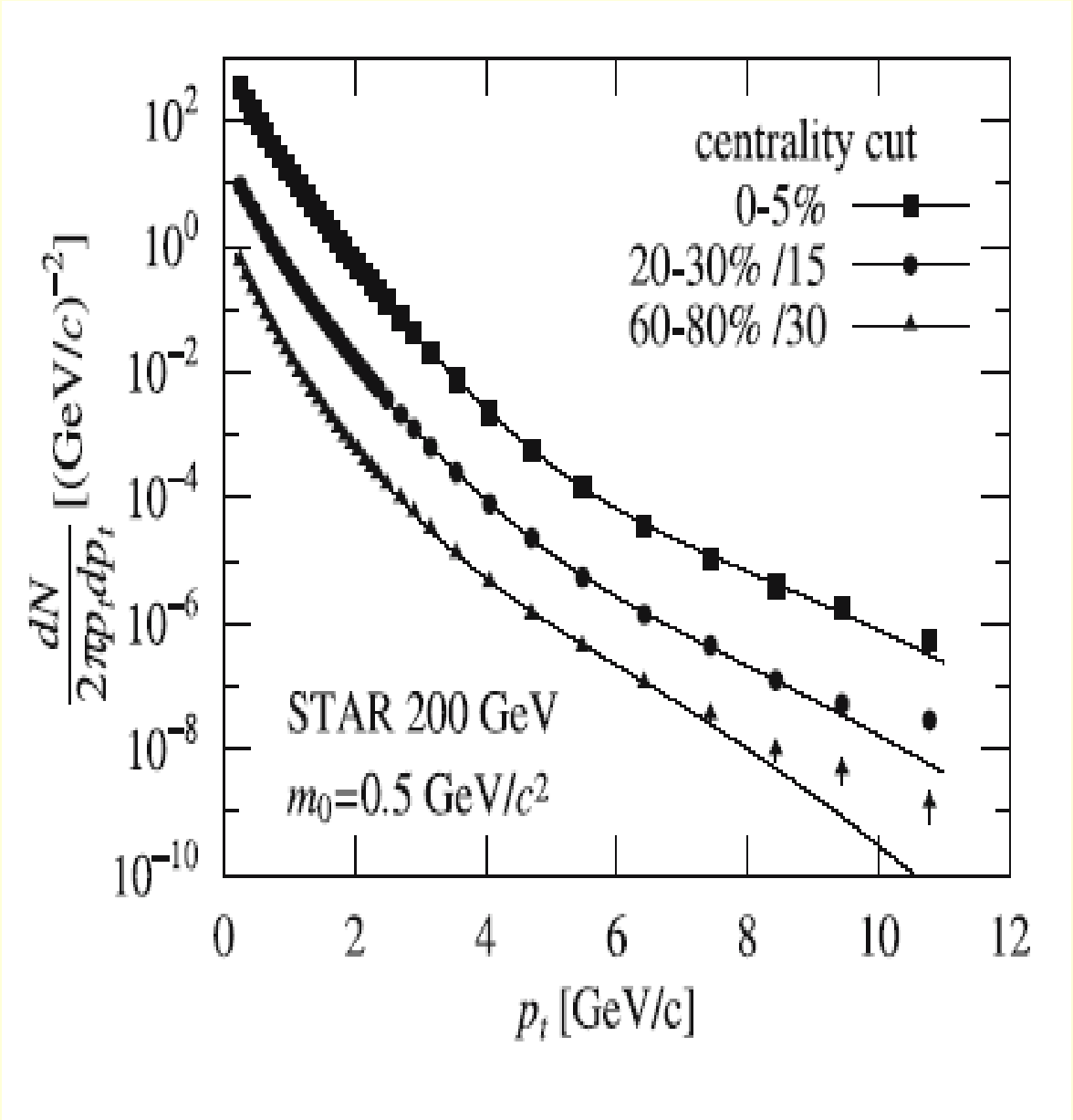
To get agreement in the whole range of transverse momenta, like this,



one needs one of the following choices,

(P) Simple power law:

$$f(p_T) = C\left(1+\frac{p_T}{p_0}\right)^{-n} \rightarrow \begin{cases} \exp\left(-\frac{n}{p_0}p_T\right) & \text{for} \quad p_T \rightarrow 0 \\ \left(\frac{p_0}{p_T}\right)^n & \text{for} \quad p_T \rightarrow \infty. \end{cases}$$

C. Michael, L. Vanryckeghem, J. Phys. G **3**, L151 (1977)
C. Michael, Prog. Part. Nucl. Phys. **2**, 1 (1979)

(T) Simple nonextensive formula

$$f(p_T) = C\int_0^\infty dp_L \left[1-(1-q)\frac{\sqrt{p_L^2+m_T}}{T_0}\right]^{\frac{1}{1-q}}$$

(H) Modified Hagedorn formula

$$f(p_T) = C\int dy\, \cosh y \int dm\rho(m)m_T \left[1-(1-q)\frac{m_T\cosh y}{T_o}\right]^{\frac{1}{1-q}}$$

What is observed:

(*) q is increasing when going from most central to most peripheral events.
(from, for example, q-1 ≈ 0.04 to q-1 ≈ 0.09 for model (T)).

This observation fits nicely the idea that q, when representing fluctuations in the system, depends (via heat capacity C) inversely on the volume of the hadronizing system, which decreases from central to peripheral collisions.

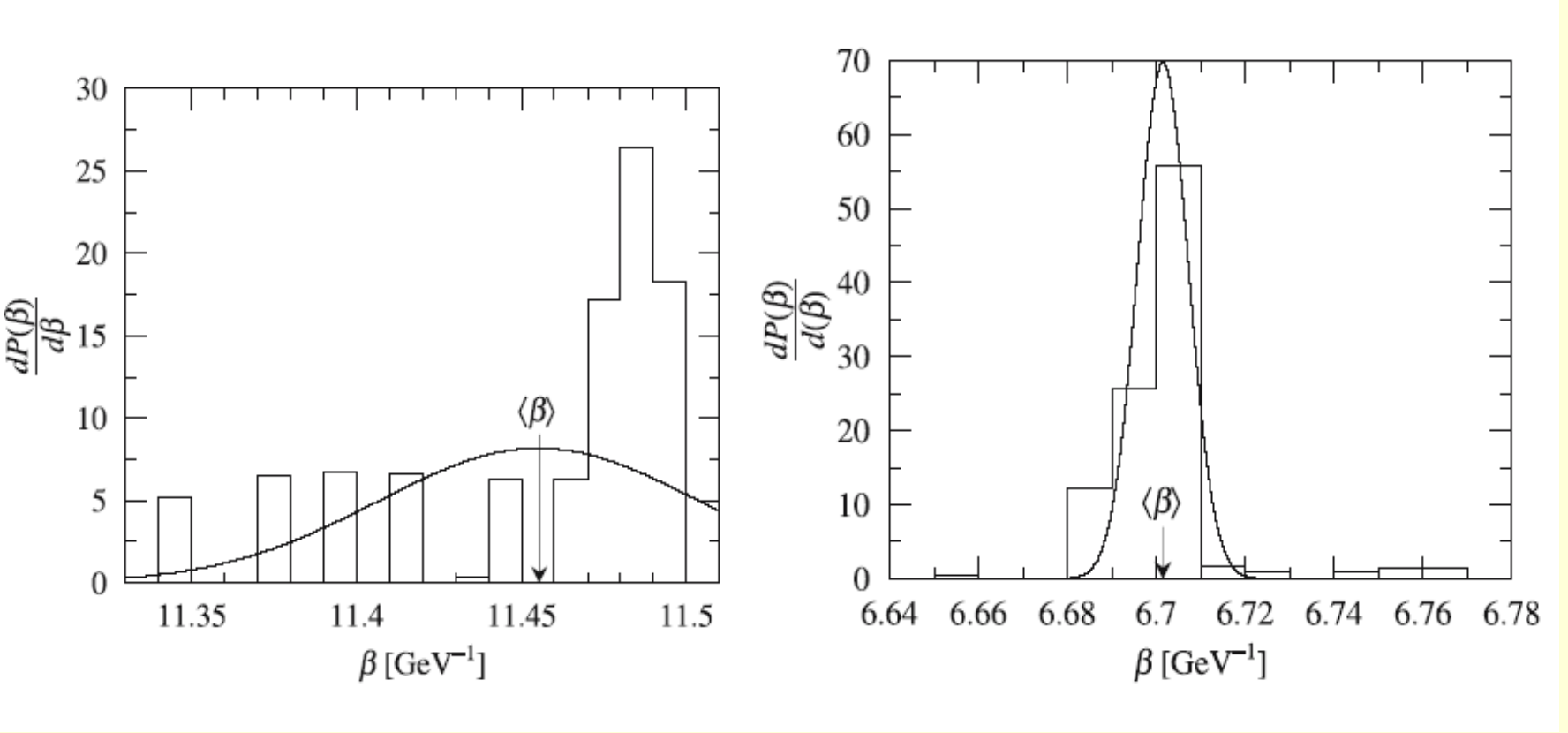
(*) Both T and q estimated by using simple nonextensive formula (T) are systematically higher than those estimated using modified Hagedorn model (H)
(from, for example, T_0≈0.2 GeV, q-1≈0.05 for model (T) to T_0≈0.15 GeV, q-1≈0.00012 for model (H) with T_H≈0.16GeV).

This observation fits nicely the idea that q represents a joint action of intrinsic fluctuations in the system. The difference between model (T) and model (H) lies in the additional fluctuations present in (H) due to the assumed spectrum mass of resonances (described by Hagedorn temperature T_H). Therefore part of fluctuations described in model (T) by parameter q is in the model (H) accounted by parameter T_H .

(*) *This is best seen below where the temperature fluctuations for q=1 and q>1 are shown.*

The centrality cut 0-5% region of STAR data was fitted (point by point, separately) using model (H) with q=1 (left panel) and with q=1.00014 (right panel). Fit was performed by fixing all parameters except β_0. It is assumed then the reciprocal of the each error bar calculated by fitting program MINUIT is proportional to the corresponding probability of this value of β. In this way a whole probability distribution for β is obtained in form of histogram – see Figure. The histogram in each panel is fitted to gamma distribution (colid curves).

NOTICE: good fit can be obtained only with q>1 – in this case distribution of temperature is very narrow, the result is very much q-sensitive!



One must be aware of the fact that:

there is still ongoing discussion on the meaning of the temperature in nonextensive systems. However, the small values of the parameter q-1 deduced from data allow us to argue that, to first approximation, our T_0 can be regarded as the hadronizing temperature in such system.

One must also remember that in general what we study here is not so much the state of equilibrium but rather some kind of stationary state.

For a thorough discussion of the temperature of nonextensive systems, see S.Abe, Physica A 368 (2006) 430 (cond-mat/0504036).

It is also worth to be aware that in addition to the possibility of long-range correlations and memory effects to be at work in relativistic heavy-ion reactions (which were so far not yet proven) one can also view $q > 1$ as a general, leading order finite-size effect, $q = 1 + O(1/N)$, see A.S. Parvan, Phys. Lett. A 350, 331 (2006) (cond-mat/0602219).

What does it mean? Our understanding of this observation is following:

(*) The more we know on the internal dynamics (for example in the form of mass spectrum of resonances in the Hagedorn model) the near we are the true BG approach. However, for a time being such knowledge is not model independent. One is therefore using q-statistics to allow for more model independent presentation of experimental results.

(*) As long as in fits to data one gets $|q-1| \neq 0$, it means that some dynamics is still there to be uncovered

(dynamics = everything outside the true BG approach).

Multiplicity fluctuations

(*) The large (and increasing) number of particles produced make it possible to study fluctuations in different physical observables on event-by-event basis

(*) Fluctuations are potentially very important source of information on the thermodynamic properties of strongly interacting systems formed in heavy ion collisions like, for example, specific heat, its chemical potential, matter compressibility,….

(*) Fluctuations of multiplicity observed in heavy ion collisions exhibit spectacular and unexpected features as function of the number of participants:
scaled variance of the multiplicity distributions, Var(N)/<N>, increases when proceeding from the central toward peripheral collisions, i.e., when the number of participants N_P decreases.

(*) So far models aimed at describing all essential features of multiparticle production processes fail to describe the above observation of behaviour of Var(N)/<N> versus N_P.

Multiplicity fluctuations

(*) **Fluctuations of multiplicity** observed in heavy ion collisions exhibit spectacular and unexpected features as function of the number of participants:

scaled variance of the multiplicity distributions, Var(N)/<N>, increases when proceeding from the central toward peripheral collisions, i.e., when the number of participants N_P decreases.

Notice:

(*) There are also other types of fluctuations concerning, for example, the whole energy available (inelasticity) but I shall not discuss them here (notice that they are fluctuations from event to event rather than in a given event).

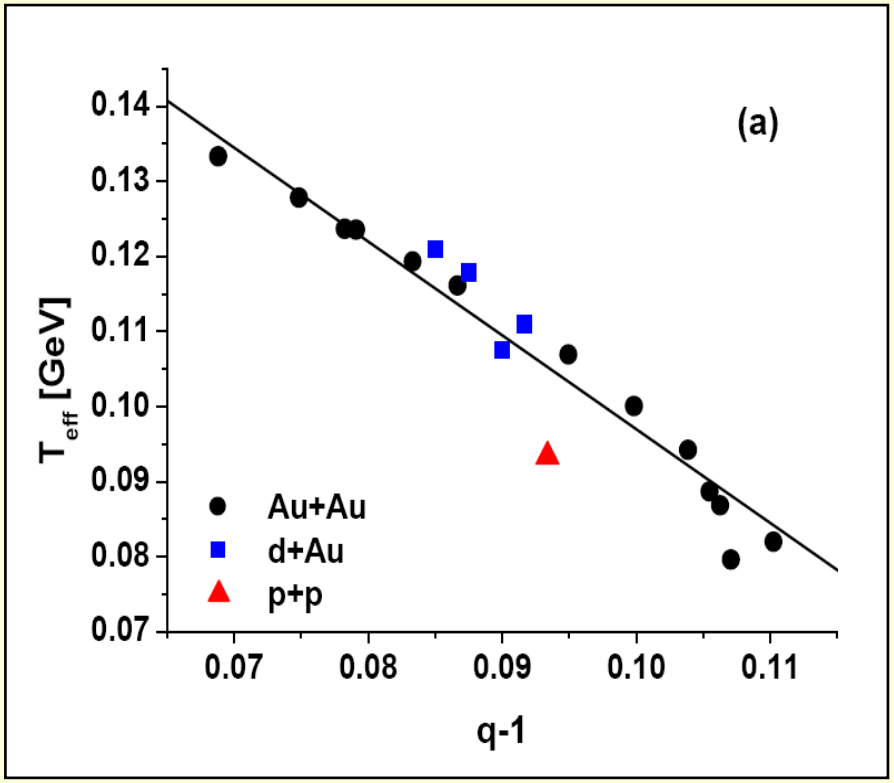
(*) In nuclear collisions there is energy exchange between the local heat bath and a still bigger environment (surroundings). Therefore in this case some energy can be transfered either

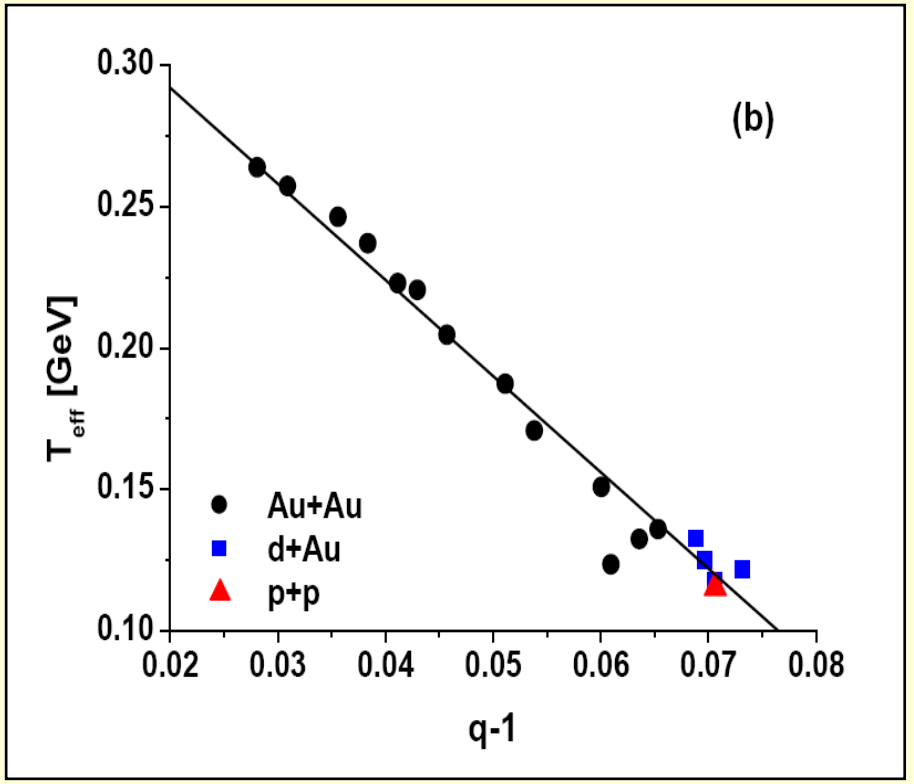
- to the system from this environment (as in cosmic rays case) or
- from the system to this environment (as in nuclear collisons case)

[WW, PRC79(2009)054903;EPJA40(2009)299; hep-ph/0904.0528-CEJP(2010)].

In this case $T \rightarrow T_{eff} = T_0 + (q-1)T_{visc}$ **with other formulae remaining intact. Notice new parameter,** T_{visc}**, here.**

$$T_{eff} = T_0 + (q-1)T_{visc}$$

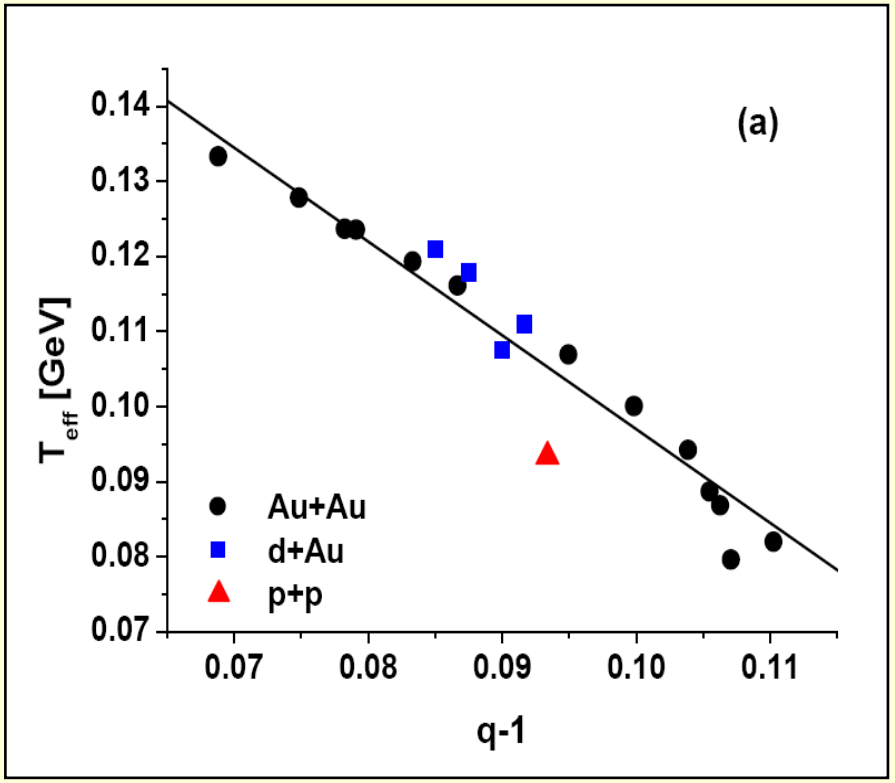


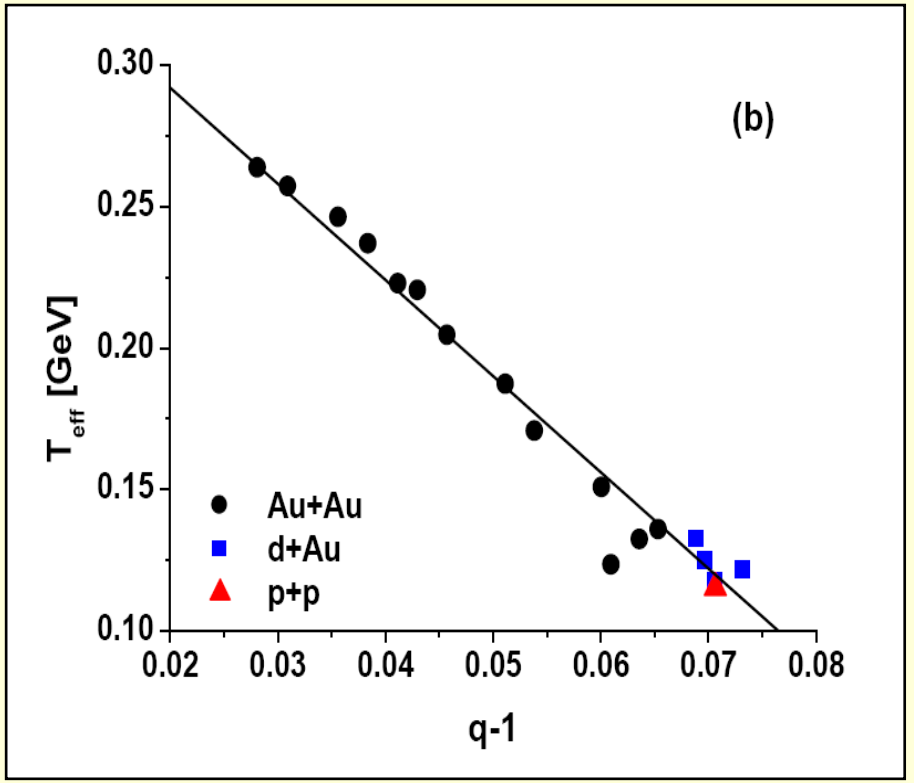


From RHIC data on 200 GeV for p_T distributions of pions (a) and antiprotons (b)

T_{eff}=0.22-1.25(q-1) for (a) and T_{eff}=0.36-3.4(q-1) for (b)

$$T_{eff} = T_0 + (q-1)T_{visc}$$

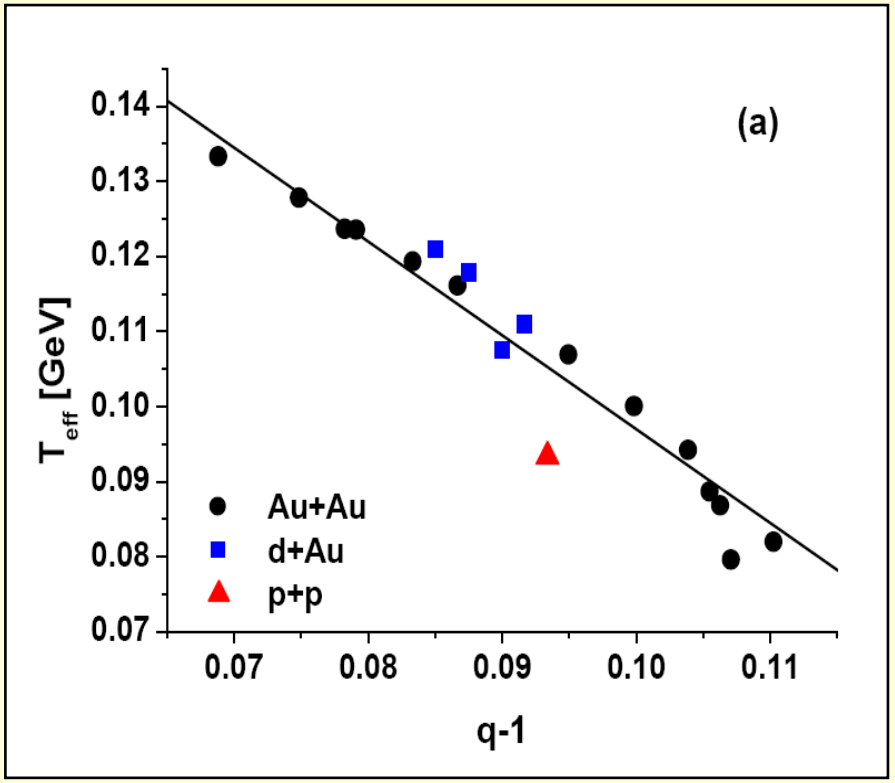


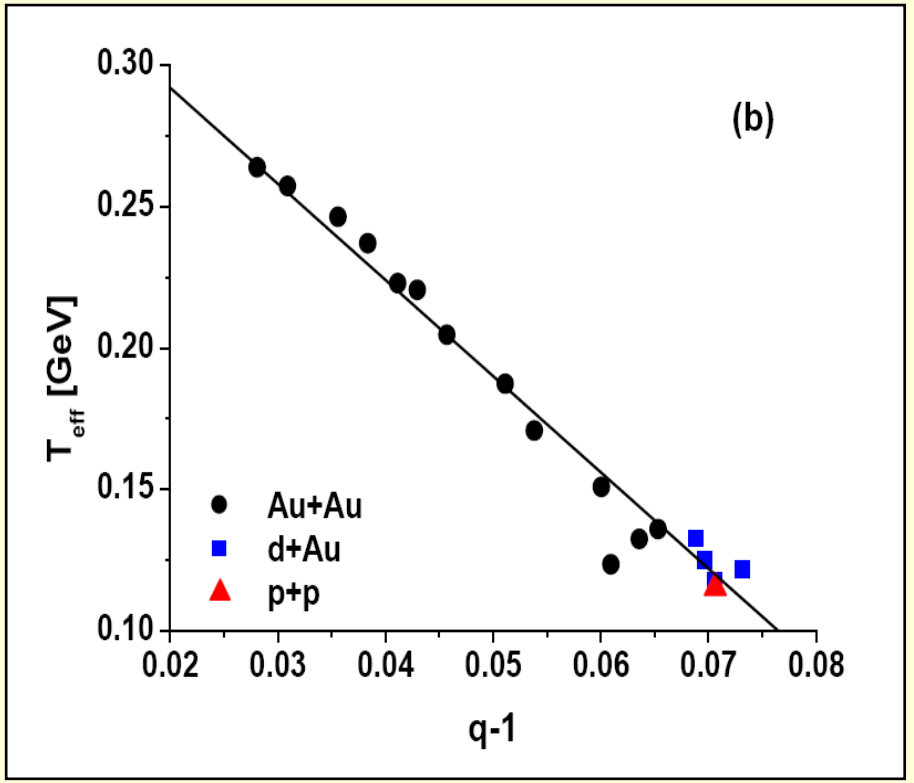


From RHIC data on p_T distributions at 200 GeV of pions (a) and antiprotons (b)

Notice that T_{visc} <0, i.e., the energy is transferred from the interaction region to the surroundings – here represented by the spectators (noninteracting nucleons)

$$T_{eff} = T_0 + (q-1)T_{visc}$$





From RHIC data on p_T distributions at 200 GeV of pions (a) and antiprotons (b)

Notice that T_{visc} <0, i.e., the energy is transferred from the interaction region to the surroundings – here represented by the spectators (noninteracting nucleons)

This will not be discussed here in detail

(cf. *G.Wilk, Z.Włodarczyk, PRC79(2009)054903; EPJA40(2009)299* **)**

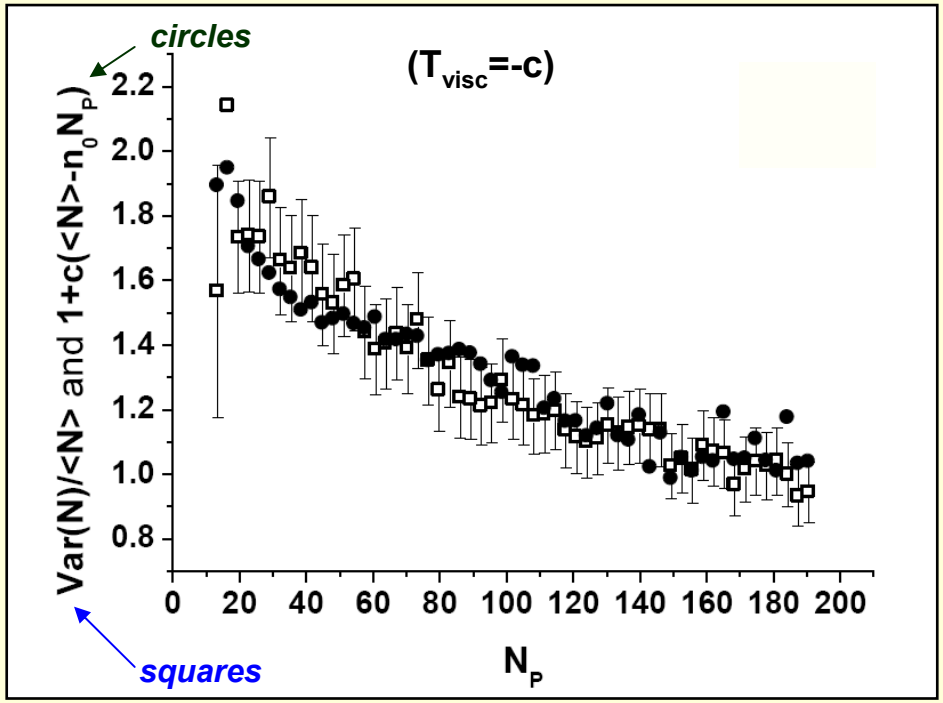
Universal participant dependence of multiplicity fluctuations

(*) We expect that: $$\mathrm{Var(N)}/\langle \mathrm{N} \rangle - 1 = \langle \mathrm{N} \rangle (q-1)$$

(*) On the other hand, if U is the accessible energy and gT_{eff} is energy per particle (with g being a parameter and $T_{eff} = T_0 + (q-1)T_{visc}$) then $\langle N \rangle = \langle U \rangle / [gT_{eff}]$ and

$$\frac{\mathrm{Var(N)}}{\langle \mathrm{N} \rangle} = 1 - T_{visc}\left(\langle \mathrm{N} \rangle - n_0 N_P\right)$$

n_0 is the multiplicity in the single NN collision measured in the region of acceptance and defined by constraint $\langle U \rangle = n_0 g T_0 N_P$



Notice:

(*) Var(N)/<N> and <N> are mutually connected

(*) Var(N)/<N> = 1 if $T_{visc} = 0$ (or $T_{eff} = T_0$) or if $\langle N \rangle = n_0 N_P$

Therefore observed decreasing of Var(N)/<N> with N_P indicates nonlinear dependence of <N> on the number of participants N_P

To continue:

For a system with finite size remaining in contact with a heat bath one has a kind of uncertainty relation (in the sense that it expresses the truth that in the case of conjugate variables one standard deviation in some measurement can only become small at the expense of the increase of some other standard deviation):

$$\mathrm{Var}(U) + C_V^2\,\mathrm{Var}(T) = \langle T \rangle^2 C_V \qquad [\bullet]$$

This is valid all the way from the canonical ensemble where

$$\mathrm{Var}(T) = 0 \quad \text{and} \quad \mathrm{Var}(U) = \langle T \rangle^2 C_V$$

to the microcanonical ensemble where

$$\mathrm{Var}(U) = 0 \quad \text{and} \quad \mathrm{Var}(T) = \langle T \rangle^2 / C_V$$

[•] expresses both the complementarity between the temperature and energy, and between the canonical and the microcanonical description of the system

(J.Uffink and J.van Lith, Found.Phys.29(1999)655, arXiv:cond-matt/9806102).

Canonically distributed system: $\mathrm{Var}(U) = \langle T \rangle^2 C_V$

Isolated system: $\mathrm{Var}(U) = 0$

Let us assume that realistic (intermediate) case interpolates linearly between these two and that the energy fluctuations in the system is equal:

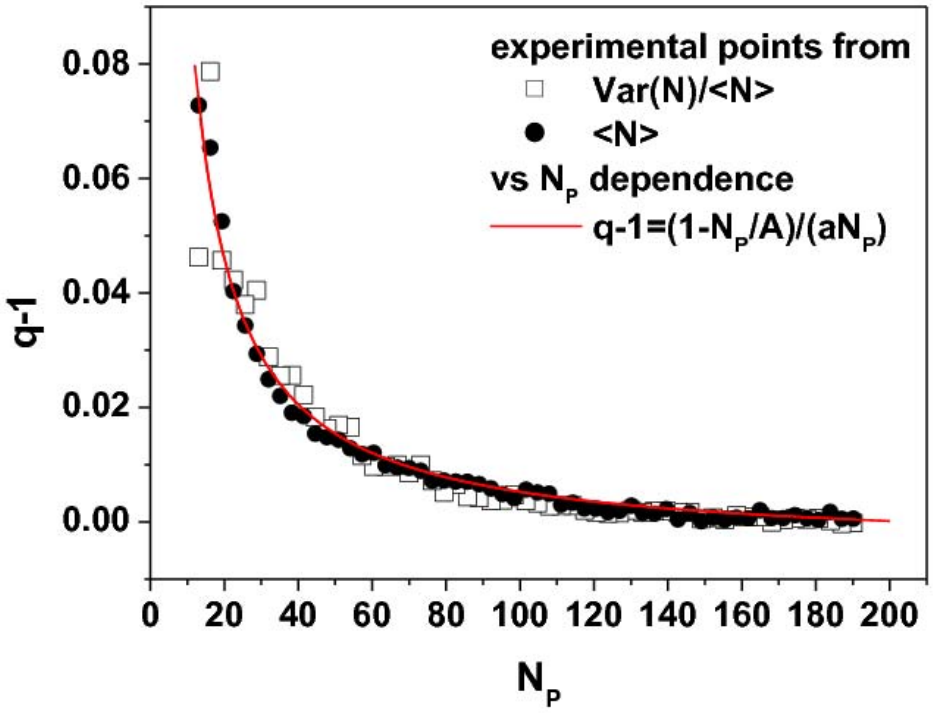
$$\mathrm{Var}(U) = \langle T \rangle^2 C_V \xi$$
$$0 \le \xi \le 1$$

Parameter ξ depends on the size of the hadronizing source and is assumed to be given by the number N_P of nucleons participating in the collision:
$\xi \approx N_P/A$.

Because $C_V \approx aN_P$ we obtain

$$q - 1 = \frac{\mathrm{Var}(T)}{\langle T \rangle^2} = \frac{1-\xi}{C_V} = \frac{1}{aN_P}\left(1 - \frac{N_P}{A}\right)$$

Centrality dependence of multiplicity fluctuations (measured by q-1)



$Var(U) = \langle T \rangle^2 C_V$

$Var(U) = 0$

$Var(U) = \langle T \rangle^2 C_V \xi$

$0 \le \xi \le 1$

$$q - 1 = \frac{1}{aN_P}(1 - \xi) \quad where \quad \xi \approx \frac{N_P}{A} \quad and \quad aN_P \approx C_V$$

To recapitulate:

(*) we know that **if U=const and T=const then P(N) is Poissonian**

(*) we know how **fluctuations of T lead to NB multiplicity distributions**

(*) we want to see how big are fluctuations of T in our hadronizing systems formed in a collision process:

Assuming that the variance expressing energy fluctuations in the system for the intermediate case is equal to

$$Var(U) = \langle T \rangle^2 C_V \xi, \quad 0 < \xi < 1,$$

where parameter ξ depends on the size of the hadronizing source, one gets finally that

$$q - 1 = (1 - \xi)/C_V$$

(*) knowing how big the fluctuations of T are, we attempt to deduce the fluctuations of the multiplicity N.

♦ Multiplicity fluctuations: - *system size dependence [1]*

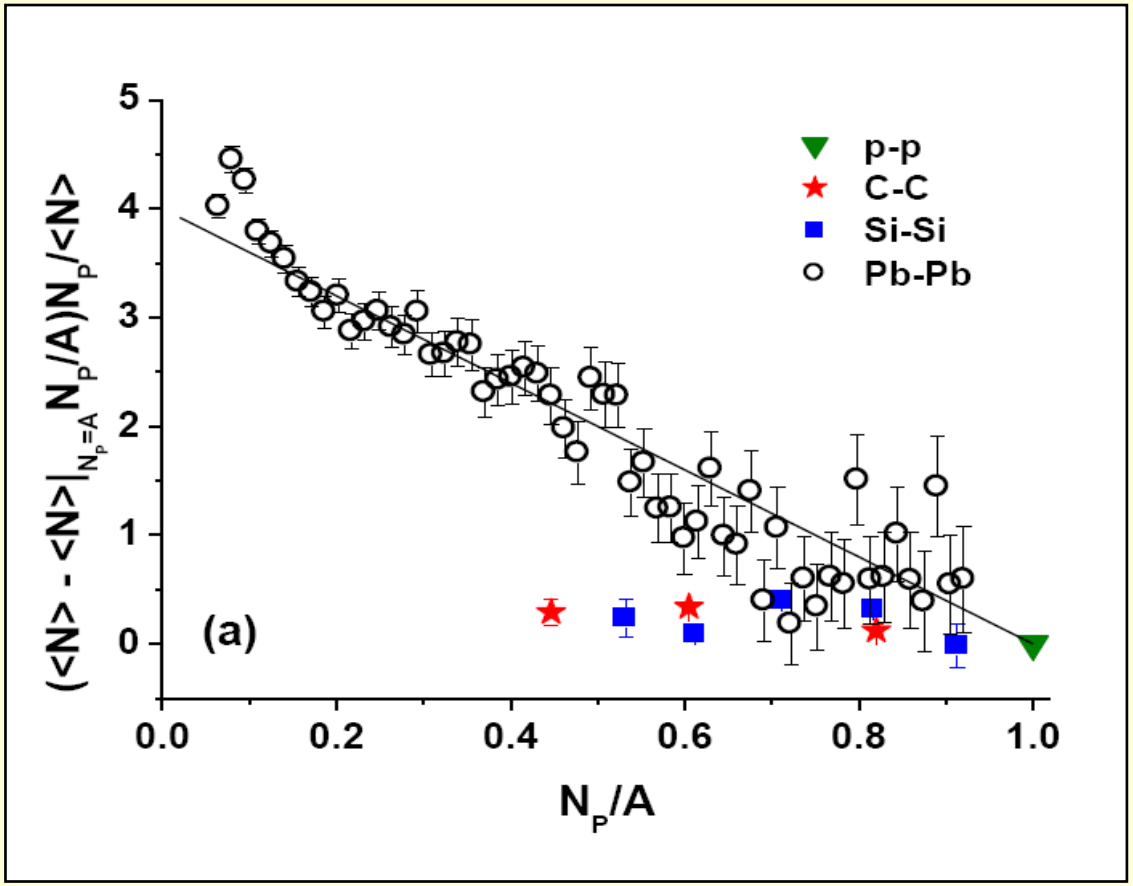
The system size enters through C_V and $N_P/A \approx \xi$:

- for $C_V = a\, N_P$

$$\left(\langle N\rangle - \frac{N_P}{A}\langle N\rangle\Big|_{N_P=A}\right)\frac{N_P}{\langle N\rangle} = \frac{c}{a}\left(1-\frac{N_P}{A}\right) = cN_P(q-1)$$

$\langle N\rangle\big|_{N_P=A} = n_0 A$ **is multiplicity extrapolated to $N_P = A$.**

Notice different dependencies for Pb+Pb collision and lighter nuclei and deviation from the linear fit for most peripheral Pb+Pb collisions.



- for $C_V = a'\langle N\rangle$

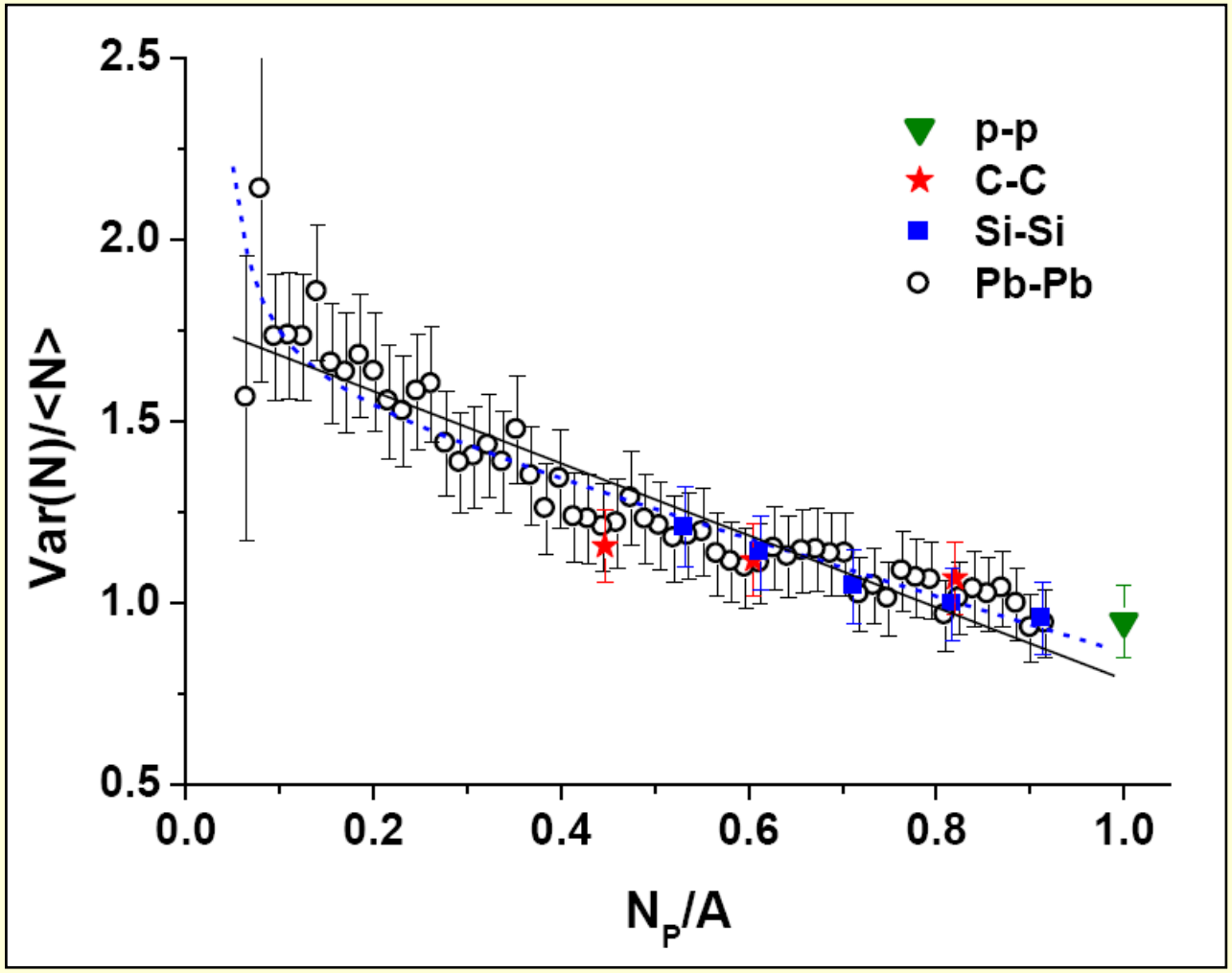
one gets similar picture

$$\left(\langle N\rangle - \frac{N_P}{A}\langle N\rangle\Big|_{N_P=A}\right) = \frac{c}{a'}\left(1-\frac{N_P}{A}\right)$$

♦ Multiplicity fluctuations: *- system size dependence [2]*

One gets in our approach simple scaling relation (dashed line):

$$\frac{\mathrm{Var(N)}}{\langle \mathrm{N} \rangle} = 1 + \langle \mathrm{N} \rangle (q-1) = 1 + \frac{n_0 \left(1 - \frac{N_P}{A}\right)}{a - \frac{c}{A}\left(\frac{A}{N_P} - 1\right)}$$

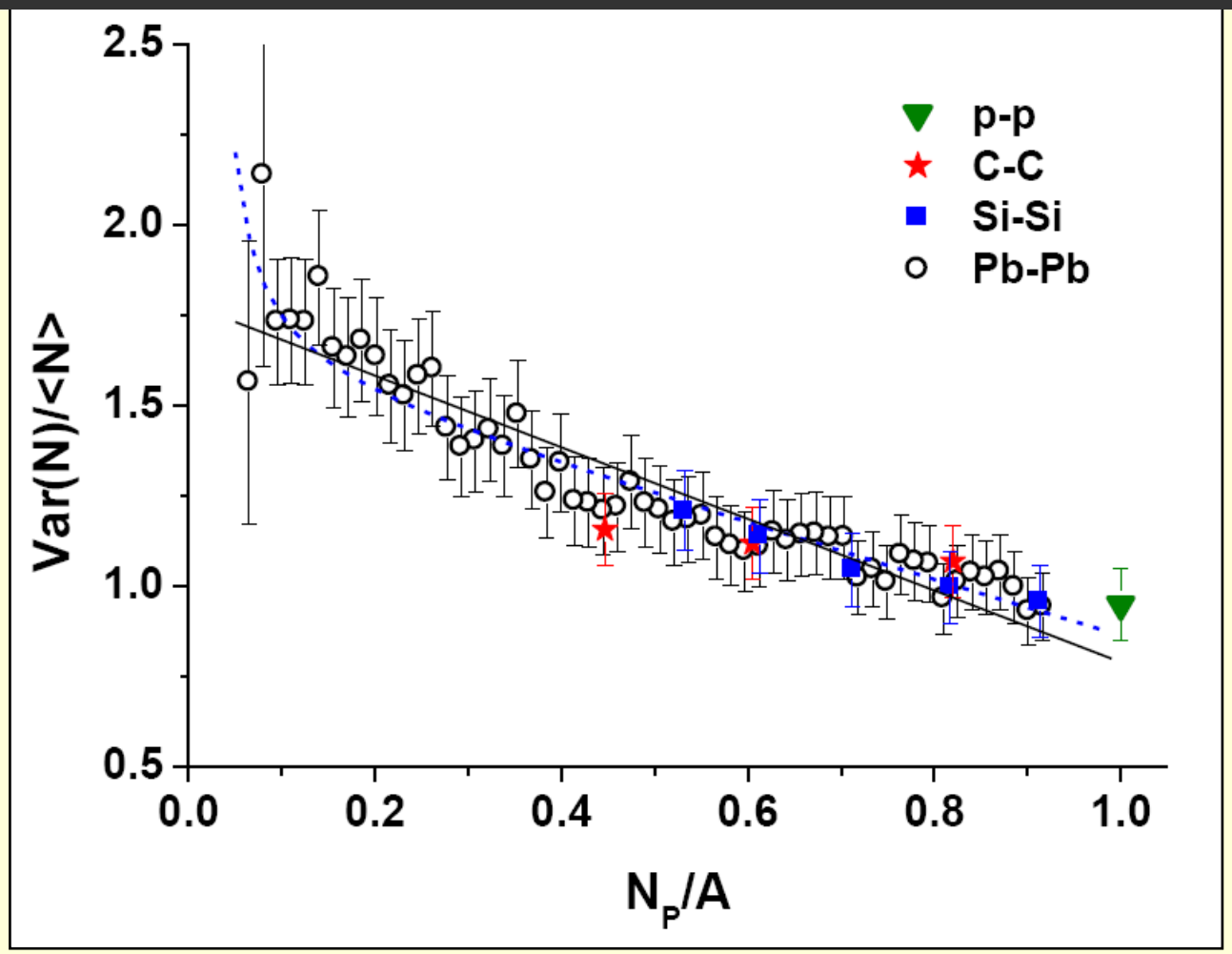


♦ Multiplicity fluctuations: - *system size dependence [2]*

The solid line represents the phenomenological fit to the simple formula (full line)

$$\frac{\mathrm{Var(N)}}{\langle \mathrm{N} \rangle} = \omega_0 + \frac{1}{a'}\left(1 - \frac{N_P}{A}\right) \quad \textit{where} \quad \omega_0 = \left[\frac{\mathrm{Var(N)}}{\langle \mathrm{N} \rangle}\right]_{N_P = A}$$

ω_0=0.79



♦ Multiplicity fluctuations: - *system size dependence [2]*

The solid line represents the phenomenological fit to the simple formula (full line) ω_0=0.79

$$\frac{\mathrm{Var(N)}}{\langle \mathrm{N} \rangle} = \omega_0 + \frac{1}{\mathrm{a'}}\left(1 - \frac{\mathrm{N_P}}{\mathrm{A}}\right) \quad \textit{where} \quad \omega_0 = \left[\frac{\mathrm{Var(N)}}{\langle \mathrm{N} \rangle}\right]_{\mathrm{N_P}=\mathrm{A}}$$

Parameter ω_0 (which within a statistical model with fixed volume varies in the range 0.5-1.0) accounts here for the possible effects expected from the following factors:

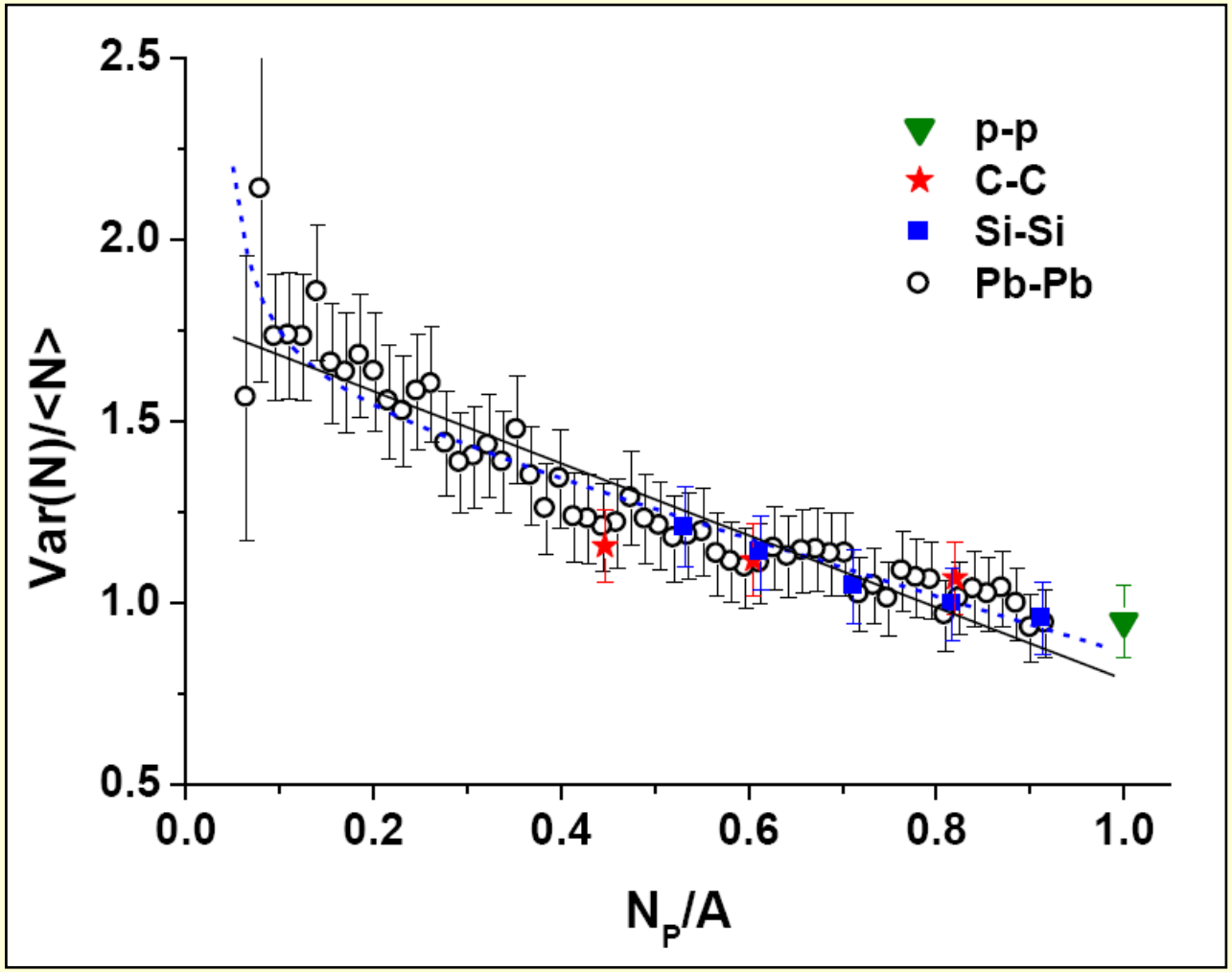
(1) The global conservation laws – when imposed on each microscopic state of the statistical system lead to suppression of the particle number fluctuation (by 0.8 for canonical ensemble down to 0.3 for microcanonical one).

(2) Effect of quantum statistics – it is observed in the primordial values of the scaled variables (i.e., calculated before decays of resonances), they enhance of particle number fluctuations by 1.05-1.06.

(3) The inclusion of known resonance decays (in either ensemble) results in the enhancement of multiplicity fluctuations by about 1.1 (depending on the acceptance).

♦ Multiplicity fluctuations: *- system size dependence [2]*

$$\frac{\mathrm{Var(N)}}{\langle \mathrm{N} \rangle} = 1 + \langle \mathrm{N} \rangle (q-1) = 1 + \frac{n_0 \left(1 - \frac{N_P}{A}\right)}{a - \frac{c}{A}\left(\frac{A}{N_P} - 1\right)}$$



♦ Multiplicity fluctuations: - *system size dependence [2]*

$$\frac{\mathrm{Var(N)}}{\langle \mathrm{N} \rangle} = 1 + \langle \mathrm{N} \rangle (q-1) = 1 + \frac{n_0 \left(1 - \frac{N_P}{A}\right)}{a - \frac{c}{A}\left(\frac{A}{N_P} - 1\right)}$$

Notice:

(*) We observe a weak dependence on the mass number A of colliding nuclei.

(*) For a small number of participants, $N_P \ll A$, we observe an additional increase of relative variance, Var(N)/<N>.

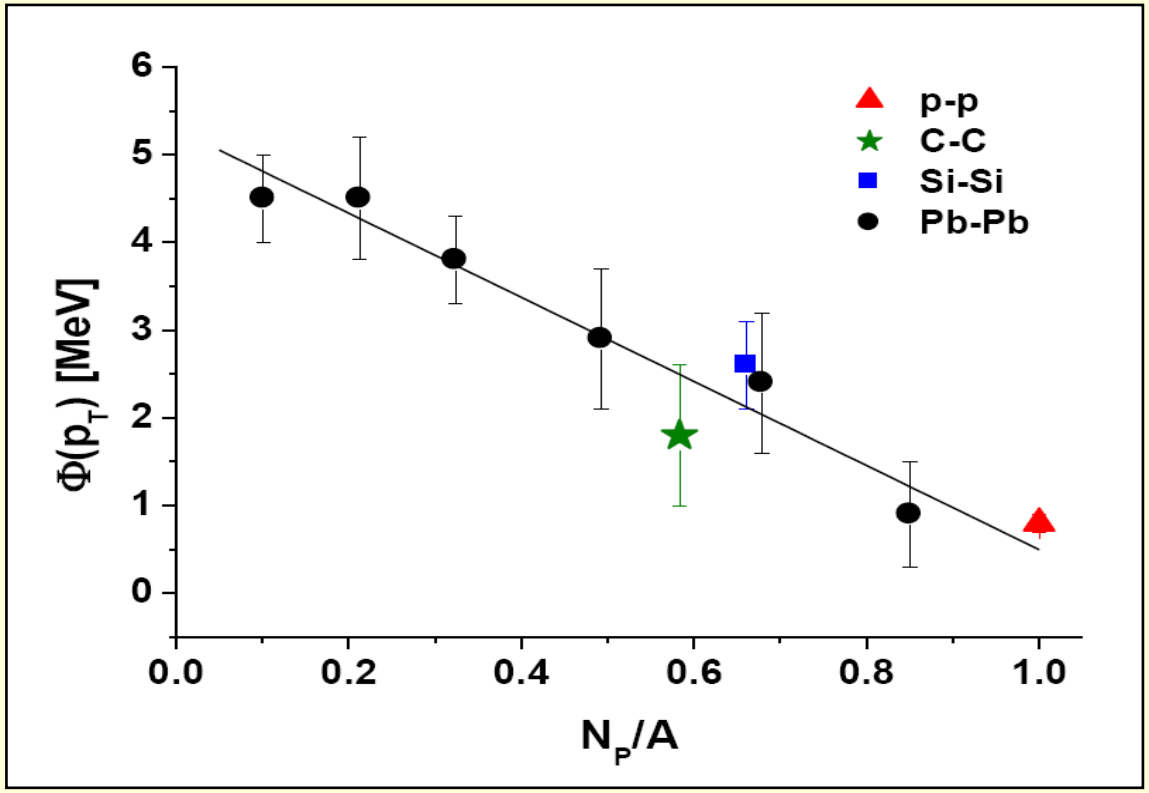
(*) However, should we use $T_{visc} > 0$ instead, one would observe the opposite trend – resulting in a nonmonotonic behavior of Var(N)/<N> with increasing number of participants N_P .

(The question about the sign of the heat source term or T_{visc} is still open because of the lack of data in the region of small number of participants).

♦ Some other results (1)

Transverse momentum fluctuations measured in nuclear collisions at 158 A GeV and quantified by the measure $\Phi(p_T)$ shows similar centrality dependence,

$$\Phi(p_T) = \Phi_{N_P=A} + b\left(1 - \frac{N_P}{A}\right) = \Phi_{N_P=A} + aN_P(q-1)$$



It is determined by the pattern of multiplicity fluctuations

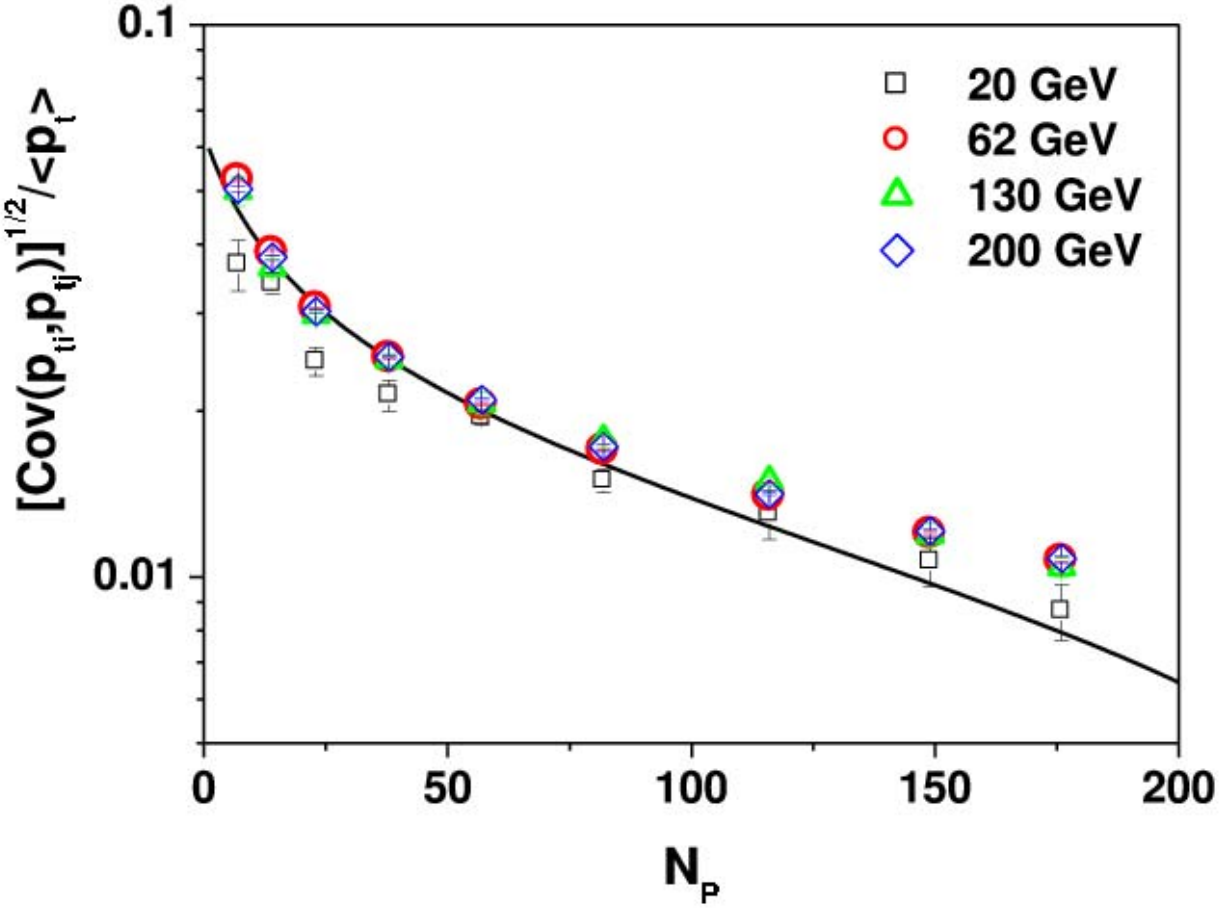
♦ Some other results (2)

$$\frac{Cov(p_{Ti},p_{Tj})}{2\sqrt{Var(p_T)}} \cong \frac{\Phi_{N_P=A} + b\left(1-\frac{N_P}{A}\right)}{n_0 N_P + \frac{c+1}{a'}\left(1-\frac{N_P}{A}\right)} = \frac{\Phi_{N_P=A} + aN_P b(q-1)}{n_0 N_P + \frac{c+1}{a'} aN_P(q-1)}$$

Our predictions (full line) for the scaled covariance for:

$Var(p_T)/\langle p_T\rangle$=0.43
$\langle p_T\rangle$=400 MeV,
b=4.8 MeV
n_0=0.642,
c=4.1,
a'=1,
$\Phi_{Np=A}$=1.44 MeV
(*PRC79(2009)054903)*

Data points are from STAR
(PRC72(2005)044902)

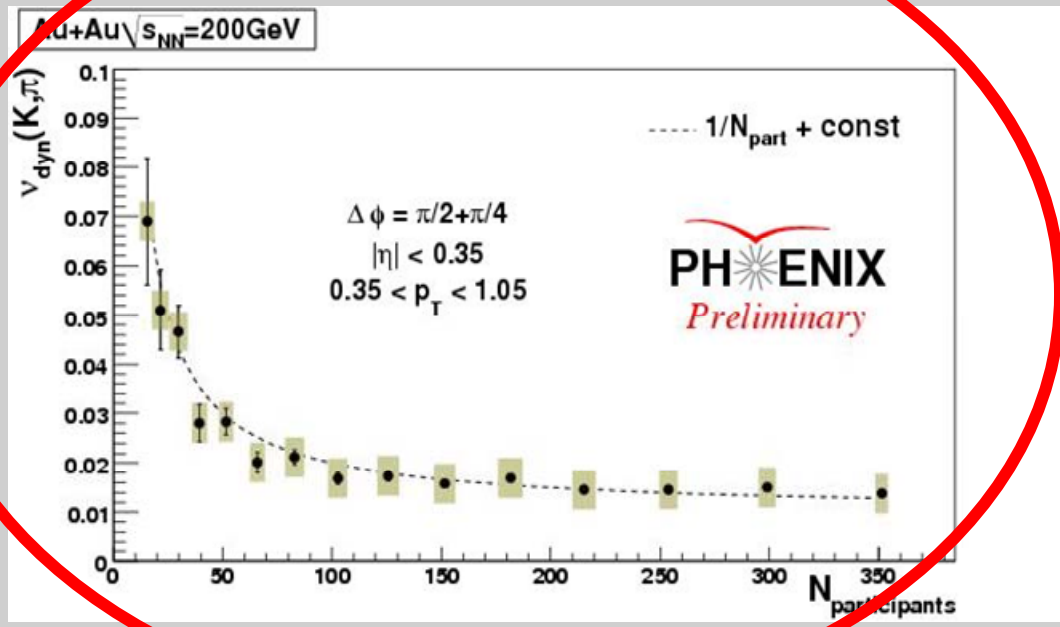


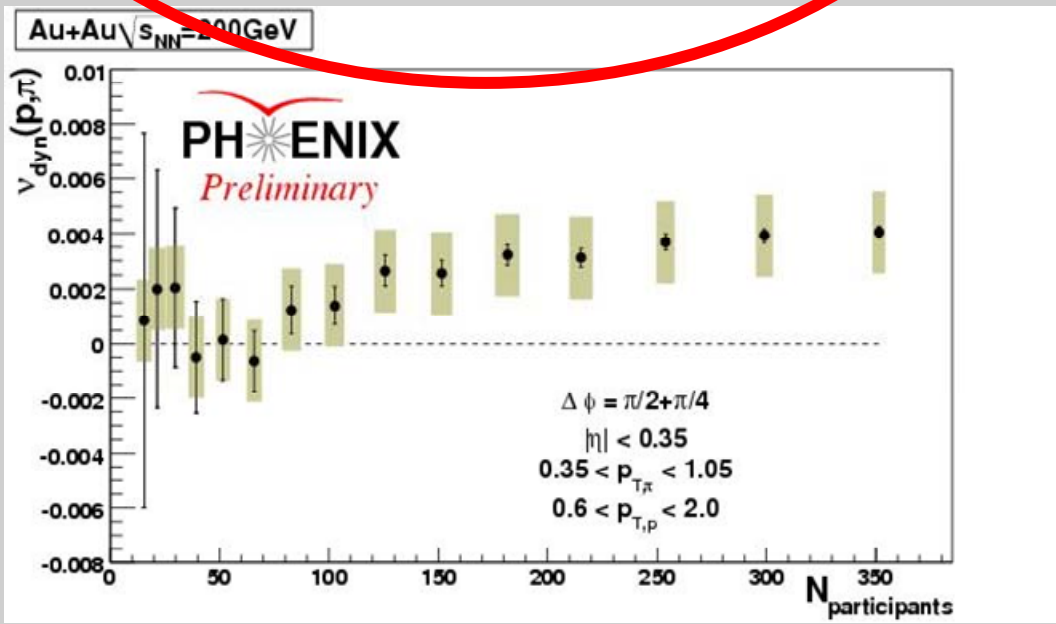
Ratio Fluctuations

- **Some other results (3) – particle ratio fluctuations**

For example – as presented at QM08

C. Pruneau, S.Gavin, S.Voloshin, Phys. Rev. C 66 044904 (2002)





Particle ratio fluctuation show no indication of critical behaviour

K.Homma, Session XVII, 02/09 15:20h
Poster by E.Haslum

♦ Some other results (3) – particle ratio fluctuations

$$\sigma^2_{N_\pi/N_K} = \frac{\left\langle (\Delta N_\pi)^2 \right\rangle}{\langle N_\pi \rangle^2} + \frac{\left\langle (\Delta N_K)^2 \right\rangle}{\langle N_K \rangle^2} - 2\frac{\langle \Delta N_\pi \Delta N_K \rangle}{\langle N_\pi \rangle \langle N_K \rangle} =$$

$$= (q_\pi - 1) + (q_K - 1) - 2\frac{\mathrm{Cov}(K\pi)}{\langle N_\pi \rangle \langle N_K \rangle} =$$

$$= \left(\frac{1}{a_K} + \frac{1}{a_\pi}\right)\frac{1}{N_P}\left(1 - \frac{N_P}{A}\right) + \kappa = \alpha\frac{1}{N_P}\left(1 - \frac{N_P}{A}\right) + \kappa$$

$$\alpha = \frac{1}{a_K} + \frac{1}{a_\pi}; \qquad \kappa = -2\frac{\mathrm{Cov}(K\pi)}{\langle N_\pi \rangle \langle N_K \rangle}; \qquad a_{\pi,K} = \frac{C_{\pi,K}}{N_P}$$

Assumption: fluctuations of both types of particles behave in the same way as function of centrality

♦ Some other results (3) – particle ratio fluctuations

$$\sigma^2_{N_\pi/N_K} = \frac{\left\langle (\Delta N_\pi)^2 \right\rangle}{\langle N_\pi \rangle^2} + \frac{\left\langle (\Delta N_K)^2 \right\rangle}{\langle N_K \rangle^2} - 2\frac{\langle \Delta N_\pi \Delta N_K \rangle}{\langle N_\pi \rangle \langle N_K \rangle} =$$

$$= (q_\pi - 1) + (q_K - 1) - 2\frac{\mathrm{Cov}(K\pi)}{\langle N_\pi \rangle \langle N_K \rangle} =$$

$$= \left(\frac{1}{a_K} + \frac{1}{a_\pi}\right)\frac{1}{N_P}\left(1 - \frac{N_P}{A}\right) + \kappa = \alpha\frac{1}{N_P}\left(1 - \frac{N_P}{A}\right) + \kappa$$

$$\alpha = \frac{1}{a_K} + \frac{1}{a_\pi}; \qquad \kappa = -2\frac{\mathrm{Cov}(K\pi)}{\langle N_\pi \rangle \langle N_K \rangle}; \qquad a_{\pi,K} = \frac{C_{\pi,K}}{N_P}$$

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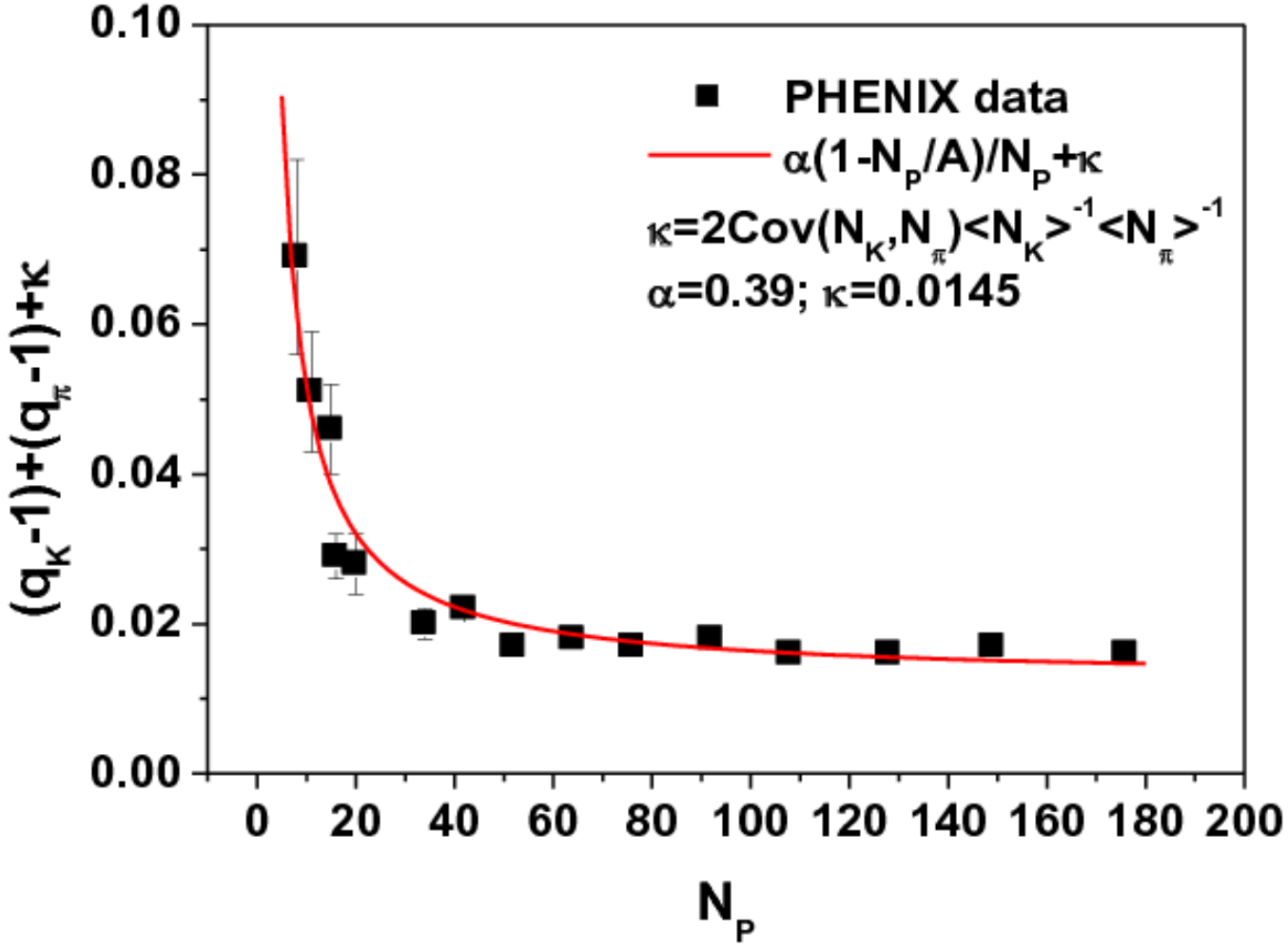

$$(q_\pi - 1) + (q_K - 1) + \kappa = \frac{\alpha}{N_P}\left(1 - \frac{N_P}{A}\right) + \kappa = \left(\frac{1}{C_\pi} + \frac{1}{C_K}\right)\left(1 - \frac{N_P}{A}\right) + \kappa$$

$C_{\pi,K}$ – heat capacities

Summary [1]:

(*) Some features of the observed multiplicity fluctuations are due to the intrinsic, nonstatistical fluctuations of the temperature of the hadronizing system.

(*) They can be described by the nonextensive Tsallis statistics with the nonextensivity parameter q, which can be associated with the number of participants N_P (assumed to represent the size of the colliding system).

(*) Data require a generalization of the notion of fluctuating temperature by adding effects of the possible energy transfer between the hadronizing source and the surroundings composed of nucleons not participating directly in the reaction.
This resulted in the introduction of a q-dependent effective temperature T_{eff}.

(*) One has to allow for the fluctuations of the full accessible energy U, otherwise Var(N)/<N> remains constant, independent of N_P/A.

Summary [2]:

(*) The above multiplicity fluctuations describe also fully:

[#] the observed transverse momentum fluctuations;

[#] the observed transverse momenta correlations;

[#] the observed pattern of particle ratios fluctuations.

In all above examples fluctuations in p_T only play minor role .

Some other examples

PHYSICAL REVIEW C **79**, 051901(R) (2009)

Spectra and radial flow in relativistic heavy ion collisions with Tsallis statistics in a blast-wave description

Zebo Tang,[1] Yichun Xu,[1] Lijuan Ruan,[2] Gene van Buren,[2] Fuqiang Wang,[3] and Zhangbu Xu[2,*]

[1]*University of Science & Technology of China, Hefei 230026, People's Republic of China*

[2]*Brookhaven National Laboratory, Upton, New York 11973, USA*

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(Received 10 February 2009; revised manuscript received 8 April 2009; published 8 May 2009)

We have implemented the Tsallis statistics in a Blast-Wave model (TBW) and applied it to midrapidity transverse-momentum spectra of identified particles measured at BNL Relativistic Heavy Ion Collider (RHIC). This new TBW function fits the RHIC data very well for $p_T < 3$ GeV/c. We observed that the collective flow velocity starts from zero in $p + p$ and peripheral Au + Au collisions and grows to $0.470 \pm 0.009c$ in central Au + Au collisions. The resulting $(q - 1)$ parameter, which characterizes the degree of nonequilibrium in a system, indicates an evolution from a highly nonequilibrated system in $p + p$ collisions toward an almost thermalized system in central Au + Au collisions. The temperature and collective velocity are well described by a quadratic dependence on $(q - 1)$. Two sets of parameters in our TBW are required to describe the meson and baryon groups separately in $p + p$ collisions while one set appears to fit all spectra in central Au + Au collisions.

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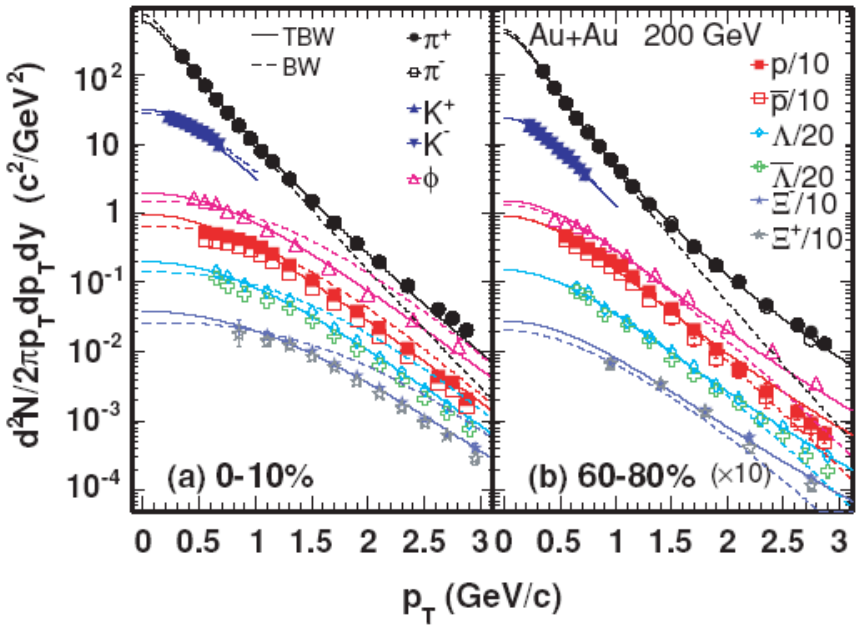


FIG. 1. (Color online) Identified particle transverse momentum spectra in Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV in 0–10% central collisions (a) and in peripheral 60–80% collisions (b). The symbols represent experiment data points. The solid curves represent the TBW fit and the dashed lines are BGBW calculations with β and T values from Ref. [5]. Only fits to the particles are shown because both models have the same spectral shapes for particles and antiparticles.

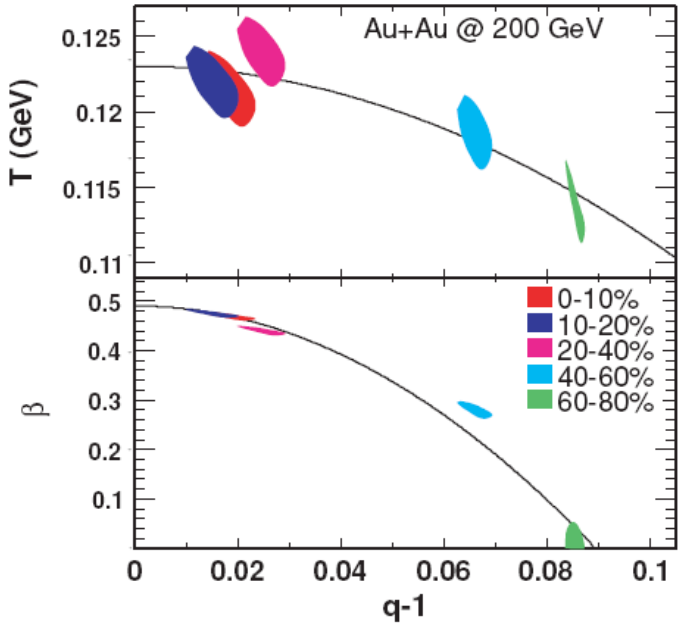


FIG. 2. (Color online) The fit parameters T and β as a function of $(q - 1)$. Each block is one-σ contour from the error matrix of the TBW fit for a given centrality of Au + Au collisions.

Historical example:

(*) observation of deviation from the expected exponential behaviour

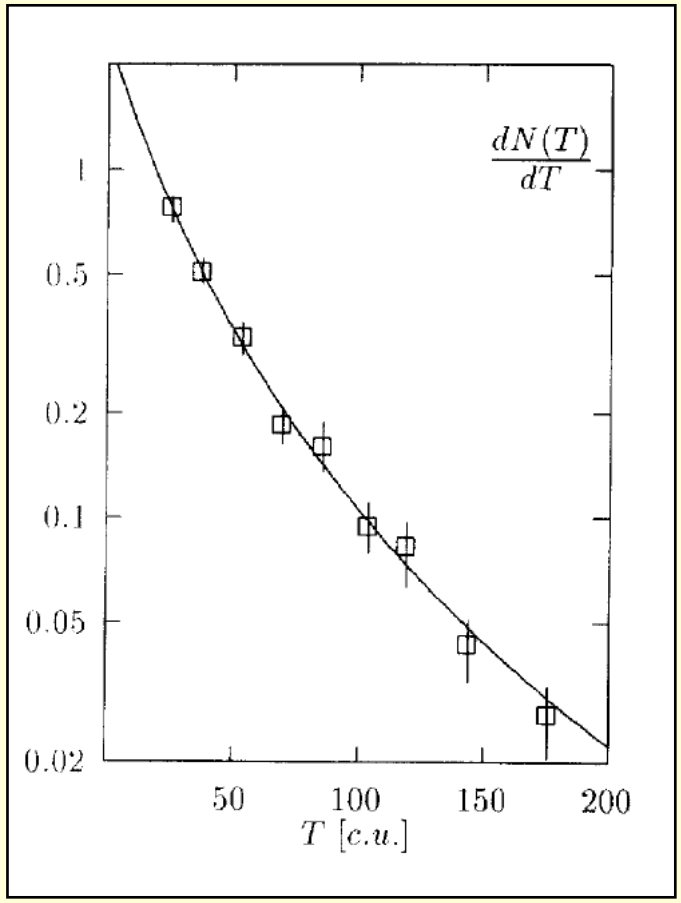
(*) successfully intrepreted (*) in terms of cross-section fluctuation:

$$\omega = \frac{< \sigma^2 > - < \sigma >^2}{< \sigma >^2} \geq 0.2$$

(*) can be also fitted by:

$$\frac{dN}{dT} = const \cdot \exp\left(-\frac{T}{\lambda}\right) \Rightarrow$$

$$\frac{dN}{dT} = const \cdot \left[1 - (1-q)\frac{T}{\lambda}\right]^{\frac{1}{1-q}}; q = 1.3$$



Depth distributions of starting points of cascades in Pamir lead chamber Cosmic ray experiment (WW, NPB (Proc.Suppl.) A75 (1999) 191

(*) immediate conjecture:

q ⇦⇨ fluctuations present in the system

(*) WW, PRD50 (1994) 2318

Fluctuations of the string tension and transverse mass distribution

Others

A. Bialas [a,b] Physics Letters B 466 (1999) 301–304

It is shown that quantum fluctuations of the string tension can account for the ''thermal'' distribution of hadrons created in decay of a colour string. © 1999 Published by Elsevier Science B.V. All rights reserved.

$$\frac{\mathrm{d}n_\kappa}{\mathrm{d}^2p_\perp} \sim \mathrm{e}^{-\pi m_\perp^2/\kappa^2},$$

+

$$P(\kappa)\,d\kappa = \sqrt{\frac{2}{\pi\langle\kappa^2\rangle}}\exp\left(-\frac{\kappa^2}{2\langle\kappa^2\rangle}\right)d\kappa,$$

where $\langle\kappa^2\rangle$ is the *average* string tension, i.e.

$$\langle\kappa^2\rangle = \int_0^\infty P(\kappa)\,\kappa^2\,d\kappa.$$

Using (1) and Eq. (4) we thus have

$$\frac{\mathrm{d}n}{\mathrm{d}^2p_\perp} \sim \int_0^\infty d\kappa\,P(\kappa)\,\mathrm{e}^{-\pi m_\perp^2/\kappa^2}$$

$$= \frac{\sqrt{2}}{\sqrt{\pi\langle\kappa^2\rangle}}\int_0^\infty d\kappa\,\mathrm{e}^{-\kappa^2/2\langle\kappa^2\rangle}\mathrm{e}^{-\pi m_\perp^2/\kappa^2}$$

$$\frac{\mathrm{d}n}{\mathrm{d}^2p_\perp} \sim \exp\left(-m_\perp\sqrt{\frac{2\pi}{\langle\kappa^2\rangle}}\right)$$

$$T = \sqrt{\frac{\langle\kappa^2\rangle}{2\pi}}$$

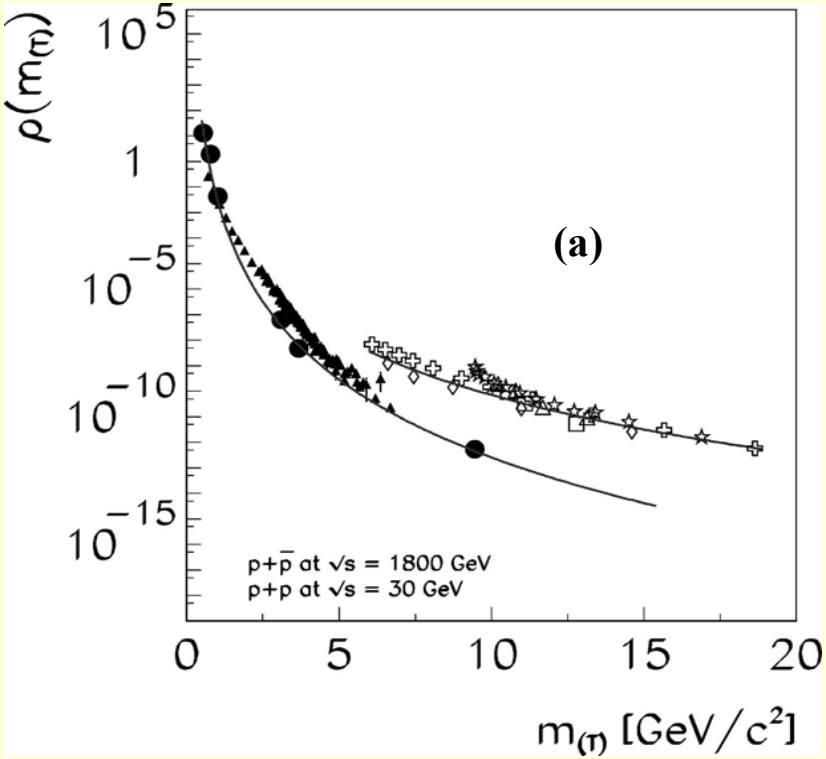
Net

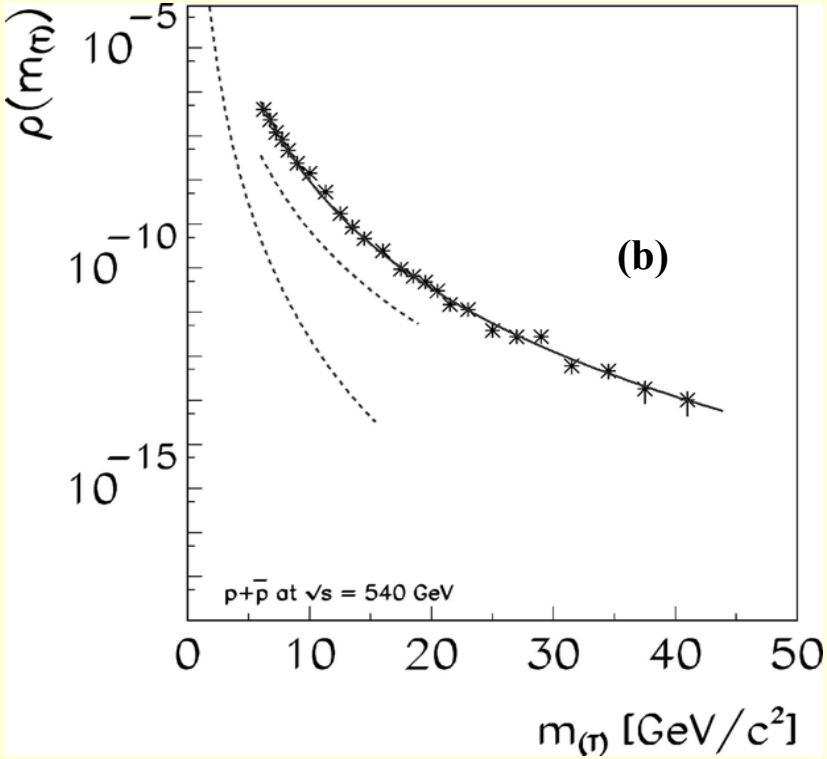
Possible connection with networks *(WW, APP B35(2004) 2141)*

(*) Prompted by the observation of the power-like behaviour of the m_T spectra

(M.Gaździcki et al.., Phys. Lett. B517 (2001) 250 : Power law in hadron production)

Net





(a) Mean multiplicity of neutral mesons (full dots) scaled to $\rho(m)=C\cdot m^{-P}$ and μ_T spectra of π^0 mesons (full triangles) scaled to $\rho(m_T)=C\cdot m_T^{-P}$ produced in pp interactions at $\sqrt{s}$=30 GeV; different quarkonia spectra at $\sqrt{s}$=1800GeV.

(b) The $\rho(m_T)=C\cdot m_T^{-P}$ spectra for pp at $\sqrt{s}$=540 GeV (solid line, dashed line show the corresponding fits to the results at 30 GeV and 1800 GeV).
P=7.7 (for quarkonia at 1800 GeV) to 10.1 (for neutral mesons at 30 GeV). In all cases considered the values of the constant C were the same (important!).

Net

Possible connection with networks *(WW, APP B35(2004) 2141)*

(*) Prompted by the observation of the power-like behaviour of the m_T spectra we have considered a possibility that :

they can be a reflection not so much of any special kind of equilibrium (resulting in some specific statistics) but rather of the formation process *resembling* the free network formation pattern discussed widely in the literature *(Barabasi et al.. RMP 74 (2002) 47; WW, APP B35 (2004) 871)*

(*) The power-law behaviour of spectra emerges naturally when:

- ***hadrons are formed in process of $q\bar{q}$ linking themselves by gluons***
- ***one identifies vertices in such network as ($q\bar{q}$) pairs and gluons as links***
- ***one assumes that the observed m_T of hadrons reflects somehow the number of links k (i.e., gluons) in such network (α - diffusion parameter):***

$$m_T \approx k^{\alpha}$$

The distribution of number of links in the network has form:

$$P(k) \approx k^{-\gamma} \qquad (\text{with } \gamma \leq 5)$$

As result one gets:

$$P(m_T) \approx m_T^{-\beta}\ ; \quad \beta = 1+(\gamma-1)/\alpha$$

where $\gamma = 1+\delta$ and δ is parameter defining the rate of growth of our network.

Net

(*) The power-like distribution $P(k) \approx k^{-\gamma}$ *corresponds to large excitations where probability of connecting a new quark to the one already existing in the system depends on the number of the actual connections realised so far* $\Rightarrow$ *large number of connections results in large excitations* $\Rightarrow$ *large emission of gluons* $\Rightarrow$ *enhances chances of connection to such a quark.*

(*) If one assumes instead that the new quark attaches itself to the already existing one with equal probability (a case of small excitations, i.e., small p_T*) then one gets instead exponential distribution of links:*

$$P(k) \approx \exp[- k/\langle k \rangle]$$

which results in $P(m_T)$ *ranging from:*

$$P(m_T) \approx \exp[- m_T^2/ \langle m_T^2 \rangle] \quad \text{for} \quad \alpha=1/2$$

when the full fledged diffusion is allowed, to

$$P(m_T) \approx \exp[- m_T/ \langle m_T \rangle] \quad \text{for} \quad \alpha=1$$

where there is no diffusion (this would be the case of quarks located on the periphery of the hadronization region in which case they could interact only with interior quarks $\Rightarrow$ $m_T \approx k$*).*

NUT

Possible thermodynamic relation between fluctuations

Suppose that in some process one has N independently produced particles with energies $\{E_{i=1,\ldots,N}\}$ distributed according to Tsallis distribution with $\beta=1/T$ and q and that all these quamtities fluctuate. Denoting:

$$U=\sum_{i=0}^{N} E_i; \qquad \omega_x^2=\frac{\mathrm{Var}(x)}{\langle x\rangle^2}$$

it can be shown that that fluctuations of N, U and T are connected in the following way ($\rho=\rho(U,T)$ is the correlation coefficient between U and T)

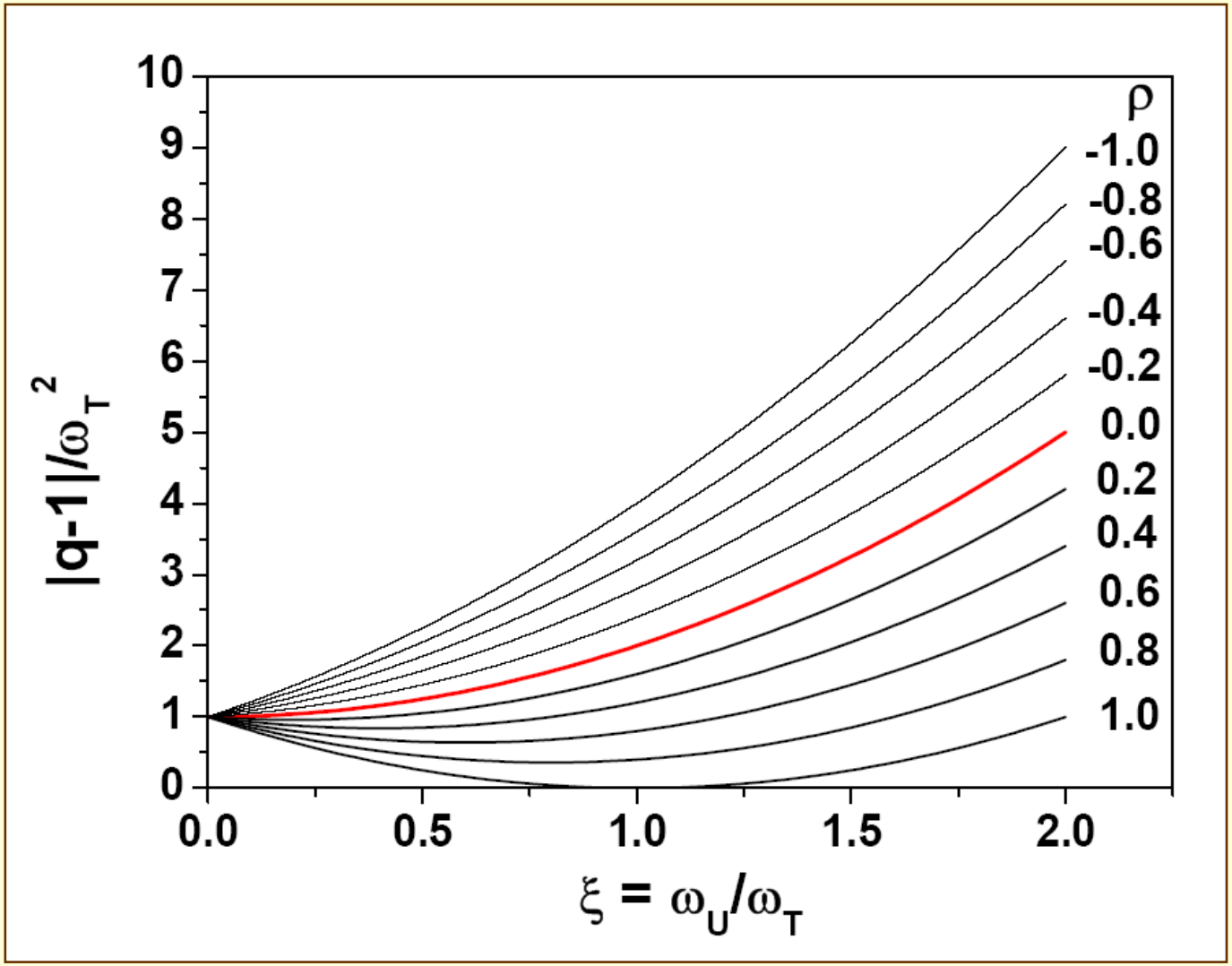
$$\left|\omega_N^2-\frac{1}{\langle N\rangle}\right|=\omega_U^2+\omega_T^2-2\rho\rho_{\ U}\omega_T=$$

$$=\left(\omega_U-\omega_T\right)^2+2\omega_U\omega_T\left(1-\rho\right)$$

and because $\omega_N^2-\frac{1}{\langle N\rangle}=q-1$ *then*

$$\left|q-1\right|=\left(\omega_U-\omega_T\right)^2+2\omega_U\omega_T\left(1-\rho\right)$$

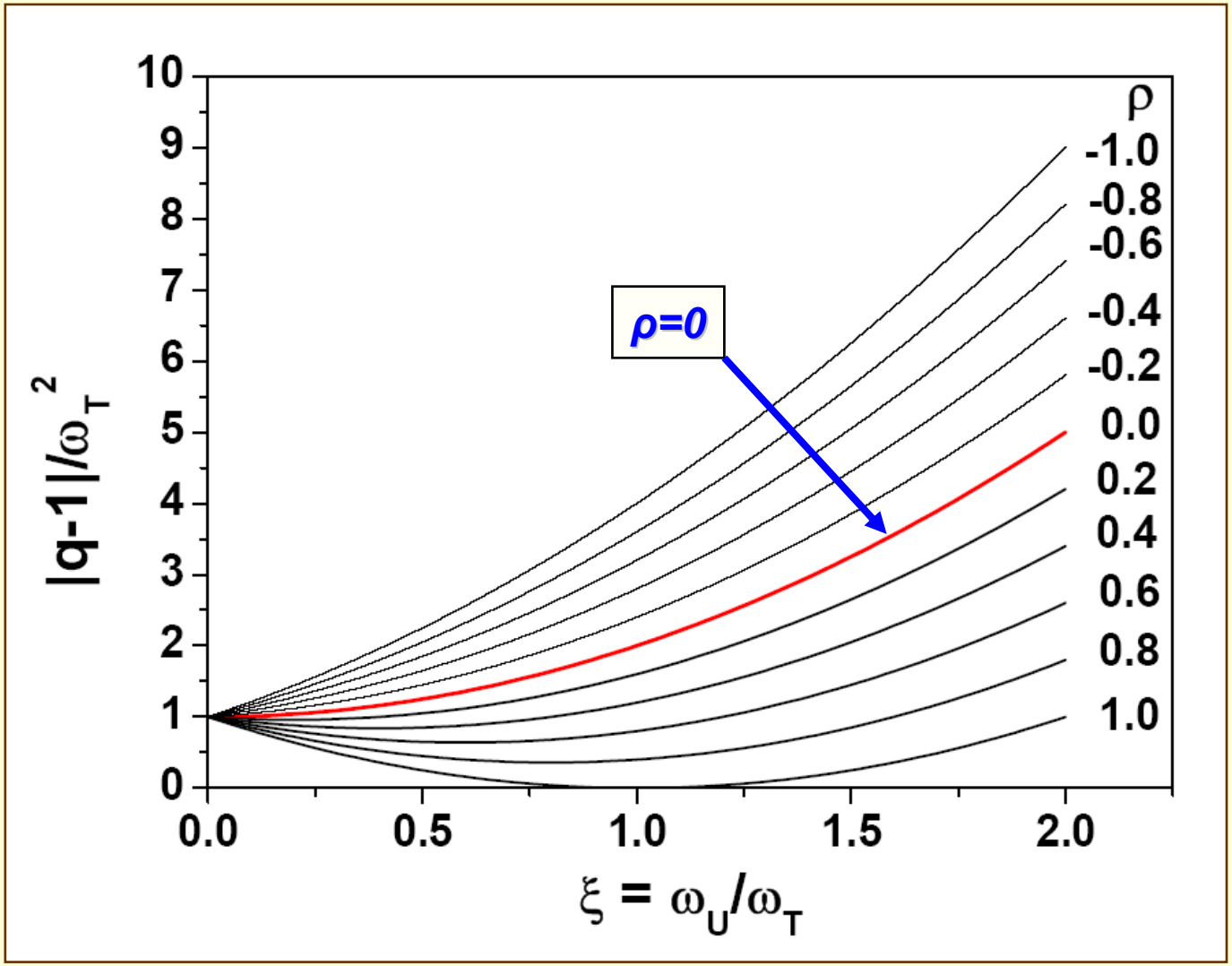
NUT



$$|q-1| = (\omega_U - \omega_T)^2 + 2\omega_U\omega_T(1-\rho)$$

$$\rightarrow \quad \frac{|q-1|}{\omega_T^2} = (1-\xi)^2 + 2(1-\rho)\xi; \qquad \xi = \frac{\omega_U}{\omega_T}$$

NUT



$$|q-1| = (\omega_U - \omega_T)^2 + 2\omega_U\omega_T(1-\rho)$$

$$\rightarrow \quad \frac{|q-1|}{\omega_T^2} = (1-\xi)^2 + 2(1-\rho)\xi; \qquad \xi = \frac{\omega_U}{\omega_T}$$

Example of q-hydrodynamics

[T.Osada,G.Wilk,PRC77(2008)044903, CEJP7(2009)432]

Physical picture:

one replaces the notion of the strict

local thermal equilibrium

by the notion of some kind of

stationary state

being formed in the collision which already includes some interactions (summarily described by the parameter q).

T.Sherman, J.Rafelski, Lecture Notes in Physics 633(2004)377.
T.S.Biro,G.Purcsel, PRL95(2005)162302; PLA372(2008)1174.
T.S.Biro, Abstract composition rule for relativistic kinetic energy in the thermodynamical limit, arXiv: 0809.4675; T.S.Biro, G.Kaniadakis, EPJB50(2006) 3; C.Beck, Physica A286(2000)164.

Basic formulas-1

For simplicity, consider the case of vanishing baryon chemical potential for the simple 1+1 dimensional system. The hydrodynamic equation of motion reflects just conservation of the energy-momentum tensor T:

$$\partial_\mu T^{\mu\nu} = 0$$

Assuming the local thermal equilibrium in the energy local frame the energy-momentum tensor for ideal (nonviscous) fluid is given by:

$$T^{\mu\nu} = (\varepsilon + P) u^\mu u^\nu - P g^{\mu\nu}$$

ε – energy density
P – pressure
u – four velocity of the fluid

P=P(ε) – equation of state (EoS)

Basic formulas-2

In the case of viscous fluid one has to replace

$$T^{\mu\nu} = (\varepsilon + P)u^{\mu}u^{\nu} - P\,g^{\mu\nu}$$

by its more general form:

$$T^{\mu\nu} = (\varepsilon + P + \Pi)u^{\mu}u^{\nu} - (P + \Pi)g^{\mu\nu} + \pi^{\mu\nu}$$

which includes now two new terms:

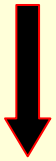
Π – bulk viscosity

$\pi^{\mu\nu}$ - shear viscosity tensor

Basic formulas-3

We propose to replace (perfect nonviscous fluid)

$$T^{\mu\nu} = (\varepsilon + P)u^{\mu}u^{\nu} - P\,g^{\mu\nu}$$



by its nonextensive version (perfect q-fluid):

$$T_q^{\mu\nu} = (\varepsilon_q + P_q)u_q^{\mu}u_q^{\nu} - P_q\,g^{\mu\nu}$$

ε_q – q-energy density
P_q – q-pressure
u_q – q-four velocity of the fluid

$P_q = P_q(\varepsilon_q)$ – q-equation of state (q-EoS)

Basic formulas-4

$$\partial_\mu T_q = 0$$

$$T_q^{\mu\nu} = \frac{1}{(2\pi)^3} \int \frac{d^3p}{p^0} p^\mu p^\nu f_q^q(x,p);$$

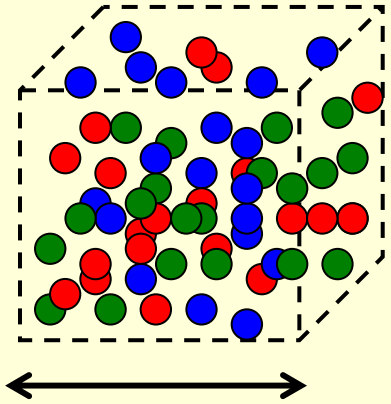
where

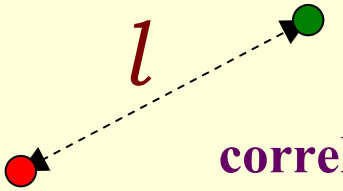
$$f_q(x,p) \equiv \exp_q\left[-\frac{p_\mu u_q^\mu(x)}{k_B T_q(x)} \right]$$

$f_q(x,p)$ is solution of the q-version of Boltzmann equation proposed by A.Lavagno, PLA301(2002)13.
For q=1 one recovers the usual formulas of kinetic theory based on BG (extensive) statistics.

Non- extensivity for system with correlations

quarks, gluons
or
hadrons



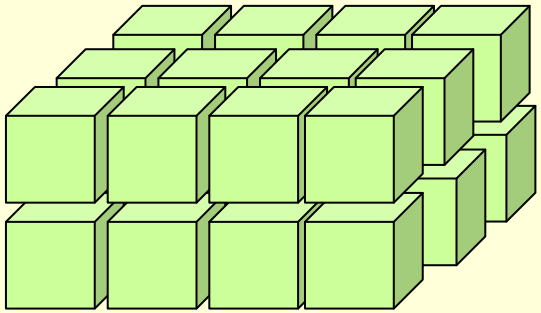


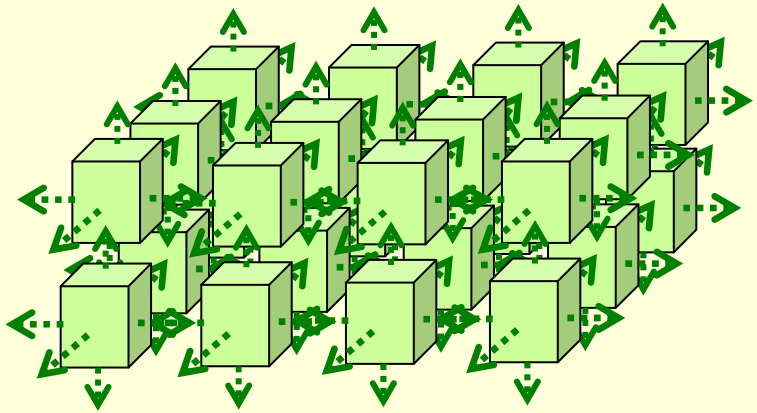
correlation and/or
fluctuation scale;
of composing particles

L_{hyd} hydro scale; definable $T^{\mu\nu}$
local equilibrium or *stationary*

correlation fluctuation	$q \sim l / L_{\text{hyd}}$	entropy S	*state*
$L_{\text{hyd}} \geq l \geq \mapsto 0$	$q = 1$	extensive	Boltzmann-Gibbs *equilibrium*
$0 \leq L_{\text{hyd}} < l$	$q > 1$	Nonextensive $S(A \oplus B) \neq S(A) + S(B)$	*stationary*

Standard hydrodynamics vs. *q*- hydrodynamics





standard hydrodynamics,
locally conserved (BG) entropy current:

$$s^{\mu}(x) \equiv -k_{\mathrm{B}} \int \frac{d^3 p}{(2\pi\hbar)^3} \frac{p^{\mu}}{p^0} \{ f(x,p) \ln f(x,p) - f(x,p) \}$$

$$S(A \oplus B) = S(A) + S(B)$$

L_{hyd} L_{hyd}

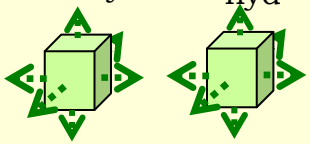
without
correlations between cells

Tsallis hydrodynamics,
locally conserved *q*- entropy current:

$$s_q^{\mu}(x) \equiv -k_{\mathrm{B}} \int \frac{d^3 p}{(2\pi\hbar)^3} \frac{p^{\mu}}{p^0} \left\{ f_q^q(x,p) \ln_q f_q(x,p) - f_q(x,p) \right\}$$

$$S_q(A \oplus B) = S_q(A) + S_q(B) + (1-q) S_q(A) S_q(B)$$

L_{hyd} L_{hyd}



including
correlations

Nonextensive/Dissipative conjecture (NexDC)-1

(*) starting from:
(ideal in a nonextensive sense - for f_q)

$$T_q^{\mu\nu} = \left(\varepsilon_q + P_q\right) u_q^{\mu} u_q^{\nu} - P_q\, g^{\mu\nu}$$

Nonextensive/Dissipative conjecture (NexDC)-2

(*) starting from:
(ideal in a nonextensive sense - for f_q)

$$T_q^{\mu\nu} = \left(\varepsilon_q + P_q\right) u_q^{\mu} u_q^{\nu} - P_q\, g^{\mu\nu}$$

(*) because usually |q-1|<<1

⟶ decomposing as:

$$\varepsilon_q\left(T_q\right) \equiv \varepsilon\left(T_q\right) + \Delta\varepsilon_q\left(T_q\right)$$

$$P_q\left(T_q\right) \equiv P\left(T_q\right) + \Delta P_q\left(T_q\right)$$

$$u_q^{\mu}(x) \equiv u^{\mu}(x) + \delta u_q^{\mu}(x)$$

Nonextensive/Dissipative conjecture (NexDC)-3

(*) starting from:
(ideal in a nonextensive sense - for f_q)

$$T_q^{\mu\nu} = \left(\varepsilon_q + P_q\right) u_q^{\mu} u_q^{\nu} - P_q\, g^{\mu\nu}$$

(*) because usually $|q-1| \ll 1$

⟶ decomposing as:

$$\varepsilon_q\left(T_q\right) \equiv \varepsilon\left(T_q\right) + \Delta\varepsilon_q\left(T_q\right)$$
$$P_q\left(T_q\right) \equiv P\left(T_q\right) + \Delta P_q\left(T_q\right)$$
$$u_q^{\mu}(x) \equiv u^{\mu}(x) + \delta u_q^{\mu}(x)$$

(*) assuming that

$$P(T) = P_q\left(T_q\right) \quad and \quad \varepsilon(T) = \varepsilon_q\left(T_q\right) + 3\Pi$$

Nonextensive/Dissipative conjecture (NexDC)-4

(*) starting from:
(ideal in a nonextensive sense - for f_q)

$$T_q^{\mu\nu} = \left(\varepsilon_q + P_q\right) u_q^\mu u_q^\nu - P_q\, g^{\mu\nu}$$

(*) because usually $|q-1| \ll 1$
⟶ decomposing as:

$$\varepsilon_q\left(T_q\right) \equiv \varepsilon\left(T_q\right) + \Delta\varepsilon_q\left(T_q\right)$$
$$P_q\left(T_q\right) \equiv P\left(T_q\right) + \Delta P_q\left(T_q\right)$$
$$u_q^\mu(x) \equiv u^\mu(x) + \delta u_q^\mu(x)$$

(*) assuming that

$$P(T) = P_q\left(T_q\right) \quad and \quad \varepsilon(T) = \varepsilon_q\left(T_q\right) + 3\Pi$$

(*) One gets :
(extensive but dissipative in the usual sense - for f)

$$T^{\mu\nu} = \varepsilon(T) u^\mu u^\nu - \left[P(T) + \Pi\right]\Delta^{\mu\nu} + 2W^{(\mu} u^{\nu)} + \pi^{\mu\nu}$$

Nonextensive/Dissipative conjecture (NexDC)-5

Nonextensive

$$T_q^{\mu\nu} = \left(\varepsilon_q + P_q\right) u_q^{\mu} u_q^{\nu} - P_q\, g^{\mu\nu}$$

+

$$P(T) = P_q(T_q) \quad and \quad \varepsilon(T) = \varepsilon_q(T_q) + 3\Pi$$

→ *Dissipative*

$$T^{\mu\nu} = \varepsilon(T) u^{\mu} u^{\nu} - \left[P(T) + \Pi\right]\Delta^{\mu\nu} + 2W^{(\mu} u^{\nu)} + \pi^{\mu\nu}$$

NexDC – some definitions

$$T^{\mu\nu} = \varepsilon(T)u^{\mu}u^{\nu} - [P(T)+\Pi]\Delta^{\mu\nu} + 2W^{(\mu}u^{\nu)} + \pi^{\mu\nu}$$

Energy/heat flow vector

$$A^{(\mu}B^{\nu)} = \frac{1}{2}\left(A^{\mu}B^{\nu} + A^{\nu}B^{\mu}\right)$$

$$a^{<\mu}b^{\nu>} = \left[\frac{1}{2}\left(\Delta^{\mu}_{\lambda}\Delta^{\nu}_{\sigma} + \Delta^{\mu}_{\sigma}\Delta^{\nu}_{\lambda}\right) - \frac{1}{3}\Delta^{\mu\nu}\Delta_{\lambda\sigma}\right]a^{\lambda}b^{\sigma}$$

$$W^{\mu} = w_q[1+\gamma]\Delta^{\mu}_{\lambda}\delta u^{\lambda}_q; \qquad w_q = \varepsilon_q + P_q$$

$$\pi^{\mu\nu} = \frac{W^{\mu}W^{\nu}}{w_q[1+\gamma]} + \Pi\Delta^{\mu\nu} = w_q\delta u_q^{<\mu}\delta u_q^{\nu>}$$

$$\Pi = \frac{1}{3}w_q\left[\gamma^2 + 2\gamma\right]; \qquad \gamma = u_{\mu}\delta u^{\nu}_q = -\frac{1}{2}\delta u_{q\mu}\delta u^{\mu}_q$$

NexDC – some definitions

$$T^{\mu\nu} = \varepsilon(T)u^{\mu}u^{\nu} - [P(T)+\Pi]\Delta^{\mu\nu} + 2W^{(\mu}u^{\nu)} + \pi^{\mu\nu}$$

Energy/heat flow vector

Shear (symmetric and traceless) pressure tensor

$$A^{(\mu}B^{\nu)} = \frac{1}{2}\left(A^{\mu}B^{\nu} + A^{\nu}B^{\mu}\right) - \frac{1}{3}\Delta^{\mu\nu}A^{\lambda}B_{\lambda}$$

$$a^{<\mu}b^{\nu>} = \left[\frac{1}{2}\left(\Delta^{\mu}_{\lambda}\Delta^{\nu}_{\sigma} + \Delta^{\mu}_{\sigma}\Delta^{\nu}_{\lambda}\right) - \frac{1}{3}\Delta^{\mu\nu}\Delta_{\lambda\sigma}\right]a^{\lambda}b^{\sigma}$$

$$W^{\mu} = w_q[1+\gamma]\Delta^{\mu}_{\lambda}\delta u^{\lambda}_q; \qquad w_q = \varepsilon_q + P_q$$

$$\pi^{\mu\nu} = \frac{W^{\mu}W^{\nu}}{w_q[1+\gamma]} + \Pi\Delta^{\mu\nu} = w_q\delta u^{<\mu}_q\delta u^{\nu>}_q$$

$$\Pi = \frac{1}{3}w_q\left[\gamma^2 + 2\gamma\right]; \qquad \gamma = u_{\mu}\delta u^{\nu}_q = -\frac{1}{2}\delta u_{q\mu}\delta u^{\mu}_q$$

NexDC – some definitions

$$T^{\mu\nu} = \varepsilon(T)u^{\mu}u^{\nu} - [P(T)+\Pi]\Delta^{\mu\nu} + 2W^{(\mu}u^{\nu)} + \pi^{\mu\nu}$$

Energy/heat flow vector

Shear (symmetric and traceless) pressure tensor

$$A^{(\mu}B^{\nu)} =$$

$$a^{<\mu}b^{\nu>} = \left[\frac{1}{2}\left(\Delta^{\mu}_{\lambda}\Delta^{\nu}_{\sigma} + \Delta^{\mu}_{\sigma}\Delta^{\nu}_{\lambda}\right) - \frac{1}{3}\Delta^{\mu\nu}\Delta_{\lambda\sigma}\right]a^{\lambda}b^{\sigma}$$

$$W^{\mu} = w_q[1+\gamma]\Delta^{\mu}_{\lambda}\delta u^{\lambda}_q; \qquad w_q = \varepsilon_q + P_q$$

q-enthalpy

$$\pi^{\mu\nu} = \frac{W^{\mu}W^{\nu}}{w_q[1+\gamma]} + \Pi\Delta^{\mu\nu} = w_q\delta u^{<\mu}_q\delta u^{\nu>}_q$$

$$\Pi = \frac{1}{3}w_q[\gamma^2 + 2\gamma]; \qquad \gamma = u_{\mu}\delta u^{\nu}_q = -\frac{1}{2}\delta u_{q\mu}\delta u^{\mu}_q$$

q-dependent bulk pressure

Consequences: production of entropy

The q-entropy current:

$$\sigma_q^{\mu}(x) = -k_B \int \frac{d^3p}{(2\pi\hbar)^3} \frac{p^{\mu}}{p^0} \left\{ f_q^q \ln_q f_q - f_q \right\};$$

We assume that q-entropy is strictly conserved:

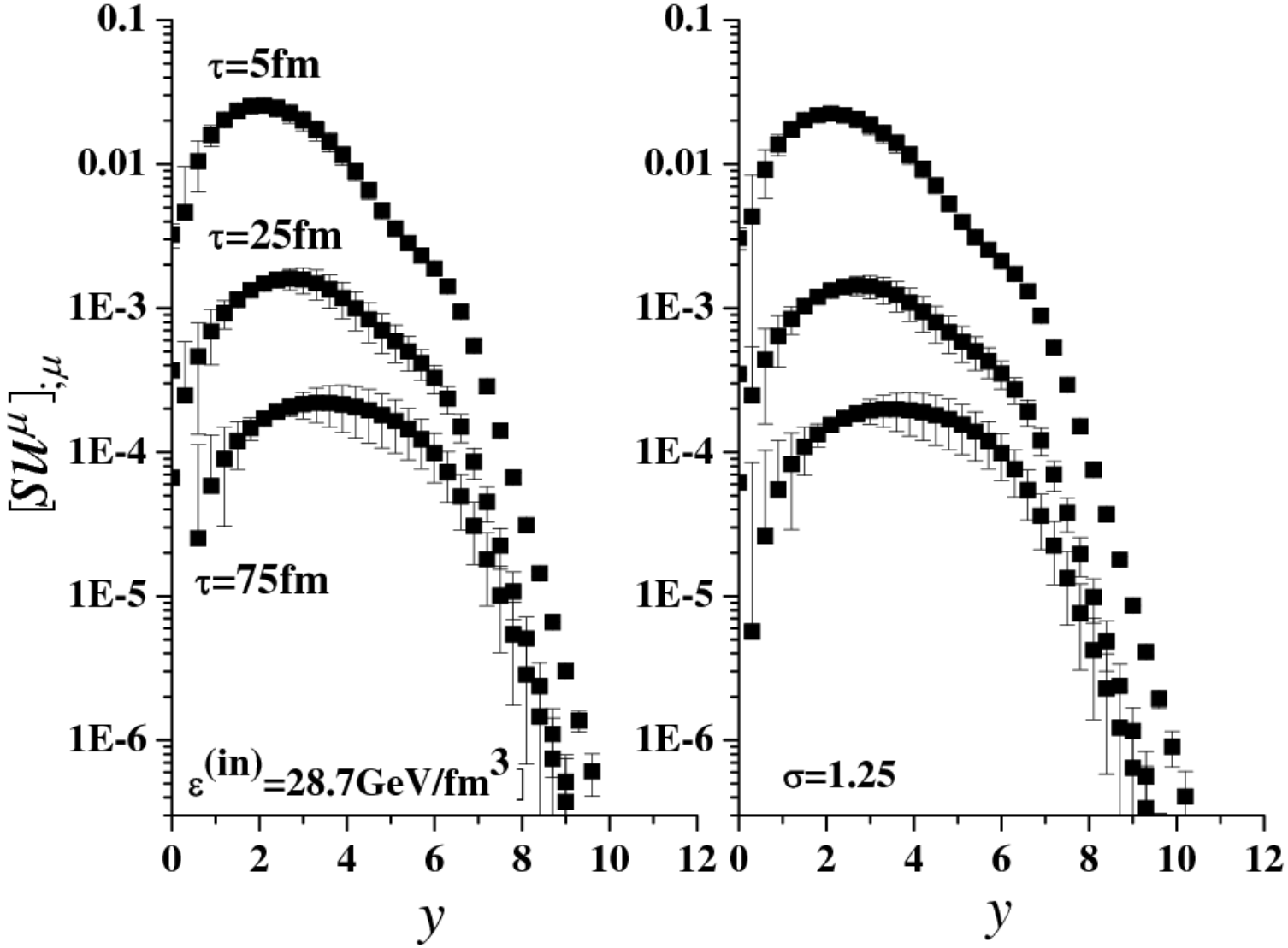
$$\sigma_q^{\mu}(x) = 0$$

Nevertheless, the usual (q=1) entropy is produced, because:

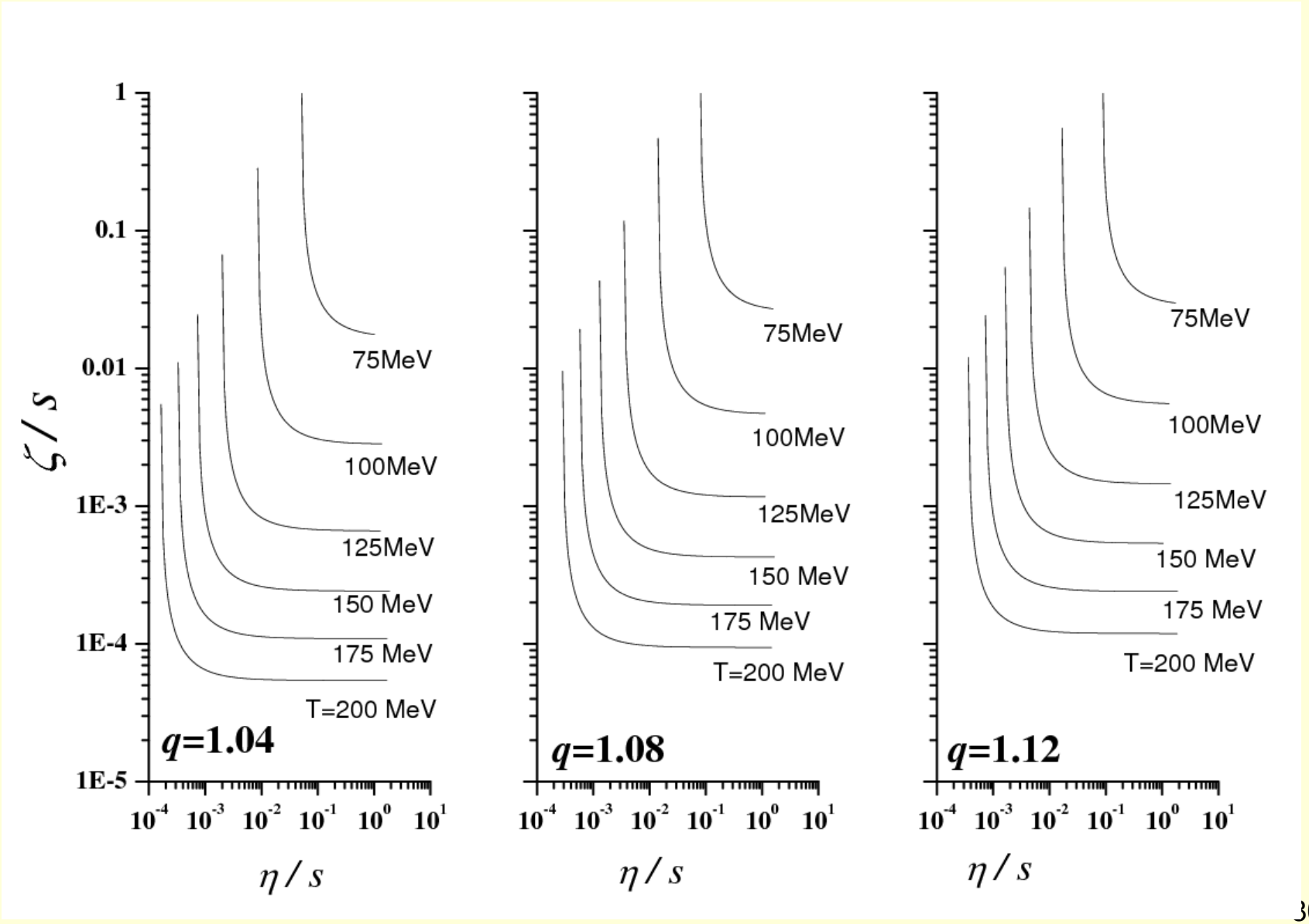
$$\left[s u^{\mu} \right]_{;\mu} = -\frac{u_{\nu}}{T} \delta T^{\mu\nu}_{;\mu} =$$

$$= -\frac{u_{\nu}}{T} \left[-\Pi \Delta^{\mu\nu} + 2W^{(\mu} u^{\nu)} + \pi^{\mu\nu} \right]_{;\mu} > 0$$

$$\ln_q X = \left(X^{(1-q)} - 1 \right) / (1-q)$$


0.1
0.01
1E-3
1E-4
1E-5
1E-6
τ=5fm
τ=25fm
τ=75fm
ε(in)=28.7GeV/fm3]
σ=1.25
[sμ]·,μ
0 2 4 6 8 10 12
y

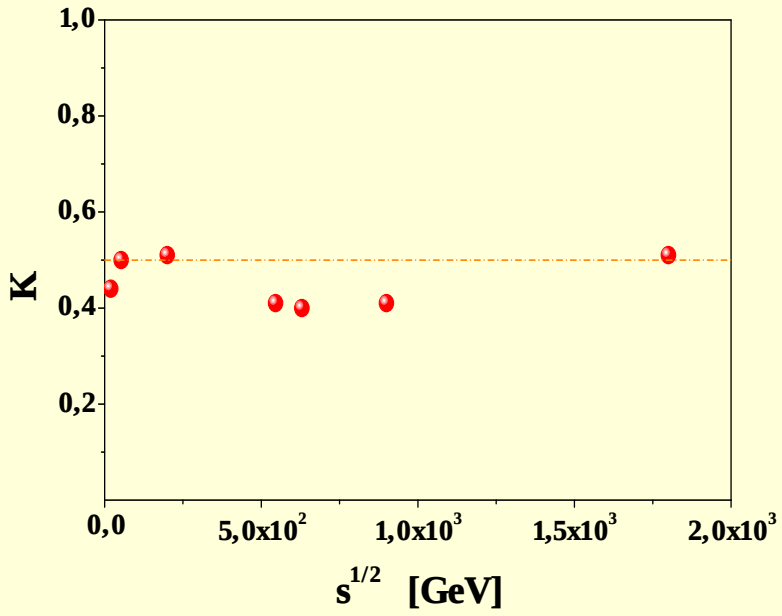
Consequences: transport coefficients

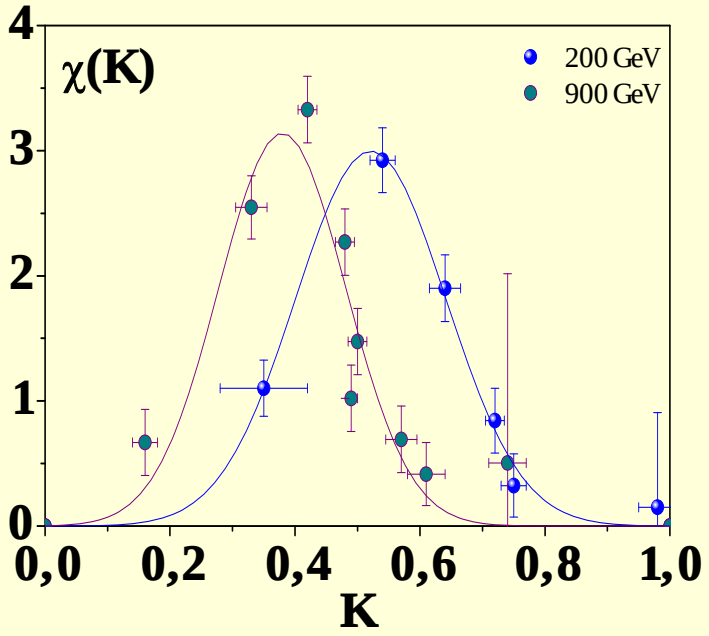


Example of small heat bath case:

EGE

For small heat baths we have results for inelasticity K:
(*) its energy dependence K(√s) and
(*) its distribution χ(K)





Extended Gaussian Esemble

$$\chi(K) = \chi_0 \exp\left[-\frac{1}{2\sigma^2}\left(K - \langle K\rangle\right)^2\right]$$

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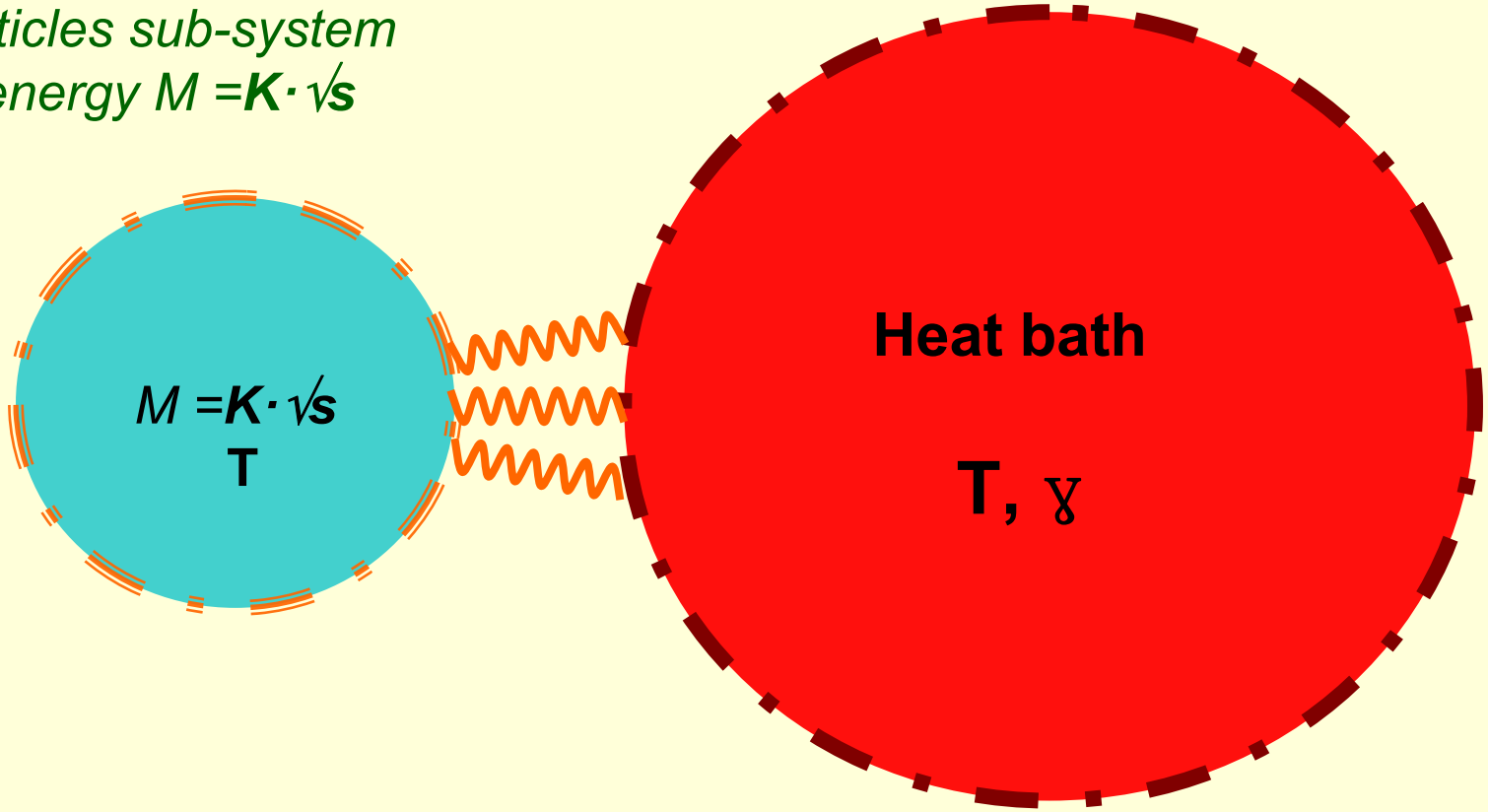
Example of small heat baths:

EGE

(N-k) –particles sub-system

*k –particles sub-system of energy M =**K**· √**s***

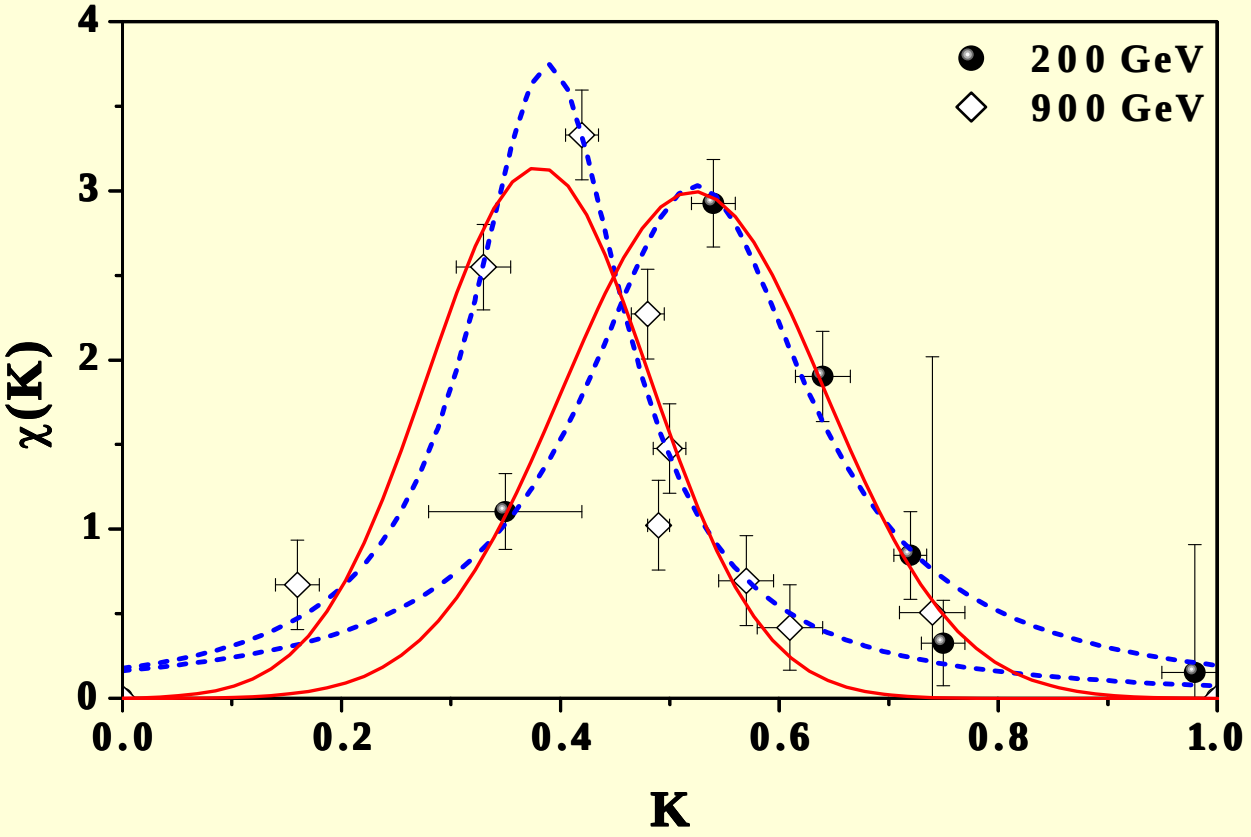


$$f(M) = C \cdot \exp\left[-\gamma\left(M - \langle M \rangle\right)^2\right]$$

EGE

Notice, however, that:

apparently better fit is obtained for Lorentzian distribution which can be regarded as q_gaussian with q=2….



…➡.. q-EGE ? …

Gaussian distribution

Lorentzian distribution

$\chi_{1<q<2}(K) \leftrightarrow$ ***Student distribution***

The End

Thank You