23.4.10

The Phase Diagram of Strongly Interacting Matter

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based on joint work with

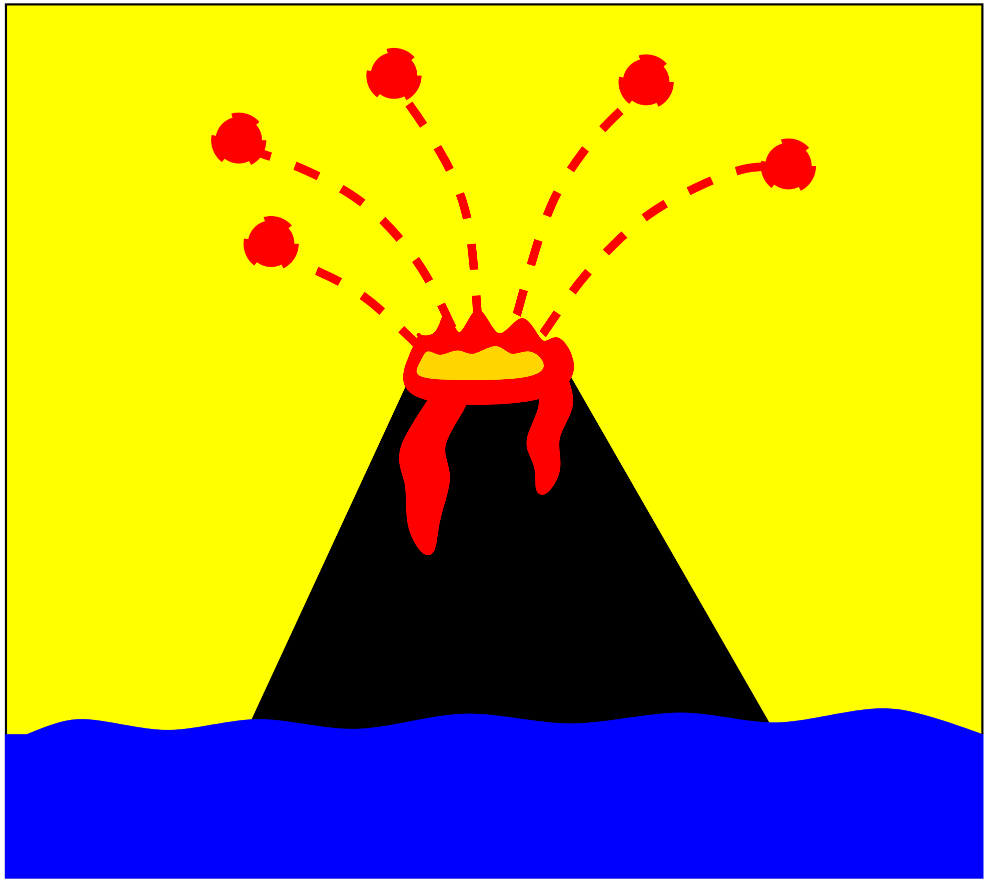
Paolo Castorina, Rajiv Gavai and Krzysztof Redlich

1. Introduction

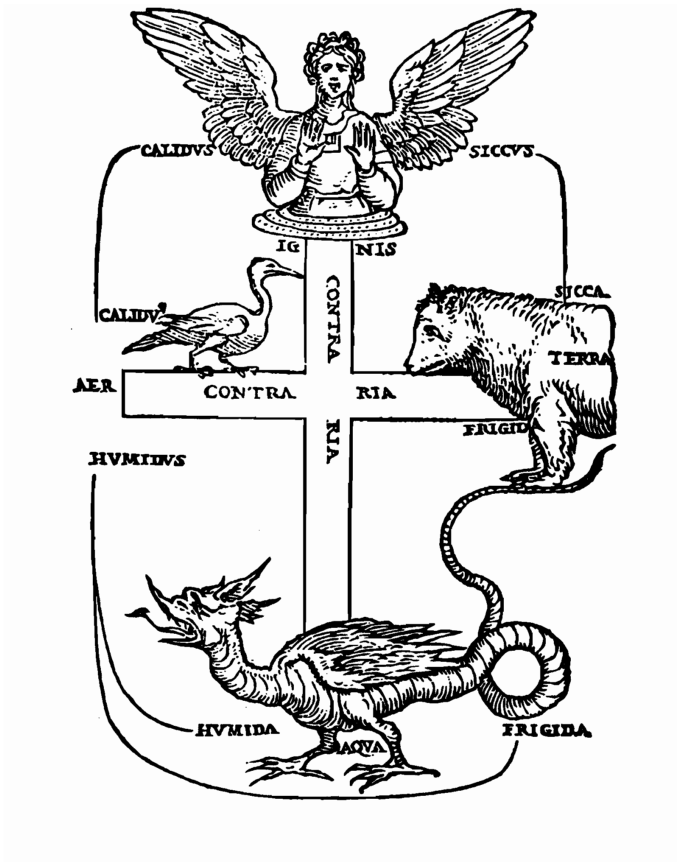
2. Hadronic Matter

3. Deconfined Quarks

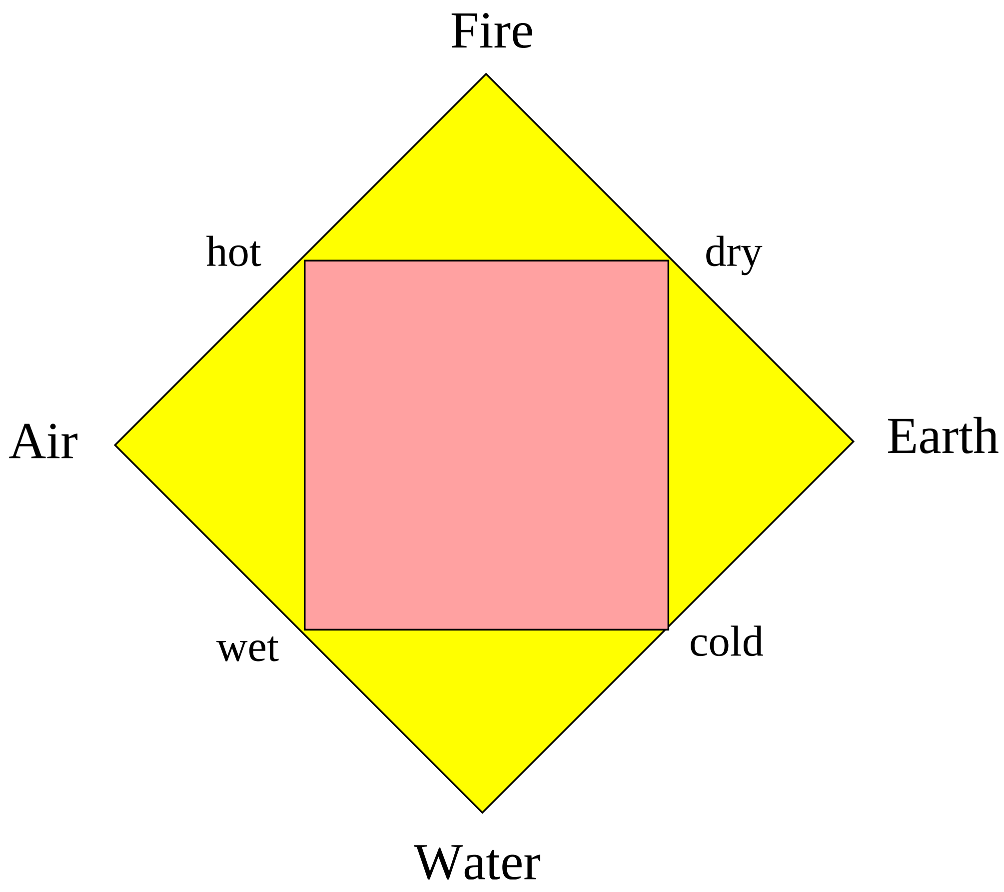
The States of Matter 500 B. C. - Experiment



The States of Matter 500 B. C. - Theory



The States of Matter 500 B. C. - Theory



Advent of strong interaction:
what happens to strongly interacting matter
as function of temperature and density?

- **I. Ya. Pomeranchuk, Doklady Akad. Nauk SSSR 1951:**

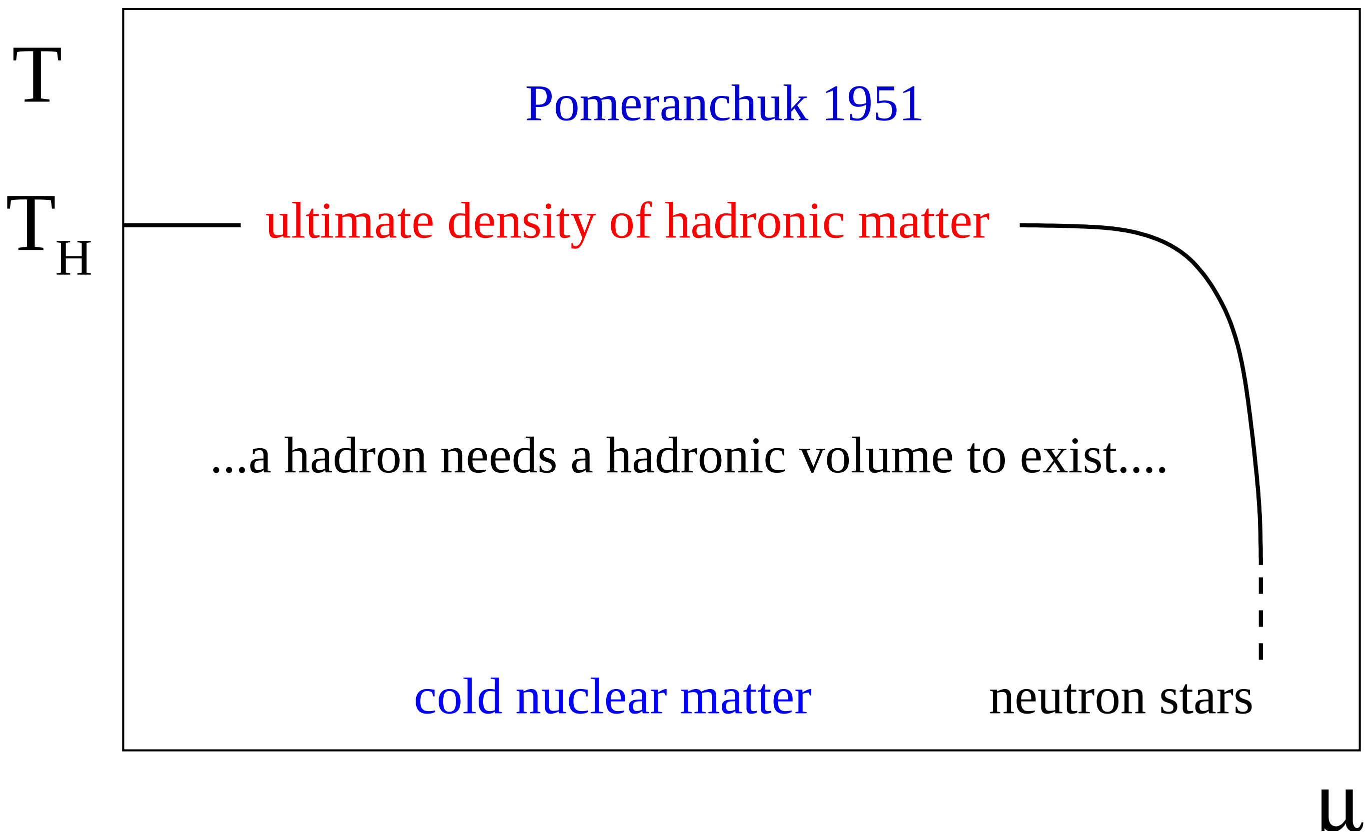
  ...the finite size of hadrons implies a density limit to hadronic matter.

- **Ya. B. Zel'dovich, JETP Letters 1959:**

  ...use the equation of state to establish how many different baryons are really elementary.

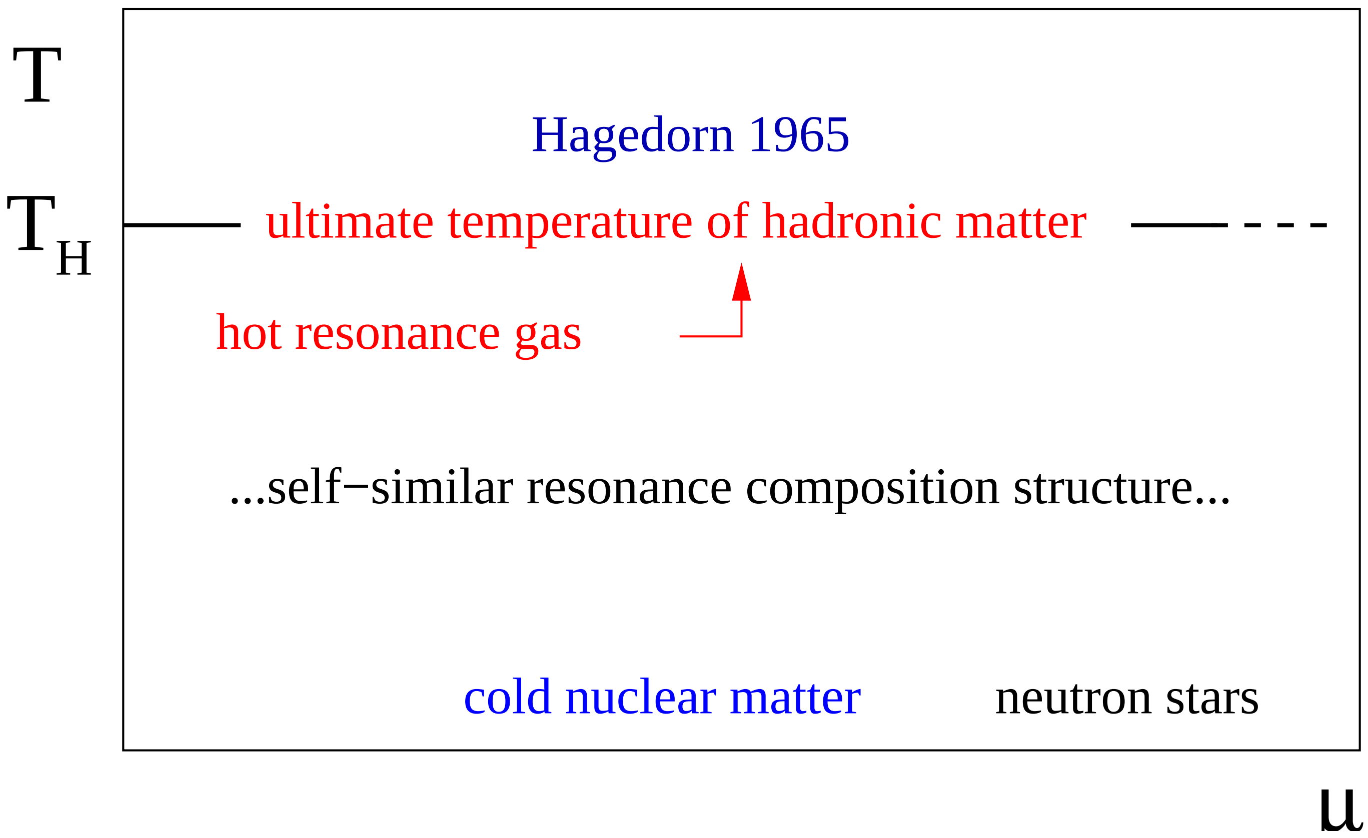
The States of Strongly Interacting Matter

1951



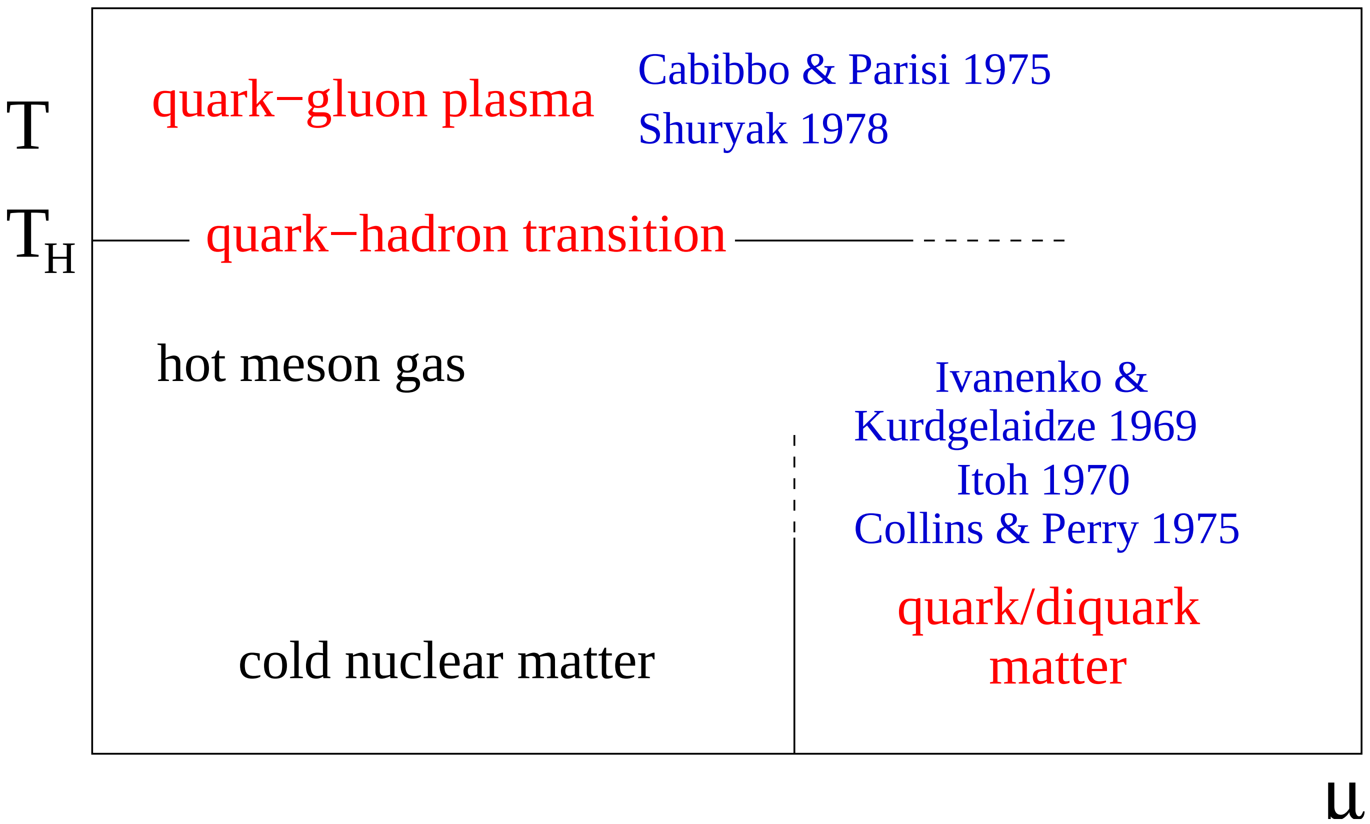
The States of Strongly Interacting Matter

1965



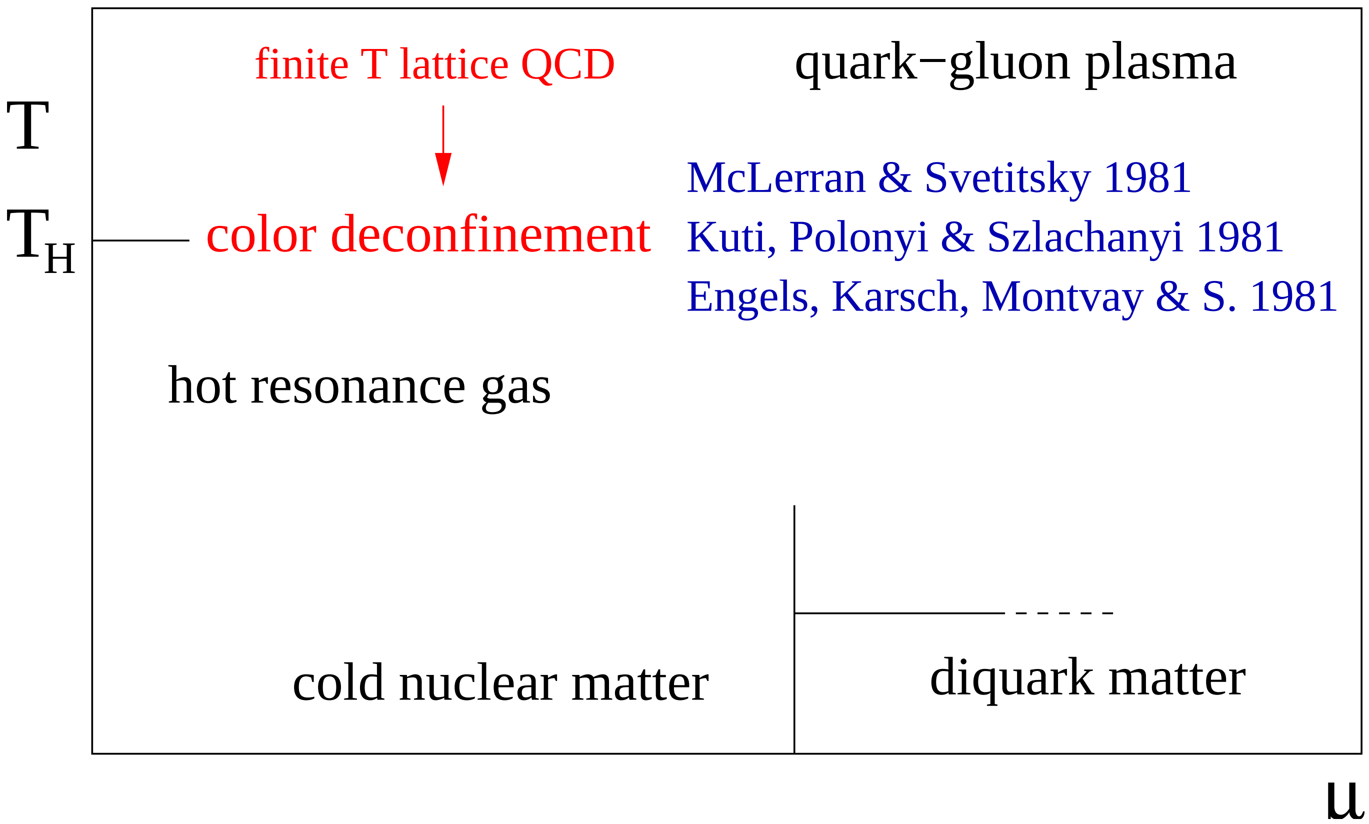
The States of Strongly Interacting Matter

1975



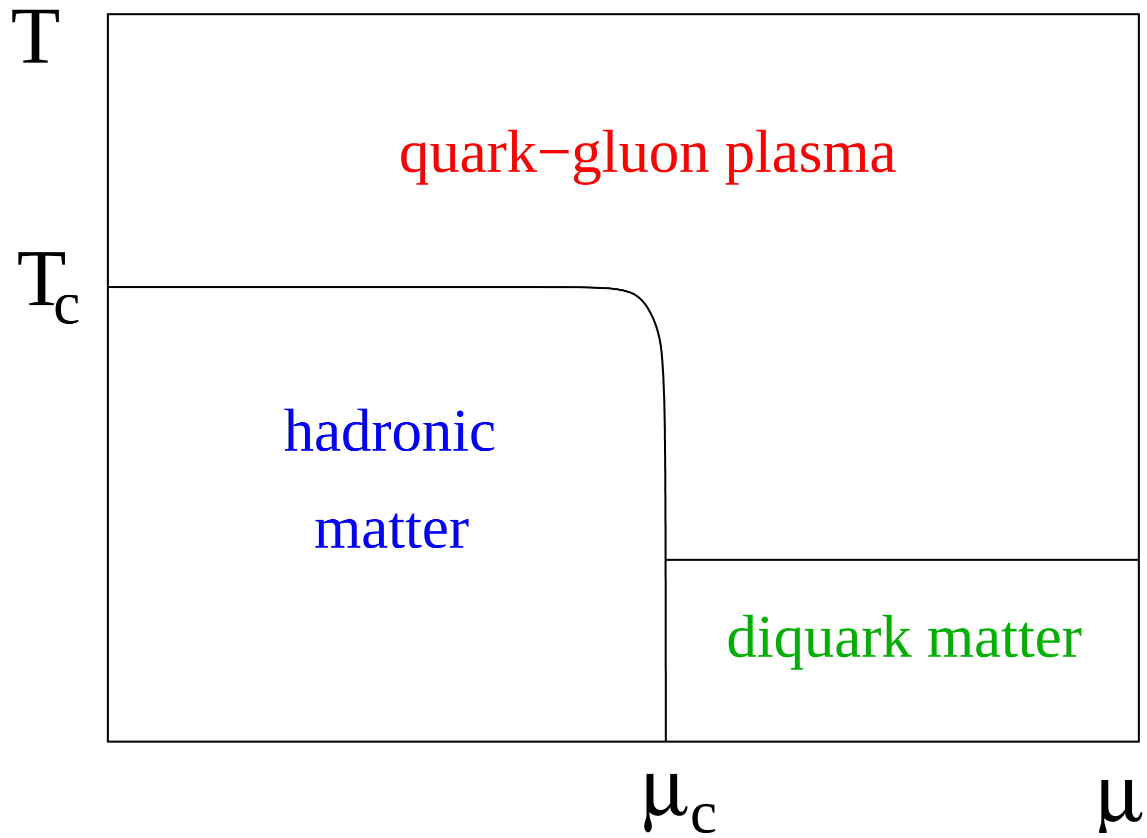
The States of Strongly Interacting Matter

1981



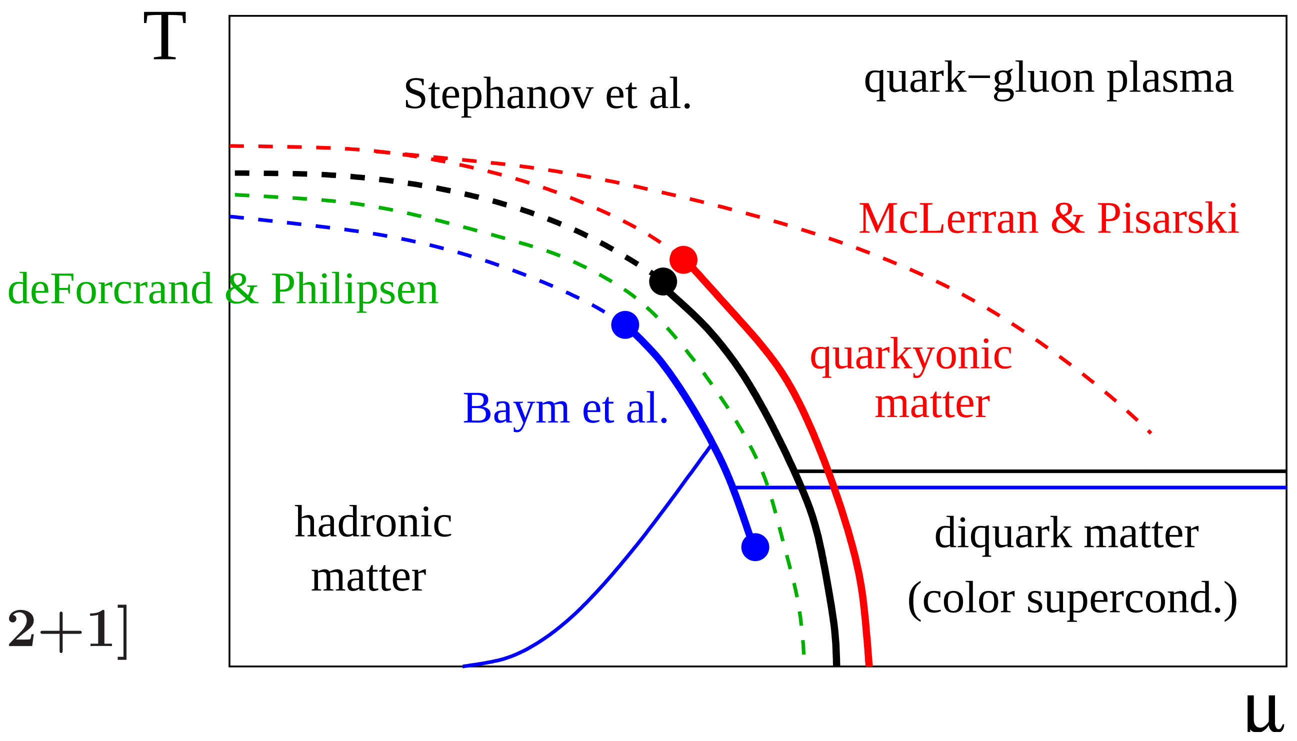
The States of Strongly Interacting Matter

1990



The States of Strongly Interacting Matter

2009



[for 2+1]

Back to basics: How does the underlying physics depend on where we are in the phase diagram?

Conventional Basis of Critical Behavior

- confinement/deconfinement ∼ spontaneous Z_2/Z_N symmetry breaking McLerran & Svetitsky 1981, Svetitsky & Yaffe 1982
- dynamical mass generation ∼ spontaneous chiral symmetry breaking Pisarski & Wilczek 1984

consider phase structure for $\mu = 0$:

genuine thermal phase transitions

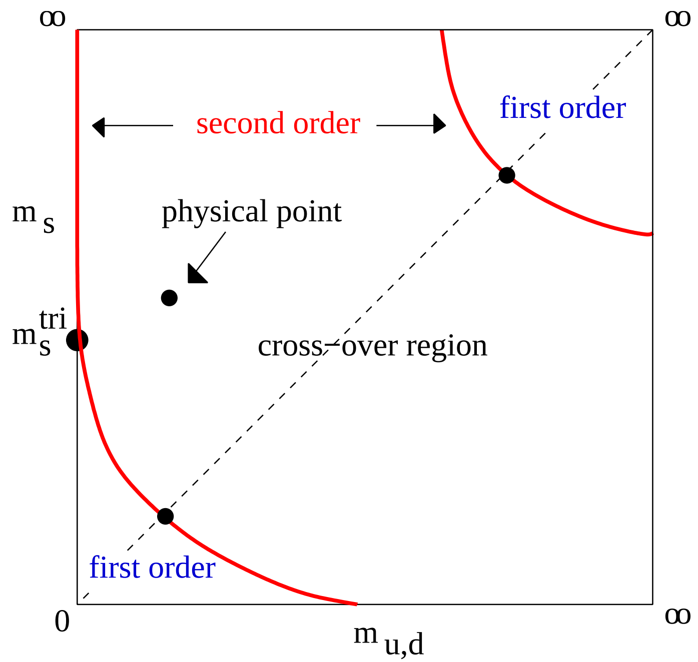
(singularities in partition function)

only for special values of $m_{u,d}, m_s$

but always ∃ "transition region"

with sharp variation of thermal

observables: "rapid cross-over"

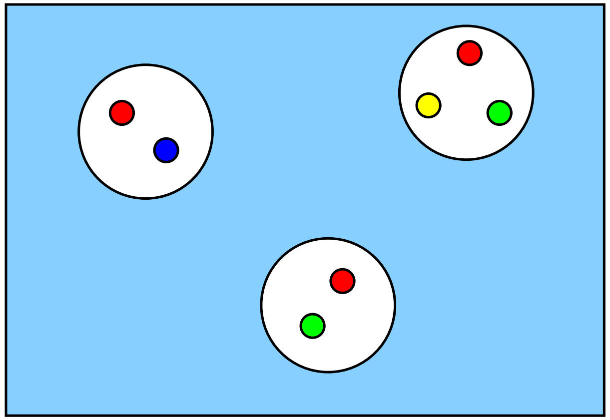


How to understand this? What about density?

What is deconfinement?

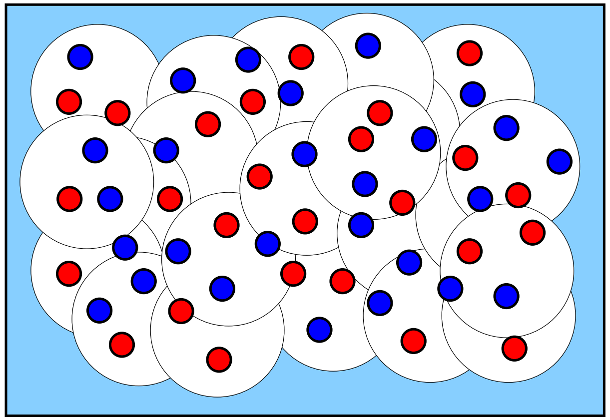
<ins>confinement</ins>:

a quark has within a range of about 1 fm one antiquark or two quarks to form a color singlet
→ low density phenomenon



<ins>deconfinement</ins>:

a quark has within a range of about 1 fm so many quarks and antiquarks that pairing becomes meaningless
→ high density phenomenon



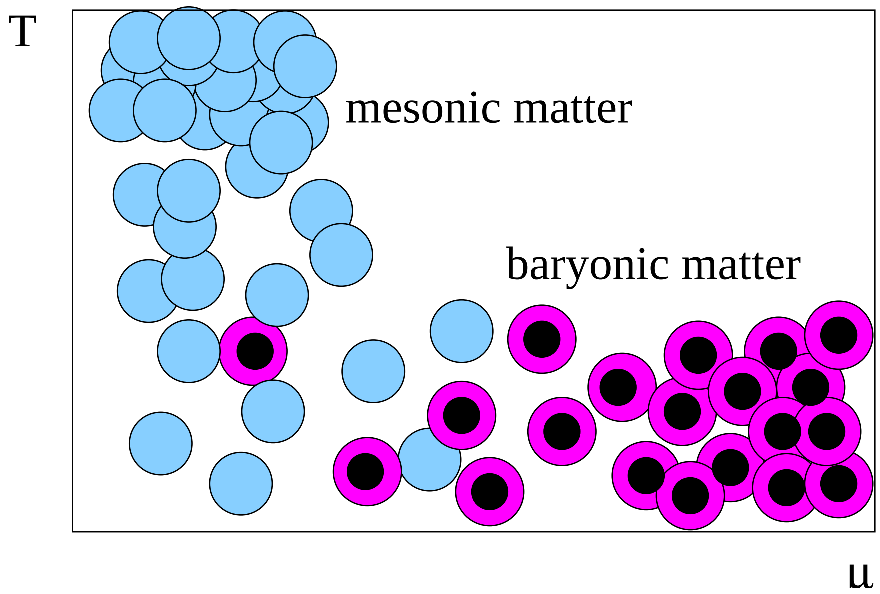
1. Introduction

2. Hadronic Matter*

3. Deconfined Quarks

* with Paolo Castorina and Krzysztof Redlich,
Eur. Phys. J. C 59 (2009) 67

Constituent Structure of Hadronic Matter



- low μ: with increasing T, mesonic medium of increasing density
  mesons experience attraction $\longrightarrow$ resonance formation
  mesons are permeable (overlap) $\longrightarrow$ resonances $\sim$ same size
- low T: with increasing μ, baryonic medium of increasing density
  nucleons experience attraction $\longrightarrow$ formation of nuclei
  nucleons repel (hard core) $\longrightarrow$ nuclei grow linearly with A

In both cases, ∃ clustering

∃ relation between clustering and critical behavior? Frenkel 1939

Essam & Fisher1963

consider spin systems, e.g., Ising model

- for $H = 0$,
  spontaneous Z_2 symmetry breaking $\longrightarrow$ magnetization transition
- but this can be translated into cluster formation and fusion

  critical behavior via cluster fusion: percolation $\equiv$
  critical behavior via spontaneous symmetry breaking

  Fisher 1967, Fortuin & Kasteleyn 1972, Coniglio & Klein 1980
- for $H \neq 0$, Isakov 1984
  partition function is analytic, no thermal critical behavior
  but clustering & percolation persists Kertész 1989

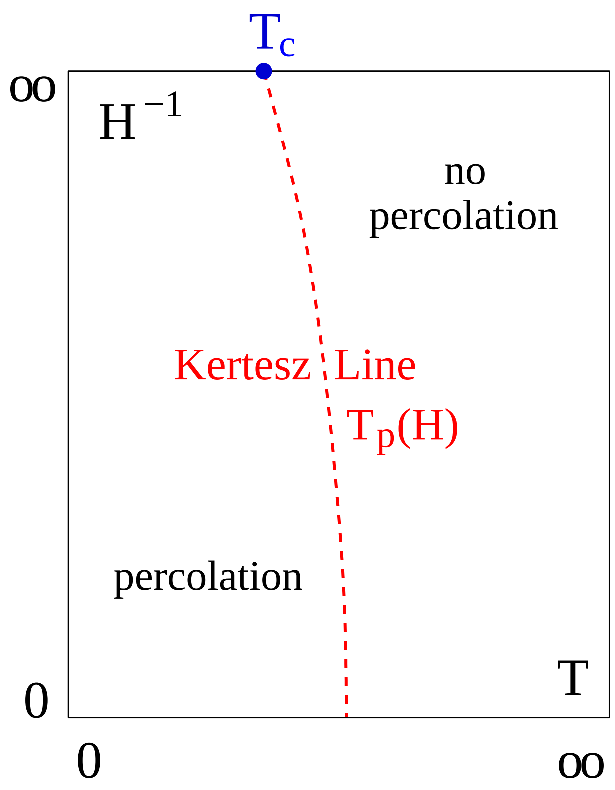
∃ geometic critical behavior

In spin systems,

$\exists$ geometric critical behavior for all values of H;

for $H = 0$, this can become identical to thermal critical behavior, with non-analytic partition function & Z_2 exponents

for $H \neq 0$, $\exists$ Kertész line geometric transition with singular cluster behavior & percolation exponents



For spin systems,

$$\text{thermal critical behavior} \subset \text{geometric critical behavior}$$

Also in QCD? Hadrons have intrinsic size, with increasing density they form clusters & eventually percolate

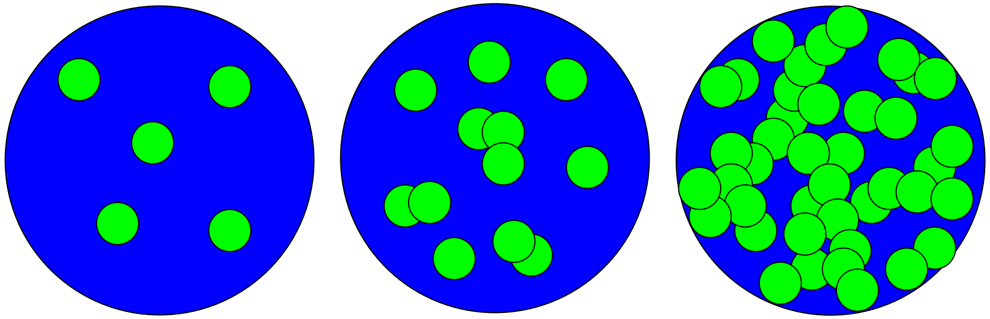
Hadron Percolation ∼ Color Deconfinement

Pomeranchuk 1951

Baym 1979, Çelik, Karsch & S. 1980

Recall percolation

- 2-d, with overlap:
  lilies on a pond



- 3-d: N spheres of volume V_h in box of volume V, with overlap
  increase density $n = N/V$ until largest cluster spans volume:
  <u>percolation</u>

  critical percolation density $n_p \simeq 0.34/V_h$

  at $n = n_P$, 30 % of space filled by overlapping spheres,
  70 % still empty

how dense is the percolating cluster? Digal, Fortunato & S. 2004

critical cluster density $n_m \simeq 1.2/V_h$

$$R_h \simeq 0.8 \text{ fm} \;\Rightarrow\; n_m \simeq \frac{0.6}{\text{fm}^3} \quad \text{as deconfinement density}$$

so far, cluster constituents were allowed arbitrary overlap

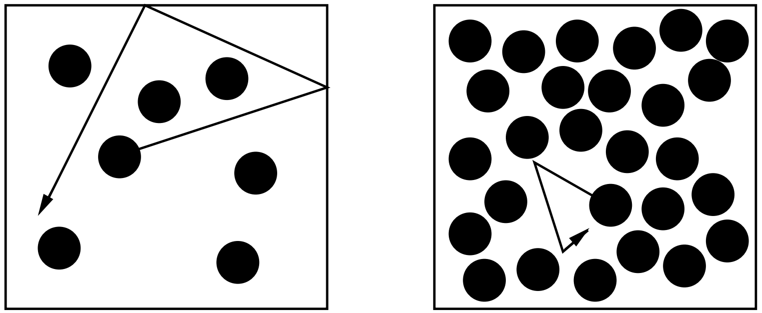
what if they have a hard core?

then ∃ jamming

at high density, constituents have restricted spatial mobility

∃ jamming transition

with mobility ∼ order parameter



Karsch & S. 1980

percolation for spheres of radius R_0
with a hard core of radius $R_{hc} = R_0/2$ Kratky 1988

hard cores tend to prevent dense clusters;

higher density needed to achieve percolating jammed clusters

$$n_b \simeq \frac{2.0}{V_0} = \frac{0.25}{V_{hc}} \simeq \frac{1.0}{\mathrm{fm}^3} \simeq 6\ \mathrm{n}_0$$

for the deconfinement density of baryonic matter

NB: additional uniform attractive potential
$\longrightarrow$ first order thermal transition

$\exists$ two percolation thresholds in strongly interacting matter:

- mesonic matter, full overlap: $n_m \simeq 0.6/\mathrm{fm}^3$
- baryonic matter, hard core: $n_b \simeq 1.0/\mathrm{fm}^3$

now apply to determine critical behavior

If interactions are resonance dominated,

interacting medium $\equiv$ ideal resonance gas

Beth & Uhlenbeck 1937; Dashen, Ma & Bernstein 1969

consider ideal resonance gas of all PDG states for $M \leq 2.5$ GeV

partition function

$$\ln\ Z(T,\mu,\mu_S,V) = \ln\ Z_M(T,\mu_S,V)\ +\ \ln\ Z_B(T,\mu,\mu_S,V)$$

with

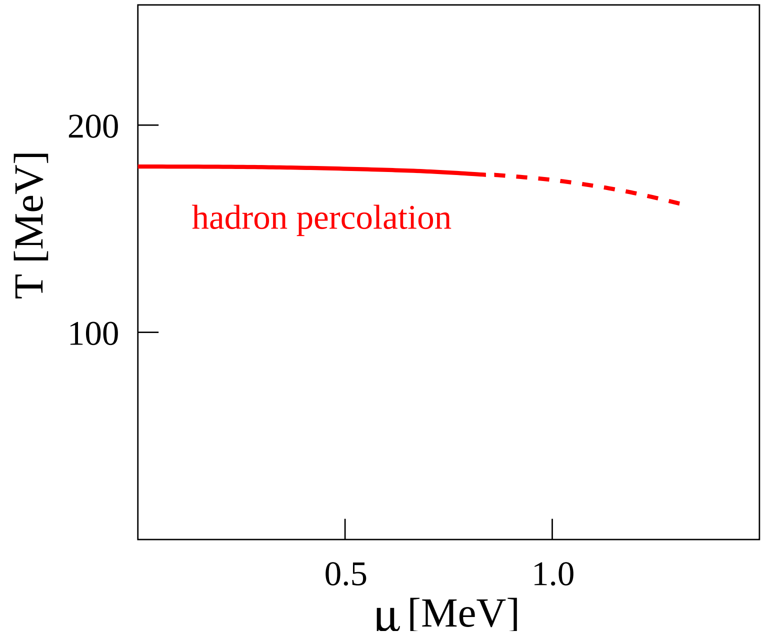
$$\ln\ Z_M(T,V,\mu_S) = \sum_{\text{mesons i}} \ln\ Z^i_M(T,V,\mu_S)$$

$$\ln\ Z_B(T,\mu,\mu_S,V) = \sum_{\text{baryons i}} \ln\ Z^i_B(T,\mu,\mu_S,V)$$

for mesonic and baryonic contributions; enforce $S=0$

- low baryon-density limit: percolation of overlapping hadrons

$$n_h(T_h, \mu) = \frac{\ln Z(T, \mu, V)}{V} = 0.6/\text{fm}^3$$



Obtain at $\mu = 0$

$$T_h \simeq 180 \text{ MeV}$$

deconfinement temperature

based on hadron percolation

baryons included, but hard core effects ignored

slow decrease of transition temperature with μ,
due to associated production

• **high baryon-density limit:**

percolation/jamming of hard-core baryons

density of pointlike baryons

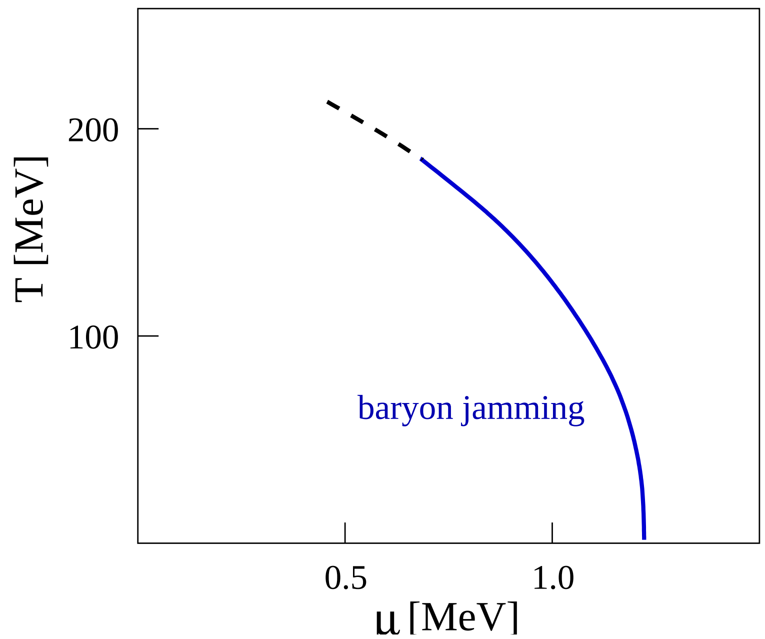
$$n_b^0 = \frac{1}{V}\left(\frac{\partial\ T \ln Z_B(T,\mu,V)}{\partial \mu}\right)$$

hard core ⇒ excluded volume
(Van der Waals)

$$n_b = \frac{n_b^0}{1 + V_{hc} n_b^0}$$

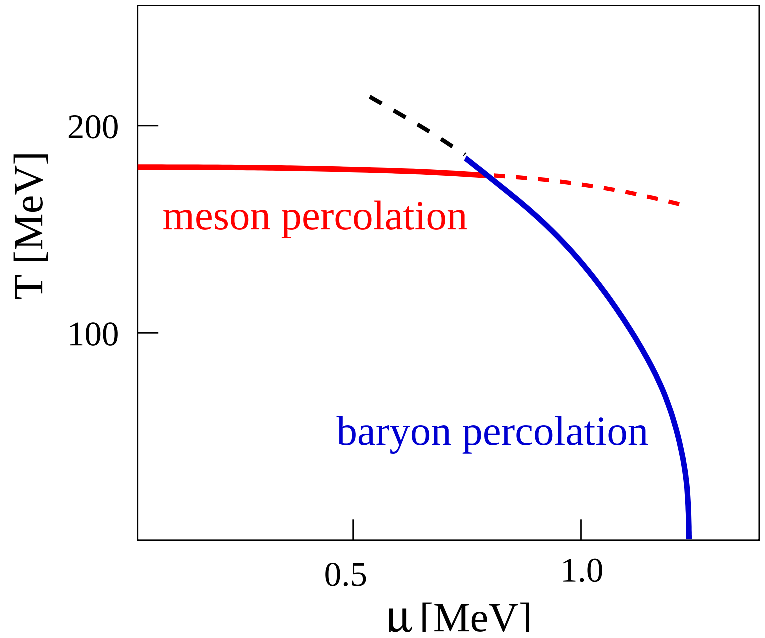
percolation threshold
⟶ transition line

$$n_b^c(T,\mu) = \frac{2.0}{V_0} = \frac{0.9}{\mathrm{fm}^3} \simeq 5\ n_0$$



combine the two mechanisms:

phase diagram of hadronic matter



• low baryon density:

percolation of overlapping hadrons

clustering $\sim$ attraction

• high baryon density:

percolation of hard-core baryons

nuclear attraction plus hard-core repulsion $\rightarrow$ 1^{st} order transition

clustering and percolation can provide

a conceptual basis for the limits of hadronic matter

in the QCD phase diagram

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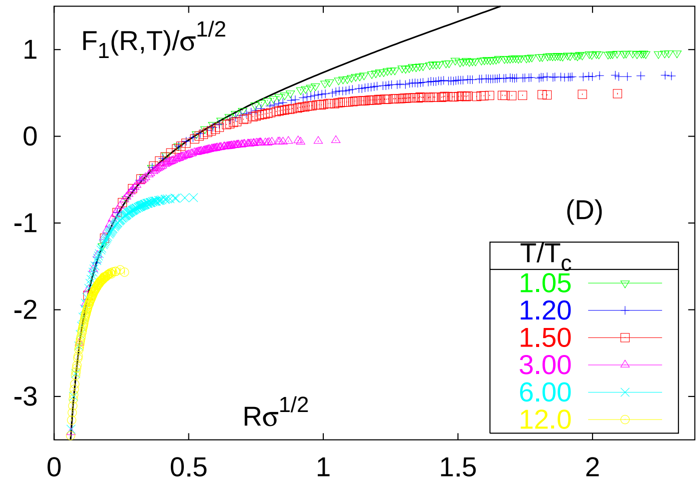
*** with Rajiv Gavai and Paolo Castorina, arXiv:1003.6078**

What happens beyond the limits?

There are two roads to deconfinement:

- Increase quark density so that several quarks/antiquarks within confinement radius $\longrightarrow$ pairing ambiguous or meaningless.
- Increase temperature so much that gluon screening forbids communication between quarks/antiquarks distance r apart.

Illustration of the second case:
heavy quark correlations



Quarks separated by about 1 fm
no longer "see" each other for $T \geq T_c$

mesonic matter:
when quark density is high enough,
gluon screening radius is short enough, so both coincide

baryonic matter?

in hadrons & in hadronic matter $\exists$ chiral symmetry breaking

$\Rightarrow$ confined quarks acquire effective mass $M_q \simeq 300$ MeV
effective size $R_q \simeq R_h/3 \simeq 0.3$ fm
through surrounding gluon cloud

what happens at deconfinement? Possible scenarios:

- plasma of massless quarks and gluons,
  ground state shift re physical vacuum $\rightarrow$ bag pressure B
- plasma of massive "constituent" quarks, all gluon effects in M_q

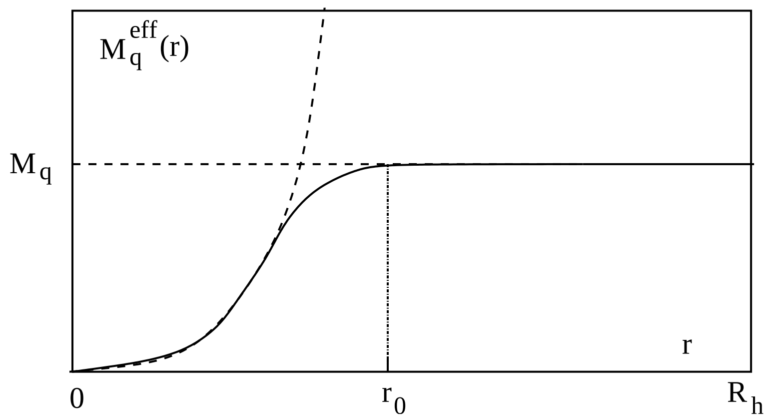
"effective" quark? $\sim$ depends on how you look:

- hadronic distances, soft probes: massive constituent quark
  (additive quark model)
- sub-hadronic distances, hard probes: bare current quark
  (deep inelastic scattering)

Origin of constituent quark mass?
quark polarizes gluon medium $\longrightarrow$ gluon cloud around quark

$$M_q \sim m_q + \epsilon_g r^3$$

where ϵ_g is the change
in energy density of the gluon field
due to the presence of the quark



QCD:
non-abelian gluon screening
limits "visibility" range to r_g

$\longrightarrow$ energy density of gluon cloud and screening radius
determine "asymptotic" constituent quark mass $\sim$ gluon cloud

relation to chiral symmetry breaking?

estimates from perturbative QCD Politzer 1976 effective quark mass $M_q^{\text{eff}}(r)$ at distance r

$$M_q^{\text{eff}}(r) = 4\ g^2(r)\ r^2 \left[\frac{g^2(r)}{g^2(r_0)}\right]^{-d} \langle\bar{\psi}\psi(r_0)\rangle$$

with reference point r_0 for determination of $\langle\bar{\psi}\psi(r_0)\rangle$; coupling is

$$g^2(r) = \frac{16\pi^2}{9}\frac{1}{\ln[1/(r^2\Lambda^2_{\text{QCD}})]}$$

for $N_f = 3,\ N_c = 3\ \rightarrow\ d = 4/9$

constituent quark mass is defined as solution of

$$M_q = M_q^{\text{eff}}(r = 1/2M_q)$$

giving M_q in terms of r_0 and $\langle\bar{\psi}\psi(r_0)\rangle$

With $r_0 = 1/2M_q$ (meeting of perturbative and non-perturbative)

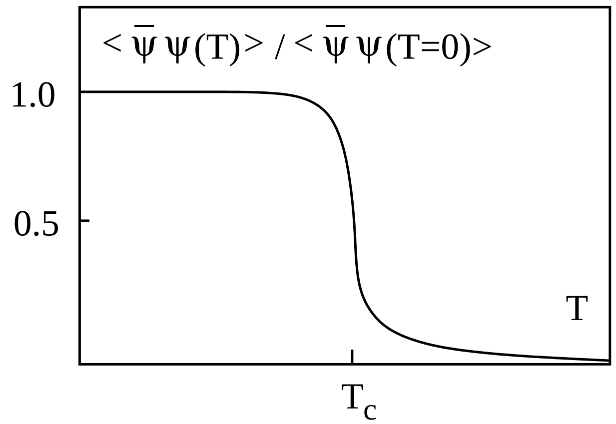
$$M_q^3 = \left\{\frac{16\pi^2}{9} \frac{1}{\ln(4M_q^2/\Lambda_{QCD}^2)}\right\} \langle\bar\psi\psi(r_0)\rangle$$

and with $\Lambda_{QCD} = 0.2$ GeV, $\langle\bar\psi\psi(r_0)\rangle^{1/3} = 0.2$ GeV

$$M_q = 375 \text{ MeV}; \quad R_q = 0.26 \text{ fm}$$

constituent quark mass determined by chiral condensate

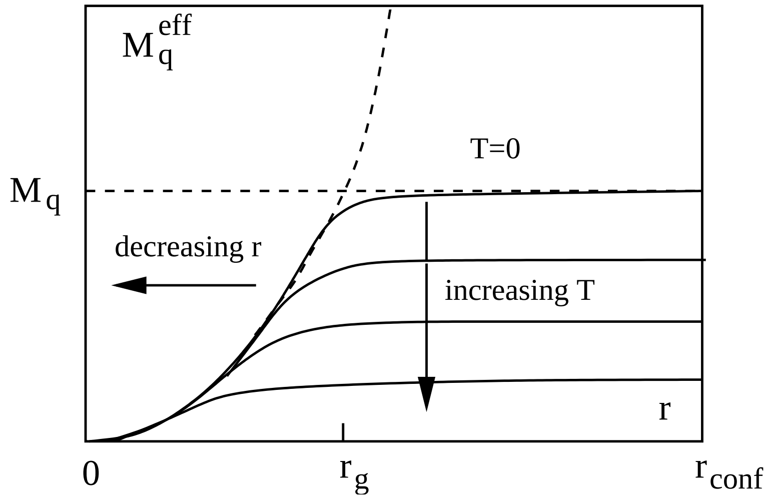
how does $\langle\bar\psi\psi(T)\rangle^{1/3}$ change with temperature?



gluon cloud evaporates,
constituent quark mass vanishes
as $T \longrightarrow T_c$

So there are two ways to make the effective quark mass vanish

- decrease interquark distance
- increase temperature

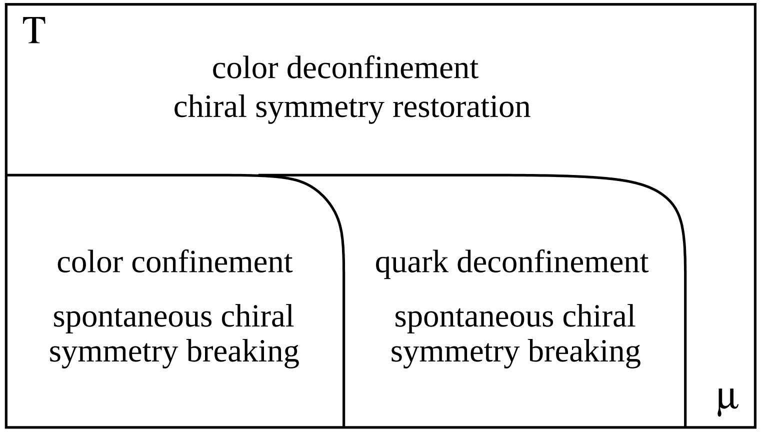


now consider different $T - \mu$ regions:

- $\mu \simeq 0,\ T \simeq T_c$: interquark distance $\sim$ 1 fm but hot medium makes gluon cloud evaporate $\Rightarrow\ M_q^{\text{eff}} \simeq 0$
- $T \simeq 0,\ \mu \simeq \mu_c$: interquark distance $\sim$ 1 fm and cold medium, gluon cloud does not evaporate $\Rightarrow\ M_q^{\text{eff}} \simeq M_q$

in cold dense matter, $M_q^{\text{eff}} \longrightarrow 0$ requires short interquark distance $\sim$ constituent quark percolation

intermediate massive quark plasma for $0.3 < r < 1$ fm and $T \lesssim T_c$

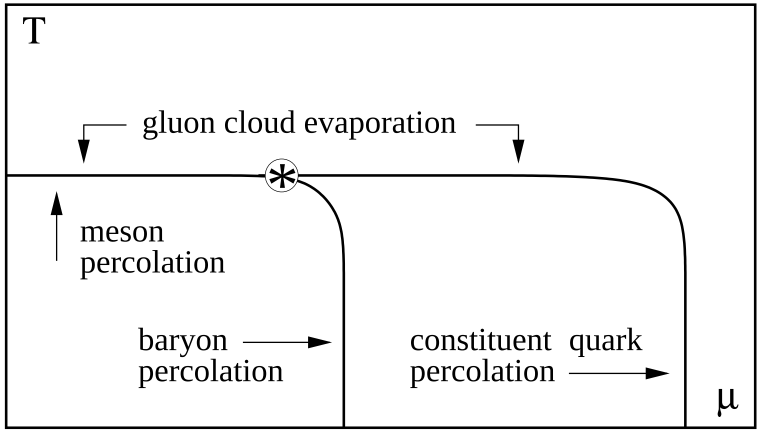


color deconfinement, but chiral symmetry remains broken;
constituents: massive colored quarks, gluons only as quark dressing

baryon density limit through quark percolation $n_b^c \simeq 3.5$ fm^{-3}

- **nuclear matter $n_b \leq 0.9$ fm^{-3}**
- **quark plasma 0.9 fm$^{-3} \leq n_b \leq 3.5$ fm^{-3}**
- **quark-gluon plasma $n_b \geq 3.5$ fm^{-3}**

Transitions:



Nature of massive quark plasma

– massive quarks and (at higher T) some massive antiquarks

– no gluons, "chiral pions"?

no color confinement, but colored bound states possible

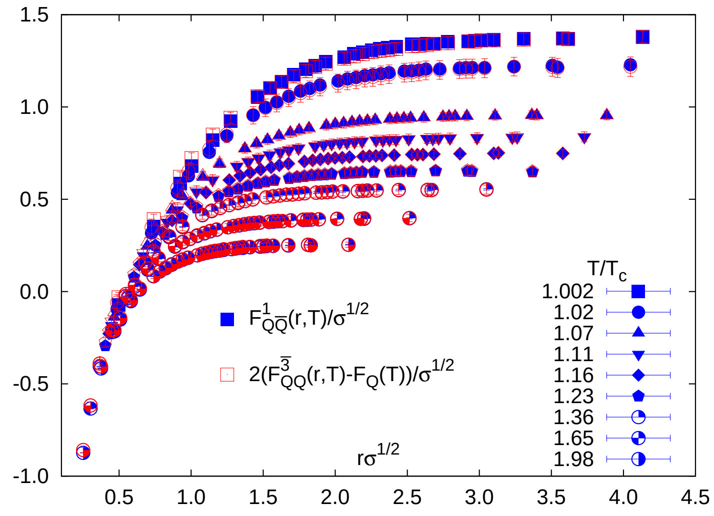
anti-triplet qq bound states = diquarks
(genuine two-body states, not Cooper pairs)

attractive interaction for

$qq \longrightarrow$ color anti-triplet,

$q\bar{q} \longrightarrow$ color singlet,

with same functional form of potential in r, T



Bielefeld Lattice Group 2002

constituent quark plasma can be structurally similar to hadron gas:

- massive quarks
- (antitriplet) diquark and (singlet) $q\bar{q}$ states
- higher excitations (colored resonance gas)
- also possible: glueballs, chiral pions
- all states have intrinsic finite size, hence $\exists$ percolation limit

quark plasma has effective color degrees of freedom

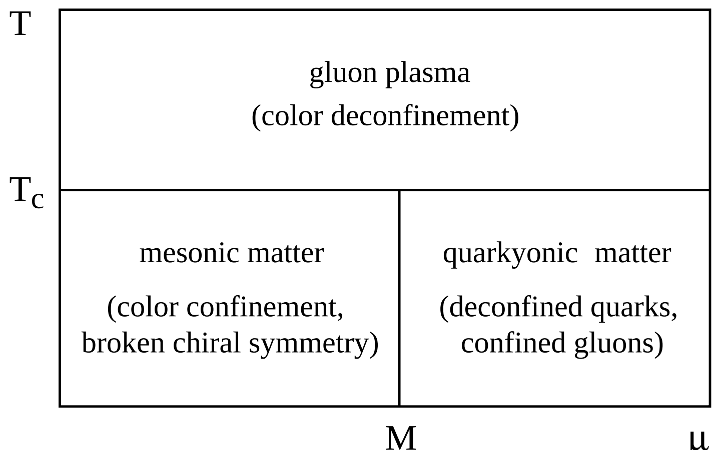
- hadron gas: $d_{\text{eff}} = 1$
- massive quark plasma: $d_{\text{eff}} = N_c$
- quark-gluon plasma: $d_{\text{eff}} = N_c^2$

relation to quarkyonic matter? McLerran & Pisarski 2007

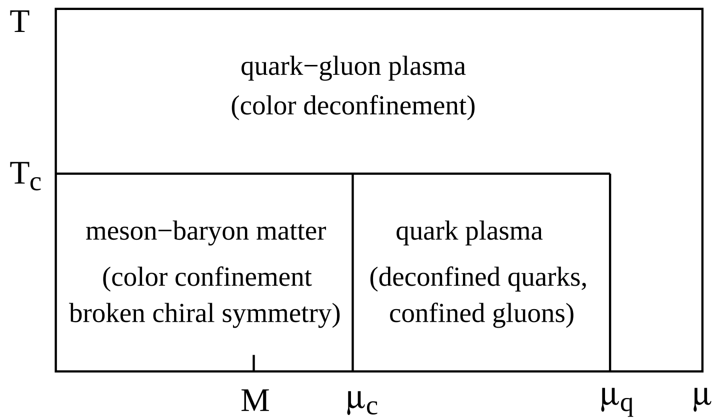
phase structure of QCD for $N_c \to \infty$:

- confined hadronic matter is purely mesonic,
  since $n_b \sim \exp\{(\mu - M)\}$, and $\mu,\ M \sim N_c$.
- quark-gluon plasma becomes gluon plasma,
  since gluon sector $\sim N_c^2$, quark sector $\sim N_c$.
- quarkyonic matter proposed to have
  color degrees of freedom $\sim N_c$, hence no “free” gluons.
- quark plasma, with $n_q \sim N_c(\mu_q^2 - M_q^2)$, contracted to $\mu_q = M_q$.

quarkyonic



quark plasma



Conclusion

- **Three State Phase Diagram** (modulo color superconductor)
- **Hadronic mattter:**
  quarks and gluons confined to hadrons, broken chiral symmetry
- **Quark plasma:**
  massive deconfined quarks, broken chiral symmetry
- **Quark-gluon plasma:**
  deconfined massless quarks and gluons, restored chiral symmetry